

Perfect matchings, Hamiltonian cycles and edge-colourings in a class of cubic graphs

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Abstract

A graph G has the Perfect-Matching-Hamiltonian property (PMH-property) if for each one of its perfect matchings, there is another perfect matching of G such that the union of the two perfect matchings yields a Hamiltonian cycle of G . The study of graphs that have the PMH-property, initiated in the 1970s by Las Vergnas and Häggkvist, combines three well-studied properties of graphs, namely matchings, Hamiltonicity and edge-colourings. In this work, we study these concepts for cubic graphs in an attempt to characterise those cubic graphs for which every perfect matching corresponds to one of the colours of a proper 3-edge-colouring of the graph. We discuss that this is equivalent to saying that such graphs are even-2-factorable (E2F), that is, all 2-factors of the graph contain only even cycles. The case for bipartite cubic graphs is trivial, since if G is bipartite then it is E2F. Thus, we restrict our attention to non-bipartite cubic graphs. A sufficient, but not necessary, condition for a cubic graph to be E2F is that it has the PMH-property. The aim of this work

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is to introduce an infinite family of E2F non-bipartite cubic graphs on two parameters, which we coin *papillon graphs*, and determine the values of the respective parameters for which these graphs have the PMH-property or are just E2F. We also show that no two papillon graphs with different parameters are isomorphic.


Keywords: Cubic graph, perfect matching, Hamiltonian cycle, 3-edge-colouring.

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Popolna prirejanja, hamiltonski cikli in barvanja povezav v družini kubičnih grafov

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Povzetek

Graf G ima lastnost hamiltonskih popolnih prirejanj (je tipa PMH), če za vsako izmed njegovih popolnih prirejanj obstaja neko drugo popolno prirejanje v grafu G , tako da unija teh dveh popolnih prirejanj da hamiltonski cikel grafa G . Raziskovanje takšnih grafov sta začela v 1970ih Las Vergnas in Häggkvist; predstavlja kombinacijo treh dobro raziskanih pojmov v zvezi z grafi, in sicer prirejanj, hamiltonskih ciklov ter barvanj povezav. V tem delu raziskujemo te koncepte za kubične grafe v želji, da bi karakterizirali tiste kubične grafe, za katere vsako popolno prirejanje ustreza eni od barv pravičnega 3-barvanja povezav grafa. Ugotovljamo, da je to enakovredno trditvi, da so taki grafi sodo-2-faktorizabilni (E2F), kar pomeni, da vsi 2-faktorji grafa vsebujejo samo sode cikle. Primer dvodelnih kubičnih grafov je trivialen, saj če je graf G dvodelen, potem je tudi tipa E2F. Zato svojo pozornost omejimo na nedvodelne kubične grafe. Zadosten, vendar ne potreben pogoj, da je kubični graf tipa E2F, je, da je tipa PMH. Namen tega dela je predstaviti neskončno

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družino nedvodelnih kubičnih grafov tipa E2F z dvema parametroma, ki smo jih poimenovali *metuljasti grafi*, in določiti vrednosti ustreznih parametrov, za katere so ti grafi tipa PMH ali pa so kar tipa E2F. Prav tako dokažemo, da nobena dva metuljasta grafa z različnimi vrednostmi parametrov nista izomorfna.

Ključne besede: Kubični grafi, popolna prirejanja, hamiltonski cikel, 3-barvanje povezav.

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