doi: 10.26529/cepsj.895

Management of Problem Solving in a Classroom Context

Eszter Kónya*[1] and Zoltán Kovács[2]

We report on the results of a professional development programme involving four Hungarian teachers of mathematics. The programme aims to support teachers in integrating problem solving into their classes. The basic principle of the programme, as well as its novelty (at least compared to Hungarian practice), is that the development takes place in the teacher's classroom, adjusted to the teacher's curriculum and in close cooperation between the teacher and researchers. The teachers included in the programme were supported by the researchers with lesson plans, practical teaching advice and lesson analyses. The progression of the teachers was assessed after the one-year programme based on a self-designed trial lesson, focusing particularly on how the teachers plan and implement problem-solving activities in lessons, as well as on their behaviour in the classroom during problem-solving activities. The findings of this qualitative research are based on video recordings of the lessons and on the teachers' own reflections. We claim that the worked-out lesson plans and the self-reflection habits of the teachers contribute to the successful management of problem-solving activities.

Keywords: classroom discussion, problem solving, professional development, reflective thinking

1 *Corresponding Author. Faculty of Science and Technology, University of Debrecen, Hungary; eszter.konya@science.unideb.hu.

2 Eszterházy Károly University, Eger, Hungary.

Vodenje reševanja problemov pri pouku

Eszter Kónya in Zoltán Kovács

V prispevku so predstavljeni rezultati programa strokovnega izpopolnjevanja, v katerem so sodelovali štirje madžarski učitelji matematike. Cilj programa je bil podpreti učitelje pri vključevanju reševanja problemov v njihovo poučevalno prakso. Osnovna načela programa pa tudi njegova novost (vsaj v primerjavi s poučevalno prakso na Madžarskem) so, da razvijanje poučevanja poteka v praksi, da je prilagojeno učiteljevemu časovnemu načrtu poučevanja ter da poteka v tesnem sodelovanju med učiteljem in raziskovalci. Učitelje, vključene v program, so raziskovalci podprli pri načrtovanju učnih ur pa tudi s praktičnimi nasveti in z analizami izvedenih učnih ur. Napredek učiteljev je bil ocenjen po enoletnem programu. Temeljil je na učiteljevi samostojno zasnovani učni uri, njihovem načrtovanju in na izvajanju dejavnosti reševanja problemov pri pouku pa tudi na njihovem odzivanju med dejavnostmi reševanja problemov. Ugotovitve kvalitativne raziskave temeljijo na videoposnetkih učnih ur in učiteljevih refleksijah opravljenega dela. Trdimo, da premišljeno načrtovanje poučevanja in učiteljeva samorefleksija prispevata k uspešnemu vodenju dejavnosti reševanja problemov pri pouku.

Ključne besede: razredna diskusija, reševanje problemov, profesionalni razvoj, refleksija

Introduction

Teaching problem solving in mathematics classrooms is not an unknown element of the tradition of teaching mathematics in Hungary and is closely related to Polya's principle of active learning (Pólya, 1981). Along with arithmetic fluency, problem-solving activities became more important in the Complex Mathematics Teaching Experiment led by Tamás Varga (1919–1987) in Hungary in the 1960s and 1970s (Varga, 1988). Nevertheless, our experience is that these activities are becoming less common in Hungarian teaching practice today: teachers often ignore problem solving as a means of achieving a better understanding of mathematical concepts.

In order to support teachers in incorporating problem solving in their classes, we elaborated on a research-based professional development (PD) programme. One of the core principles of the programme is that we only want to make incremental changes in teachers' practice, focusing on the problem-oriented approach to learning mathematics. This principle also appears in the research of Niss, who claims that "instead of making more radical changes in curricula, in teaching and learning materials, and in assessment, corresponding to the changes in the audiences, authorities have attempted to preserve the goals and the ethos of mathematics education of the past, at least in spirit, while making series of piecewise adjustments so as to avoid too drastic discontinuities in the transition from the past to the present" (Niss, 2018, p. 79).

The origin of our framework was the Japanese lesson study model (Fernandez & Yoshida, 2004), thus the PD programme is linked to the everyday teaching practices of the participants. The lesson study process includes the following three steps: (1) collaborative lesson planning; (2) one participant teaches while the others observe his/her work; (3) discussing the study lesson, reflections and suggestions. After revising the lesson, an updated version of the lesson plan is prepared. The collaboration in our model means that the teachers and researchers work together in planning the study lessons. We retain the feature of the model that the participating teachers visited and reflected upon each other's lessons. The lesson analysis was done in two steps: after the lessons and after the semester.

In line with Walsh (cited by Rott (2019)), we aimed at improving the quality of teaching within the existing curriculum, focusing on learning in the form of problems. Pehkonen et al. (2013) go one step further and emphasise the role of open problems, which also contribute to a better understanding of key principles and concepts. The open approach to teaching mathematics "leads almost automatically to problem-centred teaching and increases communication

in class, thus approaching instruction that is more open and pupil-centred" (Pehkonen et al., 2013, p. 12).

In our view, the problem-centred or problem-oriented approach to learning mathematics is characterised by three properties (Kovács & Kónya, 2019):

(1) students analyse a mathematical problem situation;
(2) students critically adapt to their own and their classmates' thinking;
(3) students learn to explain and justify their thinking.

This approach is closely related to problem-solving strategies. In this respect, we rely on our preliminary research on problem solving conducted in various student age groups from grades 4–12. We found that observing and following a pattern is a well-known and often used problem-solving strategy in the age group of 11–12 years, which is why patterning has become a feature of our programme. We also found that the further phases of the inductive thinking process, such as the formulation, testing and explaining of conjecture, are barely observable. The students usually did not feel the need for such an explanation. In our programme, item (3) is therefore given particular emphasis and is closely related to (2), as reasoning is often done in pairs or as teamwork. Consequently, the nature of students' mathematical thinking should be taken into account in the design and implementation of lessons, emphasising inductive thinking and patterning (Kónya & Kovács, 2018).

The successful implementation of problem solving in mathematics classrooms is strongly dependent on teachers' behaviour during the phases of the problem-solving process. Rott (2019) classifies teachers' behaviour as:

- closely managed, i.e., preferring only one approach, to which the students are led;
- emphasising strategies, i.e., encouraging students to pursue their ideas, aiming for strategic diversity;
- neutral, i.e., neither narrowing down students' approaches nor emphasising strategic diversity.

In order to better investigate the behavioural type of a teacher, Rott divided the problem-solving process into the four phases of Polya: (1) understanding the problem; (2) devising a plan; (3) carrying out the plan; and (4) looking back. In additional, the teacher's intervention regarding their students' problem-solving process was interpreted for each of these phases. The teachers' behaviour during these problem-solving phases was coded following Rott's grid.

The third core element of our PD programme is lesson analysis. We agree with Lee (2005) that self-monitoring and a reflective attitude help teachers to become more successful. According to Šarić and Šteh (2017), critical reflection should mean looking for new solutions and paths in teaching in order to introduce the changes that contribute to the transformation of the community for better learning. Therefore, we emphasise the importance of the teachers developing reflective thinking: why they employ certain instructional strategies, how they can improve their teaching, and how they can evaluate their work from different points of view.

Our PD programme has a narrow goal: the teacher should progress in implementing problem-oriented learning in his or her classes, i.e., s/he should be able to process the curriculum in a problem-oriented way and realise classroom discussion. Taking into consideration all of the research-based principles described above, we consider whether the PD programme we have developed is effective or not. Its effectiveness is scrutinised through the following two questions focusing on the participating teachers' progression:

Q1 Is the teacher completing the PD programme able to incorporate a problem into the lesson plan that reflects a problem-oriented processing of the curriculum?

Q2 Is the teacher able to organise classroom discussion so that the students explain and justify their ideas, while also critically adapting to their own thinking and that of their classmates?

Method

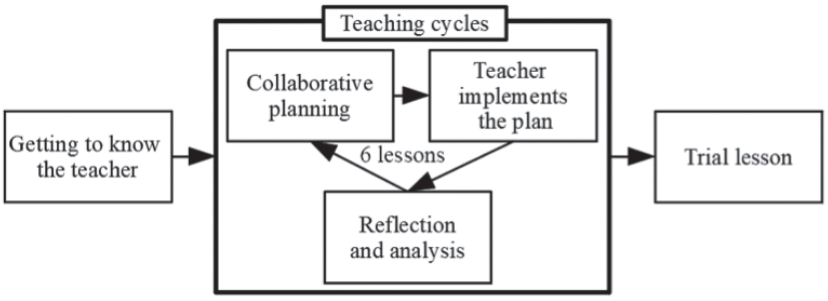
After a pilot study (Kovács & Kónya, 2019), we established the structure of our PD programme. As a first step, the researchers took part in the classes of the teachers and characterised their teaching style.

After selecting the study lessons from the teachers' agenda, the researchers developed detailed lesson plans. The teachers gave their opinions and suggestions, and determined the final lesson plan. One of the participants taught the lesson, which was video recorded, while another teacher from the PD programme took part in the class as an observer.

We organised six teaching cycles and concluded the year-long programme with one trial lesson (Figure 1).

Figure 1

The structure of the PD programme



Four teachers and two researchers (the authors of the present article) participated in the programme. The teachers had 15–20 years of experience in teaching mathematics and were employees of the same school in a Hungarian town. Having chosen one of their classes, the teachers took part in the programme together with the head of the school. Although the students of the selected classes were motivated to learn mathematics, they did not show any special interest in this school subject.

As a starting point, we visited some lessons in the experimental classes and discussed the teachers' professional views and the goal of the developmental programme with the teachers. We found that they all preferred a closely managed way of teaching: they explained the new material, asked the students direct questions and did not feel any need to initiate open classroom discussion.

After completing the last teaching cycle, the participating teachers were asked to plan a trial lesson individually in the same spirit as the previous six lessons planned by us. The lesson themes were chosen freely by the teachers from their course calendar. They had an opportunity to discuss the plan with us before its implementation. The lessons were held in the same class as the previous study lessons. After the lessons, we analysed the video recordings and a few weeks later organised a closing discussion with the teachers.

In the present paper, we focus on four trial lessons (Table 1) that were held in April 2019.

Table 1
Information about the trial lessons

	Teacher A	Teacher B	Teacher C	Teacher D
Grade	5	8	9	10
Age of students (years)	11–12	14–15	15–16	16–17
Class size	23	11	12	14
Lesson theme	Coordinate plane	Arithmetic sequence	Line reflection	Trig. functions in geometry
Lesson objective	Introduction	Introduction	Systematisation	Application
Didactical function of problem solving	Deepening new knowledge	Deepening new knowledge	Applying updated knowledge	Practising, modelling

The first research question is answered by analysing the problem situation incorporated in the lesson plan according to the following aspects: whether the problem is in line with the theme and objective of the lesson, and whether it provides an opportunity for a problem-oriented approach as outlined in the PD programme. Concerning the second research question, we use a content analysis of the transcripts made from the video recordings and evaluate the teacher's behaviour according to Rott's grid.

Results

In line with our two research questions, we present our results by first providing a brief description of the problem addressed in the trial lesson (Q1) and then highlighting some typical and informative moments of the classroom discussion during the problem-solving activity (Q2).

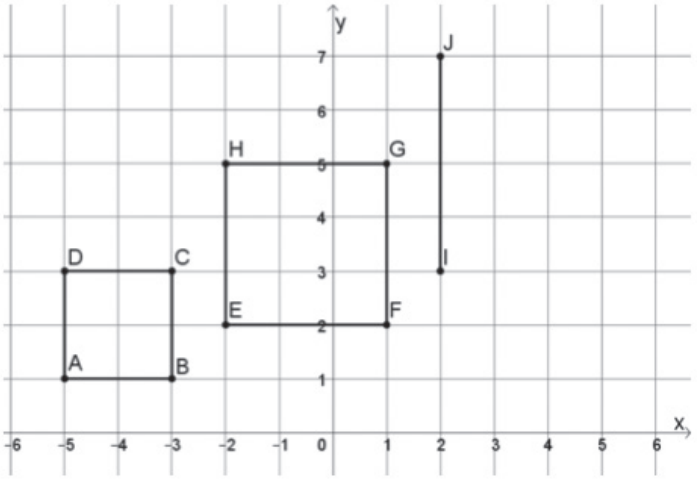
Problem A

a) Write the coordinates of the points marked on the drawing. (Figure 2)

b) Continue the sequence.

Figure 2

Diagram for Problem A



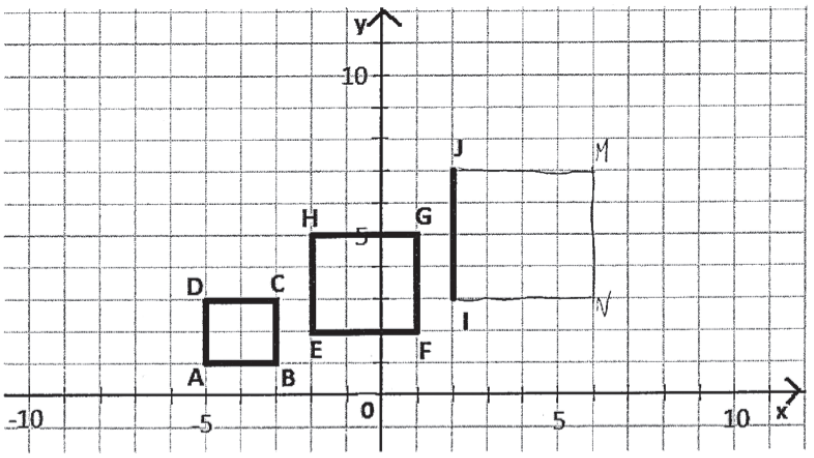
Ad Q1. The theme of the lesson in Grade 5 was to introduce the Cartesian coordinate system. The purpose of setting this complex problem was to deepen the new knowledge and practise the patterning strategy. In order to draw a new square, the students have to recognise the geometric regularity and orient themselves in the coordinate system, after which reading the coordinates of points was a straightforward application of the new material.

Ad Q2. Teacher A set this problem at the end of the lesson. The first part (writing the coordinates of the marked points) was presented to the whole class. Only those who answered the first question quickly had time to deal with the second part, i.e., the patterning problem.

From a total of 23 students, 8 answered the patterning problem and 4 solved it correctly (Figure 3).

Figure 3

A correct solution to Problem A



A(-5 ;+1)
B(-3 ;+1)
C(-3 ;+3) ✓
D(-5 ;+3) ✓
E(-2 ; +2)✓
F(+1 ;+2)✓
G(+1 ;+5)✓
H(-2 ;+5)✓
I(+2 ;+3) ✓
J(+2 ;+7) ✓
M(+6 ;+7)
N(+6 ;+3)

There was no classroom discussion on the second part of the problem.

Problem B

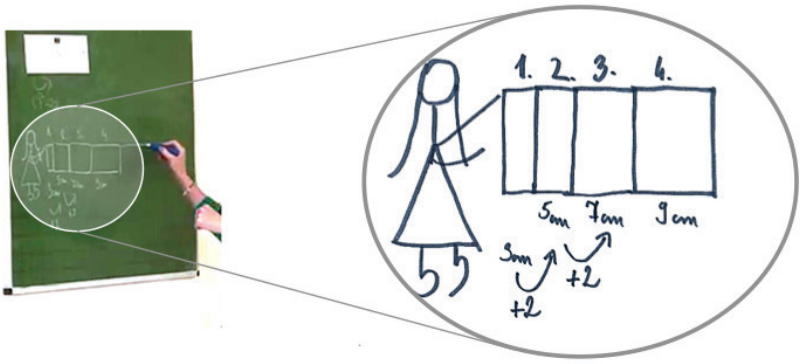
a) Zsuzsi decided to knit a scarf […] the first day she produces a 3 cm long section, […] she makes two centimetres more each day than the previous day.

b) How many centimetres will Zsuzsi knit on the 2nd, 3rd, 9th, 20th and nth day?

Ad Q1. Teacher B used the problem to introduce new material (arithmetic sequences) at the beginning of the Grade 8 lesson. The task design reflected the patterning phases (determine close members of the series, then a distant member and finally a general member). The problem in the lesson led to the formula of the general element of the sequence, knowing the first element and the difference. The rule itself was created by the students during a classroom discussion.

Ad Q2. After giving the students a few minutes to think about the solution individually, Teacher B drew and wrote on the board the students' answer regarding the 2nd, 3rd, 4th and 5th day (Figure 4).

Figure 4
Teacher B writes and interprets the students' answers on the board



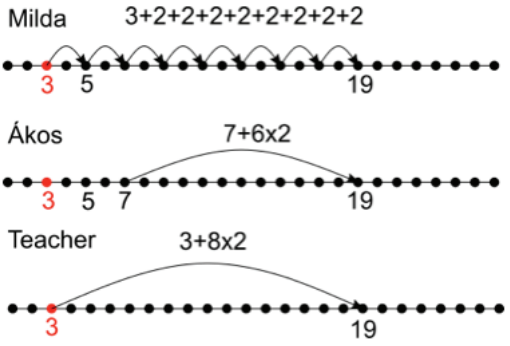
After that, the following classroom discussion was initiated.

01 Teacher B: How many centimetres will Zsuzsi knit on the 9th day?
02 Milda: 19.
03 Teacher B: The result is 19. How did you calculate it?
04 Milda: I continued the sequence.
05 Teacher B: You continued the sequence. On the 6th day, she knitted 13 cm, on the 7th day 15 cm, on the 8th day 17 cm, …, very good!
06 Ákos: (interrupts the teacher) I multiplied the difference by 6 because the 3rd member is already there, and there are six more members, and I added 12 to the 3rd member.
07 Teacher B: Ákos says he did not count one by one, but he calculated how much to add to the 5th day … to get the 9th day … 6th, 7th (she uses her fingers to count, and then she becomes uncertain) … From what did you start?
08 Ákos: From the 3rd day…
09 Teacher B: Yes, from the 3rd day. So, he calculated that from the 3rd day, he added …
10 Ákos: six …
11 Teacher B: Six times two, right and that gives the result.
12 Teacher B: Why did you start from the 3rd day? You could have started … (waiting for somebody to continue her sentence)
13 Zalán: … from the very first number (continues the teacher's thought)
14 Teacher B: … from the very first. Usually, we start from the very first, but Ákos had a very good idea … Usually, we start from the first element. After all, if we were, for example, asking for the 100th member, it would

take quite a while to determine all 99 members before it …

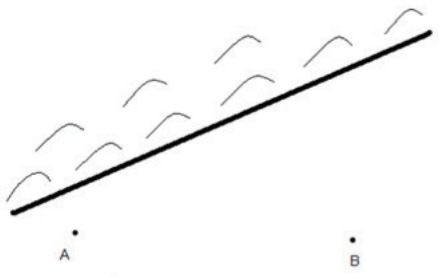
When calculating the 9th element, there was one student who enumerated all of the elements (Line 04), but another student started with the 3rd member: he already considered the previous stage and added six times the difference directly (Line 06). Teacher B did not ignore this idea but used it to steer the discussion in the direction she had imagined: to determine the elements of the sequence from the first member and the difference (Lines 12–14). We summarise the suggested solution strategies in Figure 5.

Figure 5
Different ways of calculating the 9th element of the sequence



Problem C
The man who lives in house "A" should take water from the river to the house "B" every morning. How can he make the shortest route? (Figure 6)

Figure 6
The "water from the river" problem



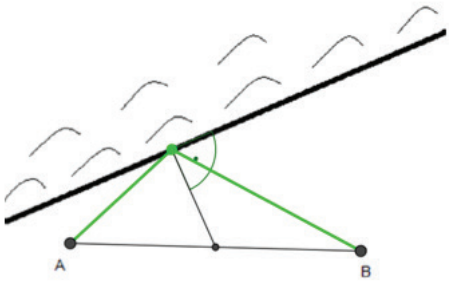
Ad Q1. Problem C is an indirect application of line reflection strongly related to the theme of the lesson. It provides an opportunity for ninth-grade students to come up with strategies to find the shortest path, as well as to discuss individual ideas. However, the problem appeared in an isolated way in the middle of the lesson, i.e., neither the previous nor the subsequent items referred to the problem.

Ad Q2. After reading the text and working on it in pairs, the classroom discussion started with the following brainstorming.

01 Teacher C: Who has some idea of how to find this shortest route?

02 Panni: He will take the water from the river at that point (she shows it with her arms) …we connect points A and B and draw a line from the midpoint perpendicular to the line [river]. (see Figure 7) … so the distances will be the same.

Figure 7
Panni's idea



03 Teacher C: Why?

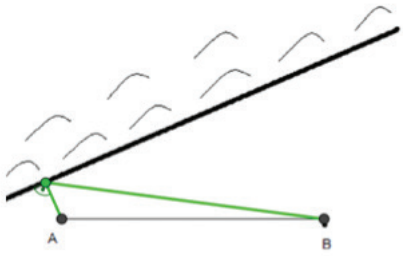
04 Panni: If he goes directly from A to the river, the route will be longer.

05 Teacher C: Please measure the distance then compare your result with those of your pair.

06 Teacher C: Are there any other ideas?

07 Tibi: We draw a line from point A to the river … (hand gestures) perpendicularly … and draw it further to point B. (see Figure 8)

Figure 8
Tibi's idea



08 Teacher C: Hmm … I'll try to give you some hints. Listen. What is the shortest way between two points?
09 The class together: A straight line.

Two students shared their ideas with the class during the brainstorming phase (Line 02 and 07), but Teacher C realised that the conversation was coming to a dead-end and gave a direct hint to move forward (Line 08). However, this hint did not guide students toward the right solution.

Problem D
What is the angle of the slope in degrees? (Figure 9)

Figure 9
Traffic sign

Ad Q1. The purpose of setting this problem was an application of trigonometric functions in an everyday situation, as well as making a mathematical model for a well-known traffic sign.
Problem D is an open problem because its starting situation is open. In the phase of planning, we decided together with Teacher D to open up the original problem by skipping the first two sentences from the text: *Before a steep street, 12% is written on a traffic sign. It means that the rise of the slope is 12% of the horizontal road.* The original form of the text was well-known by the teacher, but new (in both forms) for tenth-grade

students. The open aspect of the problem provided an opportunity to discuss the relative nature of the concept of percentages, as well as the role of the definition in both everyday life and mathematics. Problem D was well-integrated into the lesson structure along with other tasks.

Ad Q2. The classroom conversation began immediately after the students had read the text.

01 Teacher D: What does this mean? What do you think?
02 Anna: It means that the degree is the same as the percentage.
03 Teacher D: Is it that simple, the same in degrees? Hmm … it's clear to me that if it's rising by, then …
04 Juli: the road is higher by , … by …
05 Teacher D: If I run … How?
06 András: horizontally …
07 Teacher D: Originally horizontally, compared to this … (she shows the vertical direction with her arm)
08 Áron: The road is rising by of the horizontal road?
09 Teacher D: Good. Do you understand? For example, if I run metres … (waiting for Áron to complete her sentence)
10 Áron: … then it rises by metres.
11 Teacher D: I will be higher by metres. What if I want to generalize? Let's see, I run x metres?
12 Áron: then …
13 Teacher D: Very good. We helped you too much. I'll still show you what Áron said. If I run a certain number of metres, then I should calculate of that, which is the value of the rise. (She shows the triangle with the necessary data on the board.) Now tell me what the angle is in degrees? Anna said degrees.

The first answer was a typical, expected wrong answer (Line 02); the students simply tried to guess the meaning of the given data (Line 04, 08). The discussion was strongly directed by Teacher D, who closed it with a detailed solution plan.

Discussion

In this section, we discuss the findings obtained from the analysis of the presented episodes and the teachers' reflections on the lessons.

We establish that the teachers selected appropriate problems for the

chosen lesson theme and lesson objective (see Table 1). Moreover, the didactical function of the problem solving in the lessons was appropriate to the aim of teaching mathematics. The purpose of setting problems, such as deepening new knowledge or applying updated knowledge as well as modelling, is highly relevant in mathematics education today. Although methodologically not all four problems were appropriate (Problems A and C were too complex for the investigated classes), we can conclude that the participants understood the essence of the problem-oriented teaching style.

When evaluating the teachers' behaviour, we need to separate Teacher A from the other three, as she did not realise the planned classroom discussion. During the individual work of the students, Teacher A's behaviour was neutral at all stages of problem solving. She did not interpret the problem, did not provide strategic assistance, and did not check the solutions during the lesson. This corresponds to the use of the task for differentiation: only the students who had done the basic task dealt with the problem. Although the content of the problem was appropriate to the lesson theme, we agree with the reflection of Teacher A that the problem was not suitable for an introductory lesson: it was too complex for fifth graders at this time.

Many elements taught in the PD programme, such as patterning, appeared in the trial lesson of Teacher B. This teacher had previously experienced problems with her reactions in unexpected situations. On this occasion, however, she built on the students' idea. Her typical former behaviour was closely managed, while in the trial lesson she made a conscious effort to change. In the "devising the plan" phase she was neutral: her students shared their ideas freely, but the evaluation of these ideas was made by the teacher instead of the classmates. The teacher always felt a need to interpret the responses for the whole class. Nevertheless, the presented episode highlights Teacher B's professional development well.

The content of Problem C was wholly connected to the lesson theme. Teacher C's behaviour at the first two phases of the problem-solving process was appropriate, i.e., neutral or emphasising strategies, but after she realised that her students were unable to find the right solution, she suddenly switched to a closely managed style and gave them a direct hint without referring to the ideas they had previously communicated. The hint (see Line 08) was therefore useless: the students stopped thinking about the problem itself and only tried to follow the teacher's thought, giving short answers instead. The implementation of Problem C in this form was inappropriate for these ninth-grade students, as they had no chance whatsoever of finding the right solution. Teacher C's colleagues reflected that it would have been better to use the "water from the

river" problem as a worked example, and after understanding the key steps to set similar problems based on the ideas of the minimal route and line reflection.

Teacher D, too, was a closely managed type teacher before and after the PD programme. Her students are usually successful in the final exam of high school, which she regards as justification of her closely managed behaviour. Although the problem was planned as an open problem, it was implemented as a closed problem. The class did not discuss the different possible interpretations of the traffic sign and did not search for its correct meaning. Consequently, the teacher missed the opportunity to deal with the role of the definition in everyday life, as well as in mathematics. Teacher D explained the solution method while the students listened carefully. We can conclude that the teacher's behaviour hindered the independent problem-solving activity of the students.

Table 2 summarises the behaviour of the teachers in the phases of the problem-solving activities they managed in the classroom context. Teacher A did not realise classroom discussion regarding Problem A.

Table 2

Teachers' behaviour during the classroom discussion

	Teacher A	Teacher B	Teacher C	Teacher D
Understanding the problem	–	closely managed	neutral	closely managed
Devising a plan	–	neutral	1. emphasising strategies 2. closely managed	closely managed
Carrying out the plan	–	1. neutral 2. emphasising strategies	closely managed	neutral
Looking back	–	missing	missing	missing

Referring to the work of Rott (2019), where several behavioural types of teachers are listed, we can establish that Teacher D exactly fits the statements (a) "Some teachers make sure that their students understand the problem before they start working on it. This way, they take responsibility for the Understand phase, which could lead to their students not learning to analyse tasks on their own" and (f) "There are teachers that give content-related aid that directly leads to a solution very early without trying motivational or strategical aids beforehand" (Rott, 2019, p. 905).

The behaviour of Teachers B and C shows a more diverse picture. We can detect the effect of the PD programme, especially in the 2nd and 3rd phases. The

teachers tried to change their former closely managed teaching style, although this change could be considered productive only in the case of Teacher B.

A common characteristic of the lessons is the lack of the "Looking back" phase. However, the analysis of this phenomenon goes beyond the scope of the present paper.

At the end of the school year, we organised a closing discussion about the findings and their possible explanations with the four participating teachers. In line with our research questions, we highlight two topics and the teachers' brief reflections on them.

What was the novelty for you in the problem-oriented approach?

Teacher A: My students are allowed to speak in the classroom and express their opinion now.

Teacher B: I became more aware than I was before.

Teacher C: I realised that we could connect different mathematical topics by problems.

Teacher D: Now I often look for useful tasks from everyday life.

The answers strengthen our findings related to the first research question (Q1); namely, that the programme was successful from the point of view that all of the participants understood the essence of problem-oriented learning design and found appropriate problems for their lesson themes, at least regarding the content. In their reflections, the teachers pointed out some of the main characteristics of the method.

The method of implementation of the chosen problem showed a diverse picture. We found that management is strongly dependent on teachers' behaviour. We could, however, detect some indications of changes; for example, instead of a closely managed style, Teachers B and C tried to be neutral or to emphasise strategies, but after an unexpected situation they reverted into the previously used closely managed style.

One of the typical phenomena was that *often the teacher reacts to students' ideas instead of their classmates.* When asked why, they responded as follows:

Teacher B: Teachers like to correct the answer immediately; students expect it, too.

Teacher D: My students learn everything I want but they don't deal with the "Why?" questions.

Teacher A: They don't understand what the other child says. I repeat the student's idea in a simpler form.

Teacher C: The older the student, the less active he or she is. Those who don't like and know mathematics, don't like and care about classroom discussion, either.

The teachers' reflections show that they are aware of their typical "repetitive" behaviour, but in their opinion, the students expect this behaviour from them. Teachers C and D, who teach ninth- and tenth-grade classes, respectively, reported a lack of motivation of students to take part in mathematical discussions. The reason for this well-known phenomenon may be the teacher-centred teaching-learning practice to which the older students are already accustomed.

Conclusions

This paper discusses the experiences of a PD programme for teachers of mathematics elaborated by the authors. The clear goals of the programme were as follows: the teachers should (1) process the curriculum in a problem-oriented way and (2) realise classroom discussion. Two research questions were formulated:

Q1 Is the teacher completing the PD programme able to incorporate a problem into the lesson plan that reflects the problem-oriented processing of the curriculum?

Q2 Is the teacher able to organise the classroom discussion so that the students explain and justify their ideas, while also critically adapting to their own thinking and that of their classmates?

On analysing trial lessons (the outputs of the programme) and the self-reflection of the teachers related to their work, we established that the teachers selected the appropriate problems for the chosen lesson theme and lesson objective (see Table 1). Moreover, the didactical function of the problem-solving in the lessons – according to the aim of teaching mathematics – was adequate. We can therefore conclude that the participants understood the essence of a problem-oriented teaching style.

The classroom implementation of the problem-solving activities can be considered partly successful. At the beginning of our experiment, we recognised the rather rigid professional habits of all of the participating teachers and their engagement with teacher-centred instruction. After investigating their work and reflections, we can report that the teachers tried to change their habits and behaviour during the lessons as a result of the PD programme, but they

did not always succeed, often returning to their old, well-established teaching habits, especially when an unexpected situation arose.

The answers to the research questions show that the applied form of the PD programme – the essential elements of which are teacher-research cooperation, direct connection to the everyday practice of the teachers, and the reflective activity – all seem to be productive. What to expect from a programme of six teaching cycles and what realistic opportunities for improvement were available in a relatively short period became clear during the programme. In our experience, it is feasible for a teacher to develop in lesson planning and to initiate a behavioural change. Among the problems not solved by this PD programme, we can highlight the development of the ability to respond to unexpected situations. The programme needs to be adjusted to this in the future.

This phenomenon reflects the dual-process model of cognition. According to Feldon (2007), information processing occurs simultaneously on parallel pathways: controlled (high cognitive load) and automatic (low cognitive load). Controlled and automatic processes operate independently but intersect at certain points to produce human performance. When teachers process high levels of cognitive load, they are less able to dedicate working memory resources to other mental processes. Some dual-process cognitions, such as evaluation of an unexpected situation, may therefore rely almost entirely on their automatic components and operate without conscious monitoring. Studying teachers' cognitive load could therefore be a possible direction for our further research.

Another meaningful extension of our recent research to refine the presented PD programme could be the conscious development of teachers' reflective thinking. According to Lee, "… an awareness of the need for reflective thinking might be the first condition for its improvement. This should be followed by continual practice of reflection in various formats and on multiple specific issues" (Lee, 2005, p. 711).

Acknowledgement

This study was funded by the Content Pedagogy Research Programme of the Hungarian Academy of Sciences.

References

Feldon, D. F. (2007). Cognitive load and classroom teaching: The double-edged sword of automaticity. *Educational Psychologist, 42*(3), 123–137.

Fernandez, C., & Yoshida, M. (2004). *Lesson study - A Japanese approach to improving mathematics teaching and learning.* Lawrence Erlbaum.

Kónya, E., & Kovács, Z. (2018). Let's explore the solution: Look for pattern! In B. May-Tatsis, K. Tatsis, & E. Swoboda (Eds.), *Mathematics in the real world* (pp. 136–147). Wydawnictwo Uniwersytetu Rzeszowskiego.

Kovács, Z., & Kónya, E. (2019). Implementing problem solving in mathematics classes. In A. Kuzle, I. Gebel, & B. Rott (Eds.), *Implementation research on problem solving in school settings: Proceedings of the 2018 joint conference of ProMath and the GDM working group on problem solving* (pp. 121–128). WTM.

Lee, H.-J. (2005). Understanding and assessing preservice teachers' reflective thinking. *Teaching and Teacher Education, 21*(6), 699–715.

Niss, M. (2018). How can we use mathematics education research to uncover, understand and counteract mathematics specific learning difficulties? In B. May-Tatsis, K. Tatsis, & E. Swoboda (Eds.), *Mathematics in the real world* (pp. 79–99). Wydawnictwo Uniwersytetu Rzeszowskiego.

Pehkonen, E., Näveri, L., & Laine, A. (2013). On teaching problem solving in school mathematics. *CEPS Journal, 3*(4), 9–23.

Pólya, G. (1981). *Mathematical discovery on understanding, learning and teaching problem solving (combined edition).* John Wiley & Sons.

Rott, B. (2019). Teachers' behaviors, epistemological beliefs, and their interplay in lessons on the topic of problem solving. *International Journal of Science and Mathematics Education*, 18(5) 903–924.

Šarić, M., & Šteh, B. (2017). Critical reflection in the professional development of teachers: Challenges and possibilities. *CEPS Journal, 7*(3), 67–85.

Varga, T. (1988). Mathematics education in Hungary today. *Educational Studies in Mathematics, 19(3)*, 291–298.

Biographical note

Eszter Kónya, PhD, habil., is an associate professor in the field of didactics of mathematics on the Faculty of Science and Technology at the University of Debrecen, Hungary. She is involved in mathematics teacher training and professional development of practicing teachers. Her research interests include teaching and learning mathematics; problem solving and problem posing in mathematics; understanding the concepts of geometrical measurements; reasoning and proving in geometry.

Zoltán Kovács, PhD, CSc, is an associate professor at the Eszterházy Károly University, Eger, Hungary. He obtained his PhD in pure mathematics (differential geometry) and then his research interest turned to research activities in the field of mathematics education. His research topics now include the application of technology in the learning of mathematics; mathematical problem solving and problem posing; the professional development of mathematics teachers.