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Design and modeling high performance electromechanical Σ-Δ modulator

Drago Strle*

University of Ljubljana, EE Department, Tržaška 25, Ljubljana, Slovenia

Abstract: In this article we present part of the design methodology, modeling and efficient simulation of high performance micro-electromechanical ΣΔ modulator. The method is based on converting continuous-time model of the MEMS sensor and eventual analog loop filter into discrete time equivalent using impulse invariant transformation. The methodology is valid for any "MEMS based cantilever" sensor operating in a closed loop, where mechanical transfer function does not provide adequate noise shaping to reach high accuracy and resolution. Using proposed methodology makes possible to efficiently design, predict the behavior and stability of the loop and perform efficient system level simulations.

Key words: electro mechanical ΣΔ modulator, modeling, CT to DT circuit equivalence

Načrtovanje in Modeliranje elektro-mehanskega Σ-Δ modulatorja z visoko ločljivostjo

Izvleček: V članku obravnavamo metodologijo načrtovanja, modeliranje in učinkovito simulacijo visoko zmogljivega Mirko-Elektro-Mehanskega ΣΔ modulatorja. Metoda bazira na ekvivalenci odziva zveznega sistema, ki ga sestavlja zvezni model MEMS senzorja ter eventualnega zveznega filtra v zanki ob času vzorčenja in diskretnega sistema ob enakih časih z uporabo impulzno invariantne transformacije. Metoda je veljavna za kakršenkoli MEMS sensor, ki bazira na odmiku oziroma oscilaciji MEMS senzorskega elementa in deluje v zaprti zanki, kjer prevajalna funkcija mehanskega dela ne zagotavlja zadostnega slabljenja kvantizacijskega šuma, da bi dosegli veliko ločljivost in točnost. Predstavljena metodologija omogoča učinkovito načrtovanje ter predvidevanje stabilnosti ter drugih lastnosti realiziranega mikro elektro-mehanskega sistema z uporabo hitrih simulacij na visokem hierarhičnem nivoju.

Ključne besede: elektromehanski ΣΔ modulator, modeliranje, ekvivalenca vezji v zveznem in diskretnem časovnem prostoru

* Corresponding Author's e-mail: drago.strle@fe.uni-lj.si

1. Introduction

High performance micro-machined sensors based on cantilevers are usually operating in closed loop to increase dynamic range, linearity, bandwidth and other parameters. To prevent pull-in and to improve noise performances a low order force-feedback 1 bit Σ-Δ modulator can be used. Further improvements are possible if high order modulator and/or multi bit force feedback are used instead. In that case, the mechanical element transfer function together with electronic transfer function form a loop filter and in this way the influence of quantization noise is reduced and higher resolution is possible. The price paid is increased instability of the control loop and difficult design procedure. This article presents possible architecture, modeling and simulation of high performance electromechanical ΣΔ modulator operating in closed loop.

In section 2, theoretical background is given using well-known model of mechanical mass-spring system. Section 3 gives design steps needed for the efficient design of mechanical ΣΔ modulator, while simulation results for designed modulators are presented in section 4. Section 5 concludes the article.

2. Theory

Block diagram of generic electromechanical ΣΔ modulator is presented on Figure 1 ([1], [2], and [3]).

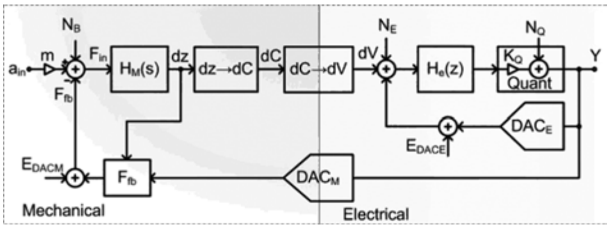

Figure 1: Generic electromechanical ΣΔ modulator

Mechanical part consists of a proof mass suspended by springs [6] (Figure 2) and can be described by (1).

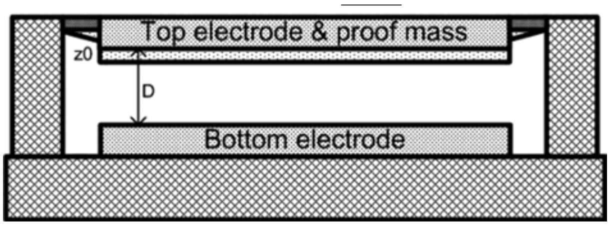

Figure 2: MEMS sensor

Mass of the sensor is m, $\omega_0 = \sqrt{(k/m)}$ is a resonance frequency of the mechanical part dependent on m and spring constant k, while δ is a damping factor (defined by friction of the media where the proof mass is mooving).

$$H_M(s) = \frac{dz(s)}{F_{in}(s)} = \frac{1/m}{s^2 + s \cdot 2\delta\omega_0 + \omega_0^2} \qquad (1)$$

Input signal is acceleration a_{in} that produce a force to the mass m. Feedback force F_{fb} tries to keep the sensor at equilibrium position, so the real input to the mechanical system is $F_{in} = m \cdot a - F_{fb}$. The sensor is prestressed by force F_0 so, that position of the mass at equilibrium is z_0, which is away from the position D. The displacement dz describes the moovement around z_0. N_B models spectral density of the Brownian noise, which enters the loop unatennuated and is dependent on sensors characteristics and presseure around it; we will neglect it in this article because it is assumed that the sensor is operating in vacuum. The displacement dz can be measured through the change of capacitance represented by block $dz \rightarrow dC$ and described by (2) (ε_0 is permitivity of free space, A is the area of the electrode, D is the distance of the electrodes with zero force, z_0 is the displacement at force F_0 and dz represent small displacement around z_0). To sense the change of the capacitance, a HF sensing signal is applied to the sense capacitors of the sensor.

$$dC = \varepsilon_0 A \left(\frac{1}{D + z_0 + dz} - \frac{1}{D + z_0} \right) \qquad (2)$$

Charge amplifier converts charges to voltage at HF and amplify it. Synchronous demodulation converts signals back to the base-band, while LP filtering removes HF mixing components; in this way the influence of 1/f noise and offset voltage is reduced. All this elements are hidden in block $dC \rightarrow dV$ of Figure 1. Part of this block is positioned in electronic part, while another part is located in the mechanical part of the model. Signal dV enters the electronic part of ΣΔ modulator with loop filter transfer function described by $H_E(z)$. Electronic noise with spectral density N_E is added at the input and consist of all electronic noise sources calculated back to that input exept the quantization noise. The quantizer is in fact a nonlinear elemet which can be modelled by simple "signum" function in nonlinear model or by gain factor K_Q and addition of quantization noise with density for linear model. The quantizer can be one bit for linearity and simplicity but can be also be multi bit if more demanding characteristics are needed. Errors caused by nonlinearity of the DAC inside the modulator are modelled by $E_{DAC\,E}$; they appear at the output unattenuated. The bit-stream signal described by $Y(s)$ for linear model is fed back and converted to voltage through block element DACM. The feedback force $dF_{fb} = F - F_0$ is dependent on V^2 and dz according to (3) where V_0 is selected in such a way that force F_0 is half of maximum force and causes prestressed position at $D + z_0$. Around that position the sensor mooves by dz. For 1 bit quantizer the equation (3)

$$dF_{fb} = -\frac{\varepsilon_0 A}{2} \left[\frac{V^2}{(D + z_0 + dz)^2} - \frac{V_0^2}{(D + z_0)^2} \right] \qquad (3)$$

simplifies to (4), where BS stands for the bit-stream $BS = \{\pm 1\}$. We can see that the force in this case is not any longer in quadratic relation with the applied voltage.

$$dF_{fb} = -\frac{\varepsilon_0 A}{2} \frac{V_0^2}{2} \left[\frac{(1 + BS)}{(D + z_0 + dz)^2} - \frac{1}{(D + z_0)^2} \right] \qquad (4)$$

For average $a_{in} = 0$ the average of $(1 + BS) = 1$ and small nonlinearity still remains due to dependency of (4) on dz. For low resolution devices this nonlinearity is of no problem but for high resolution devices the nonlinearity reduces the SNDR as shown in section 4. It can be compensated adding appropriate nonlinear electronic feedback; the solution is in developement.

To prepare the background for synthesys of the loop transfer function we need to linearise nonlinear components of the sensor. Assuming that dz is small compared to z_0, only linear term of the Taylor expansion is preserved, thus, dF_{fb} and dC are simplified to (6) and (7) respectively. The linear term in (5) is much smaller than ω_0^2, so the right summand could be neglected, which gives (6). Because linear model (6) does not contain any term in dz we can drop the line conecting that signal with block F_{fb} (Figure 1).

$$dF_{fb} \cong - \frac{\varepsilon_0 A}{2} \frac{V_0^2}{2} \left[\frac{BS}{(D+z_0)^2} - \frac{2 \cdot dz \cdot (1+BS)}{(D+z_0)^2} \right] \quad (5)$$

$$dF_{fb} \cong - \frac{\varepsilon_0 A}{2} \frac{V_0^2}{2} \left[\frac{BS}{(D+z_0)^2} \right] \quad (6)$$

$$dC \cong - \frac{\mathrm{dz} \cdot \varepsilon_0 \cdot A}{(D+z_0^2)} \quad (7)$$

Keeping the linear term of taylor expansion for *dC* gives (7).

3. Design procedure

The design procedure for an electronic ΣΔ modulator starts with synthesis of noise transfer function from required SnR and oversampling ratio, followed by selection of appropriate electronic topology to assure all requirements [7]. Because of simplicity, speed of simulation and available design tools the synthesis is usually done in z domain even for the CT-ΣΔ modulators. *NTF* (*z*) is synthesized followed by the synthesis of the loop filter *H* (*z*) for DT modulator or *H* (*s*) for CT implementation. In later case the response of DT prototype modulator and CT must be equal at $t = nT_s$ (equations (8), (9) and (10)), where $\hat{r}_{DAC}(t)$ is the impulse response of D/A converter; this is so called impulse invariant transformation [5].

$$x(nT_s) = \hat{x}_a(t)_{t=nT_s}; \qquad h_M(nT_s) = h_M(t)\big|_{t=nT_s} \quad (8)$$

$$h(nT_s) = Z^{-1}\{H(z)\} = \mathrm{L}^{-1}\{\hat{R}_{DACM}(s) \cdot \hat{H}(s)\}\big|_{t=nT_s} \quad (9)$$

$$h(nT_s) = [\,\hat{r}_{DAC}(t) * \hat{h}(t)\,]_{t=nT_s} = \left[\int_{-\infty}^{+\infty} \hat{r}_{DAC}(\tau) \cdot \hat{h}(t-\tau) d\tau \right]_{t=nT_s} \quad (10)$$

In the case of mechanical ΣΔ modulator the mechanical part of the transfer function is already present, thus part of the loop transfer function is already defined. Unfortunately, internal nodes are not available for optimization and stabilization and in addition, there is no freedom for changing parameters of that part because it is defined by required mechanical characteristics. The minimum order of the total loop transfer function is two if electronic loop filter does not exist. Unfortunately, the amount of noise shaping provided by only $H_M(s)$ is not big enough for high-resolution mechanical ΣΔ modulator; therefore, the electronic filtering is needed [4]. Because of sampling inside the loop, the system is a mixture of CT and DT system. To be able to correctly design such a system the equivalence between continuous time (CT) and discrete time (DT) system must be preserved using so-called impulse invariant transformation. We can distinguish three cases regarding loop transfer function and sampling: $H_M(s)$ and sampling in front of a quantizer, $H_M(s)$ and $H_E(z)$ with sampling in front of $H_E(z)$ and $H_M(s)$ followed by $H_E(s)$ while sampling occurs in front of a quantizer. The simplest possibility is presented on Figure 3. In this case, the only filtering element is mechanical transfer function $H_M(s)$. To be able to analytically analyze and model such modulator we have to calculate the response of the mechanical element at the sampling instances. The mechanical transfer function in z domain is than $H_M(z)$, which we obtain by impulse invariant transformation of $H_M(s)$, taking into consideration also transfer function of the D/A. Upper part of Figure 4 shows both arrangements: CT on the left and DT on the right.

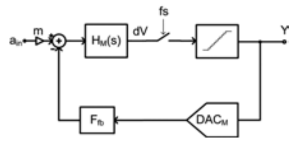

Figure 3: Simplest mechanical Σ-Δ modulator

They will behave equally if impulse responses are the same at nT_s. The CT system has three problems. The delay between sampling instant and reaction of the DAC may compromise the stability of the loop; we can solve it by taking into considerations the shape of the DAC pulse. The second problem is that the rise and the fall times of the DAC pulses are usually not equal; they are signal dependent, which degrades the SnR and causes inter-symbol interference. We can minimize it by appropriate shaping of the DAC pulse (for example RZ instead of NRZ). The third problem is that mechanical DAC generates nonlinear feedback force (3), which can be solved by appropriate electronic linearization.

For the DT implementation, the DAC is of no problem because by correct design we can assure a complete charge transfer in one sampling period.

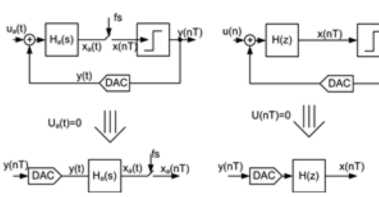

Figure 4: CT-DT equivalence

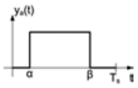

Figure 5: Shape of the DAC pulse

For CT systems, we have to implement and model the DAC correctly. Possible DAC pulse for one-bit quantizer is presented on Figure 5 and described by (11), where $0 \le \alpha < 1$, $0 \le \beta < 1$, $\alpha < \beta$ and $u(t)$ is a unit step function.

$$\hat{r}_{DACM}(t) = u(t - \alpha T_s) - u(t - \beta T_s) \quad (11)$$

The Laplace transformation of (11) gives (12):

$$L\{\hat{r}_{DACM}(t)\} = \frac{e^{-s\alpha T_s} - e^{-s\beta T_s}}{s} \quad (12)$$

The shape of the DAC pulse changes the loop transfer function. In this paper, we will treat only the case $\alpha = 0$, $\beta = 1$, while general case is under development. Mechanical properties define poles, zeroes and coefficients that can be calculated by fractional expansion of (1) using (13) and (14) for $\delta \ne 1$. Equivalent DT transfer function $\hat{H}_M(z)$ can be expressed as a sum of first order poles (15).

$$\hat{p}_{1,2} = -\delta \cdot \omega_0 \pm \sqrt{\delta^2 - 1}$$
$$\hat{a}_1 = -\hat{a}_2 = \frac{1}{m \cdot 2 \cdot \omega_0 \cdot \sqrt{\delta^2 - 1}} \quad (13)$$

$$\hat{H}_M(s) = \sum_{k=1}^{2} \frac{\hat{a}_k}{s - p_k} \quad (14)$$

$$H_M(z) = \sum_{k=1}^{2} \frac{z a_k}{z - z_k} \quad (15)$$

Parameters a_k and poles z_k are calculated by explicit evaluation of (10) equating corresponding coefficients. The results are given in (16) to (19):

$$\hat{h}(nT_s) = \sum_{k=1}^{2} \frac{\hat{a}_k}{\hat{p}_k} \left(e^{\hat{p}_k T_s (1-\beta)} - e^{\hat{p}_k T_s (1-\alpha)}\right) \quad (16)$$

$$Z^{-1}\{H(z)\} = a_1 z_1^{nT_s} u(nT_s) + a_2 z_2^{nT_s} u(nT_s) \quad (17)$$

$$a_k = \frac{\hat{a}_k}{\hat{p}_k} \left(z_k^{(1-\beta)} - z_k^{(1-\alpha)}\right); \qquad k = 1,2 \quad (18)$$

$$z_1 = e^{\hat{p}_1 T_s}; z_2 = e^{\hat{p}_2 T_s} \quad (19)$$

For $\delta = 1$ the calculation of poles and zeroes is different according to (20)

$$p_x \to 0; z_x \to 1;$$
$$a_x \to -a_x \frac{\beta - \alpha}{(1 - \beta)(1 - \alpha)} \quad (20)$$

Using transformations given in (18) and (19) the mechanical ΣΔ modulator transfer function can be translated to DT domain where simulations are faster compared to CT domain; in addition, we can easily test and predict closed loop stability characteristics and perform also other necessary simulations.

4. Simulation results

To prove the efficiency of the methodology, different MEMS electromechanical modulators have been designed using the sensor model defined in (1). Brownian noise has been neglected because it is assumed that the sensor is in a vacuum. Three different designs differ in the electronic loop filter order: zero one and two. The stability of the loop is of no concern for the first two designs, while for higher order electronic loop filter the poles and the zeroes must be optimized for stability. In our examples, the electronic loop filters are CT followed by one-bit quantizer, with sampling frequency 1*MHz*. Input acceleration is equal for all three cases. Simulation results using Matlab for three topologies are presented on Figure 6. The SnR in 1kHz bandwidth and HD are calculated for each design. As expected, increasing electronic loop-filter order increases the SnR, while HD remains the same. In addition, 2nd order electronic loop filter provides much bigger bandwidth compared to the first order structure because of more aggressive noise shaping, which is beneficial characteristics.

5. Conclusions

A design methodology and Matlab modeling of precision MEMS electromechanical ΣΔ modulators is presented in the article. We show methodology, mathematical modeling and Matlab simulation results for three different electronic loop filters. Methodology is adapted from the design procedure for the CT modulators and is applicable to general closed loop mechanical Σ-Δ modulator. In the future, the design procedure will be generalized to the synthesis of arbitrary-order electronic loop filter implemented with CT or S-C circuits with mechanical part consisting of complex poles and higher order modes to be able to control and predict the stability of the closed loop system already during the design procedure. The linearization of the feedback force is currently under development.

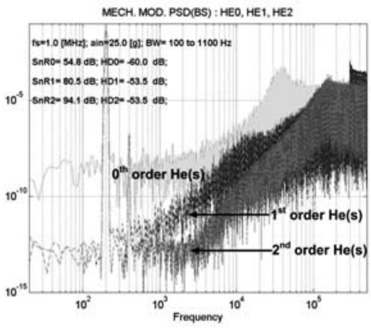

Figure 6: Spectrums of three MEMS ΣΔ modulators with different orders of electronic noise shaping filters.

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