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# Temporality in Badiou's Ontology and Greater Logic

Alain Badiou has often remarked that the single most central question of his philosophy is the following: How does a novelty inscribe itself into a specific situation? What are the strategies for delineating, within a given situation, within a given world, the new from the old? What are the necessary and sufficient conditions for the possibility of a novelty in a world? These questions hold a very specific meaning within Badiou's system of thought and in reference to his terminology. One can partly localize this meaning by an overview of Badiou's conceptualization of ontology as mathematics – should one choose to accept it[1] – in *Being and Event*[2] and partly by his construction of the Greater Logic (the logic of the transcendental) in *Logics of Worlds*.[3] This, however, does not seem to suffice. Within Badiou's work there exists an underlying theme of temporality, which seems relevant to the topic of novelty, change, and historicity, but is, as far as we can tell, never dealt with in detail.

It is difficult to conceive a thought of change without a certain immersion in time, into the before and after. An event, this paradoxical and momentary lapse of reason (in all the nuances of the word), splits the world into before and after, not into here and there. Event is, to use the language of *Being and Event*, a historical rather than, for lack of a better word, a spatial term. We are therefore tempted to ask: Why is historicity rather clearly exposed in *Being and Event*, but temporality is not? There are two possible answers to this question: either there is no need for the concept of temporality within Badiou's work or, alternatively, historicity, temporality, and the Greater Logic are indeed intertwined and we might benefit from finding a concept of temporality that sheds more light on Badiou's thought of change. In this text, we opt for the latter. We will attempt to



1 Roland Bolz, "Mathematics is Ontology? A Critique of Badiou's Ontological Framing of Set Theory", *Filozofski vestnik*, 41 (2/2020), pp. 119–142.

2 Alain Badiou, *Being and Event*, trans. O. Feltham, Continuum, New York, 2005.

3 Alain Badiou, *Logics of Worlds: Being and Event, 2*, trans. A. Toscano, Continuum, New York, 2009.

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present a specific concept of temporality that seems immanent to – or at least compliant with – Badiou's thought of change.

In order to do this, however, we must take a detour and sketch a brief overview of what we understand to be the key features of change and novelty in *Being and Event* and *Logics of Worlds*. Let us begin with a situation. A situation is defined, in the broadest sense of the term, as a consistent multiple, as a structured multiple, namely a multiple for which there exists a specific and specifiable criterion of counting its elements as one. A specific situation is always structured by the count-as-one according to a specific predicate of the situation language. Any situation can be constructed as a multiple of elements (which are themselves multiples) that can be subsumed to a specific predicate or a specific set of predicates. All other multiples, not counted-as-one by the situation's criterion of the count, are said to be subtracted from this particular situation. This subtractive multiplicity, this inconsistent multiplicity (of multiples) not counted-as-one by the situation, is where all potential novelties reside. Any potential novelty exists within a situation, but is not counted-as-one by the situation's regime of the count (*sans-papiers* in a political state are an example). Subsets, free from all specific predicates of the situation's regime of the count, are denoted (following Paul Cohen) as *generic* subsets. In *Being and Event*, this existence-without-recognition is presented in the situation with the proper name, the name of the void: ø. This generic inconsistent multiple, marked by the symbol ø, is a subset of every situation but it is not an element of every situation. This tension between inclusion and belonging provides an ontological base for the appearance of a novelty and leads to the following question: What are the conditions of the possibility of the inscription of the existent-within-the-situation in the language of the situation that does not count this existing entity as one of its elements? Obviously, one cannot fulfil these conditions within the existing language of the situation, within the existing regime of the count – otherwise the entity would already have been counted as one within the situation.



The possibility of a novelty can therefore only stem from an extension of the situation to its generic base. In other words, any possibility of a novelty depends on the possibility of the extension of the regime of the count to multiples subtracted from the current regime of the count. Novelty depends on the possibility of forcing the recognition of certain generic subsets of the situation as its elements. This, however, cannot be done without changing the regime of the

count. If the situation's regime of the count cannot change, then what is counted as one within the situation also cannot change. In such (constructible) situations there can be no novelty. Or to put it differently: any situation defined by an immutable regime of the count is incapable of novelty.

Situations with an immutable regime of the count can be very simply illustrated: these are the multiples with no tension between inclusion and belonging. Whatever belongs to such a situation is always already included in it. (Note that this property cannot be reversed since it can be shown that each multiple has at least one subset that does not belong to it as an element.) In *Being and Event* such situations are called *natural* (i.e. not *historical*). An example of a natural situation is a human cell: everything included in this specific biological multiple also belongs to a human cell as one of its elements. A cell's nucleus is, for example, a subset and an element of the cell. Furthermore, the cell's nucleus is in itself a natural situation: everything included in the nucleus also belongs to it as one of its elements. The same holds for the DNA in the nucleus – and for the phosphates and sugars in the DNA backbone, etc. This property of 'naturalness' therefore transfers from a subset to *its* subsets – for this reason such sets are called *transitive* or *normal* sets. Normal sets are, to quote Badiou, "the schema of the maximum equilibrium of presented-being".[4]

This remark is as interesting as it is unclear. It is by no means obvious what is meant here by "equilibrium". In order to proceed, we need to establish some sort of grasp of what an equilibrium might be. As we will see, introducing a strict concept of equilibrium will bring us one step closer to the concept of temporality at work in *Being and Event*. First notice that the above quote signifies a shift from a mathematically and logically dense vocabulary into physics. Equilibrium is primarily a dynamical, physical term. Furthermore, "maximum equilibrium" seems to be a pleonasm – anything less than "maximum" equilibrium is by definition not an equilibrium at all. This pleonasm does, however, give some further substantiation, perhaps inadvertently, to what will become our description of temporality within Badiou's work.

Let us begin our illustration – and not much more – of this specific notion of time with an observation that the laws of physics are time-reversible, which



4 Badiou, *Being and Event*, p. 130.

means that the equations describing these laws retain their mathematical form if we reverse the sign of the time variable. This does, however, not mean that all physical processes are easily reversable. As an illustration, imagine recording a planet circling a star for a few orbital cycles. We can now play this recording either forward or backward in time. The time-reversibility of planetary mechanics is illustrated by the fact that we cannot, in general, distinguish whether the recording of a planet circling a star was played forward or backward in time. Another similar sketch: imagine recording a collision of two billiard balls, one bouncing off the other. Like before, one cannot determine from the recording alone whether the collision was played forward or backward in time. The laws of mechanics are invariant to time-reversal, they are symmetrical under the operation of changing the direction of time. So how can the arrow of time be determined in classical systems? If fundamental laws of the world are time-reversible, why do we, macroscopic beings, perceive the flow of time at all? Why is the past so fundamentally different from the future? Why do we see traces of the former but not of the latter? How does the irreversibility related to the arrow of time arise from the essentially reversible laws of classical physics?

To illustrate how these two demands are not inconsistent requires slightly stricter definitions of reversibility and irreversibility. Imagine a droplet of ink dropped in a bowl of water. As the droplet enters the water, the molecules of water randomly collide with the molecules of ink. Mixing between the ink and water occurs. The density of the ink molecules drops and ink molecules spread over a larger and larger area in a process known as diffusion. If, however, we were to record *this* process, there could be no doubt whether the recording is being played forward or backward in time. If the ink area shrinks, the recording is being played backwards in time. If the ink area spreads, the recording is being played forward in time. Time flows forward as the ink spreads. Time flows backward if the ink shrinks (i.e. it does not).



From this dictum alone it is clear that, in the classical realm, the *direction of time* becomes reasonably well-defined only in "large-enough" macroscopic systems that contain enough particles to allow for some sort of measure of *order and disorder*. How large is "large enough"? An infinite system would be ideal but a bowl of water and a droplet of ink suffice. At the other extreme, one ink molecule and one water molecule are clearly not enough. In other words, the arrow of time is a *statistical* property of a system and not a property of its basic constituents. Time

flows in the direction in which the disorder of an isolated system increases. In an isolated system of an ink droplet in a bowl of water, the most *ordered* state of the system is the initial state where all of the ink molecules are still confined within the droplet and no mixing has yet occurred. As the diffusion mixes molecules of ink and water, the *disorder* in the system increases. If the system is left to itself, ink molecules will gradually spread over the entire volume of the bowl, reaching a state of maximum possible disorder. This, essentially, is the second law of thermodynamics, also known as the law of entropy. Its status is a bit different from other physical laws because it is, by its nature, a statistical law. Entropy is not a fundamental but rather a statistical quantity that measures the amount of disorder in a system. More precisely, the entropy of a particular state of the system can be computed from the number of ways in which we can rearrange the interchangeable constituents (ink molecules, on one hand, and water molecules, on the other) of the system and still get essentially this same system state. The law of entropy states that the entropy (the level of disorder) of an isolated system cannot decrease over time. Indeed, the entropy of the initial state (a self-contained droplet of ink in a bowl of water) of any isolated system will always be (equal to or) lower than the entropy of the final state (the fully mixed bowl of ink and water molecules). This follows from the fact that we can *construct* the initial state in *fewer ways* than the final state.

In thermodynamics, the state of a system with maximum possible entropy (maximum disorder) is defined as *a state of equilibrium*.[5] In other words: a state of equilibrium is defined as the state that can be realized in the largest number of different ways. This is a statistical definition: if left to itself, any isolated system will, through random fluctuations and basic probability, end up in the most probable state, i.e. the state that can be realized in the maximum number of ways – in a state of equilibrium. Once an isolated system reaches its equilibrium state, it will be extremely unlikely that it will ever depart far from equilibrium, simply because random fluctuations in the system will push it back towards the most probable state, which can be realized in the largest number of ways (and therefore exhibits maximum entropy) – a state of equilibrium.



5 This definition is only viable at high enough temperatures, but this need not concern us here.

Note that there is no *physical* law which prevents the system from leaving equilibrium. There is no law which makes it apodictically *impossible* for all the molecules of ink to reverse their velocities and migrate back through the bowl of water and condense themselves in the initial droplet. This reversal of diffusion is not impossible, it is merely highly improbable – one would have to wait several ages of the universe for this process to occur randomly. On the other hand, a fully mixed state of equilibrium is so probable (i.e. it can be achieved in a large number of ways / has high entropy / exhibits maximum disorder) that the system may end up in equilibrium quite rapidly.

The point of this digression was twofold: first, to show that a state of equilibrium, casually mentioned by Badiou, can be defined in relation to order and disorder; and second, to show that the arrow of time can also be defined in relation to *changes* in order and disorder. There is a connection between temporality and order. Note that even though this illustration of temporality stems from classical physics, it is profoundly un-Newtonian in nature. Time, related to a change in order, is quite different from "true and mathematical time," which in Newton's own words (from the famous Scholium to the Definitions in *Philosophiae Naturalis Principia Mathematica*) "flows equably without regard to anything external."

So, what happens with the arrow of time when an isolated system reaches equilibrium? In short: since equilibrium implies maximum disorder, *no change in the order* within the system is any longer possible (i.e. probable). In other words: the state of equilibrium is *stable*. Consequently, the arrow of time can no longer be defined in equilibrium. Equilibrium is timeless. If undisturbed, any system already in equilibrium will persist in equilibrium. A recording of a droplet of ink in a bowl of water allows for a sense of temporality only as long as the disorder in the system *increases*, i.e. as long as the ink *spreads*. Once the ink is fully mixed with the water, playing this recording forwards or backwards no longer makes any difference – there can be no meaningful introduction of the arrow of time in such a system.



Let us now return to the above-mentioned normal (transitive) sets, to "the schema of the maximum equilibrium of presented-being." One can now read this statement as the statement that natural sets are essentially timeless. They are timeless in the sense that *they do not allow for a change in order*, they are *stable*. Badiou's wording is completely consistent with our line of argument: "The

ontological criterion for natural multiples is their *stability*, their homogeneity; *that is*, as we shall see, *their immanent order*."[6] This connectedness of "maximum equilibrium" and order in natural sets can be reformulated as the following statement (which seems to be true, as far as we can tell): physical laws do not change over time. (One is almost tempted to evoke here the Leibniz criterion of the best possible world being the one with the simplest laws which allow for maximum complexity, i.e. maximum entropy – and hence stability.)

Another important consequence of the stability of natural sets, specifically of their structure, according to which every element is also a subset, directly implies that an event is not possible in natural situations. In order for an event to be *possible* in a situation, the situation must allow for the inclusion of a set whose elements do not belong to the situation. Such a situation is by definition not a transitive (normal) set. Badiou denotes such situations as *historical*, and only historical sets, with their distinction between inclusion and belonging, are those that allow for the generic extensions necessary for the occurrence of a novelty, of an event. Once one agrees to temporality as a measure of the change in a situation's order, the connection between historicity and temporality becomes quite clear: historicity is the potentiality of temporality.

Following Badiou, a novelty can only occur in a historical situation if an event, being an element of an evental site, becomes an element of the situation through an undecidable axiomatic decision of the faithful subject. This leads us to an interplay of *temporality* and *fidelity*. This interplay is not entirely unexpected: if one understands temporality in terms of a *change* in the situation's *order* and if, similarly, subjective action primarily appears as a *disruption* of this order – then temporality and subjective fidelity must be somehow related. A relation between time and fidelity is indeed mentioned (and not much more) by Badiou in the following, somewhat infamous passage:



> The real difficulty is to be found in the following: the consequences of an event, being submitted to structure, cannot be discerned as such. I have underlined this undecidability according to which the event is only possible if special procedures conserve the evental nature of its consequences. This is why its sole foundation

6 Badiou, *Being and Event*, p. 130. Emphasis added by M. L.

> lies in a *discipline* of time, which controls from beginning to end the consequences of the introduction into circulation of the paradoxical multiple, and which at any moment knows how to discern its connection to chance. I will call this organised control of time *fidelity*.[7]

The above lines perhaps become a bit more comprehensible if they are read under the assumption of the relationship between the flow of time and rising disorder. It is not the situation that unfolds *in time* – it is rather time itself that gets co-determined by the structural changes in the situational order: if the situational order does not change, the situation, even if it is itself historical, is effectively timeless. Structural changes in the situational order appear as the consequences of an event, this paradoxical presented multiple that lacks representation. The very practise of controlling the consequences of an event, and of preserving the evental nature of these consequences, is also the practise of the disruption of the situational order, of its regime of the count. Of its presentation and representation – and thus its temporality.

This operation of fidelity is twofold. First, it consists of claiming axiomatically (and undecidably) that the disorder of the situation has increased, and second, it demands that this increase in disorder, manifested as the appearance of the generic extension of the situation, must be recognized by the situation's regime of the count. The fidelity of the subject to an event is a wager that history has *already* unfolded within the situation; something undecidable has *been* decided (an event *did* happen) and this decision will now have to be formalized *retroactively* by a change in the situational order, by a change in what *will have been* admitted, anew, to the representation within the situation. In other words, the operation of the subject (the forcing) is to sustain the situation's subtraction as *belonging* to a situation, to sustain the subtraction as it will have been presented *and represented* within the situation itself. This 'will-have-been' *future anterior* structure of the work of the subject, this intervention of subjective urgency into objective impossibility, is what Badiou identifies as the fundamental law of the subject. The work of the subject is therefore closely coupled with any possible emergence of temporality within a given situation.



7 Badiou, *Being and Event*, p. 211. Emphasis in original.

One of the key tasks of *Logics of Worlds* is precisely to establish in more depth how this post-evental work of the faithful subject plays out in a *particular* situation or a *world*. Note that on the level of ontology nothing particular can be attributed to any situation. *Being and Event* does not deal with particular situations. It rather attempts to derive, from a specific minimal set of assumptions, an ontological base for the appearance of truths in *any* situation. A world, on the other hand, is a situation equipped with a *transcendental*, which must be understood as a general logical form of the objectivity of appearing[8] or, more specifically, as a hierarchical scale of the intensities of the appearance of the multiples of a particular world. The task of deriving conditions of the appearance of truths in a world is therefore focused not so much on the event itself, but rather on how evental consequences appear, through the work of the subject, as a *material* novelty in a *particular* world. Badiou's shift from the ontology of pure multiples in *Being and Event* to their localization in a particular world in *Logics of Worlds* is demanded by the fact that being itself is only thinkable insofar as it belongs to a world, i.e. a *specific* world[9], a world with a specific hierarchy of intensities of appearance, a world with a specific transcendental. Whereas a situation is a pure multiple of multiples, a world, through its transcendental, allows for a relational network of appearances. One of the driving questions behind *Logics of Worlds* is, to quote Badiou directly, a simple one[10]: "Why and how are there worlds rather than chaos?"

As remarked above, a transcendental is a specific set. More precisely: it is a set of intensities of appearances (hereinafter marked with the letters $a,b,c,...p,q,...$) in a situation. There are, of course, certain requirements that have to be met in order for a transcendental to have any specificity at all. These requirements are the formal properties required for a transcendental to be a *specific* set. We list them formally below, but in their essence they are quite literally mundane and elementary:



- A transcendental must be a partially ordered set, i.e. it needs to be reflexive (for any intensity of appearance $a$, $a \leq a$), anti-symmetric (if $a \leq b$ and $b \leq a$ then $a=b$), and transitive (if $a \leq b$ and $a \leq c$ then $a \leq c$). The property of tran-

8 Badiou, *Logics of Worlds*, p. 233.
9 *Ibid.*, p. 113.
10 *Ibid.*, p. 101.

sitivity, for instance, can literally be read simply as: if tea appears warmer than coffee, and coffee appears warmer than juice, then tea appears warmer than juice. Similar examples can be found for all other properties on this list.

- A transcendental must allow for the existence of a minimum intensity of appearance (of all appearances, one appears with the least intensity). The minimum intensity of appearance μ signifies inexistence, or, in the language of *Being and Event*, inclusion in (as a subset) but not belonging to a situation (as an element). This demand is a logical translation of an empty set being a subset of every set. *Sans-papiers* belong to the French situation, but particular people without papers do not form a subset of the situation of France, they do not *appear* formally in the citizen register or as tax payers. They do not *formally* exist.
- A transcendental must allow for a conjunction ∩ (appearances can overlap (intersect) into a new appearance) and for an envelope (appearances can combine into a union of appearances, amounting to a new appearance) of intensities of appearance.
- A transcendental must be a distributive set (an overlap of appearance $p$ with an envelope of other appearances $E(q)$ is an envelope of overlaps of appearance $p$ with each member $q$ of $E$).

Any set with this minimum set of properties is called a *Heyting algebra*. If a transcendental is to be a specific set, it must be a Heyting algebra. These properties of a transcendental are the ones which allow for a specific hierarchy, a specific network of intensities of appearances, and are thus guarantors of a specific world.



A shift from ontology to logics further requires the extension of the theory of the subject from the domain of being to the domain of appearance. The subject, as introduced in *Being and Event*, is, strictly speaking, a local state of some *specific* (local) generic procedure. The latter amounts to a finite series of evaluations that some multiples, tied to a *specific* event, belong to – and should be represented in – a *specific* situation. What has no *place here*, *should* have a place *here*. In other words: the subject is introduced strictly as a *localized* entity, but the theory of the subject in *Being and Event* does not seem to be able to respond to its own demands. It is far from clear how and through what mechanisms the localization of the subject of *Being and Event* can take place. Frank Ruda points out that *Being and Event* elaborates somewhat on the subjective process, "but it

does not give us all the conceptual tools needed to think subjectivisation proper."[11] Ruda rather describes Badiou's approach as "genericism" to emphasize the dominant role of ontology over logical localization. Consequently, an attempt was made in *Logics of Worlds* to develop formal theories of change[12] and subjective bodies.[13] These passages might to some extent address Sam Gillespie's objections in his discussion of *Being and Event*, i.e. that "Badiou's mathematical formalism, which is perfectly capable of weaving complex multiplicities and rules out of nothing, is simply an empty game of manipulating symbols. The problem is not just that of giving the operation of presentation the same ontological validity as sets; rather, what is needed is an analysis of why being must depend upon presentation as its material support, and what sort of framework may be necessary for such a dependency."[14] The importance of the translation of the theory of subject into the domain of logic was fundamental enough that *Logics of Worlds* opens precisely with a new definition of the subject: a singular subject is a material, corporeal "bearer of the dialectical overcoming of simple materialism."[15] In *Logics of Worlds*, the work of the subject therefore acquires a new signifier, materialist dialectic, which keeps affirming the following axiom: "There are only bodies and languages, except that there are truths."[16] In other words: the subject is an exception to the logic of the world. It is not difficult to locate dialectical trademarks already in what has been said above.

Firstly, we note the thought of contradiction within a single framework of unifying opposites (and also the consequences of an event through the subjective operation of fidelity). The thought of contradiction is imperative since an event is defined as a paradoxical entity that belongs to itself as an element. Secondly, the transformation of quantity into quality, which occurs through the work of the subject when a subjective body persists in fidelity to the event until a change in the regime of the count is established.



11 Frank Ruda, "To the End: Exposing the Absolute", *Filozofski vestnik*, 41 (2/2020), pp. 311–340.

12 Badiou, *Logics of Worlds*, p. 357.

13 *Ibid.*, p. 451.

14 Sam Gillespie, *The Mathematics of Novelty: Badiou's Minimalist Metaphysics*, Melbourne, re.press, 2008.

15 Badiou, *Logics of Worlds*, p. 45.

16 *Ibid.*, p. 4.

And thirdly, the negation of negation. Whenever the transcendental representation of a world is restructured, whenever a change in the regime of the count is achieved, a specific negation within this world has been itself negated. It is, however, not at all unambiguous what the negation of a specific appearance might be. As pointed out by Badiou,[17] one can define negation in relation to three fundamental principles of thought: i) the principle of identity, $p = p$; ii) the principle of the excluded middle or *tertium non datur*, $p \cup \neg p$; and iii) the principle of non-contradiction, $\neg(p \cap \neg p)$. An operation of negation that obeys the last two principles is a classical negation. Negation can, however, violate the principle of the excluded middle and obey the principle of non-contradiction – such a negation is part of the so-called intuitionistic logic (developed by Brower and Heyting). Finally, negation can also violate both the principle of the excluded middle and the principle of non-contradiction – such a negation is a paraconsistent negation. Since the logic of appearing allows for a continuous spectrum of transcendental degrees of appearance $p$, the same must hold for the negation of $p$. In other words: while the ontology is itself classical, the logic of appearance violates the principle of the excluded middle and is therefore intuitionistic.

The crucial point regarding the possibility of any dialectic is, however, that, within intuitionistic logic, the negation of negation does not, in general, lead back to the beginning but rather signifies a transgression of the original situation. This shift from the key principle of classical logic,

$$\neg\neg p = p,$$

to the new principle,

$$\neg\neg p \geq p,$$

[18] turns out to be a differentiating property of the intuitionistic logic of the transcendental, the Greater Logic. In order to see that this shift is immanent to Badiou's logic of appearing, we need to describe in more detail the concept of the negation on the level of appearance – or, as Badiou calls it, *a reverse*. What is, in other words, a reverse $\neg p$ of a specific appearance $p$? Let's jump straight to the definition[18] and state it in non-technical terms first: the reverse of a specific appearance is defined as a union of all appearances that have nothing in com-

[17] Alain Badiou, "The Three Negations", *Cardozo Law Review*, 29 (5/2007), pp. 1877–1883.

[18] Badiou, *Logics of Worlds*, p. 168.

mon[19] with the appearance undergoing reversal. Or, in the terminology of the logic of the transcendental: the reverse $\neg p$ of the specific appearance is an envelope of all (transcendental degrees of) appearances that have minimal conjunction with the (transcendental degree of) appearance undergoing reversal. Since each member of the envelope $\neg p$ has minimal conjunction (i.e. has nothing in common) with $p$, the conjunction of p with the envelope also has a minimal degree of appearance: $p \cap \neg p = \mu$.

Now remember that the envelope of $p$ is defined as a union of conjunctions of $p$ with the members of any subset of appearances of the same world.[20] Therefore, the transcendental degree of appearance of the envelope is always equal to or larger than $p$: $E(p) \geq p$. The envelope $E(p)$ of $p$ equals $p$ if and only if each member $q$ of the subset has nothing in common with $p$, or $p \cap q = \mu$. Otherwise $E(p) > p$. Let us now return to the negation of negation, or the reverse of reverse, $\neg\neg p$. The reverse of the reverse of $p$ is a (double) envelope: an outer envelope over a subset of appearances $q$ such that $q \cap \neg p = \mu$, where $q$ stems from an inner envelope of $p$ over a subset of appearances $r$ such that $r \cap p = \mu$. Now, according to the definition of the reverse, $p \cap \neg p = \mu$. Therefore, $p$ itself is also among the appearances $q$ over which the outer envelope operates. Therefore the outer envelope, or the reverse of reverse, $\neg\neg p$, is greater than or equal to $p$:

$$\neg\neg p \geq p.$$

Thus, the logic of appearance, through its violation of *tertium non datur*, allows for the dialectical transgression through negation.

What about the materialistic part of materialist dialectic? The core of Badiou's demonstration of this aspect of the work of the subject resides in his proof of the *second constitutive thesis of materialism*.[21] This thesis implies, to use his words, *that the ontological closure of the world implies its logical completeness*. The ontological closure of the world is the starting point of the proof, and consists of the following statement: if $x$ and $y$ are, in the ontological sense of the word, multiples, then their union and intersection are also multiples. From this, the



[19] *Ibid.*, p. 107.
[20] *Ibid.*, p. 163.
[21] *Ibid.*, p. 345.

following consequence can be derived[22] within the logic of the transcendental: if *x* and *y* appear in a world (with, say, intensities of appearance *p* and *q*), then all ordered pairs of their respective elements also appear in the world. In other words: if *x* and *y* appear in a world (with, say, intensities of appearance *p* and *q*), then their union and all their subsets also appear in this world. Whatever appears in a world is therefore, as proven by Lemma 1, ontologically grounded within the world. This sweeps away any possibility of transcendence and hence implies materialism. (It needs to be noted that this route from ontological closure to logical completeness is only possible if one accepts that ontology is mathematics and that a transcendental has the structure of a Heyting algebra.) So, whatever appears has an ontological ground, but the reverse is not true: being does not necessarily appear.

The logical aspect of the proof[23] requires the establishment of relations between the *objects* of a particular world. An object of a world is a multiple, appearing in that world, along with an envelope of conjunctions between this multiple and all other multiples appearing in that world. This envelope essentially measures how similar this multiple appears to all other appearing multiples of that particular world. Vaguely speaking: an object is a multiple, *localized* in a particular world through a set of similarities between itself and other multiples, appearing in that same world.[24] The logical completeness of a world is synonymous with the fact that given the relations between the *already-appearing* objects of that world, the relations between *these* objects and any *newly appearing* object are already completely determined by an existing network of relationships.

The second constitutive thesis of materialism, which establishes a connection between ontology and the logic of appearance, consequently indicates that an interruption of representation at the ontological level interrupts the consistency of relations on the level of appearance. A true change occurs as a transformation of the transcendental hierarchy on the level of appearance. The relation between temporality and order that we tried to establish at the level of ontology must therefore translate somehow also to the level of appearance. This translation takes the form of the negation of negation we referred to above: the nega-



22 *Ibid.*, p. 345.
23 *Ibid.*, pp. 345–352.
24 *Ibid.*, p. 245.

tion of a specific inexistent minimal appearance is itself negated to yield a new appearance in the world through a shift of the transcendental order – which gives rise to a temporality within the world:

> The intensities of objects and relations are measured according to a singular temporal transcendental, which objectivates in their appearing multiplicities such as 'the firm stance of the group of anarchists from one end to the other', or 'the organizing role of the rail workers' union', or 'the growing isolation of the Kurdish communists' and so on. In other words, the object absorbs, as elements of the multiplicity that it is, the modifications which include it within the time of the world, through which it only 'changes' to the extent that this 'change' is its appearing-in-the-world.[25]

If time is to exist in a world, it exists only insomuch as the transcendental of the world experiences a disruption of its order by an event, by an appearance of inexistent objects that need to be integrated, through changes in the transcendental hierarchy, into a new transcendental of the world, thus leading to a new order, a *new present*:

> The event cannot be the undivided encroachment of the past on the future or the eternally past being of the future. On the contrary, it is a separating evanescence, an atemporal instant which disjoins the previous state of an object (the site) from its subsequent state. We could also say that the event extracts from one time the possibility of another time. This other time, whose materiality envelops the consequences of the event, deserves the name of new present. The event is neither past nor future. It presents us with the present.[26]



An event presents us with the present, with what is presented but not represented, with what exists but does not appear. An event disjoins the *previous* state of an object from its *subsequent* state. It splits the world into the before and after, each with its own transcendental ordering. It is then up to the faithful subject to decide the undecidable dilemma: Did an event take place or not? Did the possibility of another time open up or not? Did the order of the world change

[25] *Ibid.*, p. 359.
[26] *Ibid.*, p. 384.

or not? Now we can follow Badiou[27] by asking: What is the outcome, for the inexistent, of this negation of negation? What is the intensity of the appearance of the inexistent in the post-evental world? There are only three possibilities: the post-evental intensity of appearance can be either maximal, intermediate, or minimal.

In the first possibility, the world experienced what Badiou calls a true event. There is no doubt that the transcendental of the world has changed and a specific temporality has emerged. In the second possibility, the post evental intensity of the inexistent is intermediate: "something happens, but without radical effects, and in the general respect of the hierarchy of degrees of appearing in the world".[28] In this case, we cannot speak of any reordering of the transcendental in any fundamental sense. The change itself was not a true event, but rather what Badiou denotes as a weak singularity. This post-evental world remains timeless. The same holds for the last possibility, where the inexistent remains inexistent. Such a change is referred to as a false event, or a simulacrum of an event, and the post-evental world is exactly the same.

Which of these three scenarios manifests itself depends ultimately on a purely subjective decision, a purely subjective choice. The assertion of the existence of this *choice* opens up a new path that leads far beyond this paper – here we can only briefly sketch one possible approach. The existence of this choice is guaranteed by the early axiomatic decision to use the axiomatic system of the Zermelo-Fraenkel set theory, supplemented by the axiom of choice.[29] The axiom of choice guarantees, informally speaking, that for any set of nonempty sets, it is possible to choose one element from each set. (This seems trivial for finite sets, but is not obvious for infinite sets.) The existence of choice is, however, guaranteed by the axiom of choice also for infinite sets. Note that the axiom says nothing about *how* the choice should be performed, it merely guarantees that there *is* a choice. Of course, one might object that the validity of the axiom of choice by definition holds over any set of *nonempty* sets (i.e. having at least one element, hence being constructible via the operation of the count) and is



27 Badiou, "The Three Negations", pp. 1877–1883.

28 *Ibid*., p. 1882.

29 See also: Oliver Feltham, "One or Many Ontologies? Badiou's Arguments for His Thesis 'Mathematics is Ontology'", *Filozofski vestnik*, 41 (2/2020), pp. 37–55.

thus inapplicable in the case of an event, which presents precisely subtracted, non-constructible, *empty* sets, for elements of which the faithful subject chooses to assert existence or not. The short answer to this objection lies in the fact that this choice is made from the position of the subject and *from the subjective position* the generic extension is not void, it is constructible.

The possibility of choice is perhaps one of the clearest demarcation lines between natural and historical situations. Choice is only given in historical situations that are capable of reinventing their own time. Out of this reordering of the world there emerges a new temporality, sustained by subjective fidelity – outside of a subjective body, there is no right or wrong answer to this choice and thus no temporality. Outside of a subjective body, there is not even any necessity for an answer to the question of whether the world can change or not – not answering this question is already an answer. But there is always a choice.

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