APEM journal

Advances in Production Engineering & Management
Volume 11 | Number 4 | December 2016 | pp 333–344
http://dx.doi.org/10.14743/apem2016.4.231

ISSN 1854-6250
Journal home: apem-journal.org
Original scientific paper

Effect of delayed differentiation on a multiproduct vendor-buyer integrated inventory system with rework

Chiu, Y.-S.P.[a], Kuo, J.-S.[a], Chiu, S.W.[b,*], Hsieh, Y.-T.[a]

[a]Department of Industrial Engineering and Management, Chaoyang University of Technology, Wufong, Taichung, Taiwan
[b]Deptartment of Business Administration, Chaoyang University of Technology, Wufong, Taichung, Taiwan

A B S T R A C T

This study explores the effect of delayed differentiation on a multiproduct vendor-buyer integrated inventory system with rework to identify its potential benefits and provide managers with in-depth information for operational decision-making. The main considerations of the proposed study include a multiproduct fabrication plan to increase machine utilization, a rework process to ensure product quality, and a multi-shipment policy to distribute the end products. In addition, these products sharing an intermediate part for which a two-stage fabrication scheme is adopted, wherein the common parts are produced at the first stage and the end products are manufactured at the second stage. The aim is to reduce the overall system costs and shorten the replenishment cycle time. Mathematical modeling and optimization methods were employed to derive the closed-form optimal replenishment cycle time and delivery decisions. We demonstrated the applicability of our research results through numerical examples and revealed that for both linear and nonlinear relationships between the common intermediate part's completion rate α and its practical value at α, our proposed two-stage production scheme with delayed differentiation is considerably beneficial vis-à-vis single-stage schemes in saving overall system costs and reducing the replenishment cycle time.

A R T I C L E   I N F O

Keywords:
Multi-product vendor-buyer system
Production-shipment decision
Rework
Common intermediate part
Delayed differentiation

**Corresponding author:*
swang@cyut.edu.tw
(Chiu, S.W.)

Article history:
Received 31 May 2016
Revised 15 November 2016
Accepted 18 November 2016

1. Introduction

Conventional economic production quantity (EPQ) model considers a single product fabrication with all items produced are of perfect quality and customer's demand satisfied by a continuous inventory issuing policy [1–3]. However, in real world supply chain systems, vendors usually adopt a *multi-product* production plan to get the most out of machine utilization and consider reworking of nonconforming items to lower their production cost. Aggarwal [4] presented a simple grouping idea under a common order cycle to resolve the multi-product inventory system. A computation procedure was also presented to derive optimal values of common order cycle. Rosenblatt and Rothblum [5] studied the multi-item inventory systems under a single–resource capacity constraint. Two solution procedures were proposed to derive optimal capacity policy. A numerical example is used to show that their solution procedures can be applied to different types of cost functions. Aliyu and Andijani [6] examined a multi–item production-inventory system with shortages, deterministic demand, deterioration, and capacity and budget constraints. Linear quadratic theory was used to solve the optimal control policy. Balkhi and Foul [7] studied a multi-product inventory model with deterministic demand, production, and deterioration rates for each product in finite time periods. Shortage is allowed and backordered for each product. For each product, they derived the optimal production and restarting times in each period that minimize the total inventory costs. Rahmani et al. [8] investigated a two-stage real capacitated production system with uncertain demand and production costs. A mixed- integer programming model was developed to the problem. An initial robust schedule was obtained and it can be improved against any possible occurrences of uncertain parameters. They provided a real case to demonstrate the practical use of their model. Chiu et al. [9] developed an exact mathematical model to simultaneously derive the production and shipment decisions for a multi-product inventory system with a rework process. A single-stage production process is considered without involving the common intermediaate part. Their results enable managers of such a specific system to better understand and control over the effects of variations in different system parameters on the optimal production-shipment policy and on the expected system costs. Additional studies related to the multi-product inventory systems can also be found elsewhere [10-14].

In multi-item production planning, if multiple products share a *common intermediate part*, vendors would always be interested in evaluating a two-stage fabrication scheme with the first stage making common intermediate parts for all products, and the second stage producing the end products to reduce overall system costs and shorten the replenishment cycle time. Gerchak et al. [15] developed a model for an arbitrary number of products with general joint demand distribution. They discussed the case of using a service-level measure where rationing of common components might be required and characterized the implied rationing rule. Garg and Tang [16] stated that practically most product families have a number of points of differentiation. They developed two models to investigate products with more than one point of differentiation. Benefits of delayed differentiation at each point in each model are examined. Necessary conditions are decided when one type of delayed differentiation is more beneficial than the other. They found that variations in demand and lead times have significant effects on determining which point of differentiation should be delayed. Graman [17] developed a two–product, single–period, order-up-to cost model to assist in deciding the inventory levels of end products and postponement capacity. A non-linear programming was used to determine the optimal solutions to inventory levels and capacity that minimize the total system cost. He indicated that altering product value, holding cost, cost of postponement, packaging cost, and fill rate can reduce expected total cost and increase postponement capacity. Other studies addressed various aspects of multi–product systems with *delayed product differentiation* can also be found elsewhere [18–21]. Also, in real manufacturing environments, due to various uncontrollable factors during production process, generation of defective items is inevitable. Quality assurances, such as inspection of product quality, rework of all repairable items, and scrapped of defective items, have been extensively studied in past decades [22–28]. Also, in contrast to the assumption of continuous issuing policy in conventional EPQ model, most nowadays supply chain systems practically adopt a periodic multi-shipment policy to distribute end products to their customers. Studies of various aspects of periodic or multi-delivery issues of vendor-buyer integrated systems have been extensively carried out during past decades [29-44].

Inspired by the potential benefits derived from applying delayed differentiation to multi-product systems, and seeking to provide managers of transnational enterprises with information to assist them in achieving the key operational goals such as maximizing machine utilization, ensuring product quality, lowering overall operating costs, and shortening response time, this study extends a prior work [9] and explores the effect of delayed differentiation on a multi-product vendor-buyer integrated inventory system with rework. Since little attention has been paid to this specific research area, the present study is intended to bridge the gap.

2. Model description and mathematical analysis

Description of the proposed multi-product vendor–buyer integrated inventory system with delayed differentiation strategy and rework using a single-machine production scheme is as follows. Consider a vendor has annual demand λ_i for L different products (where i = 1, 2,…,L) that must be satisfied. These L customized end items share a *common intermediate part* and are manufactured using a two-stage process. The first stage produces only the common intermediate components, and the second stage fabricates in sequence L different customized end products under the common production cycle time policy. The objectives of the proposed production plan are to maximize machine utilization, shorten the replenishment cycle time, and minimize total production-inventory-delivery costs. The common intermediate part is manufactured at a rate of $P_{1,0}$ in stage one. After that, L different customized end products are produced in order under a common cycle time policy in stage two (see Fig. 1), at a rate of $P_{1,i}$.

All items made are screened and unit inspection cost is included in unit production cost C_i. The production processes in each stage (either for *common intermediate part* or for customized end products) may randomly produce x_i portion of defective items at a rate of $d_{1,i}$ and $d_{1,i} = P_{1,i}x_i$ (where i = 0, 1, 2,…, L; with i = 0 denotes that it is for the production of *common intermediate part* in the stage 1). Under the ordinary assumption of the EPQ model without shortages, the constant production rate $P_{1,i}$ must be larger than the sum of demand rate λ_i and production rate of defective items $d_{1,i}$. That is: $(P_{1,i} - d_{1,i} - \lambda_i) > 0$ or $(1 - x_i - \lambda_i/P_{1,i}) > 0$. It is further assumed that all defective items can be reworked and repaired. The rework processes starts immediately after the end of regular production processes in each production cycle (see Fig. 2), at a rate of $P_{2,i}$.

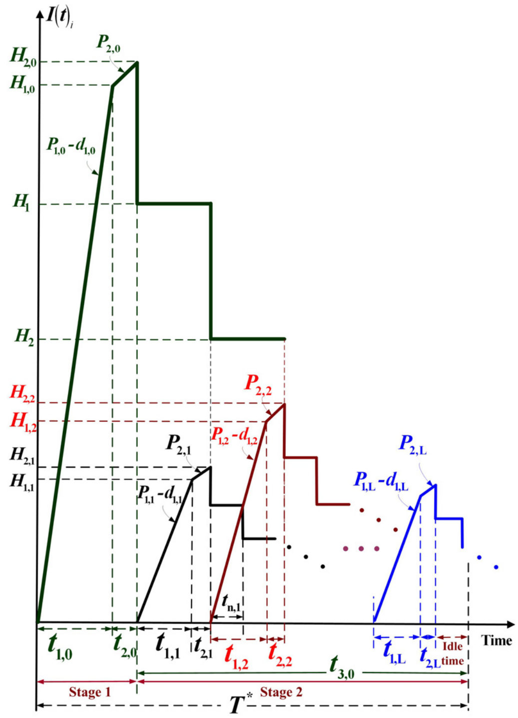

Fig. 1 Inventory level of perfect quality common intermediate parts and customized final products in the proposed two-stage multi-product vendor–buyer integrated inventory system with rework

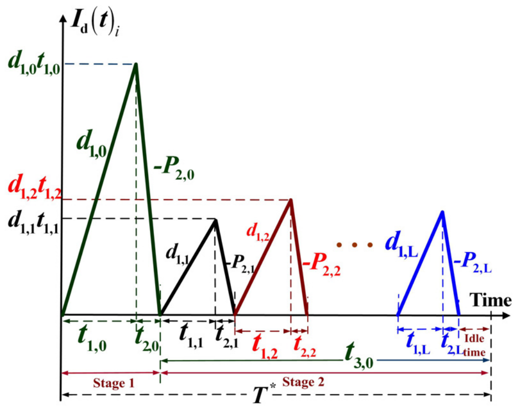

Fig. 2 Inventory level of defective items in both stages of the proposed two-stage multi-product system

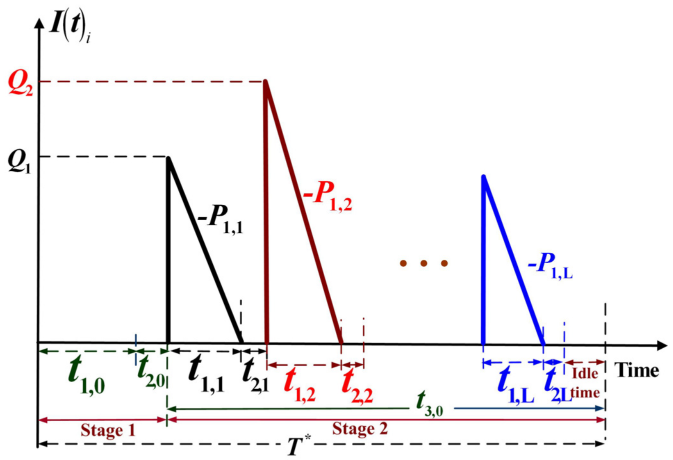

Fig. 3 Inventory level of common intermediate parts waiting to be fabricated into customized final products in the stage 2 of the proposed two-stage multi-product system

Upon completion of the production in stage 1, L different lots of *common intermediate parts* are made ready for the production in stage 2. They are fabricated in sequence into customized end products under the common production cycle time policy. The inventory level of common intermediate parts waiting to be fabricated in stage 2 is depicted in Figure 3.

In stage 2, after the completion of rework process ($t_{2,i}$) of each end product i, fixed quantity n installments of the finished batch are transported to customers at a fixed interval of time in the delivery time $t_{3,i}$ (see Fig. 1). The inventory level of end products at the buyers' side during a production cycle is depicted in Figure 4 (which is similar to Fig. 3 in [9]).

The following are additional notation used in this study (where i = 1, 2,…,L, represents L different products in stage 2; and i = 0 denotes the *common intermediate part* in stage 1):

- T – Production cycle length, one of the decision variables,
- n – Number of fixed quantity installments of the finished batch to be delivered in each cycle, the other decision variable,
- α – Completion rate of common intermediate part as compared to the finished product,
- Q_i – Production lot size for product i,
- K_i – Production setup cost for product i in a production cycle,
- C_i – Unit production cost for product i,
- $h_{1,i}$ – Unit holding cost for product i,
- $h_{2,i}$ – Holding cost per reworked item for product i,

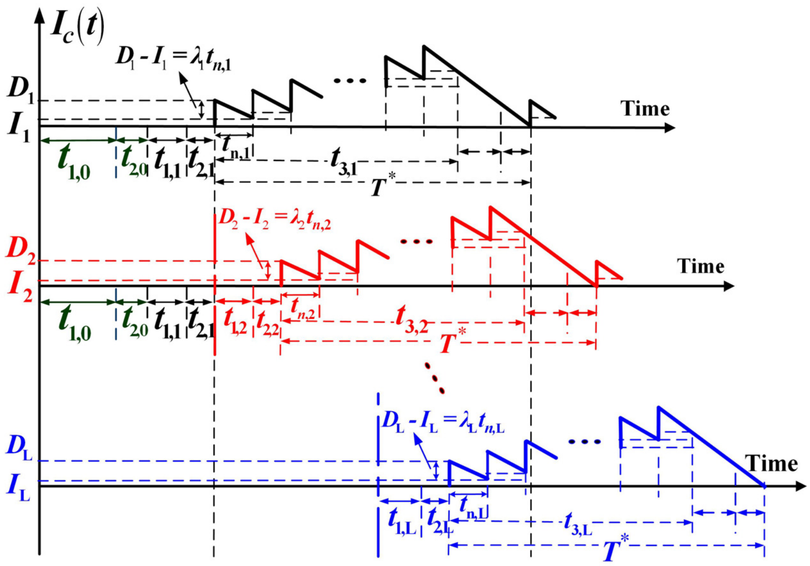

Fig. 4 Inventory level of customized final products at the buyers' side during a production cycle [9]

$h_{3,i}$ – Unit holding cost for stocks stored at customer's side,
$h_{4,i}$ – Unit holding cost for safety stocks stored at producer's side,
$C_{R,i}$ – Unit reworking cost for product i,
$t_{1,i}$ – Production uptime for product i in a production cycle,
$t_{2,i}$ – The reworking time for product i in a production cycle,
$t_{3,i}$ – Delivery time for product i in a production cycle,
H_i – Inventory level of common intermediate part at the time of producing end product i,
$H_{1,i}$ – Maximal level of perfect quality items i in the end of regular production,
$H_{2,i}$ – Maximal level of perfect quality items i in the end of rework process before delivery,
$K_{1,i}$ – Fixed delivery cost per shipment for product i,
$C_{T,i}$ – Unit delivery cost for product i,
$t_{n,i}$ – A fixed interval of time between each installment of finished items of product i to be delivered to customer during downtime $t_{3,i}$,
$I(t)_i$ – On-hand inventory level of perfect quality items i at time t,
$I_d(t)_i$ – On-hand inventory level of defective items i at time t,
$I_c(t)_I$ – On-hand inventory level of finished product i at time t, at customer's side,
I_i – The left-over number of finished items of product i in each $t_{n,i}$, at customer's side,
D_i – Number of finished items of product i to be transported to customer in each shipment,
$TC(T, n)$ – Total production-inventory-delivery cost per cycle,
$E[T]$ – The expected production cycle length,
$E[TC(T, n)]$ – The expected production-inventory-delivery cost per cycle,
$E[TCU(T, n)]$ – The long-run average costs per unit time for the proposed model.

2.1 Modeling and analysis

A two-stage EPQ-based production plan considering the postponement is proposed to satisfy annual demand λ_i of L different customized products. From Figure 1, we observe the production cycle time as

$$T = t_{1,i} + t_{2,i} + t_{3,i} = \frac{Q_i}{\lambda_i} \; for \; i = 0, 1, 2, \cdots, L \tag{1}$$

In stage 1, the production lot-size of *common intermediate parts* Q_0, depends on the sum of production lot sizes Q_i of L different products to be made in the stage 2. Therefore, we obtain the following equations (refer to Fig. 1):

$$Q_i = \lambda_i T \quad for \; i = 1, 2, \cdots, L \tag{2}$$

$$Q_0 = \sum_{i=1}^{L} Q_i = \lambda_0 T \tag{3}$$

$$t_{1,0} = \frac{Q_0}{P_{1,0}} = \frac{H_{1,0}}{P_{1,0} - d_{1,0}} \tag{4}$$

$$H_{1,0} = t_{1,0}\left(P_{1,0} - d_{1,0}\right) \tag{5}$$

$$H_{2,0} = H_{1,0} + P_{2,0} t_{2,0} = \sum_{i=1}^{L} Q_i \tag{6}$$

$$t_{2,0} = \frac{x_0 Q_0}{P_{2,0}} = \frac{d_{1,0} t_{1,0}}{P_{2,0}} = \frac{H_{2,0} - H_{1,0}}{P_{2,0}} \tag{7}$$

$$H_1 = H_{2,0} - Q_1 \tag{8}$$

$$H_i = H_{(i-1)} - Q_i \;\; for \; i = 2, 3, \cdots, L \tag{9}$$

$$H_L = H_{(L-1)} - Q_L = 0 \tag{10}$$

In stage 2, for fabrication of L different products we obtain the following equations directly from Figs. 1 to 4 (where i = 1, 2,…,L):

$$t_{1,i} = \frac{Q_i}{P_{1,i}} = \frac{H_{1,i}}{P_{1,i} - d_{1,i}} \tag{11}$$

$$H_{1,i} = \left(P_{1,i} - d_{1,i}\right) t_{1,i} \tag{12}$$

$$H_{2,i} = H_{1,i} + P_{2,i} t_{2,i} \tag{13}$$

$$t_{2,i} = \frac{x_i Q_i}{P_{2,i}} = \frac{d_{1,i} t_{1,i}}{P_{2,i}} = \frac{H_{2,i} - H_{1,i}}{P_{2,i}} \tag{14}$$

$$t_{3,i} = nt_{n,i} \tag{15}$$

$$D_i = \frac{H_{2,i}}{n} \tag{16}$$

$$I_i = D_i - \lambda_i t_{n,i} \tag{17}$$

$$nI_i = \lambda_i\left(t_{1,i} + t_{2,i}\right) \tag{18}$$

2.2 Cost analysis

Inventory holding costs for *common intermediate parts* (including perfect and imperfect quality items) during $t_{1,0}$, $t_{2,0}$, and $t_{3,0}$, are (see Figs. 1 and 2)

$$h_{1,0}\left[\frac{H_{1,0}t_{1,0}}{2} + \frac{(H_{2,0}+H_{1,0})t_{2,0}}{2} + \sum_{i=1}^{L} H_i(t_{1i} + t_{2i})\right] + h_{1,0}\left[\frac{(d_{1,0}t_{1,0})t_{1,0}}{2}\right] \tag{19}$$

In stage 2, inventory holding cost for *common intermediate part*s waiting to be fabricated into customized end products (see Fig. 3) is

$$\sum_{i=1}^{L}\left\{h_{1,i}\left[\frac{Q_i}{2}\left(t_{1,i}\right)\right]\right\} \tag{20}$$

Inventory holding costs for imperfect quality items waiting to be reworked in both stages are

$$h_{2,0}\left[\frac{d_{1,0}t_{1,0}}{2}\left(t_{2,0}\right)\right] + \sum_{i=1}^{L}\left[h_{2,\mathrm{i}}\left(\frac{P_{2,i}t_{2,i}}{2}\right)\left(t_{2,i}\right)\right] \tag{21}$$

In stage 2, fixed and variable delivery costs and inventory holding cost for finished product *i* waiting to be distributed in $t_{3,i}$ are

$$\sum_{i=1}^{L}\left[nK_{1,i} + C_{T,i}Q_i\right] + \sum_{i=1}^{L}\left\{h_{1,i}\left(\frac{n-1}{2n}\right)H_{2,i}t_{3,i}\right\} \tag{22}$$

The stock holding cost for end product *i* stored at customers' sides (see Fig. 4) is

$$\sum_{i=1}^{L}\left\{h_{3,i}\left[\frac{n(D_i-I_i)t_{n,i}}{2} + \frac{n(n+1)}{2}I_i t_{n,i} + \frac{nI_i\left(t_{1,i}+t_{2,i}\right)}{2}\right]\right\} \tag{23}$$

The overall cost per cycle $TC(T, n)$ for the proposed system, includes production setup cost, variable production cost, reworking cost, holding cost, and safety stock cost in both stages; and fixed and variable delivery costs and holding costs for stocks stored at customers' side in stage 2. Hence, $TC(T, n)$ is

$$TC(T,n) = \left\{\begin{matrix} K_0 + C_0Q_0 + C_{R,0}x_0Q_0 + h_{2,0}\left(\frac{d_{1,0}t_{1,0}}{2}\right)\left(t_{2,0}\right) + h_{4,0}(x_0Q_0)T \\ +h_{1,0}\left[\frac{H_{1,0}t_{1,0}}{2} + \frac{H_{2,0}+H_{1,0}}{2}\left(t_{2,0}\right) + \frac{d_{1,0}t_{1,0}}{2}\left(t_{1,0}\right) + \sum_{i=1}^{L} H_i(t_{1i} + t_{2i})\right]\end{matrix}\right\} +$$
$$\sum_{i=1}^{L}\left\{\begin{matrix} K_i + C_iQ_i + C_{R,i}x_iQ_i + nK_{1,i} + C_{T,i}Q_i + h_{2,i}\left(\frac{P_{2,i}t_{2,i}}{2}\right)\left(t_{2,i}\right) \\ +h_{1,i}\left[\frac{Q_i}{2}\left(t_{1,i}\right) + \frac{H_{1,i}t_{1,i}}{2} + \frac{H_{2,i}+H_{1,i}}{2}\left(t_{2,i}\right) + \left(\frac{n-1}{2n}\right)H_{2,i}t_{3,i} + \frac{d_{1,i}t_{1,i}}{2}\left(t_{1,i}\right)\right] \\ +h_{3,i}\left[\frac{n(D_i-I_i)t_{n,i}}{2} + \frac{n(n+1)}{2}I_i t_{n,i} + \frac{nI_i\left(t_{1,i}+t_{2,i}\right)}{2}\right] + h_{4,i}(x_iQ_i)T\end{matrix}\right\} \tag{24}$$

Substituting Eqs. 1 to 18 in Eq. 24 and taking randomness of defective rate into account, and the long-run average system costs $E[TCU(T, n)]$ can be derived as follows:

$$\mathrm{E}[TCU(T,n)] = \frac{\mathrm{E}[TC(T,n)]}{E[T]} =$$
$$\left\{\frac{K_0}{T} + C_0\lambda_0 + C_{R,0}\lambda_0E[x_0] + z_0T\right\} +$$
$$\sum_{i=1}^{L}\left\{\begin{matrix}\left[\frac{K_i}{T} + C_i\lambda_i + C_{R,i}\lambda_iE[x_i] + \frac{nK_{1,i}}{T} + C_{T,i}\lambda_i\right] + \frac{h_{1,i}T\lambda_i^2}{2}\left\{\delta_{2,i} - \frac{\delta_{1,i}}{n}\right\} \\ +\frac{h_{2,i}T\lambda_i^2E[x_i]^2}{2P_{2,i}} + \frac{h_{3,i}T\lambda_i^2}{2}\left[\frac{1}{P_{1,i}} + \frac{E[x_i]}{P_{2,i}} + \frac{\delta_{1,i}}{n}\right] + Th_{4,i}\lambda_iE[x_i]\end{matrix}\right\} \tag{25}$$

where

$$z_0 = \begin{Bmatrix} \frac{h_{1,0}\lambda_0^2}{2}\left[\frac{1}{P_{1,0}} + \frac{2E[x_0]}{P_{2,0}} - \frac{E[x_0]^2}{P_{2,0}}\right] + \frac{h_{2,0}\lambda_0^2 E[x_0]^2}{2P_{2,0}} \\ +h_{1,0}\sum_{i=1}^{L}\left\{\left(\frac{\lambda_i}{P_{1,i}} + \frac{\lambda_i E[x_i]}{P_{2,i}}\right)\left[\sum_{i=1}^{L}(\lambda_i) - \sum_{j=1}^{i}(\lambda_j)\right]\right\} + h_{4,0}\lambda_0 E[x_0] \end{Bmatrix};$$

$$\delta_{1,i} = \left[\frac{1}{\lambda_i} - \frac{1}{P_{1,i}} - \frac{E[x_i]}{P_{2,i}}\right]; \text{and } \delta_{2,i} = \left[\frac{1}{\lambda_i} - \frac{E[x_i]^2}{P_{2,i}} + \frac{1}{P_{1,i}} + \frac{E[x_i]}{P_{2,i}}\right] \; for\ i = 1,2,\dots,L \tag{26}$$

3. Convexity and the optimal decision

Upon obtaining the long-run average system costs $E[TCU(T, n)]$, we then prove it is a convex function by applying the Hessian matrix equations [45] to verify that Eq. 27 holds.

$$[T\ \ n]\cdot\begin{pmatrix} \frac{\partial^2 E[TCU(T,n)]}{\partial T^2} & \frac{\partial^2 E[TCU(T,n)]}{\partial T\partial n} \\ \frac{\partial^2 E[TCU(T,n)]}{\partial T\partial n} & \frac{\partial^2 E[TCU(T,n)]}{\partial n^2} \end{pmatrix}\cdot\begin{bmatrix} T \\ n \end{bmatrix} > 0 \tag{27}$$

From Eq. 25 we obtain

$$\frac{\partial E[TCU(T,n)]}{\partial T} = \left(\frac{-K_0}{T^2} + z_0\right) + \sum_{i=1}^{L}\begin{Bmatrix} \left[\frac{-K_0}{T^2} - \frac{nK_{1,i}}{T^2}\right] + \frac{h_{1,i}\lambda_i^2}{2}\left\{\delta_{2,i} - \frac{\delta_{1,i}}{n}\right\} \\ + \frac{h_{2,i}\lambda_i^2 E[x_i]^2}{2P_{2,i}} + \frac{h_{3,i}\lambda_i^2}{2}\left[\frac{1}{P_{1,i}} + \frac{E[x_i]}{P_{2,i}} + \frac{\delta_{1,i}}{n}\right] + h_{4,i}\lambda_i E[x_i] \end{Bmatrix} \tag{28}$$

$$\frac{\partial E[TCU(T,n)]}{\partial T^2} = \frac{2K_0}{T^3} + \sum_{i=1}^{L}\left\{\frac{2K_i}{T^3} + \frac{2nK_{1,i}}{T^3}\right\} \tag{29}$$

$$\frac{\partial E[TCU(T,n)]}{\partial n} = \sum_{i=1}^{L}\left\{\frac{K_{1,i}}{T} + \frac{T\lambda_i^2}{2n^2}\left[\left(h_{1,i} - h_{3,i}\right)\delta_{1,i}\right]\right\} \tag{30}$$

$$\frac{\partial E[TCU(T,n)]}{\partial n^2} = \sum_{i=1}^{L}\left\{\frac{T\lambda_i^2}{n^3}\left[\left(h_{3,i} - h_{1,i}\right)\delta_{1,i}\right]\right\} \tag{31}$$

$$\frac{\partial^2 E[TCU(T,n)]}{\partial T\partial n} = \sum_{i=1}^{L}\left\{-\frac{K_{1,i}}{T^2} + \frac{T\lambda_i^2}{2n^2}\left[\left(h_{1,i} - h_{3,i}\right)\delta_{1,i}\right]\right\} \tag{32}$$

Substituting Eqs. 29, 31, and 32 in Eq. 27, we obtain

$$[T\quad n]\cdot\begin{pmatrix} \frac{\partial^2 E[TCU(T,n)]}{\partial T^2} & \frac{\partial^2 E[TCU(T,n)]}{\partial T\partial n} \\ \frac{\partial^2 E[TCU(T,n)]}{\partial T\partial n} & \frac{\partial^2 E[TCU(T,n)]}{\partial n^2} \end{pmatrix}\cdot\begin{bmatrix} T \\ n \end{bmatrix} = \frac{2K_0}{T} + \sum_{i=1}^{L}\frac{2K_i}{T} > 0 \tag{33}$$

Because K_0, K_i, and T are all positive, we find Eq. 33 is positive. Hence, $E[TCU(T, n)]$ is a strictly convex function for all T and n different from zero. In order to simultaneously determine production-shipment decision for the proposed system, we can solve the linear system of first derivatives of $E[TCU(T, n)]$ with respect to T and n, respectively, by setting these partial derivatives equal to zero. With further derivations we find

$$T^* = \sqrt{\frac{K_0 + \sum_{i=1}^{L}\left(K_i + nK_{1,i}\right)}{z_0 + \sum_{i=1}^{L}\begin{Bmatrix} \frac{h_{1,i}\lambda_i^2}{2}\left\{\delta_{2,1} - \frac{\delta_{1,i}}{n}\right\} + \frac{h_{2,i}\lambda_i^2 E[x_i]^2}{2P_{2,i}} \\ + \frac{h_{3,i}\lambda_i^2}{2}\left[\frac{1}{P_{1,i}} + \frac{E[x_i]}{P_{2,i}} + \frac{\delta_{1,i}}{n}\right] + h_{4,i}\lambda_i E[x_i] \end{Bmatrix}}} \tag{34}$$

and

$$n^* = \sqrt{\frac{\left(K_0 + \sum_{i=1}^{L}K_i\right)\sum_{i=1}^{L}\left[\frac{\lambda_i^2}{2}\left(h_{3,1} - h_{1,1}\right)\left(\delta_{1,i}\right)\right]}{\left(\sum_{i=1}^{L}K_{1i}\right)\left\{z_0 + \sum_{i=1}^{L}\begin{Bmatrix} \frac{h_{1,i}\lambda_i^2}{2}\left(\delta_{2,i}\right) + \frac{h_{2,1}\lambda_i^2 E[x_i]^2}{2P_{2,i}} + h_{4,i}\lambda_i E[x_i] \\ + \frac{h_{3,i}\lambda_i^2}{2}\left[\frac{1}{P_{1,i}} + \frac{E[x_i]}{P_{2,i}} + \frac{\delta_{1,i}}{n}\right] \end{Bmatrix}\right\}}} \tag{35}$$

4. Numerical example and discussion

The following numerical example is used to show the practical uses of research results obtained in the previous section. Consider a manufacturer must fabricate five different products and they share a *common intermediate part* that has completion rate α = 0.5 (i.e., halfway done). To ease comparison efforts for readers, we reconsider a numerical example used in a prior study [9] regarding optimization of a single-stage multi-product system without adopting postponement in its production. Annual production rates of five end products $P_{1,i}$ = 58,000, 59,000, 60,000, 61,000, and 62,000 units, respectively; annual demands λ_i = 3,000, 3,200, 3,400, 3,600, and 3,800 units, respectively; annual reworking rates $P_{2,i}$ = 46,400, 47,200, 48,000, 48,800, and 49,600 units, respectively; setup costs K_i = \$17,000, \$17,500, \$18,000, \$18,500, and \$19,000, respectively; unit fabrication costs C_i = \$80, \$90, \$100, \$110, and \$120, respectively; the defective rates x_i follow uniform distribution over the intervals [0, 0.05], [0, 0.10], [0, 0.15], [0, 0.20], and [0, 0.25], respectively; and unit reworking costs $C_{R,i}$ = \$50, \$55, \$60, \$65, and \$70, respectively. Based on *common intermediate part's* completion rate α = 0.5, a straightforward relationship $1/\alpha$ is assumed for its relevant production rates. Hence, in the proposed two-stage single-machine production scheme we have $P_{1,0}$ = $(1/\alpha)$*(the mean of $P_{1,i}$'s) = 120,000 and $P_{2,0}$ = $(1/\alpha)$*(the mean of $P_{2,i}$'s) = 96,000.

The relationship between *common intermediate part's* relevant costs and its completion rate α can either be linear or nonlinear. Both cases are investigated in the following subsections.

4.1 Case 1: Analysis of linear relationship of cost relevant variables

If the relationship between practical fabrication related cost of *common intermediate part* (or called 'the value' of common part) and its completion rate α is linear, then for α = 0.5 we have the following linear-based relevant values of variables in our proposed system:

- C_0 – \$40, unit fabrication cost for common intermediate part,
- K_0 – \$8,500, setup cost for common intermediate part,
- $C_{R,0}$ – \$25, unit reworking cost for common intermediate part,
- $h_{1,0}$ – \$5, unit holding cost for common intermediate part,
- $h_{4,0}$ – \$5, unit safety stock cost for common intermediate part,
- $h_{2,0}$ – \$15, unit holding cost for common intermediate part during the reworking processes,
- K_i – Setup costs of end products are \$8,500, \$9,000, \$9,500, \$10,000, and \$10,500 respectively,
- x_0 – [0, 0.04], the interval uniformly distributed defective rate in the production of common intermediate part,
- C_i – Unit production costs of end products are \$40, \$50, \$60, \$70, and \$80, respectively,
- $h_{1,i}$ – Unit holding costs of end products are \$10, \$15, \$20, \$25, and \$30, respectively,
- $P_{1,i}$ – Annual production rates of five end products are 112,258, 116,066, 120,000, 124,068, and 128,276 units, respectively; they are simply calculated by $P_{1,i} = 1/(1/P_{1,i} - 1/P_{1,0})$,
- x_i – End items' defective rates follow the uniform distribution over the intervals [0, 0.01], [0, 0.06], [0, 0.11], [0, 0.16], and [0, 0.21], respectively,
- $C_{R,i}$ – Unit reworking costs of end products are \$25, \$30, \$35, \$40, and \$45, respectively,
- $P_{2,i}$ – Annual reworking rates of five end products are 89,806, 92,852, 96,000, 99,254, and 102,621 units, respectively; they are simply calculated by $P_{2,i} = 1/(1/P_{2,i} - 1/P_{2,0})$,
- $h_{2,i}$ – Unit holding cost per reworked items of end products are \$30, \$35, \$40, \$45, and \$50, respectively,
- $K_{1,i}$ – Fixed delivery costs per shipment: \$1,800, \$1,900, \$2,000, \$2,100, and \$2,200, respectively,
- $C_{T,i}$ – Unit delivery costs of end items are \$0.1, \$0.2, \$0.3, \$0.4, and \$0.5, respectively,
- $h_{3,i}$ – Unit holding costs at the customer's side are \$70, \$75, \$80, \$85, and \$90, respectively,
- $h_{4,0}$ – Unit safety stock costs of end products are \$10, \$15, \$20, \$25, and \$30, respectively.

First, the annual demand for *common intermediate parts* λ_0 = 17,000 can be obtained by applying Eqs. 2 and 3. Then, by calculating Eqs. 34, 35, and 25, we derive the optimal number of deliveries n^* = 3, optimal production cycle time T^* = 0.4614 (years), and the expected system costs per unit time $E[TCU(T^*, n^*)]$ = $2,145,834. Figure 5 depicts the effects of variations of the production cycle time T on the expected system costs $E[TCU(T, n)]$.

The behavior of $E[TCU(T, n)]$ with respect to the common intermediate part's completion rate α is exhibited in Figure 6. It can be seen that as the completion rate α increases, the long- run expected system costs $E[TCU(T, n)]$ decreases, and the proposed model realizes a system cost savings of 3.76 % at α = 0.5 (i.e., system costs decreased from $2,229,658 [9] to $2,145,834) as compared to that in prior study which used a single-stage production scheme. This analytical result demonstrates that the proposed two-stage multi-item production scheme with delayed differentiation is a considerably beneficial model for manufacturers who must meet demands for multiple products that share a common intermediate part.

Figure 7 shows the effects of variations of common part's completion rate α on the optimal production cycle time T^*. As the completion rate α increases, the optimal cycle time T^* decreases significantly, and in the proposed model optimal cycle time T^* is reduced by 25.5 % at α = 0.5 (i.e., it decreases from 0.6193 [9] to 0.4614 (years)) as compared to that in prior study which used a single-stage production scheme. Such an analytical result indicates our proposed two-stage multi–item production scheme with delayed differentiation provides a shorter cycle time (or faster response time) than that in a conventional one-stage multi-item system [9].

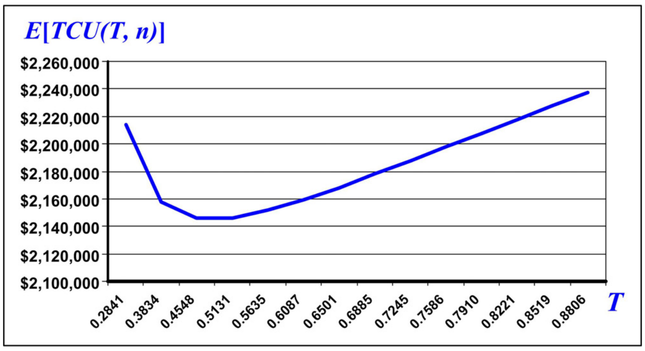

Fig. 5 The effects of variations of the production cycle time T on the expected system costs $E[TCU(T, n)]$

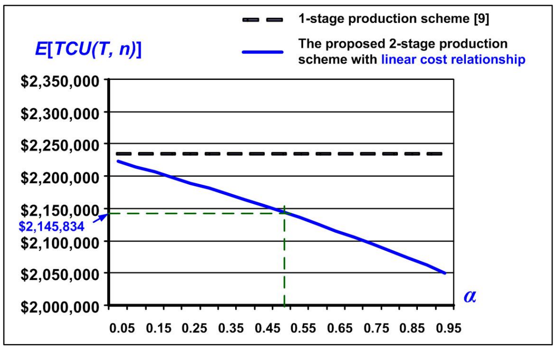

Fig. 6 The behavior of $E[TCU(T, n)]$ with respect to the common intermediate part's completion rate α

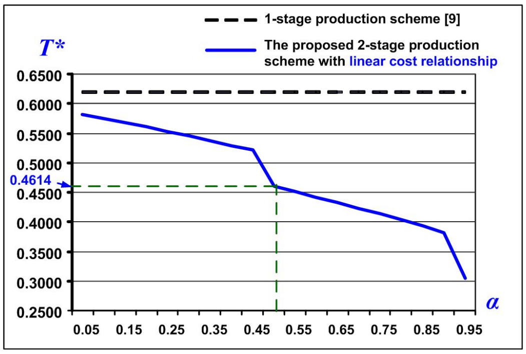

Fig. 7 The effects of variations of common part's completion rate α on the optimal production cycle time T^*

4.2 Case 2: Analysis of nonlinear relationship of cost relevant variables

In this section, we demonstrate that the proposed model is capable of analyzing any given non-linear relationship between the *common part's* relevant costs and its completion rate α. For instance, if a nonlinear relationship of 'α^(1/3)' between *common part's* relevant costs and α is known, then C_0 = [α^(1/3)]C_1 = [(0.5)^(1/3)]$80 = $63, so it obviously has higher production cost (or called value) than that in the linear relationship case (which is \$40). Apply the similar computation we have the following values of other relevant parameters: $C_{R,0}$ = \$40, K_0 = \$13,493, $h_{1,0} = h_{4,0}$ = \$8, and $h_{2,0}$ = \$24. Assume the following parameters' values remain the same as stated in subsection 4.1: $P_{1,0}$ = 120,000, $P_{2,0}$ = 96,000, and x_0 = [0, 0.04]. Accordingly, in stage 2 we obtain the values of other variables as follows: $C_{,i}$ = \$17, \$27, \$37, \$47, and \$57, respectively; K_i = \$3,507, \$4,007, \$4,507, \$5,007, and \$5,507, respectively; $C_{R,i}$ = \$10, \$15, \$20, \$25, and \$30, respectively; and x_i follows the uniform distribution over the intervals [0, 0.01], [0, 0.06], [0, 0.11], [0, 0.16], and [0, 0.21], respectively

We apply Eqs. 34, 35, and 25 to obtain the optimal number of shipments n^* = 3, the optimal production cycle time T^* = 0.4005 (years), and the expected system costs $E[TCU(T^*, n^*)]$ = \$2,093,253. Figure 8 depicts the behavior of $E[TCU(T, n)]$ with respect to the common part's completion rate α under both linear and nonlinear relationships. In nonlinear relationship case, as the common part's completion rate α increases, the expected system costs $E[TCU(T, n)]$ decreases, and it indicates that $E[TCU(T, n)]$ is decreased by 2.45 % at α = 0.5 (i.e., system costs declined from \$2,145,834 to \$2,093,253) compared to that in the earlier linear case. The analytical results demonstrate that the proposed two-stage multi-item production scheme with delayed differentiation is a greatly beneficial model to manufacturers who have to meet demands for multiple products that share a common intermediate part.

Figure 9 illustrates the behavior of the optimal production cycle time T^* with respect to the common part's completion rate α under both linear and nonlinear relationships. As completion rate α increases, the optimal production cycle time T^* decreases significantly, and in the nonlinear case, the optimal cycle time T^* is shortened by 13.20 % at α = 0.5 (i.e., it reduces from 0.4614 to 0.4005) compared to that in the earlier linear case. Therefore, it demonstrates that the proposed two-stage multi-item production scheme with delayed differentiation is a considerably beneficial model (in terms of faster response cycle time) for manufacturers who have to meet demands for multiple products that share a common intermediate part.

Furthermore, the analytical results reveals that if the *common part's* relevant costs are higher (e.g., having a nonlinear relationship α^(1/3) rather than the linear one), then the optimal cycle time T^* reduces significantly compared to that in the linear case.

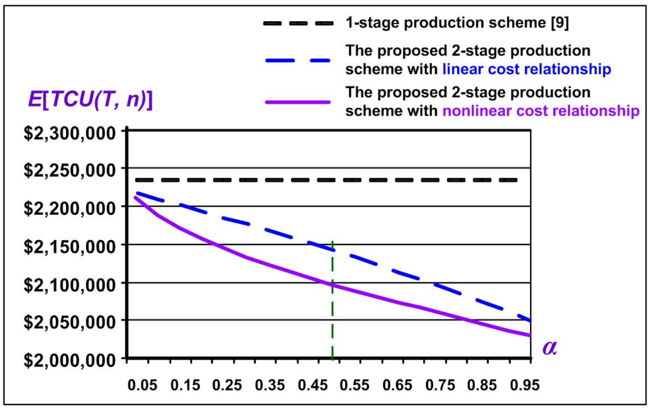

Fig. 8 The behavior of $E[TCU(T, n)]$ with respect to the common intermediate part's completion rate α under both linear and nonlinear relationships

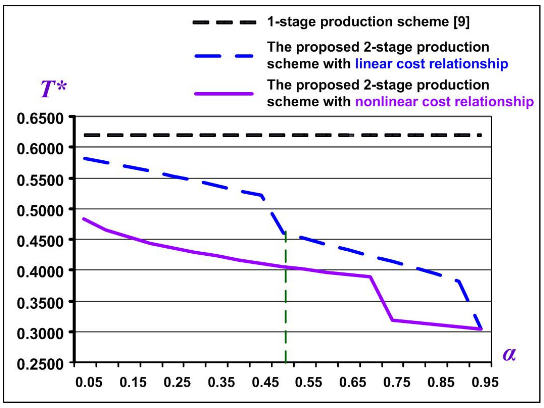

Fig. 9 The behavior of the optimal production cycle time T^* with respect to the common intermediate part's completion rate α under both linear and nonlinear relationships

5. Conclusion

Inspired by the potential benefits derived from applying delayed differentiation to multi-product systems, and with the aim of providing managers of transnational enterprises with information to assist them in achieving the key operational goals such as maximizing machine utilization, ensuring product quality, lowering overall operating costs, and shortening response time, this study explores the effect of delayed differentiation on a multi-product vendor-buyer integrated inventory system with rework, using a single machine production scheme.

Using mathematical modeling and optimization methods, we derive the closed-form optimal replenishment cycle time and delivery decisions and demonstrate the practical use of our results through a numerical example. The results reveal that our proposed multi-product fabrication scheme with delayed differentiation strategy is considerably beneficial in saving expected system costs and reducing replenishment cycle time. Further analysis also indicates that when the *common intermediate part's* value is higher (e.g., having a nonlinear relationship α^(1/3) rather than the linear one), both the expected system costs and production cycle time reduces significantly compared to that in the linear case. For future study, to explore and compare the effects of the dual-machine production scheme on the optimal operating policies of the same system would be an interesting direction.

Acknowledgement

Authors would like to express their gratefulness to the Ministry of Science and Technology of Taiwan for sponsor of this research (under grant no.: MOST 102-2410-H-324-015-MY2).

References

[1] Hadley, G., Whitin, T.M. (1963). *Analysis of Inventory Systems*, Prentice-Hall, New Jersey, USA.

[2] Silver, E.A., Pyke, D.F., Peterson, R. (1998). *Inventory Management and Production Planning and Scheduling*, John Wiley & Sons, New York, USA.

[3] Nahmias, S. (2009). *Production & Operations Analysis*, McGraw-Hill, New York, USA.

[4] Aggarwal, V. (1984). Grouping multi-item inventory using common cycle periods, *European Journal of Operational Research*, Vol. 17, No. 3, 369-372, doi: 10.1016/0377-2217(84)90132-2.

[5] Rosenblatt, M.J., Rothblum, U.G. (1990). On the single resource capacity problem for multi-item inventory systems, *Operations Research*, Vol. 38, No. 4, 686-693, doi: 10.1287/opre.38.4.686.

[6] Aliyu, M.D.S., Andijani, A.A. (1999). Multi-item-multi-plant inventory control of production systems with shortages/backorders, *International Journal of Systems Science*, Vol. 30, No. 5, 533-539, doi: 10.1080/002077299292272.

[7] Balkhi, Z.T., Foul, A. (2009). A multi-item production lot size inventory model with cycle dependent parameters, *International Journal of Mathematical Models and Methods in Applied Sciences*, Vol. 3, No. 2, 94-104.

[8] Rahmani, D., Ramezanian, R., Fattahi, P., Heydari, M. (2013). A robust optimization model for multi-product two-stage capacitated production planning under uncertainty, *Applied Mathematical Modelling*, Vol. 37, No. 20-21, 8957-8971, doi: 10.1016/j.apm.2013.04.016.

[9] Chiu, Y-S.P., Chiang, K-W., Chiu, S.W., Song, M-S. (2016). Simultaneous determination of production and shipment decisions for a multi-product inventory system with a rework process, *Advances in Production Engineering & Management*, Vol. 11, No. 2, 141-151, doi: 10.14743/apem2016.2.216.

[10] Caggiano, K.E., Jackson, P.L., Muckstadt, J.A., Rappold, J.A. (2009). Efficient computation of time-based customer service levels in a multi-item, multi-echelon supply chain: A practical approach for inventory optimization, *European Journal of Operational Research*, Vol. 199, No. 3, 744-749, doi: 10.1016/j.ejor.2008.08.002.

[11] Björk, K.-M. (2012). A multi-item fuzzy economic production quantity problem with a finite production rate, *International Journal of Production Economics*, Vol. 135, No. 2, 702-707, doi: 10.1016/j.ijpe.2011.10.003.

[12] Ma, W.-N., Gong, D.-C., Lin, G.C. (2010). An optimal common production cycle time for imperfect production processes with scrap, *Mathematical and Computer Modelling*, Vol. 52, No. 5-6, 724-737, doi: 10.1016/j.mcm.2010.04.024.

[13] Guerrero, W.J., Yeung, T.G., Guéret, C. (2013). Joint-optimization of inventory policies on a multi-product multi-echelon pharmaceutical system with batching and ordering constraints, *European Journal of Operational Research*, Vol. 231, No. 1, 98-108, doi: 10.1016/j.ejor.2013.05.030.

[14] Chiu, Y.-S.P., Sung, P.-C., Chiu, S.W., Chou, C.-L. (2015). Mathematical modeling of a multi-product EMQ model with an enhanced end items issuing policy and failures in rework, *SpringerPlus*, Vol. 4, No. 1, 679, doi: 10.1186/s40064-015-1487-4.

[15] Gerchak, Y., Magazine, M.J., Gamble, B.A. (1988). Component commonality with service level requirements, *Management Science*, Vol. 34, No. 6, 753-760, doi: 10.1287/mnsc.34.6.753.

[16] Garg, A., Tang, C.S. (1997). On postponement strategies for product multiple points of differentiation, *IIE Transactions,* Vol. 29, No. 8, 641-650, doi: 10.1080/07408179708966374.

[17] Graman, G.A. (2010). A partial-postponement decision cost model, *European Journal of Operational Research*, Vol. 201, No. 1, 34-44, doi: 10.1016/j.ejor.2009.03.001.

[18] Collier, D.A. (1982). Aggregate safety stock levels and component part commonality, *Management Science*, Vol. 28, No. 11, 1296-1303, doi: 10.1287/mnsc.28.11.1296.

[19] Swaminathan, J.M., Tayur, S.R. (1998). Managing broader product lines through delayed differentiation using vanilla boxes, *Management Science*, Vol. 44, No. 12 (Part 2), S161-S172.

[20] Swaminathan, J.M., Tayur, S.R. (2003). Models for supply chains in e-business, *Management Science,* Vol. 49, No. 10, 1387-1406, doi: 10.1287/mnsc.49.10.1387.17309.

[21] Bernstein, F., DeCroix, G.A., Wang, Y. (2011). The impact of demand aggregation through delayed component allocation in an assemble-to-order system, *Management Science,* Vol. 57, No. 6, 1154-1171, doi: 10.1287/mnsc.1110.1340.

[22] Shih, W. (1980). Optimal inventory policies when stock-outs result from defective products, *International Journal of Production Research*, Vol. 18, No. 6, 677-686, doi: 10.1080/00207548008919699.

[23] Makis, V. (1998). Optimal lot sizing and inspection policy for an EMQ model with imperfect inspections, *Naval Research Logistics*, Vol. 45, No. 2, 165-186, doi: 10.1002/(SICI)1520-6750(199803)45:2<165::AID-NAV3>3.0.CO;2-6.

[24] Ojha, D., Sarker, B.R., Biswas, P. (2007). An optimal batch size for an imperfect production system with quality assurance and rework, *International Journal of Production Research*, Vol. 45, No. 14, 3191-3214, doi: 10.1080/00207540600711853.

[25] Chiu, Y.-S.P., Wu, M.-F., Cheng, F.-T., Hwang, M.-H. (2014). Replenishment lot sizing with failure in rework and an enhanced multi-shipment policy, *Journal of Scientific & Industrial Research*, Vol. 73, 648-652.

[26] Khan, M., Jaber, M.Y., Ahmad, A.-R. (2014). An integrated supply chain model with errors in quality inspection and learning in production, *Omega*, Vol. 42, No. 1, 16-24, doi: 10.1016/j.omega.2013.02.002.

[27] Yeh, T.-M., Pai, F.-Y., Liao, C.-W. (2014) Using a hybrid MCDM methodology to identify critical factors in new product development, *Neural Computing and Applications*, Vol. 24, No. 3, 957-971, doi: 10.1007/s00521-012-1314-6.

[28] Pal, S., Mahapatra, G.S., Samanta, G.P. (2015). A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness, *Economic Modelling*, Vol. 46, 334-345, doi: 10.1016/j.econmod.2014.12.031.

[29] Schwarz, L.B. (1973). A simple continuous review deterministic one-warehouse N-retailer inventory problem, *Management Science*, Vol. 19, No. 5, 555-566, doi: 10.1287/mnsc.19.5.555.

[30] Goyal, S.K., Gupta, Y.P. (1989). Integrated inventory models: The buyer-vendor coordination, *European Journal of Operational Research*, Vol. 41, No. 3, 261-269, doi: 10.1016/0377-2217(89)90247-6.

[31] Hill, R.M. (1996). Optimizing a production system with a fixed delivery schedule, *Journal of the Operational Research Society*, Vol. 47, No. 7, 954-960, doi: 10.1057/jors.1996.121.

[32] Thomas, D.J., Hackman, S.T. (2003). A committed delivery strategy with fixed frequency and quantity, *European Journal of Operational Research*, Vol. 148, No. 2, 363-373, doi: 10.1016/S0377-2217(02)00398-3.

[33] Çömez, N., Stecke, K.E., Çakanyildirim, M. (2012). Multiple in-cycle transshipments with positive delivery times, *Production and Operations Management*, Vol. 21, No. 2, 378-395, doi: 10.1111/j.1937-5956.2011.01244.x.

[34] Wu, M.-F., Chiu, Y.-S.P., Sung, P.-C. (2014). Optimization of a multi-product EPQ model with scrap and an improved multi-delivery policy, *Journal of Engineering Research*, Vol. 2, No. 4, 103-118, doi: 10.7603/s40632-014-0027-7.

[35] Safaei, M. (2014). An integrated multi-objective model for allocating the limited sources in a multiple multi-stage lean supply chain, *Economic Modelling*, Vol. 37, 224-237, doi: 10.1016/j.econmod.2013.10.018.

[36] Chiu, S.W., Sung, P.-C., Tseng, C.-T., Chiu, Y.-S.P. (2015). Multi-product FPR model with rework and multi-shipment policy resolved by algebraic approach, *Journal of Scientific & Industrial Research*, Vol. 74, No. 10, 555-559.

[37] Tseng, C.-T., Wu, M.-F., Lin, H.-D., Chiu,Y.-S.P. (2014). Solving a vendor-buyer integrated problem with rework and a specific multi-delivery policy by a two-phase algebraic approach, *Economic Modelling*, Vol. 36, 30-36, doi: 10.1016/j.econmod.2013.09.013.

[38] Hishamuddin, H., Sarker, R.A., Essam, D. (2014). A recovery mechanism for a two echelon supply chain system under supply disruption, *Economic Modelling*, Vol. 38, 555-563, doi: 10.1016/j.econmod.2014.02.004.

[39] Chiu, S.W., Huang, C.-C., Chiang, K.-W., Wu, M.-F. (2015). On intra-supply chain system with an improved distribution plan, multiple sales locations and quality assurance. *SpringerPlus*, Vol. 4, No. 1, 687, doi: 10.1186/ s40064-015-1498-1.

[40] Pai, F.-Y. (2015). How supplier exercised power affects the cooperative climate, trust and commitment in buyer-supplier relationships, *International Journal of Business Excellence*, Vol. 8, No. 5, 662-673, doi: 10.1504 /IJBEX.2015.071275.

[41] Ocampo, L.A. (2015). A hierarchical framework for index computation in sustainable manufacturing, *Advances in Production Engineering & Management*, Vol. 10, No. 1, 40-50, doi: 10.14743/apem2015.1.191.

[42] Šarić, T., Šimunović, K., Pezer, D., Šimunović, G. (2014). Inventory classification using multi-criteria ABC analysis, neural networks and cluster analysis, *Technical Gazette – Tehnički vjesnik*, Vol. 21, No. 5, 1109-1115.

[43] Kaveh, M., Dalfard, V.M. (2012). A study on the effect of inflation and time value of money on lot sizing in spite of reworking in an inventory control model, *Technical Gazette – Tehnički vjesnik*, Vol. 19, No. 4, 819-826.

[44] Al-Hawari, T., Ahmed, A., Khrais, S., Mumani, A. (2013). Impact of assignment, inventory policies and demand patterns on supply chain performance, International Journal of Simulation Modelling, Vol. 12, No. 3, 164-177, doi: 10.2507/IJSIMM12(3)3.235

[45] Rardin, R.L. (1998). *Optimization in Operations Research*, Prentice-Hall, New Jersey, USA.