BLED WORKSHOPS
IN PHYSICS
VOL. 14, NO. 2

Proceedings to the 16th Workshop
What Comes Beyond . . . *(p. 82)*
Bled, Slovenia, July 14-21, 2013

8 Neutrino Masses and Mixing Within a $SU(3)$ Family Symmetry Model With One or Two Light Sterile Neutrinos

A. Hernández-Galeana⋆

Departamento de Física, Escuela Superior de Física y Matemáticas, I.P.N.,
U. P. "Adolfo López Mateos". C. P. 07738, México, D.F., México

Abstract. We report a global fit of parameters for fermion masses and mixing, including light sterile neutrinos, within a local vector $SU(3)$ family symmetry model. In this scenario, ordinary heavy fermions, top and bottom quarks and tau lepton, become massive at tree level from **Dirac See-saw** mechanisms implemented by the introduction of a new set of $SU(2)_L$ weak singlet vector-like fermions, U, D, E, N, with N a sterile neutrino. The $N_{L,R}$ sterile neutrinos allow the implementation of a 8×8 general tree level See-saw Majorana neutrino mass matrix with four massless eigenvalues. Hence, light fermions, including light neutrinos obtain masses from one loop radiative corrections mediated by the massive $SU(3)$ gauge bosons. This BSM model is able to accommodate the known spectrum of quark masses and mixing in a 4×4 non-unitary V_{CKM} as well as the charged lepton masses. The explored parameter space region provide the vector-like fermion masses: $M_D \approx 914.365$ GeV, $M_U \approx 1.5$ TeV, $M_E \approx 5.97$ TeV, $SU(3)$ family gauge boson masses of $\mathcal{O}(1-10)$ TeV, the neutrino masses $(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0.0085, 0.049, 0.22, 3.21, 1749.96, 1 \times 10^8, 1 \times 10^9)$ eV, with the squared neutrino mass differences: $m_2^2 - m_1^2 \approx 7.23 \times 10^{-5}$ eV^2, $m_3^2 - m_1^2 \approx 2.4 \times 10^{-3}$ eV^2, $m_4^2 - m_1^2 \approx 0.049$ eV^2, $m_5^2 - m_1^2 \approx 10.3$ eV^2. We also show the corresponding U_{PMNS} lepton mixing matrix. However, the neutrino mixing angles are extremely sensitive to parameter space region, and an improved and detailed analysis is in progress.

Povzetek. V modelu, kjer družinsko kvantno število določa grupa SU[3], poskrbi za maso obeh težkih kvarkov (t in b) ter za maso težkega leptona (tau) tako imenovani „Diracov sea-saw" mehanizem že na drevesnem nivoju. To dosežem tako, da predpostavim eksistenco novih kvarkov U in D ter novih leptonov E in N. Vsi so levoročni in brez šibkega naboja ter nosijo tripletni naboj SU(2). Nevtrini $N_{L,R}$, ki ne nosijo nobenega naboja, določajo na drevesnem nivoju masno matriko 8×8 Majoraninega tipa, ki ima štiri lastne vrednosti enake nič. Za maso lahkih fermionov, vključno z nevtrini, poskrbijo v popravkih z eno zanko masivni umeritveni bozoni družinskega kvantnega števila grupe SU(3). S primerno izbiro parametrov dosežem, da se lastnosti kvarkov in (z elektromagnetnim nabojem) nabitih leptonov ujemajo z izmerjenimi. Kvarkovska mešalna matrika 4×4 ni unitarna. Nova družina kvarkov in nabitih leptonov ima mase okoli TeV ali več ($M_D \approx 914.365$ TeV, $M_U \approx 1.5$ TeV, $M_E \approx 5.97$ TeV), mase umeritvenih bozonov z družinskimi kvantnimi števili grupe SU(3) pa so $\mathcal{O}(1-10)$ TeV. Za mase nevtrinov najdem $(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0.0085, 0.049, 0.22, 3.21, 1749.96, 1 \times 10^8, 1 \times 10^9)$ eV, za kvadrate razlik njihovih mas

---

⋆ e-mail: albino@esfm.ipn.mx

pa: $m_2^2 - m_1^2 \approx 7.23 \times 10^{-5}$ eV^2, $m_3^2 - m_1^2 \approx 2.4 \times 10^{-3}$ eV^2, $m_4^2 - m_1^2 \approx 0.049$ eV^2, $m_5^2 - m_1^2 \approx 10.3$ eV^2. Izračunam tudi leptonsko mešalno matriko. Ker je le-ta močno odvisna od izbire parametrov, bo vanjo potrebno vložiti še nekaj truda. Delo je v teku.

8.1 Introduction

The standard picture of three flavor neutrinos has been successful to account for most of the neutrino oscillation data. However, several experiments have reported new experimental results, on neutrino mixing[1], on large θ_{13} mixing from Daya Bay[2], T2K[3], MINOS[4], DOUBLE CHOOZ[5], and RENO[6], implying a deviation from TBM[7] scenario. In addition, the recent experimental results from the LSND and MiniBooNe short-baseline neutrino oscillation experiments, provide indications in favor of the existence of light sterile neutrinos in the eV-scale, in order to explain the tension in the interpretation of these data[8,9].

The strong hierarchy of quark and charged lepton masses and quark mixing have suggested to many model building theorists that light fermion masses could be generated from radiative corrections[10], while those of the top and bottom quarks and the tau lepton are generated at tree level. This may be understood as the breaking of a symmetry among families , a horizontal symmetry . This symmetry may be discrete [11], or continuous, [12]. The radiative generation of the light fermions may be mediated by scalar particles as it is proposed, for instance, in references [13,14] and the author in [15], or also through vectorial bosons as it happens for instance in "Dynamical Symmetry Breaking" (DSB) and theories like " Extended Technicolor " [16].

In this report, we address the problem of fermion masses and quark mixing within an extension of the SM introduced by the author in [17], which includes a vector gauged SU(3)[18] family symmetry commuting with the SM group. In previous reports[19] we showed that this model has the properties to accommodate a realistic spectrum of charged fermion masses and quark mixing. We introduce a hierarchical mass generation mechanism in which the light fermions obtain masses through one loop radiative corrections, mediated by the massive bosons associated to the SU(3) family symmetry that is spontaneously broken, while the masses for the top and bottom quarks as well as for the tau lepton, are generated at tree level from "Dirac See-saw"[20] mechanisms implemented by the introduction of a new generation of $SU(2)_L$ weak singlets vector-like fermions.

Recently, some authors have pointed out interesting features regarding the possibility of the existence of vector-like matter, both from theory and current experiments[23]. From the fact that the vector-like quarks do not couple to the W boson, the mixing of U and D vector-like quarks with the SM quarks gives rise to an extended 4×4 non-unitary CKM quark mixing matrix. It has pointed out for some authors that these vector-like fermions are weakly constrained from Electroweak Precision Data (EWPD) because they do not break directly the custodial symmetry, then main experimental constraints on the vector-like matter come from the direct production bounds, and their implications on flavor physics. See the ref. [23]

for further details on constraints for vector-like fermions. Theories and models with extra matter may also provide attractive scenarios for present cosmological problems, such as candidates for the nature of the Dark Matter ([21],[22]).

In this article, we report for the first time a global fit of the free parameters of the $SU(3)$ *family symmetry model to accommodate quark and lepton masses and mixing, including light sterile neutrinos.*

8.2   Model with $SU(3)$ flavor symmetry

8.2.1   Fermion content

We define the gauge group symmetry $G \equiv SU(3) \otimes G_{SM}$, where $SU(3)$ is a flavor symmetry among families and $G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the "Standard Model" gauge group, with g_s, g and g' the corresponding coupling constants. The content of fermions assumes the ordinary quarks and leptons assigned under G as:

$$\psi_q^o = (3,3,2,\tfrac{1}{3})_L \quad , \quad \psi_u^o = (3,3,1,\tfrac{4}{3})_R \quad , \quad \psi_d^o = (3,3,1,-\tfrac{2}{3})_R$$

$$\psi_l^o = (3,1,2,-1)_L \quad , \quad \psi_e^o = (3,1,1,-2)_R \ ,$$

where the last entry corresponds to the hypercharge Y, and the electric charge is defined by $Q = T_{3L} + \tfrac{1}{2}Y$. The model also includes two types of extra fermions: Right handed neutrinos $\Psi_\nu^o = (3,1,1,0)_R$, and the $SU(2)_L$ singlet vector-like fermions

$$U_{L,R}^o = (1,3,1,\tfrac{4}{3}) \quad , \quad D_{L,R}^o = (1,3,1,-\tfrac{2}{3}) \tag{8.1}$$

$$N_{L,R}^o = (1,1,1,0) \quad , \quad E_{L,R}^o = (1,1,1,-2) \tag{8.2}$$

The transformation of these vector-like fermions allows the mass invariant mass terms

$$M_U \, \bar{U}_L^o \, U_R^o + M_D \, \bar{D}_L^o \, D_R^o + M_E \, \bar{E}_L^o \, E_R^o + h.c. \ , \tag{8.3}$$

and

$$m_D \, \bar{N}_L^o \, N_R^o + m_L \, \bar{N}_L^o \, (N_L^o)^c + m_R \, \bar{N}_R^o \, (N_R^o)^c + h.c \tag{8.4}$$

The above fermion content make the model anomaly free. After the definition of the gauge symmetry group and the assignment of the ordinary fermions in the usual form under the standard model group and in the fundamental 3-representation under the $SU(3)$ family symmetry, the introduction of the right-handed neutrinos is required to cancel anomalies[24]. The $SU(2)_L$ weak singlets vector-like fermions have been introduced to give masses at tree level only to the third family of known fermions through Dirac See-saw mechanisms. These vector like fermions play a crucial role to implement a hierarchical spectrum for quarks and charged lepton masses, together with the radiative corrections.

8.3   SU(3) **family symmetry breaking**

To implement a hierarchical spectrum for charged fermion masses, and simultaneously to achieve the SSB of SU(3), we introduce the flavon scalar fields: η_i, $i = 2, 3$, transforming under the gauge group as $(3, 1, 1, 0)$ and taking the "Vacuum Expectation Values" (VEV's):

$$\langle \eta_3 \rangle^\mathsf{T} = (0, 0, \Lambda_3) \quad , \quad \langle \eta_2 \rangle^\mathsf{T} = (0, \Lambda_2, 0) \,. \tag{8.5}$$

The above scalar fields and VEV's break completely the SU(3) flavor symmetry. The corresponding SU(3) gauge bosons are defined in Eq.(8.29) through their couplings to fermions. Thus, the contribution to the horizontal gauge boson masses from Eq.(8.5) read

- η_3 : $\frac{g_{H_3}^2 \Lambda_3^2}{2}(Y_2^+ Y_2^- + Y_3^+ Y_3^-) + g_{H_3}^2 \Lambda_3^2 \frac{Z_2^2}{3}$
- η_2 : $\frac{g_{H_2}^2 \Lambda_2^2}{2}(Y_1^+ Y_1^- + Y_3^+ Y_3^-) + \frac{g_{H_2}^2 \Lambda_2^2}{4}(Z_1^2 + \frac{Z_2^2}{3} - 2Z_1 \frac{Z_2}{\sqrt{3}})$

Therefore, neglecting tiny contributions from electroweak symmetry breaking, we obtain the gauge boson mass terms

$$M_1^2 Y_1^+ Y_1^- + \frac{M_1^2}{2} Z_1^2 + (\frac{4}{3} M_2^2 + \frac{1}{3} M_1^2)\frac{Z_2^2}{2} - \frac{M_1^2}{\sqrt{3}} Z_1 Z_2$$
$$+ M_2^2 Y_2^+ Y_2^- + (M_1^2 + M_2^2) Y_3^+ Y_3^- \tag{8.6}$$

$$M_1^2 = \frac{g_{H_2}^2 \Lambda_2^2}{2} \quad , \quad M_2^2 = \frac{g_{H_3}^2 \Lambda_3^2}{2} \quad , \quad M_3^2 = M_1^2 + M_2^2 \tag{8.7}$$

From the diagonalization of the $Z_1 - Z_2$ squared mass matrix, we obtain the eigenvalues

$$M_-^2 = \frac{2}{3}\left(M_1^2 + M_2^2 - \sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2} \right),$$
$$M_+^2 = \frac{2}{3}\left(M_1^2 + M_2^2 + \sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2} \right) \tag{8.8}$$

$$M_1^2 Y_1^+ Y_1^- + M_-^2 \frac{Z_-^2}{2} + M_+^2 \frac{Z_+^2}{2} + M_2^2 Y_2^+ Y_2^- + (M_1^2 + M_2^2) Y_3^+ Y_3^- \tag{8.9}$$

where

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} \tag{8.10}$$

$$\cos\phi \sin\phi = \frac{\sqrt{3}}{4} \frac{1}{\sqrt{(\frac{M_2^2}{M_1^2} - 1)^2 + \frac{M_2^2}{M_1^2}}}$$

with the hierarchy $M_1, M_2 \gg M_W$.

	Z_1	Z_2
Z_1	M_1^2	$-\frac{M_1^2}{\sqrt{3}}$
Z_2	$-\frac{M_1^2}{\sqrt{3}}$	$\left(\frac{4}{3}M_2^2 + \frac{1}{3}M_1^2\right)$

8.4   Electroweak symmetry breaking

Recently ATLAS[25] and CMS[26] at the Large Hadron Collider announced the discovery of a Higgs-like particle, whose properties, couplings to fermions and gauge bosons will determine whether it is the SM Higgs or a member of an extended Higgs sector associated to a BSM theory. The electroweak symmetry breaking in the $SU(3)$ family symmetry model involves the introduction of two triplets of $SU(2)_L$ Higgs doublets.

To achieve the spontaneous breaking of the electroweak symmetry to $U(1)_Q$, we introduce the scalars: $\Phi^u = (3, 1, 2, -1)$ and $\Phi^d = (3, 1, 2, +1)$, with the VEVs: $\langle\Phi^u\rangle^T = (\langle\Phi_1^u\rangle, \langle\Phi_2^u\rangle, \langle\Phi_3^u\rangle)$, $\langle\Phi^d\rangle^T = (\langle\Phi_1^d\rangle, \langle\Phi_2^d\rangle, \langle\Phi_3^d\rangle)$, where T means transpose, and

$$\langle\Phi_i^u\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_i \\ 0 \end{pmatrix} \qquad , \qquad \langle\Phi_i^d\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ V_i \end{pmatrix} . \tag{8.11}$$

The contributions from $\langle\Phi^u\rangle$ and $\langle\Phi^d\rangle$ generate the W and Z gauge boson masses

$$\frac{g^2}{4}\left(v_u^2 + v_d^2\right)W^+W^- + \frac{(g^2 + g'^2)}{8}\left(v_u^2 + v_d^2\right)Z_o^2 \tag{8.12}$$

$v_u^2 = v_1^2 + v_2^2 + v_3^2$, $v_d^2 = V_1^2 + V_2^2 + V_3^2$. Hence, if we define as usual $M_W = \frac{1}{2}gv$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246\,\text{GeV}$.

8.5   Tree level neutrino masses

Now we describe briefly the procedure to get the masses for ordinary fermions. The analysis for quarks and charged leptons has already discussed in [19]. Here, we introduce the procedure for neutrinos.

Before "Electroweak Symmetry Breaking"(EWSB) all ordinary, "Standard Model"(SM) fermions remain massless, and the quarks and leptons global symmetry is:

$$SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{e_R} \tag{8.13}$$

8.5.1 Tree level Dirac neutrino masses

With the fields of particles introduced in the model, we may write the Dirac type gauge invariant Yukawa couplings

$$h_D \, \Psi^o_l \, \Phi^u \, N^o_R \;\; + \;\; h_2 \, \Psi^o_\nu \, \eta_2 \, N^o_L \;\; + \;\; h_3 \, \Psi^o_\nu \, \eta_3 \, N^o_L \;\; + \;\; M_D \, \bar{N}^o_L \, N^o_R \;\; + h.c \quad (8.14)$$

h_D, h_2 and h_3 are Yukawa couplings, and M_D a Dirac type, invariant neutrino mass for the sterile neutrinos $N^o_{L,R}$. After electroweak symmetry breaking, we obtain in the interaction basis $\Psi^{o\,T}_{\nu\,L,R} = (\nu^o_e, \nu^o_\mu, \nu^o_\tau, N^o)_{L,R}$, the mass terms

$$h_D \, \left[\nu_1 \, \bar{\nu}^o_{eL} + \nu_2 \, \bar{\nu}^o_{\mu L} + \nu_3 \, \bar{\nu}^o_{\tau L} \right] N^o_R + \left[h_2 \, \Lambda_2 \, \bar{\nu}^o_{\mu R} + h_3 \, \Lambda_3 \, \bar{\nu}^o_{\tau R} \right] N^o_L$$
$$+ M_D \, \bar{N}^o_L \, N^o_R + h.c. \, , \quad (8.15)$$

8.5.2 Tree level Majorana masses:

Since $N^o_{L,R}$, Eq.(8.2), are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L \, \bar{\Psi}^o_l \, \Phi^u (N^o_L)^c \quad + \quad m_L \, \bar{N}^o_L \, (N^o_L)^c \quad (8.16)$$

and

$$h_{2R} \, \bar{\Psi}^o_\nu \, \eta_2 \, (N^o_R)^c \quad + \quad h_{3R} \, \bar{\Psi}^o_\nu \, \eta_3 \, (N^o_R)^c \quad + \quad m_R \, \bar{N}^o_R \, (N^o_R)^c + h.c \, , \quad (8.17)$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L \, \left[\nu_1 \, \bar{\nu}^o_{eL} + \nu_2 \, \bar{\nu}^o_{\mu L} + \nu_3 \, \bar{\nu}^o_{\tau L} \right] (N^o_L)^c \quad + \quad m_L \, \bar{N}^o_L \, (N^o_L)^c + h.c. \, , \quad (8.18)$$

$$+ \left[h_{2R} \, \Lambda_2 \, \bar{\nu}^o_{\mu R} + h_{3R} \, \Lambda_3 \, \bar{\nu}^o_{\tau R} \right] (N^o_R)^c \quad + \quad m_R \, \bar{N}^o_R \, (N^o_R)^c + h.c. \, , \quad (8.19)$$

Thus, in the basis $\Psi^{o\,T}_\nu = \left(\nu^o_{eL}, \, \nu^o_{\mu L}, \, \nu^o_{\tau L}, \, (\nu^o_{eR})^c, \, (\nu^o_{\mu R})^c, \, (\nu^o_{\tau R})^c, \, N^o_L, \, (N^o_R)^c \right)$, the Generic 8×8 tree level Majorana mass matrix for neutrinos $\mathcal{M}^o_\nu$, from Table 8.1, $\bar{\Psi}^o_\nu \, \mathcal{M}^o_\nu \, (\Psi^o_\nu)^c \;$ + h.c., read

$$\mathcal{M}^o_\nu = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \alpha_1 & a_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_2 & a_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & a_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b_2 & \beta_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & b_3 & \beta_3 \\ \alpha_1 & \alpha_2 & \alpha_3 & 0 & b_2 & b_3 & m_L & m_D \\ a_1 & a_2 & a_3 & 0 & \beta_2 & \beta_3 & m_D & m_R \end{pmatrix} \quad (8.20)$$

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	$(N^o_L)^c$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	0	0	0	$h_L \nu_1$	$h_D \nu_1$
$\overline{\nu^o_{\mu L}}$	0	0	0	0	0	0	$h_L \nu_2$	$h_D \nu_2$
$\overline{\nu^o_{\tau L}}$	0	0	0	0	0	0	$h_L \nu_3$	$h_D \nu_3$
$\overline{(\nu^o_{eR})^c}$	0	0	0	0	0	0	0	0
$\overline{(\nu^o_{\mu R})^c}$	0	0	0	0	0	0	$h_2 \Lambda_2$	$h_{2R} \Lambda_2$
$\overline{(\nu^o_{\tau R})^c}$	0	0	0	0	0	0	$h_3 \Lambda_3$	$h_{3R} \Lambda_3$
$\overline{N^o_L}$	$h_L \nu_1$	$h_L \nu_2$	$h_L \nu_3$	0	$h_2 \Lambda_2$	$h_3 \Lambda_3$	m_L	M_D
$\overline{(N^o_R)^c}$	$h_D \nu_1$	$h_D \nu_2$	$h_D \nu_3$	0	$h_{2R} \Lambda_2$	$h_{3R} \Lambda_3$	M_D	m_R

Table 8.1. Tree Level Majorana masses

Diagonalization of the $\mathcal{M}_\nu^{(o)}$, Eq.(8.20), yields four zero eigenvalues, associated to the neutrino fields: $ap = \sqrt{a_1^2 + a_2^2}$

$$\frac{a_2}{ap}\,\nu^o_{eL} - \frac{a_1}{ap}\,\nu^o_{\mu L} \quad , \quad \frac{a_1\,a_3}{ap\,a}\,\nu^o_{eL} + \frac{a_2\,a_3}{ap\,a}\,\nu^o_{\mu L} - \frac{ap}{a}\,\nu^o_{\tau L},$$

$$\nu^o_{eR} \quad , \quad \frac{b_3}{b}\,\nu^o_{\mu R} - \frac{b_2}{b}\,\nu^o_{\tau R}$$

Assuming for simplicity, $\frac{h_2}{h_{2R}} = \frac{h_3}{h_{3R}}$, the Characteristic Polynomial for the nonzero eigenvalues of $\mathcal{M}_\nu^o$ reduce to the one of the matrix m_4, Eq.(8.21), where

$$m_4 = \begin{pmatrix} 0 & 0 & \alpha & a \\ 0 & 0 & b & \beta \\ \alpha & b & m_L & m_D \\ a & \beta & m_D & m_R \end{pmatrix} \quad , \quad U_4 = \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix} \tag{8.21}$$

$$a = \sqrt{a_1^2 + a_2^2 + a_3^2}, \; \alpha = \sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}\,,$$
$$b = \sqrt{b_2^2 + b_3^2}\,, \qquad \beta = \sqrt{\beta_2^2 + \beta_3^2}$$

$$U_4^\mathsf{T}\, m_4\, U_4 = \mathrm{Diag}(m_5^o, m_6^o, m_7^o, m_8^o) \equiv d_4 \quad , \quad m_4 = U_4\, d_4\, U_4^\mathsf{T} \tag{8.22}$$

Eq.(8.22) impose the constrains

$$u_{11}^2\, m_5^o + u_{12}^2\, m_6^o + u_{13}^2\, m_7^o + u_{14}^2\, m_8^o = 0 \tag{8.23}$$

$$u_{21}^2\, m_5^o + u_{22}^2\, m_6^o + u_{23}^2\, m_7^o + u_{24}^2\, m_8^o = 0 \tag{8.24}$$

$$u_{11}u_{21}\, m_5^o + u_{12}u_{22}\, m_6^o + u_{13}u_{23}\, m_7^o + u_{14}u_{24}\, m_8^o = 0\,, \tag{8.25}$$

corresponding to the $(m_4)_{11} = (m_4)_{22} = (m_4)_{12} = 0$ zero entries, respectively.

In this form, we diagonalize $\mathcal{M}_\nu^o$ by using the orthogonal matrix

$$U_\nu^o = \begin{pmatrix}
\frac{a_2}{ap} & \frac{a_1\,a_3}{a\,ap} & 0 & 0 & \frac{a_1}{a}u_{11} & \frac{a_1}{a}u_{12} & \frac{a_1}{a}u_{13} & \frac{a_1}{a}u_{14} \\[2mm]
-\frac{a_1}{ap} & \frac{a_2\,a_3}{a\,ap} & 0 & 0 & \frac{a_2}{a}u_{11} & \frac{a_2}{a}u_{12} & \frac{a_2}{a}u_{13} & \frac{a_2}{a}u_{14} \\[2mm]
0 & -\frac{ap}{a} & 0 & 0 & \frac{a_3}{a}u_{11} & \frac{a_3}{a}u_{12} & \frac{a_3}{a}u_{13} & \frac{a_3}{a}u_{14} \\[2mm]
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\[2mm]
0 & 0 & 0 & \frac{b_3}{b} & \frac{b_2}{b}u_{21} & \frac{b_2}{b}u_{22} & \frac{b_2}{b}u_{23} & \frac{b_2}{b}u_{24} \\[2mm]
0 & 0 & 0 & -\frac{b_2}{b} & \frac{b_3}{b}u_{21} & \frac{b_3}{b}u_{22} & \frac{b_3}{b}u_{23} & \frac{b_3}{b}u_{24} \\[2mm]
0 & 0 & 0 & 0 & u_{31} & u_{32} & u_{33} & u_{34} \\[2mm]
0 & 0 & 0 & 0 & u_{41} & u_{42} & u_{43} & u_{44}
\end{pmatrix} \tag{8.26}$$

$$(U_\nu^o)^T\, \mathcal{M}_\nu^o\, U_\nu^o = \mathrm{Diag}(0,0,0,0,m_5^o,m_6^o,m_7^o,m_8^o) \tag{8.27}$$

Notice that the first four columns in U_ν^o correspond to the four massless eigenvectors. Hence, the tree level mixing, U_ν^o, depends on the ordering we define for these four degenerated massless eigenvectors. However, it turns out that the final mixing product $U_\nu^o\, U_\nu$, as well as the final mass eigenvalues are independent of the choice of this ordering.

8.6 One loop neutrino masses

After tree level contributions the fermion global symmetry is broken down to:

$$SU(2)_{q_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_L} \otimes SU(2)_{\nu_R} \otimes SU(2)_{e_R} \tag{8.28}$$

Therefore, in this scenario light neutrinos may get extremely small masses from radiative corrections mediated by the SU(3) heavy gauge bosons.

8.6.1 One loop Dirac Neutrino masses

After the breakdown of the electroweak symmetry, neutrinos may get tiny Dirac mass terms from the generic one loop diagram in Fig. 8.1, The internal fermion line in this diagram represent the tree level see-saw mechanism, Eq.(8.15). The vertices read from the $SU(3)$ flavor symmetry interaction Lagrangian

$$i\mathcal{L}_{int} = \frac{g_H}{2}\left(\bar{e}^o\gamma_\mu e^o - \bar{\mu}^o\gamma_\mu\mu^o\right)Z_1^\mu + \frac{g_H}{2\sqrt{3}}\left(\bar{e}^o\gamma_\mu e^o + \bar{\mu}^o\gamma_\mu\mu^o - 2\bar{\tau}^o\gamma_\mu\tau^o\right)Z_2^\mu$$

$$+ \frac{g_H}{\sqrt{2}}\left(\bar{e}^o\gamma_\mu\mu^o Y_1^+ + \bar{e}^o\gamma_\mu\tau^o Y_2^+ + \bar{\mu}^o\gamma_\mu\tau^o Y_3^+ + h.c.\right), \quad (8.29)$$

The contribution from these diagrams may be written as

$$c_Y\frac{\alpha_H}{\pi}\,m_\nu(M_Y)_{ij} \quad , \quad \alpha_H = \frac{g_H^2}{4\pi}, \quad (8.30)$$

$$m_\nu(M_Y)_{ij} \equiv \sum_{k=5,6,7,8} m_k^o\, U_{ik}^o U_{jk}^o\, f(M_Y, m_k^o) \quad (8.31)$$

and $f_{Y_k} = \frac{M_Y^2}{M_Y^2 - m_k^{o\,2}}\ln\frac{M_Y^2}{m_k^{o\,2}} \approx \ln\frac{M_Y^2}{m_k^{o\,2}}$

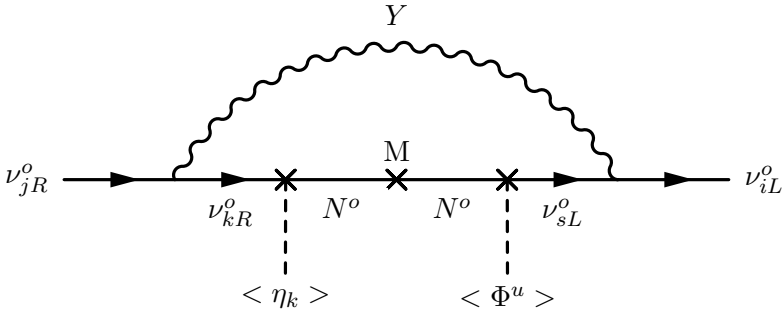**Fig. 8.1.** Generic one loop diagram contribution to the Dirac mass term $m_{ij}\,\bar{\nu}_{iL}^o\nu_{jR}^o$. $M = M_D, m_L, m_R$

	ν_{eR}^o	$\nu_{\mu R}^o$	$\nu_{\tau R}^o$	N_R^o
$\bar{\nu}_{eL}^o$	$D_{\nu\,14}$	$D_{\nu\,15}$	$D_{\nu\,16}$	0
$\bar{\nu}_{\mu L}^o$	0	$D_{\nu\,25}$	$D_{\nu\,26}$	0
$\bar{\nu}_{\tau L}^o$	0	$D_{\nu\,35}$	$D_{\nu\,36}$	0
$\bar{N}_L^o$	0	0	0	0

Table 8.2. One loop Dirac mass terms $m_{ij}\,\bar{\nu}_{iL}^o\,\nu_{jR}^o$

$$m_\nu(M_{Z_1})_{ij} = \cos\phi\, m_\nu(M_-)_{ij} - \sin\phi\, m_\nu(M_+)_{ij}$$

$$m_\nu(M_{Z_2})_{ij} = \sin\phi\, m_\nu(M_-)_{ij} + \cos\phi\, m_\nu(M_+)_{ij}$$

$$G_{\nu,m\,ij} = \frac{\sqrt{\alpha_2\alpha_3}}{\pi}\,\frac{1}{2\sqrt{3}}\,\cos\phi\,\sin\phi\,[m_\nu(M_-)_{ij} - m_\nu(M_+)_{ij}]$$

$$\mathcal{F}(M_Y) = m_5^o\, u_{11}^2\, f_{Y_5} + m_6^o\, u_{12}^2\, f_{Y_6} + m_7^o\, u_{13}^2\, f_{Y_7} + m_8^o\, u_{14}^2\, f_{Y_8} \qquad (8.32)$$

$$\mathcal{G}(M_Y) = m_5^o\, u_{21}^2\, f_{Y_5} + m_6^o\, u_{22}^2\, f_{Y_6} + m_7^o\, u_{23}^2\, f_{Y_7} + m_8^o\, u_{24}^2\, f_{Y_8} \qquad (8.33)$$

$$\mathcal{H}(M_Y) = m_5^o\, u_{11}u_{21}\, f_{Y_5} + m_6^o\, u_{12}u_{22}\, f_{Y_6} + m_7^o\, u_{13}u_{23}\, f_{Y_7}$$
$$+ m_8^o\, u_{14}u_{24}\, f_{Y_8} \quad (8.34)$$

$$m_\nu(M_Y)_{15} = \frac{a_1 b_2}{ab}\mathcal{H}(M_Y)\,; \quad m_\nu(M_Y)_{16} = \frac{a_1 b_3}{ab}\mathcal{H}(M_Y)$$

$$m_\nu(M_Y)_{25} = \frac{a_2 b_2}{ab}\mathcal{H}(M_Y)\,; \quad m_\nu(M_Y)_{26} = \frac{a_2 b_3}{ab}\mathcal{H}(M_Y)$$

$$m_\nu(M_Y)_{35} = \frac{a_3 b_2}{ab}\mathcal{H}(M_Y)\,; \quad m_\nu(M_Y)_{36} = \frac{a_3 b_3}{ab}\mathcal{H}(M_Y)$$

$$D_{\nu\,14} = \frac{1}{2}\left[\frac{a_2 b_2}{ab}\mathcal{H}(M_1) + \frac{a_3 b_3}{ab}\mathcal{H}(M_2)\right],$$

$$D_{\nu\,15} = \frac{a_1 b_2}{ab}\left[-\frac{1}{4}\mathcal{H}(M_{Z_1}) + \frac{1}{12}\mathcal{H}(M_{Z_2})\right],$$

$$D_{\nu\,25} = \frac{a_2 b_2}{ab}\left[\frac{1}{4}\mathcal{H}(M_{Z_1}) + \frac{1}{12}\mathcal{H}(M_{Z_2}) - \mathcal{H}(G_{\nu,m})\right] + \frac{1}{2}\frac{a_3 b_3}{ab}\mathcal{H}(M_3),$$

$$D_{\nu\,36} = \frac{1}{2}\frac{a_2 b_2}{ab}\mathcal{H}(M_3) + \frac{1}{3}\frac{a_3 b_3}{ab}\mathcal{H}(M_{Z_2}),$$

$$D_{\nu\,16} = \frac{a_1 b_3}{ab}\left[-\frac{1}{6}\mathcal{H}(M_{Z_2}) - \mathcal{H}(G_{\nu,m})\right],$$

$$D_{\nu\,26} = \frac{a_2 b_3}{ab}\left[-\frac{1}{6}\mathcal{H}(M_{Z_2}) + \mathcal{H}(G_{\nu,m})\right],$$

$$D_{\nu\,35} = \frac{a_3 b_2}{ab}\left[-\frac{1}{6}\mathcal{H}(M_{Z_2}) + \mathcal{H}(G_{\nu,m})\right],$$

$$\mathcal{H}(G_{\nu,m}) = \frac{\sqrt{\alpha_2 \alpha_3}}{\pi}\frac{1}{2\sqrt{3}}\cos\phi\,\sin\phi\,[\mathcal{H}(M_-) - \mathcal{H}(M_+)]$$

8.6.2   One loop L-handed Majorana masses

Neutrinos also obtain one loop corrections to L-handed and R-handed Majorana masses from the diagrams of Fig. 8.2 and Fig. 8.3, respectively.

A similar procedure as for Dirac Neutrino masses, leads to the one loop Majorana mass terms

$$m_\nu(M_Y)_{11} = \frac{a_1^2}{a^2}\mathcal{F}(M_Y)\,; \quad m_\nu(M_Y)_{12} = \frac{a_1 a_2}{a^2}\mathcal{F}(M_Y)$$

$$m_\nu(M_Y)_{13} = \frac{a_1 a_3}{a^2}\mathcal{F}(M_Y)\;; \quad m_\nu(M_Y)_{22} = \frac{a_2^2}{a^2}\mathcal{F}(M_Y)$$

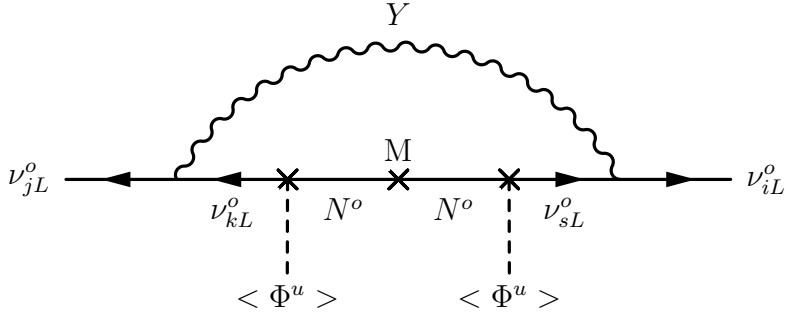$$m_\nu(M_Y)_{23} = \frac{a_2 a_3}{a^2}\mathcal{F}(M_Y)\;; \quad m_\nu(M_Y)_{33} = \frac{a_3^2}{a^2}\mathcal{F}(M_Y)$$

Fig. 8.2. Generic one loop diagram contribution to the L-handed Majorana mass term $m_{ij} \, \bar{\nu}^o_{iL} (\nu^o_{jL})^\mathsf{T}$. $M = M_D, m_L, m_R$

	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	$L_{\nu\,11}$	$L_{\nu\,12}$	$L_{\nu\,13}$	0
$\nu^o_{\mu L}$	$L_{\nu\,12}$	$L_{\nu\,22}$	$L_{\nu\,23}$	0
$\nu^o_{\tau L}$	$L_{\nu\,13}$	$L_{\nu\,23}$	$L_{\nu\,33}$	0
N^o_L	0	0	0	0

Table 8.3. One loop L-handed Majorana mass terms $m_{ij} \, \bar{\nu}^o_{iL} \, (\nu^o_{jL})^\mathsf{T}$

$$L_{\nu\,11} = \frac{a_1^2}{a^2} \left[\frac{1}{4} \mathcal{F}(M_{Z_1}) + \frac{1}{12} \mathcal{F}(M_{z_2}) + \mathcal{F}(G_{\nu,m}) \right] \,,$$

$$L_{\nu\,22} = \frac{a_2^2}{a^2} \left[\frac{1}{4} \mathcal{F}(M_{Z_1}) + \frac{1}{12} \mathcal{F}(M_{z_2}) - \mathcal{F}(G_{\nu,m}) \right] \,,$$

$$L_{\nu\,33} = \frac{1}{3} \frac{a_3^2}{a^2} \mathcal{F}(M_{z_2}) \,,$$

$$L_{\nu\,12} = \frac{a_1 a_2}{a^2} \left[-\frac{1}{4} \mathcal{F}(M_{Z_1}) + \frac{1}{2} \mathcal{F}(M_1) + \frac{1}{12} \mathcal{F}(M_{z_2}) \right] \,,$$

$$L_{\nu\,13} = \frac{a_1 a_3}{a^2} \left[-\frac{1}{6} \mathcal{F}(M_{Z_2}) + \frac{1}{2} \mathcal{F}(M_2) - \mathcal{F}(G_{\nu,m}) \right] \,,$$

$$L_{\nu\,23} = \frac{a_2 a_3}{a^2} \left[-\frac{1}{6} \mathcal{F}(M_{Z_2}) + \frac{1}{2} \mathcal{F}(M_3) + \mathcal{F}(G_{\nu,m}) \right]$$

$$\mathcal{F}(G_{\nu,\mathfrak{m}}) = \frac{\sqrt{\alpha_2\,\alpha_3}}{\pi}\,\frac{1}{2\sqrt{3}}\,\cos\phi\,\sin\phi\,[\mathcal{F}(M_-) - \mathcal{F}(M_+)] \qquad (8.35)$$
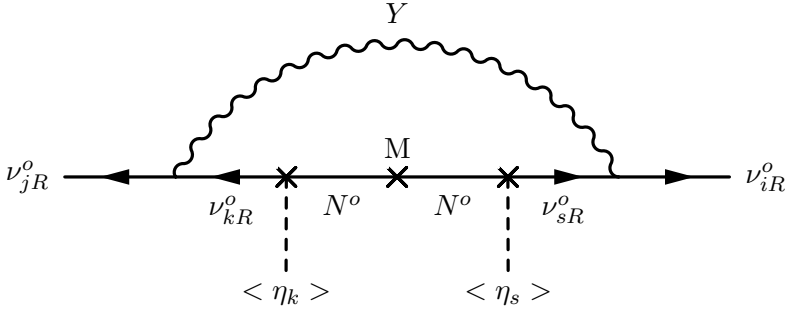
8.6.3   One loop R-handed Majorana masses

Fig. 8.3. Generic one loop diagram contribution to the R-handed Majorana mass term $\mathfrak{m}_{ij}\,\bar{\nu}^o_{iR}(\nu^o_{jR})^\mathsf{T}$. $M = M_D, \mathfrak{m}_L, \mathfrak{m}_R$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	0	0	0	0
$\nu^o_{\mu R}$	0	$R_{\nu\,55}$	$R_{\nu\,56}$	0
$\nu^o_{\tau R}$	0	$R_{\nu\,56}$	$R_{\nu\,66}$	0
N^o_R	0	0	0	0

Table 8.4. One loop R-handed Majorana mass terms $\mathfrak{m}_{ij}\,\bar{\nu}^o_{iR}\,(\nu^o_{jR})^\mathsf{T}$

$$\mathfrak{m}_\nu(M_Y)_{55} = \frac{b_2^2}{b^2}\mathcal{G}(M_Y) \;; \quad \mathfrak{m}_\nu(M_Y)_{66} = \frac{b_3^2}{b^2}\mathcal{G}(M_Y)$$

$$\mathfrak{m}_\nu(M_Y)_{56} = \frac{b_2 b_3}{b^2}\mathcal{G}(M_Y)$$

$$R_{\nu\,55} = \frac{b_2^2}{b^2}\left[\frac{1}{4}\mathcal{G}(M_{Z_1}) + \frac{1}{12}\mathcal{G}(M_{z_2}) - \mathcal{G}(G_{\nu,m})\right],$$

$$R_{\nu\,66} = \frac{1}{3}\frac{b_3^2}{b^2}\mathcal{G}(M_{z_2}),$$

$$R_{\nu\,56} = \frac{b_2 b_3}{b^2}\left[-\frac{1}{6}\mathcal{G}(M_{z_2}) + \frac{1}{2}\mathcal{G}(M_3) + \mathcal{G}(G_{\nu,m})\right]$$

$$\mathcal{G}(G_{\nu,m}) = \frac{\sqrt{\alpha_2 \alpha_3}}{\pi}\frac{1}{2\sqrt{3}}\cos\phi\,\sin\phi\,[\mathcal{G}(M_-) - \mathcal{G}(M_+)]$$

Thus, in the Ψ_ν^o basis, we may write the one loop contribution for neutrinos as

$$\mathcal{M}_{1\,\nu}^o =$$

$$
\begin{pmatrix}
L_{\nu\,11} & L_{\nu\,12} & L_{\nu\,13} & D_{\nu\,14} & D_{\nu\,15} & D_{\nu\,16} & 0 & 0 \\
L_{\nu\,12} & L_{\nu\,22} & L_{\nu\,23} & 0 & D_{\nu\,25} & D_{\nu\,26} & 0 & 0 \\
L_{\nu\,13} & L_{\nu\,23} & L_{\nu\,33} & 0 & D_{\nu\,35} & D_{\nu\,36} & 0 & 0 \\
D_{\nu\,14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
D_{\nu\,15} & D_{\nu\,25} & D_{\nu\,35} & 0 & R_{\nu\,55} & R_{\nu\,56} & 0 & 0 \\
D_{\nu\,16} & D_{\nu\,26} & D_{\nu\,36} & 0 & R_{\nu\,56} & R_{\nu\,66} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\tag{8.36}
$$

8.6.4   Neutrino mass matrix up to one loop

Finally, we obtain the Majorana mass matrix for neutrinos up to one loop

$$\mathcal{M}_\nu = (U_\nu^o)^{\mathsf{T}}\,\mathcal{M}_{1\,\nu}^o\,U_\nu^o + \mathrm{Diag}(0,0,0,0,m_5^o,m_6^o,m_7^o,m_8^o),\tag{8.37}$$

where explicitly

$$\mathcal{M}_\nu =$$

$$
\begin{pmatrix}
N_{11} & N_{12} & N_{13} & N_{14} & N_{15} & N_{16} & N_{17} & N_{18} \\
N_{12} & N_{22} & N_{23} & N_{24} & N_{25} & N_{26} & N_{27} & N_{28} \\
N_{13} & N_{23} & 0 & 0 & N_{35} & N_{36} & N_{37} & N_{38} \\
N_{14} & N_{24} & 0 & N_{44} & N_{45} & N_{46} & N_{47} & N_{48} \\
N_{15} & N_{25} & N_{35} & N_{45} & N_{55} + m_5^o & N_{56} & N_{57} & N_{58} \\
N_{16} & N_{26} & N_{36} & N_{46} & N_{56} & N_{66} + m_6^o & N_{67} & N_{68} \\
N_{17} & N_{27} & N_{37} & N_{47} & N_{57} & N_{67} & N_{77} + m_7^o & N_{78} \\
N_{18} & N_{28} & N_{38} & N_{48} & N_{58} & N_{68} & N_{78} & N_{88} + m_8^o
\end{pmatrix}
\tag{8.38}
$$

Majorana L-handed:

$$N_{11} = \frac{a_1^2 a_2^2}{a_p^2 a^2}(F_{Z_1} - F_1) \tag{8.39}$$

$$N_{12} = -\frac{a_1 a_2 a_3}{2 a^3}\left[\frac{a_2^2 - a_1^2}{a_p^2}(F_{Z_1} - F_1) + F_2 - F_3 - 6F_m\right] \tag{8.40}$$

$$
\begin{aligned}
N_{22} = \frac{a_3^2}{a^2}\Bigg[&\frac{1}{4}\frac{(a_2^2 - a_1^2)^2}{a_p^2\,a^2}(F_{Z_1} - F_1) + \frac{a_2^2}{a^2}(F_2 - F_3) \\
&+ \frac{a_p^2}{4\,a^2}(F_1 + 3F_{Z_2} - 4F_2) - 3\frac{a_2^2 - a_1^2}{a^2}F_m\Bigg]
\end{aligned}
\tag{8.41}
$$

Dirac:

$$N_{13} = \frac{a_2}{2\,a_p}\left(\frac{a_2\,b_2}{a\,b}H_1 + \frac{a_3\,b_3}{a\,b}H_2\right) = q_{11} \tag{8.42}$$

$$N_{14} = -\frac{a_1\,b_3}{2\,a_p\,b}\left(\frac{a_2\,b_2}{a\,b}H_{Z_1} + \frac{a_3\,b_3}{a\,b}H_3 - 6\frac{a_2\,b_2}{a\,b}H_m\right) = q_{12} \tag{8.43}$$

$$N_{23} = \frac{a_1\,a_3}{2\,a_p\,a}\left(\frac{a_2\,b_2}{a\,b}H_1 + \frac{a_3\,b_3}{a\,b}H_2\right) = q_{21} \tag{8.44}$$

$$
\begin{aligned}
N_{24} = &\frac{a_2\,(a_p^2\,b_2^2 + a_3^2\,b_3^2)}{2\,a_p\,a^2\,b^2}H_3 + \frac{(a_2^2 - a_1^2)a_3\,b_2\,b_3}{4\,a_p\,a^2\,b^2}H_{Z_1} \\
&+ \frac{3\,a_p\,a_3\,b_2\,b_3}{4\,a^2\,b^2}H_{Z_2} - \frac{3\,a_2^2\,a_3\,b_2\,b_3}{a_p\,a^2\,b^2}H_m = q_{22}
\end{aligned}
\tag{8.45}
$$

Majorana R-handed:

$$N_{44} = \frac{b_2^2 \, b_3^2}{4 \, b^4} \left(G_{Z_1} + 3 G_{Z_2} - 4 G_3 - 12 G_m \right) \tag{8.46}$$

Majorana L-handed and Dirac:

$$N_{15} = -F_{15} \, u_{11} + q_{13} \, u_{21} \quad ; \quad N_{16} = -F_{15} \, u_{12} + q_{13} \, u_{22} \tag{8.47}$$

$$N_{17} = -F_{15} \, u_{13} + q_{13} \, u_{23} \quad ; \quad N_{18} = -F_{15} \, u_{14} + q_{13} \, u_{24} \tag{8.48}$$

$$F_{15} = \frac{a_1 \, a_2}{2 \, a_p \, a} \left[\frac{a_2^2 - a_1^2}{a^2} (F_{Z_1} - F_1) + \frac{a_3^2}{a^2} (F_3 - F_2) \right.$$
$$\left. +2 \, \frac{(2 \, a_3^2 - a_p^2)}{a^2} \, F_m \right]$$

$$q_{13} = -\frac{a_1 \, b_2}{2 a_p \, b} \left[\frac{a_2 \, b_2}{a \, b} H_{Z_1} + \frac{a_3 \, b_3}{a \, b} H_3 - 2 \frac{a_2}{a} \frac{b_2^2 - 2 b_3^2}{b_2 \, b} H_m \right]$$

$$N_{25} = F_{25} \, u_{11} + q_{23} \, u_{21} \quad ; \quad N_{26} = F_{25} \, u_{12} + q_{23} \, u_{22} \tag{8.49}$$

$$N_{27} = F_{25} \, u_{13} + q_{23} \, u_{23} \quad ; \quad N_{28} = F_{25} \, u_{14} + q_{23} \, u_{24} \tag{8.50}$$

$$F_{25} =$$
$$\frac{a_3}{4 \, a_p \, a^4} \left[(a_2^2 - a_1^2)^2 \, (F_{Z_1} - F_1) + 2 \, a_2^2 (a_3^2 - a_p^2) \, (F_3 - F_2) - a_p^4 \, (F_{Z_2} - F_1) \right.$$
$$\left. -2 \, a_p^2 (a_3^2 - a_p^2) \, (F_{Z_2} - F_2) + 4 \, (a_2^2 - a_1^2) \, (a_3^2 - 2 a_p^2) \, F_m \right]$$

$$q_{23} = \frac{a_2 \, (a_3^2 - a_p^2) \, b_2 \, b_3}{2 \, a_p \, a^2 \, b^2} H_3 + \frac{(a_2^2 - a_1^2) a_3 \, b_2^2}{4 \, a_p \, a^2 \, b^2} H_{Z_1} + \frac{a_p \, a_3 \, (b_2^2 - 2 \, b_3^2)}{4 \, a^2 \, b^2} H_{Z_2}$$
$$- \frac{a_3 \, [a_p^2 \, b^2 + a_2^2 (b_2^2 - 2 \, b_3^2)]}{a_p \, a^2 \, b^2} H_m$$

Dirac:

$$N_{35} = q_{31} \, u_{11} \, , \; N_{36} = q_{31} \, u_{12} \, , \; N_{37} = q_{31} \, u_{13} \, , \; N_{38} = q_{31} \, u_{14} \tag{8.51}$$

$$q_{31} = \frac{a_1}{2 a} \left(\frac{a_2 \, b_2}{a \, b} H_1 + \frac{a_3 \, b_3}{a \, b} H_2 \right)$$

Dirac and Majorana R-handed:

$$N_{45} = q_{32}\, u_{11} + G_{45}\, u_{21} \quad , \quad N_{46} = q_{32}\, u_{12} + G_{45}\, u_{22} \tag{8.52}$$

$$N_{47} = q_{32}\, u_{13} + G_{45}\, u_{23} \quad , \quad N_{48} = q_{32}\, u_{14} + G_{45}\, u_{24} \tag{8.53}$$

$$q_{32} = -\frac{a_2\, a_3\, (b_2^2 - b_3^2)}{2\, a^2\, b^2}\, H_3 + \frac{(a_2^2 - a_1^2)\, b_2\, b_3}{4\, a^2\, b^2}\, H_{Z_1} - \frac{(2a_3^2 - a_p^2)b_2\, b_3}{4\, a^2\, b^2}\, H_{Z_2}$$
$$+ \frac{(a_3^2 + a_1^2 - 2a_2^2)\, b_2\, b_3}{a^2\, b^2}\, H_m$$

$$G_{45} = \frac{b_2\, b_3}{4\, b^2}\left[\frac{b_2^2 - 2b_3^2}{b^2}\, (G_{Z_2} - G_3) + \frac{b_2^2}{b^2}\, (G_{Z_1} - G_3) \right.$$
$$\left. -4\frac{(2\, b_2^2 - b_3^2)}{b^2}\, G_m \right]$$

Majorana L-handed, Dirac and Majorana R-handed:

$$N_{55} = F_{55}\, u_{11}^2 + 2\, q_{33}\, u_{11}\, u_{21} + G_{55}\, u_{21}^2 \tag{8.54}$$

$$N_{56} = F_{55}\, u_{11}\, u_{12} + q_{33}\, (u_{11}\, u_{22} + u_{12}\, u_{21}) + G_{55}\, u_{21}\, u_{22} \tag{8.55}$$

$$N_{57} = F_{55}\, u_{11}\, u_{13} + q_{33}\, (u_{11}\, u_{23} + u_{13}\, u_{21}) + G_{55}\, u_{21}\, u_{23} \tag{8.56}$$

$$N_{58} = F_{55}\, u_{11}\, u_{14} + q_{33}\, (u_{11}\, u_{24} + u_{14}\, u_{21}) + G_{55}\, u_{21}\, u_{24} \tag{8.57}$$

$$N_{66} = F_{55}\, u_{12}^2 + 2\, q_{33}\, u_{12}\, u_{22} + G_{55}\, u_{22}^2 \tag{8.58}$$

$$N_{67} = F_{55}\, u_{12}\, u_{13} + q_{33}\, (u_{13}\, u_{22} + u_{12}\, u_{23}) + G_{55}\, u_{22}\, u_{23} \tag{8.59}$$

$$N_{68} = F_{55}\, u_{12}\, u_{14} + q_{33}\, (u_{14}\, u_{22} + u_{12}\, u_{24}) + G_{55}\, u_{22}\, u_{24} \tag{8.60}$$

$$N_{77} = F_{55}\, u_{13}^2 + 2\, q_{33}\, u_{13}\, u_{23} + G_{55}\, u_{23}^2 \tag{8.61}$$

$$N_{78} = F_{55}\, u_{13}\, u_{14} + q_{33}\, (u_{14}\, u_{23} + u_{13}\, u_{24}) + G_{55}\, u_{23}\, u_{24} \tag{8.62}$$

$$N_{88} = F_{55}\, u_{14}^2 + 2\, q_{33}\, u_{14}\, u_{24} + G_{55}\, u_{24}^2 \tag{8.63}$$

$$F_{55} = \frac{a_1^2\, a_2^2}{a^4}\, F_1 + \frac{a_1^2\, a_3^2}{a^4}\, F_2 + \frac{a_2^2\, a_3^2}{a^4}\, F_3 + \frac{(a_2^2 - a_1^2)^2}{4\, a^4}\, F_{Z_1} + \frac{(2a_3^2 - a_p^2)^2}{12\, a^4}\, F_{Z_2}$$
$$+ \frac{(a_2^2 - a_1^2)\, (2\, a_3^2 - a_p^2)}{a^4}\, F_m$$

$$q_{33} = \frac{a_2\, a_3\, b_2\, b_3}{a^2\, b^2}\, H_3 + \frac{(a_2^2 - a_1^2)\, b_2^2}{4\, a^2\, b^2}\, H_{Z_1} - \frac{(2a_3^2 - a_p^2)\, (b_2^2 - 2b_3^2)}{12\, a^2\, b^2}\, H_{Z_2}$$
$$+ \frac{a_3^2\, b_2^2 - a_p^2\, b_3^2 - a_2^2\, (b_2^2 - 2\, b_3^2)}{a^2\, b^2}\, H_m$$

$$G_{55} = \frac{b_2^2\, b_3^2}{b^4}\, G_3 + \frac{b_2^4}{4\, b^4}\, G_{Z_1} + \frac{(b_2^2 - 2b_3^2)^2}{12\, b^4}\, G_{Z_2} - \frac{b_2^2\, (b_2^2 - 2b_3^2)}{b^4}\, G_m$$

8.6.5 $(V_{CKM})_{4\times 4}$ and $(V_{PMNS})_{4\times 8}$ mixing matrices

Within this SU(3) family symmetry model, the transformation from massless to physical mass fermions eigenfields for quarks and charged leptons is

$$\psi_L^o = V_L^o \, V_L^{(1)} \, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o \, V_R^{(1)} \, \Psi_R \,,$$

and for neutrinos $\Psi_\nu^o = U_\nu^o \, U_\nu \, \Psi_\nu$. Recall now that vector like quarks, Eq.(8.1), are $SU(2)_L$ weak singlets, and hence, they do not couple to W boson in the interaction basis. In this way, the interaction of L-handed up and down quarks; $f_{uL}^{o\,T} = (u^o, c^o, t^o)_L$ and $f_{dL}^{o\,T} = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$\frac{g}{\sqrt{2}} \, \bar{f}^o{}_{uL} \gamma_\mu f_{dL}^o W^{+\mu} = \frac{g}{\sqrt{2}} \, \bar{\Psi}_{uL} \, [(V_{uL}^o \, V_{uL}^{(1)})_{3\times4}]^T \, (V_{dL}^o \, V_{dL}^{(1)})_{3\times4} \, \gamma_\mu \Psi_{dL} \, W^{+\mu} \,,$$
(8.64)

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times4} = [(V_{uL}^o \, V_{uL}^{(1)})_{3\times4}]^T \, (V_{dL}^o \, V_{dL}^{(1)})_{3\times4} \qquad (8.65)$$

Similar analysis of the couplings of active L-handed neutrinos and L-handed charged leptons to W boson, leads to the lepton mixing matrix

$$(U_{PMNS})_{4\times8} = [(V_{eL}^o \, V_{eL}^{(1)})_{3\times4}]^T \, (U_\nu^o \, U_\nu)_{3\times8} \qquad (8.66)$$

8.7   Numerical results

To illustrate the spectrum of masses and mixing, let us consider the following fit of space parameters at the M_Z scale [27]

Using the strong hierarchy for quarks and charged leptons masses[15], here we report the fermion masses and mixing, coming out from a global fit of the parameter space.

In the approach $\alpha_2 \approx \alpha_3 = \alpha_H$, we take the input values

$$M_1 = 10 \, \text{TeV} \quad , \quad M_2 = 1 \, \text{TeV} \quad , \quad \frac{\alpha_H}{\pi} = 0.05$$

for the M_1, M_2 horizontal boson masses, Eq.(8.7), and the $SU(3)$ coupling constant, respectively, and the ratio of electroweak VEV's: V_i from Φ^d, and v_i from Φ^u

$$\frac{V_1}{V_2} = 0.09981 \quad , \quad \frac{\sqrt{V_1^2 + V_2^2}}{V_3} = 0.54326$$

$$\frac{v_1}{v_2} = 0.1 \quad , \quad \frac{\sqrt{v_1^2 + v_2^2}}{v_3} = 0.5$$

8.7.1   Quark masses and mixing

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 7933.76 \\ 0 & 0 & 0 & 79337.6 \\ 0 & 0 & 0 & 159467. \\ 0 & 1.18613 \times 10^6 & -841128. & 374542. \end{pmatrix} \text{MeV}, \tag{8.67}$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_u = \begin{pmatrix} -1.40509 & 187.442 & -66.8139 & -255.74 \\ -0.125675 & -609.844 & 408.793 & 1564.71 \\ -0.062809 & -1197.67 & -172100. & 1825.79 \\ -0.001885 & -35.9461 & 14.3165 & 1.502 \times 10^6 \end{pmatrix} \text{MeV} \tag{8.68}$$

and the u-quark masses

$$(\, m_u \,,\, m_c \,,\, m_t \,,\, M_U \,) =$$
$$(\, 1.3802 \,,\, 640.801 \,,\, 172105 \,,\, 1.502 \times 10^6 \,) \,\text{MeV} \tag{8.69}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 1740.94 \\ 0 & 0 & 0 & 17442.3 \\ 0 & 0 & 0 & 32265.8 \\ 0 & 70019.9 & -41383.4 & 910004 \end{pmatrix} \text{MeV} \tag{8.70}$$

$$\mathcal{M}_d = \begin{pmatrix} 3.09609 & 28.1593 & -47.4565 & -4.23475 \\ 0.271539 & -40.5966 & 215.617 & 19.2404 \\ 0.147401 & -176.235 & -2846.26 & 37.484 \\ 0.005900 & -7.05504 & 16.8159 & 914365. \end{pmatrix} \text{MeV} \tag{8.71}$$

$$(\, m_d \,,\, m_s \,,\, m_b \,,\, M_D \,) = (\, 2.82 \,,\, 61.9998 \,,\, 2860 \,,\, 914365 \,) \,\text{MeV} \tag{8.72}$$

and the quark mixing

$$V_{CKM} = \begin{pmatrix} 0.974352 & 0.225001 & 0.003647 & 0.000410 \\ -0.224958 & 0.973502 & 0.041031 & -0.001417 \\ -0.005632 & 0.040662 & -0.997868 & -0.039994 \\ 0.000576 & -0.002325 & 0.031130 & 0.001251 \end{pmatrix} \tag{8.73}$$

8.7.2 Charged leptons:

$$\mathcal{M}_e^o = \begin{pmatrix} 0 & 0 & 0 & 28340.3 \\ 0 & 0 & 0 & 283940. \\ 0 & 0 & 0 & 525249. \\ 0 & 17105.4 & -11570.9 & 5.94752 \times 10^6 \end{pmatrix} \text{ MeV} \qquad (8.74)$$

$$\mathcal{M}_e = \begin{pmatrix} -0.499137 & 29.7086 & -43.9181 & -0.15097 \\ -0.043776 & -72.8148 & 238.953 & 0.821414 \\ -0.023663 & -183.913 & -1720.65 & 1.18425 \\ -0.002378 & -18.4839 & 34.6241 & 5.977 \times 10^6 \end{pmatrix} \text{ MeV} \qquad (8.75)$$

fit the charged lepton masses:

$$(m_e , \, m_\mu , \, m_\tau , \, M_E) = (0.486 , \, 102.7 , \, 1746.17 , \, 5.977 \times 10^6) \, \text{MeV}$$

and the mixing

$$V_{e\,L}^o \, V_{e\,L}^{(1)} =$$

$$\begin{pmatrix} 0.968866 & 0.24054 & -0.0584594 & 0.00474112 \\ 0.205175 & -0.912554 & -0.350561 & 0.0475013 \\ -0.138557 & 0.330471 & -0.929446 & 0.0878703 \\ -0.00217545 & 0.0132348 & 0.0990967 & 0.994987 \end{pmatrix} \qquad (8.76)$$

8.7.3 Neutrinos:

$$\mathcal{M}_\nu^o = \text{eV}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 53594.6 & 44137.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 535946. & 441372. \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.07 \times 10^6 & 887147. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.80 \times 10^6 & 1.49 \times 10^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & -886604. & -730152. \\ 53594.6 & 535946. & 1.07 \times 10^6 & 0 & 1.8097 \times 10^6 & -886604. & 1.97 \times 10^8 & 4.88 \times 10^8 \\ 44137.2 & 441372. & 887147. & 0 & 1.49 \times 10^6 & -730152. & 4.88 \times 10^8 & 7.02 \times 10^8 \end{pmatrix} \qquad (8.77)$$

$\mathcal{M}_\nu = \text{eV}$

$$
\begin{pmatrix}
-0.0119 & 0.0527 & 0.0227 & -0.0878 & -0.0693 & 0.1674 & -0.0016 & 0.0004 \\
0.0527 & -0.036 & 0.002 & 0.068 & 0.043 & -0.748 & 0.007 & -0.002 \\
0.0227 & 0.002 & 0. & 0. & 0.0008 & 0.0005 & -5.2 \times 10^{-6} & 1.5 \times 10^{-6} \\
-0.0878 & 0.068 & 0. & -0.125 & -0.1218 & 1.282 & -0.012 & 0.003 \\
-0.0693 & 0.043 & 0.0008 & -0.121 & 3.206 & -0.7430 & 0.0074 & -0.0021 \\
0.1674 & -0.748 & 0.0005 & 1.282 & -0.7430 & 1749.96 & 0.0003 & -0.0001 \\
-0.0016 & 0.007 & -5.2 \times 10^{-6} & -0.012 & 0.0074 & 0.0003 & -1. \times 10^8 & 1.1 \times 10^{-6} \\
0.0004 & -0.002 & 1.5 \times 10^{-6} & 0.003 & -0.0021 & -0.0001 & 1.1 \times 10^{-6} & 1. \times 10^9
\end{pmatrix}
$$

$$(8.78)$$

generates the neutrino mass eigenvalues

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = \text{eV}$$
$$(0, -0.0085, 0.049, -0.22, 3.21, 1749.96, -1 \times 10^8, 1 \times 10^9) \quad (8.79)$$

the squared mass differences

$$m_2^2 - m_1^2 \approx 0.0000723 \, \text{eV}^2 \quad , \quad m_3^2 - m_1^2 \approx 0.0024 \, \text{eV}^2 \quad (8.80)$$

$$m_4^2 - m_1^2 \approx 0.0492 \, \text{eV}^2 \quad , \quad m_5^2 - m_1^2 \approx 10.3182 \, \text{eV}^2 \quad (8.81)$$

and the lepton mixing matrix

$$U_{PMNS} =$$

$$
\begin{pmatrix}
0.2104 & 0.3520 & 0.8658 & -0.2861 & 0.0060 & 0.0053 & 0.00005 & 0.00001 \\
-0.8282 & 0.0030 & 0.0186 & -0.5478 & 0.1038 & -0.0507 & -0.0005 & -0.0001 \\
0.0807 & 0.0041 & 0.0052 & 0.0881 & 0.8475 & -0.5074 & -0.0050 & -0.0014 \\
0.0034 & 0.0003 & 0.0011 & -0.0021 & -0.0857 & 0.0512 & 0.0005 & 0.0001
\end{pmatrix}
\quad (8.82)
$$

8.8   Conclusions

We have reported a low energy parameter space, within a local SU(3) Family symmetry model, which combines tree level "Dirac See-saw" mechanisms and radiative corrections to implement a successful hierarchical spectrum, for charged fermion masses and quark mixing. In section 8.7 we illustrated the predicted values for quark and charged lepton masses at the the M_Z scale[27], and a non-unitary quark mixing matrix $(V_{CKM})_{4\times4}$ within allowed values reported in PDG 2012 [28], coming from a parameter space with the horizontal gauge boson masses within (1-10) TeV, the SU(2)$_L$ weak singlet vector-like fermion masses $M_D \approx$

914.365 GeV, $M_U \approx 1.5$ TeV, $M_E \approx 5.97$ TeV, the neutrino masses in Eq.(8.79), including two light sterile neutrinos, and the squared neutrino mass differences: $m_2^2 - m_1^2 \approx 7.23 \times 10^{-5}$ eV^2, $m_3^2 - m_1^2 \approx 2.4 \times 10^{-3}$ eV^2, $m_4^2 - m_1^2 \approx 0.049$ eV^2, $m_5^2 - m_1^2 \approx 10.3$ eV^2.

Hence the new particles introduced in this model are within reach at the current LHC and neutrino oscillation experiments.

It is worth to comment here that the symmetries and the transformation of the fermion and scalar fields, all together, forbid tree level Yukawa couplings between ordinary standard model fermions. Consequently, the flavon scalar fields introduced to break the symmetries: Φ^u, Φ^d, η_2 and η_3, couple only ordinary fermions to their corresponding vector like fermion at tree level. Thus, FCNC scalar couplings to ordinary fermions are suppressed by light-heavy mixing angles, which as is shown in $(V_{CKM})_{4 \times 4}$, Eq.(8.73), may be small enough to suppress properly the FCNC mediated by the scalar fields within this scenario.

Acknowledgements

References

1. G. Altarelli, *An Overview of Neutrino Mixing*, arXiv: 1210.3467 [hep-ph]; A. Yu Smirnov, *Neutrino 2012: Outlook - Theory*, arXiv: 1210.4061 [hep-ph].
2. DAYA-BAY Collaboration, F. P. An *et. al.*, *Observation of Electron-Antineutrino Disappearance at the Daya Bay*, arXiv: 1203.1669.
3. T2K Collaboration, K. Abe *et. al.*, *Indication of Electron Neutrino Appearance from an Accelerator-Produced Off-Axis Muon Neutrino Beam*, Phys. Rev. Lett. **107** (2011 041801).
4. MINOS Collaboration, P. Adamson *et. al.*, *Improved search for muon-neutrino to electron-neutrino oscillations in MINOS*, Phys. Rev. Lett. **107** (2011) 181802.
5. DOUBLE-CHOOZ Collaboration, Y. Abe *et. al.*, *Indication for the Disappearance of Reactor Electron Antineutrinos in the Double Chooz Experiment*, arXiv: 1207.6632.
6. RENO Collaboration, J. K. Ahn *et. al.*, *Observation of Reactor Electron Antineutrino Disappearance in the Reno Experiment*, arXiv: 1204.0626.
7. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. **B 530** (2002) 167; See also: R. Gaitan, A. Hernandez-Galeana, J.M. Rivera-Rebolledo and P. Fernandez de Cordoba, "Neutrino mixing and masses in a left-right model with mirror fermions", European Physical Journal C **72** , (2012) 1859-1866 ; arXiv:1201.3155[hep-ph].
8. **MiniBooNE** Collaboration, A. A. Aguilar-Arevalo *et. al.*, *A Combined* $\nu_\mu \to \nu_e$ *and* $\bar{\nu}_\mu \to \bar{\nu}_e$ *Oscillation Analysis of the MiniBooNE Excesses*, arXiv: 1207.4809.
9. J.M. Conrad, W.C. Louis, and M.H. Shaevitz, arXiv:1306.6494; I. Girardi, A. Meroni, and S.T. Petcov, arXiv:1308.5802; M. Laveder and C. Giunti, arXiv:1310.7478; A. Palazzo, arXiv:1302.1102; O. Yasuda, arXiv:1211.7175; J. Kopp, M. Maltoni and T. Schwetz, Phys. Rev. Lett. **107** (2011) 091801; C. Giunti, arXiv:1111.1069; 1107.1452; F. Halzen, arXiv:1111.0918; Wei-Shu Hou and Fei-Fan Lee, arXiv:1004.2359; O. Yasuda,

arXiv:1110.2579; Y.F. Li and Si-shuo Liu, arXiv:1110.5795; B. Bhattacharya, A. M. Thalapillil, and C. E. M. Wagner, arXiv:1111.4225; J. Barry, W. Rodejohann and He Zhang, arXiv:1110.6382[hep-ph]; JHEP **1107** (2011) 091; F.R. Klinkhamer, arXiv:1111.4931[hep-ph].

10. S. Weinberg, Phys. Rev. Lett.**29** (1972) 388; H. Georgi and S.L. Glashow, Phys. Rev. **D 7** (1973) 2457; R.N. Mohapatra, Phys. Rev. **D 9** (1974) 3461; S.M. Barr and A. Zee, Phys. Rev. **D 15** (1977) 2652; H. Georgi, "Fermion Masses in Unified models", in *First Workshop in Grand Unification*, ed. P.H. Frampton, S.L. Glashow, and A. Yildiz(1980, Math Sci Press, Brookline, MA); S.M. Barr ,Phys. Rev. **D 21** (1980) 1424; R. Barbieri and D.V. Nanopoulos, Phys. Lett. **B 95** (1980) 43; S.M. Barr ,Phys. Rev. **D 24** (1981) 1895; L.E. Ibanez, Phys. Lett. **B 177** (1982) 403; B.S. Balakrishna, A.L. Kagan and R.N. Mohapatra, Phys. Lett. **B 205** (1988) 345; B.S. Balakrishna, Phys. Rev. Lett. **60** (1988) 1602; K.S. Babu and E. Ma, Mod. Phys. Lett. **A 4** (1989) 1975; H.P. Nilles, M. Olechowski and S. Pokorski, Phys. Lett. **B 248** (1990) 378; R. Rattazzi, Z. Phys.**C 52** (1991) 575; K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. **64** (1990) 2747; X.G. He, R.R. Volkas, and D.D. Wu, Phys. Rev. **D 41** (1990) 1630; Ernest Ma, Phys. Rev. Lett.**64** (1990) 2866; B.A. Dobrescu and P.J. Fox, JHEP **0808** (2008) 100; S.M. Barr, Phys. Rev. **D 76** (2007) 105024; S.M. Barr and A. Khan, Phys. Rev. **D 79** (2009) 115005.

11. Sandip Pakvasa and Hirotaka Sugawara, Phys. Lett. **B 73** (1978) 61; Y. Yamanaka, H. Sugawara, and S. Pakvasa, Phys. Rev. **D 25** (1982) 1895; K. S. Babu and X.G. He, ibid. **D 36** (1987) 3484; Ernest Ma, Phys. Rev. Lett. **B 62** (1989) 61.

12. A. Davidson, M. Koca, and K. C. Wali, Phys. Rev. Lett. **B 43** (1979) 92, Phys. Rev. **D 20** (1979) 1195; C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B 147** (1979) 277; A. Sirlin, Phys. Rev. **D 22** (1980) 971; A. Davidson and K. C. Wali, ibid. **D 21** (1980) 787.

13. X.G. He, R. R: Volkas, and D. D. Wu, Phys. Rev. **D 41** (1990) 1630; Ernest Ma, Phys. Rev. Lett. **64** (1990) 2866.

14. E. Garcia, A. Hernandez-Galeana, D. Jaramillo, W. A. Ponce and A. Zepeda, Revista Mexicana de Fisica Vol. **48(1)** (2002) 32; E. Garcia, A. Hernandez-Galeana, A. Vargas and A. Zepeda, hep-ph/0203249.

15. A.Hernandez-Galeana, Phys.Rev. **D 76** (2007) 093006.

16. N. Chen, T. A. Ryttov, and R. Shrock, arXiv:1010.3736 [hep-ph]; C. T. Hill and E. H. Simmons, Phys. Rept. **381** (2003) 235; *Workshop on Dynamical Electroweak Symmetry Breaking*, Southern Denmark Univ. 2008 (http://hep.sdu.dk/dewsb); R.S. Chivukula, M. Narain, and J. Womersley, in Particle Data Group, J. Phys. G **37** (2010) 1340; F. Sannino, Acta Phys. Polon. **B 40** (2009) 3533 (arXiv:0911.0931).

17. A. Hernandez-Galeana, Rev. Mex. Fis. **Vol. 50(5),** (2004) 522. hep-ph/0406315.

18. For some references on $SU(3)$ family symmetry see:
J.L. Chkareuli, C.D. Froggatt, and H.B. Nielsen, Nucl. Phys. B **626** (2002) 307;T. Appelquist, Y. Bai, and M. Piai, Phys. Lett. B **637** (2006) 245; T. Appelquist, Y. Bai, and M. Piai, Phys. Rev. D **74** (2006) 076001, and references therein.

19. A. Hernandez-Galeana, Bled Workshops in Physics, (ISSN:1580-4992),**Vol. 13, No. 2,** (2012) Pag. 28; arXiv:1212.4571[hep-ph]; **Vol. 12, No. 2,** (2011) Pag. 41; arXiv:1111.7286[hep-ph]; **Vol. 11, No. 2,** (2010) Pag. 60; arXiv:1012.0224[hep-ph]; Bled Workshops in Physics,**Vol. 10, No. 2,** (2009) Pag. 67; arXiv:0912.4532[hep-ph];

20. Z.G.Berezhiani and M.Yu.Khlopov, Sov.J.Nucl.Phys. **51** (1990) 739; 935; Sov.J.Nucl.Phys. **52** (1990) 60; Z.Phys.C- Particles and Fields **49** (1991) 73; Z.G.Berezhiani, M.Yu.Khlopov and R.R.Khomeriki, Sov.J.Nucl.Phys. **52** (1990) 344; A.S.Sakharov and M.Yu.Khlopov, Phys.Atom.Nucl. **57** (1994) 651; M.Yu. Khlopov: *Cosmoparticle physics*, World Scientific, New York -London-Hong Kong - Singapore, 1999; M.Yu. Khlopov: *Fundamentals of Cosmoparticle physics*, CISP-Springer, Cambridge, 2011; Z.G. Berezhiani, J.K. Chkareuli,

JETP Lett. **35** (1982) 612; JETP Lett. **37** (1983) 338; Z.G. Berezhiani, Phys. Lett. **B 129** (1983) 99.

21. N.S. Mankoc-Borstnik, arXiv: 1011.5765; Bled Workshops in Physics,**Vol. 12, No. 2**, (2011) Pag. 112; A. Hernandez-Galeana and N.S. Mankoc-Borstnik,Bled Workshops in Physics,**Vol. 12, No. 2**, (2011) Pag. 55: arXiv:1112.4368[hep-ph]; **Vol. 11, No. 2**, (2010) Pag. 89 , Pag. 105; G. Bregar, R.F. Lang and N.S. Mankoc-Borstnik, Pag.161; M. Y. Khlopov and N.S. Mankoc-Borstnik, Pag.177; arXiv:1012.0224[hep-ph]; N.S. Mankoc-Borstnik, Bled Workshops in Physics, **Vol. 10, No. 2**, (2009) Pag. 119; G. Bregar and N.S. Mankoc-Borstnik, Pag. 149; arXiv:0912.4532[hep-ph]

22. M. Yu Khlopov, A. G. Mayorov, and E. Yu Soldatov, Bled Workshops in Physics, **Vol. 12, No. 2**, (2011) Pag. 94; arXiv:1111.3577[hep-ph]; **Vol. 11, No. 2**, (2010) Pag. 73; Pag. 185; arXiv:1012.0224[hep-ph]; **Vol. 10, No. 2**, (2009) Pag. 79; M.Y. Khlopov, Pag. 155; arXiv:0912.4532[hep-ph];

23. J.A. Aguilar-Saavedra, R. Benbrik, S. Heinemeyer, and M. Pérez-Victoria, arXiv:1306.0572; J.A. Aguilar-Saavedra, arXiv:1306.4432; Jonathan M. Arnold, Bartosz Fornal and Michael Trott, JHEP 1008:059, 2010, arXiv:1005.2185 and references therein.

24. T. Yanagida, Phys. Rev. D **20** (1979) 2986.

25. G. Aad *et. al.*, ATLAS Collaboration, Phys. Lett. **B 716** (2012) 1, arXiv: 1207.7214.

26. S. Chatrchyan *et. al.*, CMS Collaboration, Phys. Lett. **B 716** (2012) 30, arXiv: 1207.7235.

27. Zhi-zhong Xing, He Zhang and Shun Zhou, Phys. Rev. D **86** (2012) 013013.

28. J. Beringer *et al.*, (Particle Data Group), Phys. Rev. D **86** (2012) 010001.