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Nucleons or diquarks: Competition between clustering and color superconductivity in quark matter

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Baryonic matter at high densities could be very different from matter in ordinary nuclei: manifest chiral symmetry might be restored, baryons might dissolve into a fluid of free quarks, and quark pairs might condense to form a colour superconductor. Do these changes occur at sharp phase transitions or smooth cross-overs? How many phase transitions are there? At what densities do they occur? So far lattice QCD has provided no answers to these questions and we must rely on models to explore the possible behaviour of strongly interacting matter at high densities.

The possibility of colour superconductivity has recently attracted considerable attention following suggestions by Wilczek, Shuryak and others that strongly attractive quark-quark forces could lead to such a state with a large gap, of the order of 100 MeV. (For reviews of this idea, see [1,2].) In recent work [3], we have examined the competition between this pairing of quarks and the three-quark clustering responsible for forming baryons at low densities. We have used a generalized Nambu–Jona-Lasinio model, which provides a quark-quark interaction which is similar to that in the instanton-liquid model often used in studies of colour superconductivity. Since there is no simple analogue of the BCS state for composite fermions, we have gone back to a Cooper-type treatment, looking for the instabilities of a Fermi gas of quarks.

The model includes interactions that generate bound diquarks states in both scalar and axial diquark channels. The energies of these are found by solving the quark-quark Bethe-Salpeter equations, which are straightforward algebraic equations in this kind of model. The relativistic Faddeev equations for the nucleon and Δ are constructed using the methods of Refs. [4,5] and, in particular, [6]. These are integral equations which we solve iteratively using the method of Malfleit and Tjon [7].

A sharp 3-momentum cut-off of about 600 MeV was used to regulate the contact interaction since this is easy to combine with the effects of the Fermi sea. However it should be noted that this choice is not covariant. The couplings in the scalar and axial diquark channels are chosen to give the observed N and Δ masses in vacuum. For a parameter set which gives a quark mass of 450 MeV, the scalar and axial diquarks have masses of 635 MeV and 700 MeV.

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At finite density, the quark mass is reduced and vanishes at the chiral phase transition, which occurs at a Fermi momentum of about 370 MeV for the parameter sets we use. (For comparison, nuclear-matter density corresponds to $k_F = 270$ MeV.) In addition, Pauli blocking provides a lower cut-off on the momenta of the quarks in the Bethe-Salpeter and Faddeev equations.

We find that the nucleon remains bound with respect to the quark-diquark threshold only up to nuclear matter density. Morover, except for densities below about a quarter of that of nuclear matter, we find that it is energetically much more favorable to form three diquarks rather than two nucleons. Hence in models of this type, quark matter is more unstable against pairing (leading to a colour superconductor) than it is against three-quark clustering, even at the density of nuclear matter.

Clearly something important is missing from models of this type: confinement. We need to examine whether extending the model to include confinement at low densities also affects behaviour at high densities. Indeed, is it possible to get a realistic phase diagram without confinement?

References

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