



New resonances and spectroscopy at Belle

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Abstract. The Belle experiment at the KEKB asymmetric-energy electron-positron collider has proven to be an excellent environment for a wide variety of measurements. Besides its main goal – measurements of CP violation in the system of B mesons – other most important achievements are observations of several yet undiscovered particles and measurements of their properties. The discoveries were often surprising, since only some of the observed states were predicted in various models, while others were not. The existence of these resonances therefore still imposes theoretical questions regarding their nature and also represents a challenge for a proper description in terms of QCD. Selected experimental results together with their possible interpretations are reviewed in this paper.

1 Introduction

The Belle detector [1] at the asymmetric-energy e^+e^- collider KEKB [2] has accumulated around 630 fb^{-1} by July 2006. The KEKB collider is often called the *B-factory*, since it operates at the energy of the $\Upsilon(4S)$ resonance, slightly above the $B\bar{B}$ -production threshold, and thus the accumulated data set contains a large number of $B\bar{B}$ pairs. While the main goal of both B-factories* are measurements of CP violation in the B-meson system, the excellent detector performances also enable searches for new hadronic (bound) states as well as studies of their properties. There are several possible mechanisms of the particle production at B-factories: production in the B-meson decays, fragmentation of quarks in e^+e^- annihilation or creation of C-even states in two photon processes. In this paper, we address some interesting discoveries of new hadronic states, produced by different mechanisms and observed by the Belle collaboration.

2 The X, Y, Z story

Several charmonium-like new states have been recently observed by Belle, namely: X(3872), Z(3930), Y(3940) and X(3940). The naming convention indicates the lack of knowledge about the structure and properties of these particles at the time of their discovery.

* Besides KEKB in Japan, there is a similar collider called PEP-II in the USA, delivering data to the *BABAR* [3] detector.

2.1 Observation and properties of X(3872)

In 2003 Belle reported on the analysis of $B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$ decays, where a narrow charmonium-like state (X(3872)) decaying to $\pi^+ \pi^- J/\psi$ was discovered [4]. For the most recent update from Belle [5], the fitted yield of both, charged and neutral B mesons reconstructed in $B \rightarrow KX(3872)(\rightarrow \pi^+ \pi^- J/\psi)$ decay mode is shown in Fig. 1(a) as a function of the $\pi^+ \pi^- J/\psi$ invariant mass. The observation of X(3872) resonance was later confirmed by the CDF [6], D0 [7] and BABAR [8] experiments. Currently, the world average of the mass is $M(X(3872)) = (3871.2 \pm 0.5) \text{ MeV}/c^2$ [9] and the upper limit on its width, as measured by Belle, is $\Gamma(X(3872)) < 2.3 \text{ MeV}$ [4].

Several interpretations of X(3872) resonance have been suggested, including charmonium hypothesis [10,?], $D^0 \bar{D}^{*0}$ molecule [12] and tetraquarks [13]. Various dedicated studies were performed at Belle in order to determine possible quantum numbers of X(3872) and its nature. In 2005 Belle reported a strong evidence for the radiative decay of $X(3872) \rightarrow \gamma J/\psi$ [14]. The fitted yield of reconstructed B mesons, as obtained from the simultaneous fit to the ΔE and M_{bc} distributions** for $B \rightarrow K\gamma J/\psi$ decay candidates is shown in Fig. 1(b) as a function of the $M(\gamma J/\psi)$. The observed signal with a significance above 4σ can be converted to $\mathcal{B}(X(3872) \rightarrow \gamma J/\psi)/\mathcal{B}(X(3872) \rightarrow \pi^+ \pi^- J/\psi) = 0.14 \pm 0.05$, which is not in agreement with the expectations for charmonium interpretation of X(3872). However, the observation of this radiative decay establishes even charge-conjugation parity (C) of X(3872).

Furthermore, Belle examined possible J^{PC} quantum number assignments of X(3872) by studying angular correlations between the final-state particles in $X(3872) \rightarrow \pi^+ \pi^- J/\psi$ decays [5]. An example is presented in Fig. 1(c): the measured distribution of the angle between the negative of the B meson flight direction and π^+ momentum from X(3872) in the X(3872) frame, is in agreement with the expectation for the 1^{++} state. Additionally, the $\pi^+ \pi^-$ invariant mass distribution for the events in the $X(3872) \rightarrow \pi^+ \pi^- J/\psi$ signal region, shown in Fig 1(d), peaks at the upper kinematic limit indicating the $\rho^0 J/\psi$ intermediate state and favours S-wave over P-wave as the relative orbital angular momentum between the final-state dipion and J/ψ . As a consequence of these studies, $J^{PC} = 1^{++}$ is strongly favoured for the X(3872), but the 2^{++} can not be completely ruled out.

The latter possibility could be ruled out by the recent study [15] of $B \rightarrow K D^0 \bar{D}^0 \pi^0$ decays, where a near-threshold enhancement at the $(3875.4 \pm 0.7(\text{stat.}) \pm 1.1(\text{syst.})) \text{ MeV}/c^2$ for the invariant mass of the $D^0 \bar{D}^0 \pi^0$ system was observed (see Fig. 1(e, f)). If the observed enhancement – whose invariant mass is however about 2σ higher than the world average value for X(3872) – is indeed due to the X(3872), the $J^{PC} = 1^{++}$ quantum number assignment for the X(3872) would again be favoured, since near-threshold decays $X(3872) \rightarrow D^0 \bar{D}^{*0}/D^0 \bar{D}^0 \pi^0$ are expected to be strongly suppressed for $J = 2$.

** Two kinematic variables are used to identify B-meson candidates: $\Delta E \equiv E_B - E_{\text{beam}}$ and $M_{bc} \equiv 1/c^2 \sqrt{E_{\text{beam}}^2 - (p_{bc})^2}$, where E_B and p_B are the reconstructed energy and momentum of the B candidate, and E_{beam} is the beam energy, all expressed in the centre-of-mass (CM) frame.

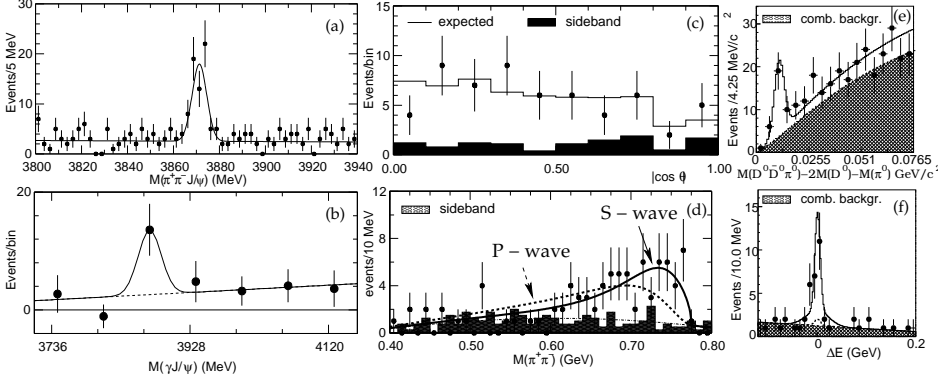


Fig. 1. (a) $\pi^+\pi^-J/\psi$ invariant mass for $B \rightarrow K\pi^+\pi^-J/\psi$ decays [5]; (b) Yield of B mesons for $B \rightarrow K\gamma J/\psi$ decay candidates as a function of $M(\gamma J/\psi)$ [14]; (c) Distribution of angle in X(3872) decays described in the text. The full histogram represents the expectation for $J^{PC} = 1^{++}$ assignment and the hatched histogram is the contribution of background as obtained from the scaled sidebands of $M(\pi^+\pi^-J/\psi)$; (d) $M(\pi^+\pi^-)$ distribution for events in X(3872) signal region. The histogram again indicates the sideband-determined background, while the solid (dashed) curve shows the result of the fit using Breit-Wigner function for $\rho^0 \rightarrow \pi^+\pi^-$, and assuming J/ψ and ρ^0 to be in a relative S-wave (P-wave); (e) and (f) $\Delta M \equiv M(D^0\bar{D}^0\pi^0) - 2M(D^0) - M(\pi^0)$ and ΔE distributions for near-threshold $D^0\bar{D}^0\pi^0$ enhancement in $B \rightarrow KD^0\bar{D}^0\pi^0$ decay [15].

While currently available X(3872) data – the mass, possible 1^{++} quantum numbers and observed decay modes – are in agreement with the hypothesis that X(3872) is a $D^0\bar{D}^{*0}$ molecule [12], some spin assignments corresponding to more conventional interpretations can still not be ruled out (see for example Ref. [16]). Further experimental results and theoretical calculations are thus needed to resolve the puzzle about the nature of the X(3872) resonance.

2.2 Z(3930) resonance

A search for the χ'_{cJ} ($J = 0$ or $J = 2$) states and other C-even charmonium states in the mass range of $3.73 \text{ GeV}/c^2 - 4.3 \text{ GeV}/c^2$ was performed for the two-photon production of $D\bar{D}$ pairs, $\gamma\gamma \rightarrow D\bar{D}$ [17]. The two-photon process was studied in the non-tagged mode, where final-state electron and positron produced in the reaction $e^+e^- \rightarrow e^+e^-D\bar{D}$ are not detected, and the $D\bar{D}$ system has a very small transverse momentum w.r.t. the e^+e^- axis. These requirements help selecting $D\bar{D}$ pairs produced exclusively in collisions of two quasi-real photons. The D mesons were reconstructed in decays of $D^0 \rightarrow K^-\pi^+$, $K^-\pi^+\pi^0$, $K^-\pi^+\pi^-\pi^+$ and $D^+ \rightarrow K^-\pi^+\pi^+$ (and their charge conjugated modes). The obtained $D\bar{D}$ invariant-mass distribution is shown in Fig. 2(a). A clear peak with 5.3 σ significance denoted as Z(3930) was observed with mass $(3929 \pm 5(\text{stat.}) \pm 2(\text{syst.})) \text{ MeV}/c^2$ and width $(29 \pm 10(\text{stat.}) \pm 2(\text{syst.})) \text{ MeV}$. A product of the two-photon decay width and branching fraction of the Z(3930) is found to be $\Gamma(Z(3930))\mathcal{B}(Z(3930) \rightarrow D\bar{D}) = 0.18 \pm 0.05 \pm 0.03 \text{ keV}$.

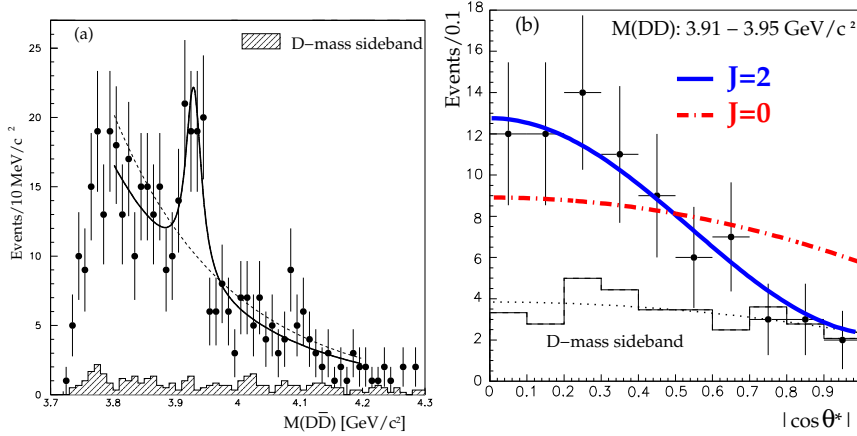


Fig. 2. (a) Invariant mass of $D\bar{D}$ pairs produced in non-tagged two-photon reactions. The curves indicate the result of the fit with a resonant component (solid) and without it (dashed). (b) The $|\cos \theta^*|$ distribution for $Z(3930) \rightarrow D\bar{D}$ decays. Expected predictions for $J = 2$ and $J = 0$ are shown as a solid and a dash-dotted line, respectively, and contain the non-peaking background shown separately by the dotted curve.

An angular analysis was also performed by Belle collaboration [17]. Efficiency corrected $\cos \theta^*$ distribution, where θ^* is the angle between D meson and the beam axis in the $\gamma\gamma$ rest frame, shows that the spin-2 assignment for the observed resonance is strongly favoured over spin-0 assignment. All performed $Z(3930)$ measurements are thus consistent with the expectations for the χ'_{c2} , a radial excitation of 3P_2 charmonium.

2.3 Two new states at $M \approx 3940 \text{ MeV}/c^2$

After the observation of a sub-threshold decay of $X(3872) \rightarrow \omega J/\psi$ [14], using $B \rightarrow KJ/\psi\pi^+\pi^-\pi^0$ decays in a similar way as described for $B \rightarrow KJ/\psi\pi^+\pi^-$ decays in Sec. 2.1, Belle performed an analysis of the $\omega J/\psi$ system produced in exclusive $B \rightarrow K\omega J/\psi$ decays [18], selecting events with $M(\pi^+\pi^-\pi^0) \approx m_\omega$. Events with $M(K\omega) < 1.6 \text{ GeV}/c^2$ are rejected in order to suppress $K_X \rightarrow K\omega$ contribution, where K_X denotes resonances such as $K_1(1270)$, $K_1(1400)$, and $K_2(1400)$ that are known to decay to $K\omega$. The events clustering near the bottom of the Dalitz plot shown in Fig. 3(a) are responsible for a strong enhancement above the phase space expectation, which can be observed in the plot of signal yield of B decays, as obtained from the fit to the M_{bc} distribution, in bins of $M(\omega J/\psi)$ (see Fig. 3(b)). The fit with an S-wave Breit-Wigner function yields (58 ± 11) events with a statistical significance above 8σ , corresponding to a new resonance named $Y(3940)$ with a mass of $(3943 \pm 11(\text{stat.}) \pm 13(\text{syst.})) \text{ MeV}/c^2$ and a total width $\Gamma = (87 \pm 22(\text{stat.}) \pm 26(\text{syst.})) \text{ MeV}$. The measured fraction for this state is $\mathcal{B}(B \rightarrow KY(3940))\mathcal{B}(Y(3940) \rightarrow \omega J/\psi) = (7.1 \pm 1.3(\text{stat.}) \pm 3.1(\text{syst.})) \cdot 10^{-5}$.

Due to rather intriguing properties, the nature of $Y(3940)$ is still mysterious. Namely, any charmonium state with a mass around $3940 \text{ MeV}/c^2$ is expected to

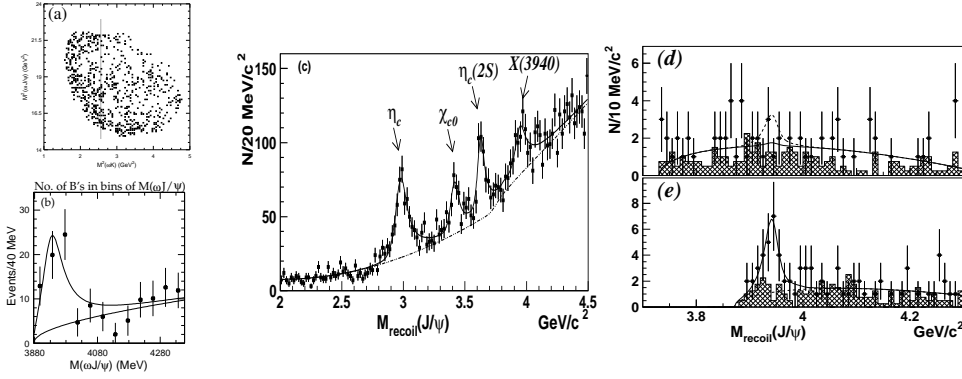


Fig. 3. (a) $M^2(\omega J/\psi)$ vs. $M^2(\omega K)$ Dalitz plot for $B \rightarrow K\omega J/\psi$ decays. Vertical line indicates the region selected by the requirement $M(K\omega) \geq 1.6 \text{ GeV}/c^2$. (b) Yield of B mesons in $B \rightarrow K\omega J/\psi$ decays as a function of $M(\omega J/\psi)$. (c) Spectrum of mass recoiling against the J/ψ . Same recoil mass for events tagged as (d) $J/\psi D\bar{D}$ and (e) $J/\psi D\bar{D}^*$.

dominantly decay to $D\bar{D}$ and/or $D\bar{D}^*$, which for $Y(3940)$ have not been observed yet. Adding that for a $c\bar{c}$ charmonium the hadronic transition to $\omega J/\psi$ should be heavily suppressed, one can conclude that the $Y(3940)$ resonance is probably not a conventional radially excited P-wave charmonium state. As an alternative interpretation, it has been suggested that $Y(3940)$ is one of $c\bar{c}$ -gluon hybrid charmonium states that were first predicted in 1978 [19] and are expected to be produced in B meson decays [20]. It has been shown that $D^{(*)}\bar{D}^{(*)}$ decays for these exotic states are forbidden or heavily suppressed [21], so that such a hybrid state with a mass equal to that of the $Y(3940)$ would have a large branching fraction for decays to J/ψ or ψ' plus light hadrons [22]. However, while this interpretation is able to explain $Y(3940)$ decay modes, predicted masses for $c\bar{c}$ -gluon hybrid states are between 4300 and 4500 GeV/c^2 [23], substantially higher than the measured $Y(3940)$ mass.

Another resonance with a similar mass above $D\bar{D}^{(*)}$ threshold – denoted as $X(3940)$ – was also discovered by the Belle collaboration. This state was observed in the J/ψ recoil mass spectrum for inclusive $e^+e^- \rightarrow J/\psi X$ processes [24]. The mass recoiling against the $J/\psi \rightarrow \ell^+\ell^-$ is determined as $M_{\text{recoil}}(J/\psi) = \sqrt{(E_{\text{CM}} - E_{J/\psi}^*)^2 - (cp_{J/\psi}^*)^2}/c^2$, where E^* is the J/ψ CM energy and E_{CM} is the CM energy of the event. The new peak can be seen in a recoil mass spectrum at about 3940 MeV/c^2 , together with three known peaks corresponding to η_c , χ_{c0} and $\eta_c(2S)$ (see Fig. 3(c)).

Searches for two exclusive decay modes of this newly observed state were performed: $X(3940) \rightarrow D\bar{D}^{(*)}$ and $X(3940) \rightarrow \omega J/\psi$. For the former search, only a single D meson besides the J/ψ was reconstructed to increase the efficiency. Only events with the recoil mass $M_{\text{recoil}}(DJ/\psi)$ close to $D^{(*)}$ mass were retained for the analysis. The resulting mass recoiling against the J/ψ – corresponding to the invariant mass of the $D\bar{D}$ and $D\bar{D}^*$ system – is shown in Fig. 3(d, e)). While no significant signal was observed at the mass of about 3940 MeV/c^2 for the $e^+e^- \rightarrow$

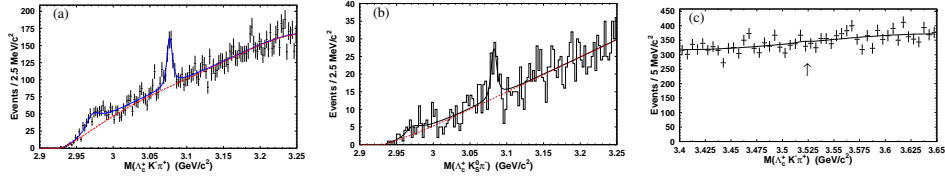


Fig. 4. Distributions of the invariant masses (a) $M(\Lambda_c^+ K^- \pi^+)$ and (b) $M(\Lambda_c^+ K_S^0 \pi^-)$ (both shown as points with error bars) together with the fitting function (the solid line). The dashed lines indicate the background contributions in both cases. (c) The distribution of $\Lambda_c^+ K_S^0 \pi^-$ invariant mass (points with error bars) in the region around the value of 3.52 GeV (marked with an arrow), shown together with the fit result (depicted as a solid line).

$J/\psi D\bar{D}$ events, there is a clear peak for events tagged as $e^+e^- \rightarrow J/\psi D\bar{D}^*$. The mass of the $X(3940)$ resulting from the fit shown in Fig. 3(e) is $(3943 \pm 6(\text{stat.}) \pm 6(\text{syst.})) \text{ MeV}/c^2$ and the upper limit on the $X(3940)$ total width is 52 MeV at the 90% confidence level.

No significant signal was found for $X(3940) \rightarrow \omega J/\psi$ decays. Since $X(3940)$ state does not share decay modes with the $Y(3940)$, these two states appear not to be the same. A possible interpretation is that $X(3940)$ state is a radially excited charmonium state $\eta_c(3S)$.

3 Observation of $\Xi_{cx}(2980)$ and $\Xi_{cx}(3077)$

Early this year, using the data sample of 461.5 fb^{-1} the Belle collaboration reported the first observation of two charmed baryons [25]. These two baryons, denoted as $\Xi_{cx}(2980)^+$ and $\Xi_{cx}(3077)^+$, are found to be decaying into a $\Lambda_c^+ K^- \pi^+$ final state (see Fig. 4 (a)). Assuming that these states carry charm and strangeness, the above observation would represent the first example of a baryonic decay in which the initial c and s quarks are carried away by two different final state particles. Most naturally, these two states could be interpreted as excited charm-strange baryons Ξ_c . This interpretation could be further justified by positive results of the search for neutral isospin related partners of the above states, performed in events with the $\Lambda_c^+ K_S^0 \pi^-$ final state. The latter search results in the observation of the $\Xi_{cx}(3077)^0$ together with a broad enhancement near the threshold, i.e. in the mass region corresponding to the $\Xi_{cx}(2980)^0$ (see Fig. 4 (b)). Preliminary values of properties for the four observed states are collected in Table 1.

In the $\Lambda_c^+ K^- \pi^+$ final state, the SELEX collaboration reported the observation of a double charmed baryon Ξ_{cc}^+ at the mass of about $3520 \text{ MeV}/c^2$ [26], which has not been confirmed by other experiments. This result – together with the observation of new states mentioned in the previous section of this paper, and the surprisingly large cross section for double charmonium production at B-factories [27,?] – have generated renewed theoretical interest in the spectroscopy, decays and production of charmonium. It has been suggested that the comparison with the production of double charm tetraquark $T_{cc} = cc\bar{u}\bar{d}$ [29] could shed some light on these experimental results. To search for the Ξ_{cc}^+ state, Belle extended

New State	Mass (MeV/c ²)	Width (MeV/c ²)	Yield (events)	Signif. (σ)
$\Xi_{cx}(2980)^+$	$2978.5 \pm 2.1 \pm 2.0$	$43.5 \pm 7.5 \pm 7.0$	405.3 ± 50.7	6.3
$\Xi_{cx}(3077)^+$	$3076.7 \pm 0.9 \pm 0.5$	$6.2 \pm 1.2 \pm 0.8$	326.0 ± 39.6	9.7
$\Xi_{cx}(2980)^0$	$2977.1 \pm 8.8 \pm 3.5$	43.5 (fixed)	42.3 ± 23.8	2.0
$\Xi_{cx}(3077)^0$	$3082.8 \pm 1.8 \pm 1.5$	$5.2 \pm 3.1 \pm 1.8$	67.1 ± 19.9	5.1

Table 1. Summary of the parameters of the new states in the $\Lambda_c^+ K^- \pi^+$ and $\Lambda_c^+ K_s^0 \pi^+$ final states: masses, widths, yields and statistical significance.

the range of $M(\Lambda_c^+ K^- \pi^+)$ search to include the region around 3520 MeV/c² (see Fig. 4 (c)). However, the study shows no evidence for this state, and as a result only an upper limit on the ratio of cross-sections for exclusive $\Xi_{cc}(3520)^+$ production and inclusive Λ_c^+ production is given at the 90% confidence level: $\sigma(\Xi_{cc}(3520)^+) \times \mathcal{B}(\Xi_{cc}(3520)^+ \rightarrow \Lambda_c^+ K^- \pi^+) / \sigma(\Lambda_c^+) < 1.5 \cdot 10^{-4}$.

4 Conclusion

The large data sample collected by the Belle experiment at KEKB provides an excellent opportunity for the search of new particles. During the Belle operation more than ten new states have been discovered. In this paper we report on some of the most exciting, like X(3872), Y(3940), X(3940) and Z(3930). The latter two resonances can be interpreted as charmonium states, $\eta_c(3S)$ and χ'_{c2} , respectively. None of the existing measurements contradicts the X(3872) interpretation as a $D^0 \bar{D}^{*0}$ molecule. The nature of Y(3940) remains to be addressed.

Recently, new charmed baryons, $\Xi_{cx}(2980)$ and $\Xi_{cx}(3077)$, have been observed by the Belle collaboration. These states are most naturally interpreted as the excited charmed strange baryons, Ξ_c . However, in contrast to known excited Ξ_c baryons, the observed new states decay into separate charmed (Λ_c^\pm) and strange (K) hadrons. As further studies of all new states are ongoing, more interesting results on charm spectroscopy are expected soon from the Belle experiment.

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