ECONOMIC AND BUSINESS REVIEW

Volume 11 | Issue 4 Article 4

12-31-2009

The value of sharing advance capacity information under "zero-full" supply capacity availability

Marko Jakšič

Borut Rusjan

Recommended Citation

Jakšič, M., & Rusjan, B. (2009). The value of sharing advance capacity information under "zero-full" supply capacity availability. *Economic and Business Review, 11*(4). https://doi.org/10.15458/2335-4216.1273

THE VALUE OF SHARING ADVANCE CAPACITY INFORMATION UNDER "ZERO-FULL" SUPPLY CAPACITY AVAILABILITY

MARKO JAKŠIČ*
BORUT RUSJAN**

ABSTRACT: *The importance of sharing information within modern supply chains has been established by both practitioners and researchers. Accurate and timely information helps firms effectively reduce the uncertainties of a volatile and uncertain business environment. We model periodic review, single-item, stochastic demand and stochastic supply where, in a given period, supply is either available or completely unavailable. In addition, a supply chain member has the ability to obtain advance capacity information ('ACI') about the future supply capacity availability. We show that the optimal ordering policy is a base stock policy with the optimal base stock level being a function of future capacity availability or unavailability given through ACI. In a numerical experiment we quantify the value of ACI and provide relevant managerial insights.*

Keywords: *Operational research; Inventory; Stochastic models; Value of information; Advance capacity information*

UDC: 519.8:005.745

JEL classification: C61, M11

1. INTRODUCTION

Particularly in the last two decades, companies working in the global business environment have realised the critical importance of effectively managing the flow of materials across the supply chain. Industry experts estimate not only that total supply chain costs represent the majority of most organisations' operating expenses but also that, in some industries, these costs approach 75% of the total operating budget (Monczka et al., 2009). Inventory and hence inventory management play a central role in the operational

* University of Ljubljana, Faculty of Economics, Kardeljeva ploščad 5, 1000 Ljubljana, Slovenia, Email: marko.jaksic@ef.uni-lj.si

** University of Ljubljana, Faculty of Economics, Kardeljeva ploščad 5, 1000 Ljubljana, Slovenia, Email: borut.rusjan@ef.uni-lj.si

behaviour of a production system or supply chain. The fact is that the average cost of managing and holding inventory in the United States is 30% to 35% of its value, inventory represents about a third of the current assets and up to 90% of the working capital of a typical company in the United States is invested in inventories (Jacobs et al., 2008). Due to the complexities of modern production processes and the extent of global supply chains, inventory appears in different forms at each level of the supply chain. A supply chain member needs to control its inventory levels by applying some sort of inventory control mechanism. The appropriate selection of this mechanism may significantly impact on the customer service level and the member's inventory cost, as well as supply chain system-wide costs.

Due to the focus on providing a quality service to the customer, it is no surprise that demand uncertainties attracted the initial attention. However, through time companies have realised that the effective management of supply is equally important. A look at the supply chain's production and supply capacities allocated to produce or deliver a certain product reveals that these are generally far from stable over time. On the contrary, supply capacity may be highly variable for several reasons, like frequent changes in the product mix, particularly in a setting where multiple products share the same capacity, changes in the workforce (e.g. holiday leave), a machine breakdown and repair, preventive maintenance etc. To compensate for these uncertainties extra inventory needs to be kept.

However, there is another, perhaps even more appealing way to tackle uncertainties in supply. Foreknowledge of future supply availability can help managers anticipate possible future supply shortages, while also allowing them to react in a timely manner by either building up stock to prevent future stockouts or reducing stock in the case of favourable supply conditions. Thus, system costs can be reduced by carrying less safety stock while still achieving the same level of performance. These benefits should encourage the parties in a supply chain to formalise their co-operation to enable the requisite information exchange by either implementing necessary information sharing concepts like Electronic Data Interchange ('EDI') and Enterprise Resource Planning ('ERP') or by using formal supply contracts. We could argue that extra information is always beneficial, but further thought has to be put into investigating in which situations the benefits of information exchange are substantial and when it is only marginally useful. Many companies nowadays have invested millions to improve inventory management through modern planning systems like ERP systems, which allow them to use one software package with a number of integrated modules, rather than multiple, conflicting systems with different operating platforms and data formats (Scott, 2000). While a decade ago Bush and Cooper (1988) and Metters (1997) attributed problems in determining the appropriate inventory levels to the fact that companies typically do not use any formal analytic approach, the problem persists as companies do not realise that an ERP system is only a management tool, a framework that must be effectively applied and integrated to achieve success (Scott, 2000; Schwarz, 2005).

In this paper, we explore the benefit of using available advance capacity information ('ACI') about future uncertain supply capacity conditions to improve the performance of the inventory control policy and reduce the relevant inventory cost. We study a periodic review, single product, single location inventory model where both the demand and supply capacity are assumed to be stochastic. We assume that due to their intimate knowledge of the production process the supplier is able to provide upfront information on its capacity availability to the retailer. This is done for a certain limited number of future periods where in some periods the capacity is completely unavailable, while in other periods there is complete availability of the capacity. Through ACI, the retailer can anticipate near future supply shortages with certainty and prepare for periods of overall product unavailability by building up inventory in advance.

The supply setting described above can also be observed in practice. We give two examples in which a retailer would be facing periods of full or zero supply capacity availability and could take use of ACI if available. When a supplier/manufacturer is facing high set-up costs in his production process, which are due to setting up the production process to meet particular customer's/retailer's needs, he would be reluctant to frequently change production programme. A situation like this can be observed in many capital-intensive industries where production resources are usually quite inflexible, resulting in high production switching costs (metal industry, food processing etc.). This results in a highly irregular supply to a particular retailer. Since it would be too costly for a manufacturer to fulfil each separate order from a retailer directly from a production line, while at the same time the option of storing high inventories of products is also not possible for longer periods (particularly in food processing industry), a relatively low service level to the retailer is acceptable. However, to optimise its production costs the manufacturer already considers high set-up costs in its master production scheduling by determining the production plan for several production periods in advance. Thus, it can communicate information about future supply availability to the retailer and, while it cannot reduce the irregularity of supply, it can reduce its uncertainty in a number of near-future periods. Advance information about future supplier capacity availability enables a retailer to prepare for periods of supply unavailability in advance and by doing so to significantly decrease both inventory holding costs (resulting from the high inventory levels needed to cope with supply uncertainty) and backorder costs (mainly a consequence of high supply variability). A similar practical setting is when the manufacturer's transportation costs of supplying the products to the retailer are high and manufacturer resorts to less frequent supplies to the retailer.

Throughout the paper we address two main research questions: (1) What is the best way to integrate ACI into the inventory control policy and how will ACI affect the optimal inventory levels? (2) What is the value of ACI and in which settings does ACI turn out to be particularly important?

Our contributions in this paper can be summarised as follows: (1) We present a new model that incorporates ACI within a limited supply capacity setting, where supply capacity is modelled as a Bernoulli process. (2) We derive the structure of the optimal policy and show that it fits in the group of base stock policies, where the base stock level is a function of the revealed capacity realisations in the near future. (3) Finally, we establish the value of ACI, plus recognise and describe the system settings in which ACI brings considerable savings.

We proceed with a brief review of the relevant literature where the focus is on presenting the different ways suggested by researchers to tackle the problem of supply uncertainty. Supply uncertainty is commonly attributed to one of two sources: yield randomness and the randomness of the available capacity. The problem of either fully available or unavailable supply can be related to both of these categories. More specifically, Henig and Gerchak (1990) analyse the random yield case where the creation of good products is a Bernoulli process. Hence, the number of good products depends on the order size and the probability of generating a good product from one unit of output, and follows a binomial distribution. Ciarallo et al. (1994) analyse an inventory model with a stochastic limited supply. In this case, the actual order realisation is the minimum of the initial order given and the realised random supply capacity. They show that the optimal policy continues to be a base stock policy as in the case of unlimited supply. The optimal base stock level must be increased to account for possible capacity shortfalls in future periods. This work is extended by Jakšič et al. (2008) where the notion of *advance capacity information* is introduced and the value of ACI in the presence of non-stationary stochastic demand and limited supply is assessed. The optimal ordering policy in this case is a state-dependent base stock policy where the base stock level is a function of ACI. Wang and Gerchak (1990) analyse both uncertainty effects by extending the uncertain capacity setting by also incorporating the effect of yield uncertainty. Work closely related to the topic of this paper is the research presented by Güllü et al. (1997, 1999). In a deterministic demand setting, they show that the optimality of the base stock policy also applies in the case of a Bernoulli-type supply process. They obtain a newsboy-like formula to characterise the optimal base stock levels. We extend this work by imposing no restrictions on the characteristics of demand process characteristics, modelling demand as a non-stationary stochastic process and, in addition, we are primarily interested in the effect of ACI on the optimal performance of the inventory system. Finally, we refer the reader to conceptually similar work of Tan et al. (2007), where they tackle the demand side of the supply chain rather than supply side. They analyse the structure of the optimal policy for the case with imperfect advance demand information and the demand process is modelled as a Bernoulli process.

The remainder of the paper is organised as follows. We present the detailed model formulation and the structural characteristics of the optimal policy in Section 2. In Section 3, we present the results of a numerical study to assess the value of ACI and provide relevant managerial insights. Finally, we summarise our findings and suggest possible extensions in Section 4.

2. MODEL AND OPTIMAL POLICY PRESENTATION

In this section, we give the notation and describe the model. In addition, we derive the structure of the optimal policy and characterise the optimal base stock levels. We model supply uncertainty as a Bernoulli process, where p_t, $0 \le p_t \le 1$ denotes the probability of full capacity availability in period t. We introduce the parameter of supply capacity availability a_t, where $a_t = 0$ denotes the zero availability case and $a_t = 1$ the full availability case. In period t, the manager obtains ACI a_{t+n} on the supply capacity availability in period $t+n$, where parameter n denotes the length of the ACI horizon. Thus, in period t the supply capacity availability for n future periods is known and we record it in the ACI vector $\vec{a}_t = (a_{t+1}, a_{t+2}, \ldots, a_{t+n})$. Note that the capacity availabilities in periods $t+n+1$ and later are still uncertain.

Presuming that unmet demand is fully backlogged, the goal is to find an optimal policy that would minimise the inventory holding costs and backorder costs over a finite planning horizon T. The demand is generally modelled to be stochastic non-stationary with known distributions in each time period, while still being independent from period to period. For the simplicity of presentation a case of zero supply lead time was chosen; however, the model could easily be extended to consider positive supply lead times. The major notation is summarised in Table 1 and some extra notation is introduced as needed.

TABLE 1: *Summary of the notation.*

T	:	number of periods in the planning horizon
n	:	length of the advance capacity information horizon, $n \ge 0$
h	:	inventory holding cost per unit per period
b	:	backorder cost per unit per period
x_t	:	inventory position at time t before ordering
y_t	:	inventory position at time t after ordering
$\hat{x}_t$	:	net inventory at time t
z_t	:	order size at time t
a_t	:	parameter of the supply capacity availability, denoting either full or zero supply capacity availability in period t
p_t	:	probability of full supply capacity availability in period t
d_t	:	actual demand as the realisation of random demand in period t

We assume the following sequence of events. (1) At the start of period t, the manager reviews the inventory position before ordering x_t and ACI a_{t+n} on the supply capacity availability in period $t+n$ is received, which could potentially limit the order z_{t+n} that will be given in period $t+n$. (2) In the case of $a_t = 1$, the ordering decision z_t is made and correspondingly the inventory position is raised to the inventory position after ordering y_t, $y_t = x_t + z_t$. (3) The order placed at the start of the period is received. (4) At the end of the period, demand d_t is observed and satisfied through on-hand inventory; otherwise it is backordered. Inventory holding and backorder costs are incurred based on the end-of-period net inventory.

To follow the evolution of the inventory system through time we need to keep track of starting inventory position x_t, current supply capacity availability a_t, and the vector of ACI $\vec{a}_t$. The state space can therefore be described by an $n+2$-dimensional vector $(x_t, a_t, \vec{a}_t)$ and becomes updated in period $t+1$ in the following manner:

$$\begin{aligned} x_{t+1} &= x_t + z_t - d_t, \\ \vec{a}_{t+1} &= (a_{t+2}, a_{t+3}, \ldots, a_{t+n+1}). \end{aligned} \tag{1}$$

Finally, we can derive the minimal discounted expected cost function $f_t(x_t, a_t, \vec{a}_t)$ (also later referred to as the optimal cost function) that optimises the cost over finite planning horizon T from time t onward, starting in the initial state $(x_t, a_t, \vec{a}_t)$ and which is given in equation .

$$f_t(x_t, a_t, \vec{a}_t) = \begin{cases} \min\limits_{y_t \geq x_t} \{C_t(y_t) + \alpha E_{d_t} f_{t+1}(y_t - d_t, a_{t+1}, \vec{a}_{t+1})\}, & \text{if } T-n \leq t \leq T, \\ \min\limits_{y_t \geq x_t} \{C_t(y_t) + \alpha E_{d_t, a_{t+n+1}} f_{t+1}(y_t - d_t, a_{t+1}, \vec{a}_{t+1})\}, & \text{if } 1 \leq t \leq T-n-1, \end{cases} \tag{2}$$

where α is a discount factor, $C_t(y_t) = h\max(0, y_t - d_t) + b\max(0, d_t - y_t)$ represents the single period cost function, and the ending condition is defined as $f_{T+1}(\cdot) \equiv 0$.

The single period cost function $C_t(y_t)$ gives the total inventory holding plus the backorder cost in each period, where the inventory holding costs are incurred if the inventory position after ordering y_t is higher than subsequent realisation of demand d_t. The opposite is true for backorder costs as they are incurred when the demand exceeds the available inventory. As we are interested in minimizing the costs over a finite planning horizon, from period *t* up to period *T*, we do not take into account the costs that would occur in subsequent periods beyond period *T*. We denote this by writing the ending condition $f_{T+1}(\cdot) \equiv 0$

The solution to this dynamic programming formulation minimises the cost of managing the system for a finite horizon problem with *T-t* periods remaining until termination. Cost function f_t is a function of the inventory position before ordering and all the supply capacity information available before ordering, that is, current supply capacity realisation ACI on future supply capacity realisations. The optimal y_t is determined or, equivalently, the ordering decision z_t is made by minimizing the sum of period *t* single period cost and the discounted expected cost of period *t+1* onward, where the order can only be filled if supply capacity is available in the current period. Observe that going backward from period *T* we start to build up the vector of ACI $\vec{a}_t$ by adding a new component to the vector in each period, up to period *T–n*, where the vector has all *n* components. This means that the decision-maker can now optimise his ordering decision based on the full extent of ACI available in period *T–n*. Going back another period to *T–n–1,* we eliminate the last component a_{t+n+1} of the ACI vector, by taking the expectation over all possible realisations of the available supply capacity (full or zero capacity) in period *t+n+1*, while also accounting for all possible realisations of the demand in period *t* (the expectation is denoted as $E_{d_t, a_{t+n+1}}$). Due to the limited length of the ACI horizon, we are now faced with the full extent of the state space over which the minimisation is made. In this manner we proceed backwards in time until we end in the starting period. In other words, the optimisation is made for all possible combinations of y_t and all possible supply capacity realisations given by the ACI vector, where y_t values are restricted by the supply capacity availability. Thus, the optimal y_t is determined for each of these combinations by minimizing the sum of a single period cost function $C_t(y_t)$ and the cost of managing the system in the remaining periods *t+1* to *T*, given by f_{t+1}.

We proceed with the characterisation of the optimal solution given by the dynamic programming formulation in equation . The characterisation of the optimal policy is based on establishing the convexity of the optimal cost function f_t. Note that the single period cost function $C_t(y)$ is convex in y since it is the usual Newsboy cost function (Porteus, 2002). Based on the convexity results proven in the Appendix, we show the structure of the optimal policy in the following theorem.

Theorem 1. Let $\hat{y}_t(\vec{a}_t)$ be the smallest minimiser of function $g_t(y_t, \vec{a}_t)$. For any $\vec{a}_t$, the following holds for all t:

1. *The optimal ordering policy under ACI is a state-dependent base stock policy with the optimal base stock level $\hat{y}_t(\vec{a}_t)$.*
2. *Under the optimal policy, the inventory position after ordering $y_t(\vec{a}_t)$, in the case of supply capacity availability $a_t = 1$, is given by:*

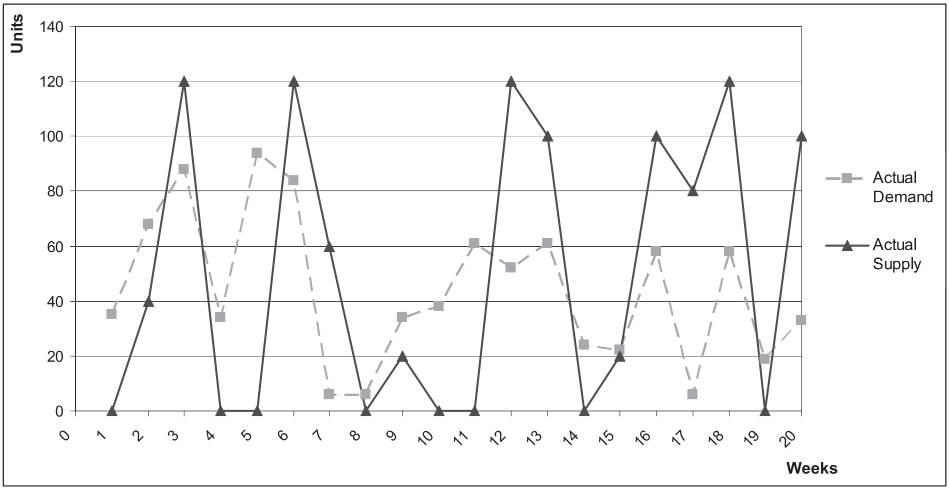
$$y_t(\vec{a}_t) = \begin{cases} \hat{y}_t(\vec{a}_t), & x_t \le \hat{y}_t(\vec{a}_t), \\ x_t, & x_t > \hat{y}_t(\vec{a}_t). \end{cases} \tag{3}$$

The base stock policy obtained is characterised by a single base stock level $\hat{y}_t(\vec{a}_t)$, which determines the optimal level of the inventory position after ordering. The base stock level $\hat{y}_t(\vec{a}_t)$ is a function of future supply availability given by the vector of ACI $\vec{a}_t$. The optimal inventory policy instructs the manager to raise the inventory position up to the base stock level in the case where the inventory position before ordering is below the base stock level. However, if the inventory position exceeds the base stock level the order should not be placed.

3. NUMERICAL EXAMPLE

In this section we present the results of the numerical analysis which was carried out to quantify the value of ACI and to gain insights into how the value of ACI changes with a change in the relevant system parameters. Numerical calculations were done by solving the dynamic programming formulation given in equation . In Figure 1 we present an example of an end-customer demand and supply pattern faced by a retailer. Weekly demand roughly varies between 0 and 100, with an average of 45 and a coefficient of variation of 0.6. The actual supply process is highly irregular with random periods of zero supply and a probability of full supply availability of *p*=0.6. A gross packaging size from a manufacturer to the retailer contains 20 units.

FIGURE 1: *Example of end-customer demand and the retailer's supply pattern.*



Based on this setting we selected the following set of input parameters: $T = 20$, $n = (0, 1, 2, 3, 4, 5)$, the probability of supply capacity availability $p = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1)$ is assumed to be constant over the whole planning horizon, the cost structure $h = 1$, $b = (5, 20, 100)$, discount factor $\alpha = 0.99$, uniformly distributed demand with the expected value of $E(d) = 5$ and coefficient of variation $CV_d = (0, 0.3, 0.6)$. The chosen parameter values do not correspond directly to monetary units (the cost structure above), although they are chosen in line with what researchers have observed in practice: backorder costs are prevalent over inventory holding costs, and the demand distribution with *CV*s, which are commonly observed in practice. The results are presented in Figures 2-3 and Tables 1-3.

To determine the value of ACI, we undertake a performance comparison between the no information case, $n = 0$, and the case where ACI is given for a certain number of future periods, $n > 0$. We define the *relative value* of ACI for $n > 0$ as:

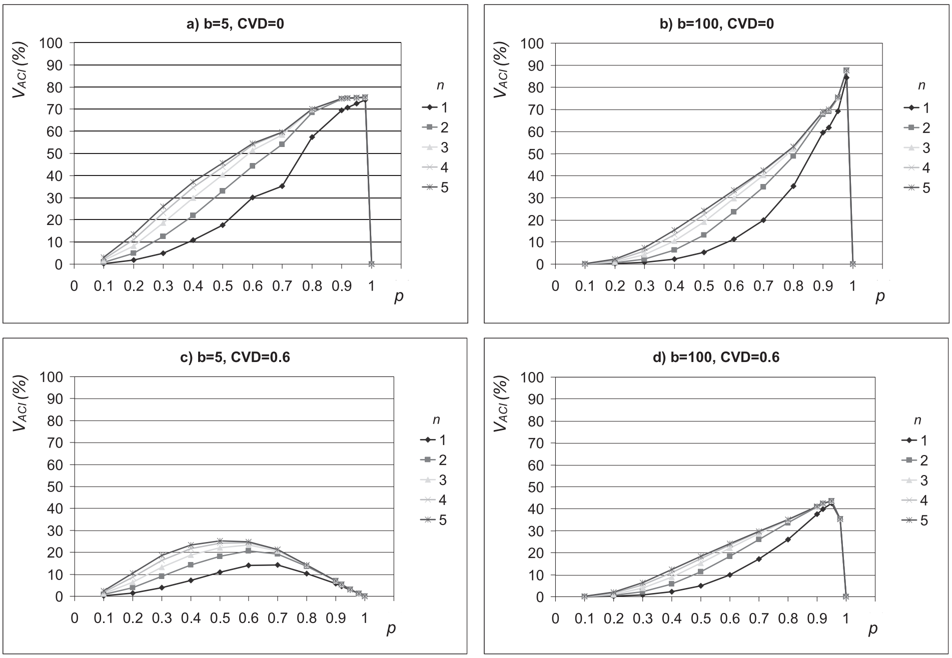
$$\%V_{ACI}(n>0) = \frac{f_t^{(n=0)} - f_t^{(n>0)}}{f_t^{(n=0)}}. \tag{4}$$

We also define the *absolute change* in the value of ACI with which we measure the extra benefit gained by extending the length of the ACI horizon by one time period, from n to $n+1$:

$$\Delta V_{ACI}(n+1) = f_t^{(n)} - f_t^{(n+1)}. \tag{5}$$

Let us first observe the effect of changing the system parameters on the total cost. Obviously, decreasing the extent of supply capacity availability by decreasing the value of p will increase the costs. Due to the increased probability of multiple consecutive periods of zero capacity, the likelihood of backorders occurring will rise dramatically and the costs will grow particularly in the case of a high b/h cost ratio. In addition, costs rise when one has to deal with the effect of growing demand uncertainty. The demand uncertainty causes deviations in the actual inventory levels from the desired target levels set by the manager. This results in frequent mismatches between the demand and the available on-hand inventory and, consequently, high costs.

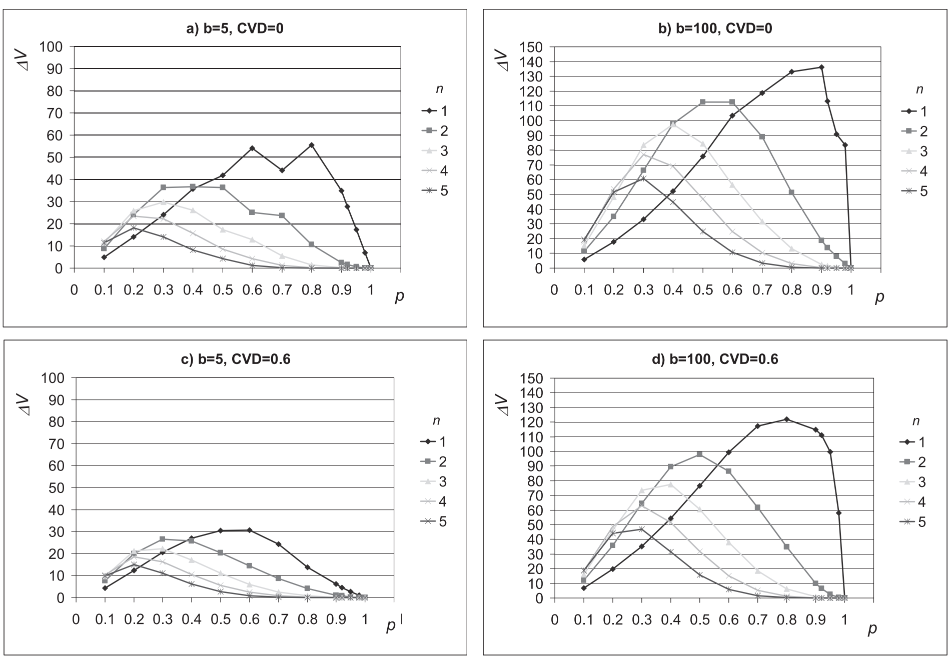
FIGURE 2: *Relative value of ACI*



These costs can be effectively reduced when ACI is available. Extending the ACI horizon obviously also increases the extent of the cost savings. However, the marginal gain of increasing n by 1 period varies substantially depending on the particular setting. When we consider the case of low supply capacity unavailability (p close to 1), we observe a surprisingly high relative decrease in costs measured through $\%V_{ACI}$. This can be attributed to the fact that ACI enables us to anticipate and prepare for the rare periods of complete capacity unavailability. Thus, we can avoid backorders and at the same time lower the inventory levels that we would otherwise need to compensate for the event of multiple successive periods with zero capacity. Especially in the case of low demand uncertainty, and also a high b/h ratio, $\%V_{ACI}$ can reach levels above 80%, even close to 90% (Figures 1a and 1b). When the manager wants to gain the most from anticipating future supply capacity unavailability, it would be helpful if no additional uncertainties were present that would prevent it meeting the desired target inventory level. We also observe that these high relative cost savings are already gained with a short ACI horizon. Extending n above 1 only leads to small further cost reductions. This is an important insight regarding the practical use of ACI, when the majority of gains are already possible with limited future visibility it is more likely that the manager will be able to obtain ACI (possibly also more accurate information) from his supplier. While the short ACI horizon is sufficient in the case of p being close to 1, we see that a longer ACI horizon is needed in a setting with high supply capacity unavailability. Observe that for low p values the relative marginal savings are actually increasing. When multiple periods of zero capacity can occur one after the other it is particularly important to anticipate the extent of future capacity unavailability. In such a setting, it is very important if one can have an additional period of future visibility. Several researchers who have studied a conceptually similar problem of sharing advance demand information suggest that prolonging the information horizon has diminishing returns (Özer and Wei, 2004). Although we consider a special case of zero or full supply availability in this paper, this result actually shows that this does not hold in general.

While we have observed a large relative decrease in costs in some settings, it may be more important for a particular company to determine the potential decrease in absolute cost figures. Intuitively, we would expect that the biggest absolute gains would occur in a setting where the uncertainty of supply is high, and the possible shortage anticipation through ACI would be the most beneficial. We confirm this in Tables 1-3 (the shaded areas under Average change) where we see that the biggest absolute savings are attained for an availability probability of between 0.2 and 0.4. Here, the cost decrease is bigger for a higher n (Figure 2). In fact, the lower the availability the more we gain by prolonging the ACI horizon. In the case of extremely low levels of capacity availability the system becomes too hard to manage due to an extremely long inventory pre-build phase, and the gains from using ACI are limited.

FIGURE 3: *Absolute change in the value of ACI*



The effect of demand uncertainty on an absolute decrease in costs is not as obvious as it was in the relative case. This can be attributed to the interaction of two factors. While stronger demand uncertainty intensifies the difficulties of managing inventories as described above, it also contributes to higher costs and thus provides more potential for savings through ACI.

5. CONCLUSIONS

In this paper, we study a periodic review inventory model in the presence of stochastic demand and limited supply availability. The supply capacity is modelled as a Bernoulli processes, meaning that there are randomly interchanging periods of complete capacity unavailability and full availability. We upgrade the base case with no information on future supply capacity availability by considering the possibility that a supply chain member can obtain advance capacity information ('ACI') from its upstream partner. We develop an optimal policy and show that it is a base stock policy with a state-dependent base stock level. The optimum base stock level is determined by the currently available ACI where, in the case of information about unfavourable supply capacity conditions in future periods, the base stock level is raised sufficiently to avoid the probable stock-outs. By means of a numerical analysis, we quantify the benefit of ACI and determine the situations when obtaining ACI can be particularly important. While the relative cost savings are highest for the case of close to full availability due to the fact that one can completely avoid backorders with only a small extra inventory, the cost reduction in absolute terms is greater for cases with medium to low supply capacity availability. Further, we show that in most cases having only a little future visibility already offers considerable savings, although when one faces the possibility of consecutive periods of supply unavailability it can be very beneficial to extend the ACI horizon. In general, managers should recognise that the extent of savings shown clearly indicates that sharing ACI should be encouraged in supply chains with unstable supply conditions. In our experience, the current dynamic programming cost formulation is manageable in terms of the complexity of the calculations and can also be used for larger practical problems. However, a natural extension of this work would involve developing a simpler, preferably also optimal, inventory policy that would capture the effect of sharing ACI.

TABLE 2: *Value of ACI for b=5*

			p									
		n	**1**	**0.9**	**0.8**	**0.7**	**0.6**	**0.5**	**0.4**	**0.3**	**0.2**	**0.1**
CV_D=0	Cost	0	0.00	50.27	96.94	125.47	179.06	237.05	330.42	483.98	788.55	1605.02
		1	0.00	15.35	41.34	81.37	124.98	195.20	294.66	459.92	774.45	1600.15
		2	0.00	12.80	30.64	57.70	99.81	158.87	257.85	423.57	750.38	1591.42
		3	0.00	12.63	29.13	52.10	86.90	141.55	231.73	393.87	724.36	1580.07
		4	0.00	12.62	28.94	50.95	82.67	132.92	216.10	371.69	700.84	1568.04
		5	0.00	12.62	28.93	50.75	81.53	128.60	208.02	357.55	682.65	1556.62
	Relative value (%)	1	0.00	69.47	57.35	35.15	30.20	17.65	10.82	4.97	1.79	0.30
		2	0.00	74.54	68.39	54.01	44.26	32.98	21.96	12.48	4.84	0.85
		3	0.00	74.87	69.95	58.47	51.47	40.29	29.87	18.62	8.14	1.55
		4	0.00	74.89	70.14	59.39	53.83	43.93	34.60	23.20	11.12	2.30
		5	0.00	74.89	70.16	59.55	54.47	45.75	37.04	26.12	13.43	3.02
	Absolute change	1	0.00	**34.93**	**55.59**	**44.10**	**54.08**	**41.85**	35.76	24.07	14.10	4.87
		2	0.00	2.55	10.71	23.67	25.17	36.34	**36.81**	**36.35**	24.07	8.73
		3	0.00	0.17	1.51	5.60	12.91	17.31	26.12	29.70	**26.01**	11.35
		4	0.00	0.01	0.19	1.15	4.23	8.64	15.63	22.18	23.52	**12.03**
		5	0.00	0.00	0.02	0.20	1.14	4.31	8.08	14.14	18.19	11.43
CV_D=0,3	Cost	0	36.42	66.32	106.95	143.63	188.38	250.56	341.67	493.68	797.66	1612.92
		1	36.42	51.05	72.54	103.78	148.55	212.64	310.01	470.80	784.39	1608.29
		2	36.42	49.49	66.80	91.44	128.42	185.71	278.23	439.62	762.32	1600.09
		3	36.42	49.34	65.69	87.86	120.28	171.24	257.05	413.51	738.37	1589.57
		4	36.42	49.32	65.47	86.80	117.10	164.06	243.97	394.21	717.10	1578.25
		5	36.42	49.32	65.43	86.50	115.89	160.60	236.26	380.68	699.76	1567.28
	Relative value (%)	1	0.00	23.02	32.17	27.74	21.14	15.13	9.27	4.64	1.66	0.29
		2	0.00	25.38	37.54	36.34	31.83	25.88	18.57	10.95	4.43	0.80
		3	0.00	25.61	38.58	38.83	36.15	31.66	24.77	16.24	7.43	1.45
		4	0.00	25.64	38.78	39.56	37.84	34.52	28.60	20.15	10.10	2.15
		5	0.00	25.64	38.82	39.77	38.48	35.90	30.85	22.89	12.27	2.83
	Absolute change	1	0.00	**15.27**	**34.41**	**39.85**	**39.83**	**37.92**	31.66	22.89	13.27	4.64
		2	0.00	1.56	5.74	12.34	20.13	26.93	**31.78**	**31.18**	22.07	8.20
		3	0.00	0.16	1.11	3.58	8.14	14.47	21.18	26.11	**23.95**	10.52
		4	0.00	0.01	0.22	1.05	3.18	7.18	13.08	19.30	21.27	**11.32**
		5	0.00	0.00	0.04	0.30	1.21	3.45	7.71	13.53	17.34	10.97
CV_D=0,6	Cost	0	82.77	103.57	131.77	170.95	218.26	278.82	369.59	521.15	824.76	1638.21
		1	82.77	97.48	118.05	146.71	187.68	248.42	342.67	500.58	812.40	1633.80
		2	82.77	96.36	113.87	138.02	173.37	227.97	316.89	474.04	792.54	1626.11
		3	82.77	96.27	113.09	135.46	167.47	216.94	299.75	451.86	771.42	1616.46
		4	82.77	96.25	112.93	134.67	165.06	211.39	289.30	435.63	752.82	1606.21
		5	82.77	96.25	112.90	134.45	164.14	208.71	283.18	424.42	737.78	1596.43
	Relative value (%)	1	0.00	5.89	10.41	14.18	14.01	10.90	7.28	3.95	1.50	0.27
		2	0.00	6.96	13.58	19.27	20.57	18.24	14.26	9.04	3.91	0.74
		3	0.00	7.05	14.17	20.76	23.27	22.19	18.90	13.30	6.47	1.33
		4	0.00	7.07	14.29	21.22	24.37	24.18	21.73	16.41	8.72	1.95
		5	0.00	7.07	14.32	21.35	24.80	25.14	23.38	18.56	10.55	2.55
	Absolute change	1	0.00	**6.10**	**13.72**	**24.24**	**30.59**	**30.40**	**26.92**	20.57	12.35	4.41
		2	0.00	1.11	4.17	8.70	14.30	20.45	25.78	**26.54**	19.86	7.69
		3	0.00	0.10	0.78	2.56	5.90	11.04	17.14	22.18	**21.12**	9.66
		4	0.00	0.01	0.16	0.79	2.41	5.55	10.45	16.23	18.60	**10.25**
		5	0.00	0.00	0.03	0.22	0.92	2.67	6.11	11.21	15.05	9.78

TABLE 3: *Value of ACI for b=20*

			p									
		n	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
CV_D=0	Cost	0	0.00	108.32	179.28	252.26	358.94	508.71	751.09	1202.76	2258.01	5569.65
		1	0.00	36.62	86.10	169.13	277.04	442.65	704.27	1171.85	2241.12	5563.93
		2	0.00	23.29	60.49	119.59	215.45	371.59	630.89	1115.29	2208.71	5553.19
		3	0.00	21.99	54.15	103.61	186.30	324.44	572.82	1055.23	2166.73	5538.60
		4	0.00	21.87	52.65	98.54	172.62	299.17	532.90	1005.40	2123.84	5521.71
		5	0.00	21.86	52.32	96.96	167.01	286.37	508.52	968.87	2085.40	5504.13
	Relative value (%)	1	0.00	66.20	51.97	32.96	22.82	12.98	6.23	2.57	0.75	0.10
		2	0.00	78.50	66.26	52.59	39.98	26.95	16.00	7.27	2.18	0.30
		3	0.00	79.70	69.80	58.93	48.10	36.22	23.73	12.27	4.04	0.56
		4	0.00	79.81	70.63	60.94	51.91	41.19	29.05	16.41	5.94	0.86
		5	0.00	79.82	70.82	61.56	53.47	43.71	32.29	19.45	7.64	1.18
	Absolute change	1	0.00	**71.70**	**93.18**	**83.13**	**81.90**	66.06	46.82	30.91	16.90	5.72
		2	0.00	13.33	25.61	49.54	61.59	**71.07**	**73.38**	56.56	32.40	10.73
		3	0.00	1.30	6.35	15.98	29.15	47.15	58.07	**60.06**	41.99	14.59
		4	0.00	0.12	1.50	5.06	13.68	25.27	39.92	49.83	**42.89**	16.90
		5	0.00	0.01	0.33	1.58	5.61	12.80	24.37	36.53	38.43	**17.57**
CV_D=0,3	Cost	0	36.42	135.85	197.09	275.72	378.34	527.83	778.85	1222.38	2279.71	5593.61
		1	36.42	68.81	120.85	195.27	304.22	466.15	733.12	1192.13	2263.06	5587.95
		2	36.42	63.45	102.20	161.33	254.05	403.82	669.31	1139.93	2232.48	5577.69
		3	36.42	62.96	98.60	150.50	232.04	367.95	621.02	1086.73	2193.87	5564.05
		4	36.42	62.91	97.90	147.27	222.98	349.05	590.23	1044.31	2155.37	5548.48
		5	36.42	62.91	97.77	146.31	219.35	339.39	571.58	1013.44	2121.60	5532.50
	Relative value (%)	1	0.00	49.35	38.68	29.18	19.59	11.69	5.87	2.47	0.73	0.10
		2	0.00	53.29	48.15	41.49	32.85	23.49	14.06	6.75	2.07	0.28
		3	0.00	53.66	49.97	45.41	38.67	30.29	20.26	11.10	3.77	0.53
		4	0.00	53.69	50.33	46.59	41.06	33.87	24.22	14.57	5.45	0.81
		5	0.00	53.69	50.39	46.93	42.02	35.70	26.61	17.09	6.94	1.09
	Absolute change	1	0.00	**67.04**	**76.24**	**80.45**	**74.12**	61.68	45.73	30.25	16.65	5.66
		2	0.00	5.36	18.65	33.93	50.17	**62.33**	**63.81**	52.20	30.58	10.25
		3	0.00	0.50	3.60	10.83	22.01	35.87	48.29	**53.20**	**38.61**	13.64
		4	0.00	0.04	0.70	3.23	9.06	18.90	30.79	42.42	38.50	15.57
		5	0.00	0.00	0.13	0.96	3.63	9.65	18.66	30.87	33.77	**15.98**
CV_D=0,6	Cost	0	91.05	172.61	244.32	319.62	423.43	574.03	817.42	1272.77	2335.62	5652.68
		1	91.05	125.66	176.03	249.67	356.90	516.59	773.42	1243.09	2319.01	5646.91
		2	91.05	122.83	164.82	225.10	317.03	464.83	717.15	1195.14	2289.68	5636.73
		3	91.05	122.53	162.49	217.56	300.44	435.73	675.66	1148.12	2253.91	5623.40
		4	91.05	122.50	162.04	215.35	293.76	420.89	649.29	1111.34	2219.02	5608.53
		5	91.05	122.50	161.96	214.72	291.19	413.61	633.51	1085.03	2189.11	5593.55
	Relative value (%)	1	0.00	27.20	27.95	21.88	15.71	10.01	5.38	2.33	0.71	0.10
		2	0.00	28.84	32.54	29.57	25.13	19.02	12.27	6.10	1.97	0.28
		3	0.00	29.02	33.49	31.93	29.04	24.09	17.34	9.79	3.50	0.52
		4	0.00	29.03	33.68	32.62	30.62	26.68	20.57	12.68	4.99	0.78
		5	0.00	29.04	33.71	32.82	31.23	27.95	22.50	14.75	6.27	1.05
	Absolute change	1	0.00	**46.95**	**68.29**	**69.95**	**66.53**	**57.44**	44.01	29.68	16.61	5.77
		2	0.00	2.83	11.21	24.57	39.87	51.77	**56.27**	**47.95**	29.33	10.19
		3	0.00	0.31	2.33	7.53	16.58	29.10	41.49	47.02	**35.77**	13.33
		4	0.00	0.03	0.45	2.21	6.69	14.84	26.37	36.78	34.88	14.87
		5	0.00	0.00	0.09	0.63	2.56	7.27	15.78	26.31	29.91	**14.99**

TABLE 4: *Value of ACI for b=100*

			p									
		n	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
CV_D=0	Cost	0	0.00	228.71	378.08	593.46	917.66	1428.07	2357.22	4301.25	9380.07	26309.55
		1	0.00	92.58	244.86	474.87	814.32	1352.24	2305.11	4268.18	9362.27	26303.63
		2	0.00	73.78	193.67	386.03	701.78	1239.56	2207.25	4201.86	9327.24	26292.43
		3	0.00	71.44	180.58	354.33	645.25	1155.18	2109.67	4118.46	9278.93	26277.00
		4	0.00	71.17	177.43	343.94	620.31	1108.20	2040.57	4041.37	9225.11	26258.77
		5	0.00	71.14	176.79	340.62	609.62	1083.50	1995.91	3980.49	9173.66	26239.36
	Relative value (%)	1	0.00	59.52	35.24	19.98	11.26	5.31	2.21	0.77	0.19	0.02
		2	0.00	67.74	48.77	34.95	23.52	13.20	6.36	2.31	0.56	0.07
		3	0.00	68.76	52.24	40.29	29.69	19.11	10.50	4.25	1.08	0.12
		4	0.00	68.88	53.07	42.04	32.40	22.40	13.43	6.04	1.65	0.19
		5	0.00	68.90	53.24	42.61	33.57	24.13	15.33	7.46	2.20	0.27
	Absolute change	1	0.00	**136.14**	**133.22**	**118.59**	103.34	75.83	52.11	33.07	17.79	5.93
		2	0.00	18.80	51.19	88.84	**112.54**	**112.68**	**97.86**	66.33	35.04	11.20
		3	0.00	2.33	13.09	31.70	56.53	84.38	97.58	**83.40**	48.31	15.43
		4	0.00	0.27	3.15	10.39	24.93	46.98	69.10	77.08	**53.81**	18.23
		5	0.00	0.03	0.64	3.33	10.70	24.71	44.66	60.88	51.46	**19.40**
CV_D=0,3	Cost	0	36.42	242.76	405.88	623.04	943.58	1461.26	2392.68	4343.79	9435.50	26375.73
		1	36.42	131.39	279.88	503.45	843.04	1385.07	2339.59	4309.98	9417.12	26369.44
		2	36.42	116.14	238.30	431.75	746.22	1278.49	2245.67	4244.88	9382.14	26357.98
		3	36.42	114.73	229.71	408.36	700.99	1208.93	2159.98	4167.18	9335.43	26342.61
		4	36.42	114.59	227.95	400.95	681.75	1171.12	2100.90	4098.11	9285.20	26324.81
		5	36.42	114.58	227.61	398.74	673.90	1151.68	2063.92	4045.61	9238.70	26306.24
	Relative value (%)	1	0.00	45.88	31.04	19.20	10.66	5.21	2.22	0.78	0.19	0.02
		2	0.00	52.16	41.29	30.70	20.92	12.51	6.14	2.28	0.57	0.07
		3	0.00	52.74	43.40	34.46	25.71	17.27	9.73	4.07	1.06	0.13
		4	0.00	52.80	43.84	35.65	27.75	19.86	12.20	5.66	1.59	0.19
		5	0.00	52.80	43.92	36.00	28.58	21.19	13.74	6.86	2.09	0.26
	Absolute change	1	0.00	**111.37**	**126.00**	**119.60**	**100.54**	76.19	53.09	33.82	18.38	6.28
		2	0.00	15.25	41.58	71.70	96.82	**106.58**	**93.92**	65.10	34.98	11.47
		3	0.00	1.41	8.58	23.39	45.24	69.55	85.70	**77.70**	46.71	15.36
		4	0.00	0.13	1.77	7.40	19.24	37.81	59.08	69.07	**50.23**	17.80
		5	0.00	0.01	0.34	2.22	7.85	19.44	36.98	52.51	46.49	**18.57**
CV_D=0,6	Cost	0	91.05	305.27	466.78	687.14	1010.66	1532.47	2470.47	4432.75	9543.15	26482.59
		1	91.05	190.60	344.85	569.93	911.29	1455.87	2416.16	4397.50	9523.54	26475.69
		2	91.05	180.61	310.01	508.33	825.00	1357.91	2326.60	4332.98	9487.67	26463.49
		3	91.05	179.79	303.93	489.69	787.16	1297.26	2249.08	4259.50	9441.23	26447.36
		4	91.05	179.71	302.77	484.30	771.89	1265.60	2197.44	4196.44	9392.55	26428.97
		5	91.05	179.70	302.56	482.77	765.97	1249.83	2165.92	4149.50	9348.54	26410.08
	Relative value (%)	1	0.00	37.56	26.12	17.06	9.83	5.00	2.20	0.80	0.21	0.03
		2	0.00	40.84	33.59	26.02	18.37	11.39	5.82	2.25	0.58	0.07
		3	0.00	41.10	34.89	28.73	22.11	15.35	8.96	3.91	1.07	0.13
		4	0.00	41.13	35.14	29.52	23.62	17.41	11.05	5.33	1.58	0.20
		5	0.00	41.13	35.18	29.74	24.21	18.44	12.33	6.39	2.04	0.27
	Absolute change	1	0.00	**114.66**	**121.94**	**117.21**	**99.37**	76.60	54.31	35.26	19.61	6.90
		2	0.00	10.00	34.84	61.60	86.29	**97.96**	**89.55**	64.52	35.87	12.19
		3	0.00	0.81	6.08	18.64	37.84	60.65	77.52	**73.48**	46.44	16.13
		4	0.00	0.08	1.15	5.39	15.26	31.65	51.64	63.06	**48.69**	18.39
		5	0.00	0.01	0.21	1.53	5.93	15.77	31.52	46.93	44.00	**18.88**

Appendix

In the Appendix we show and prove the necessary convexity results that lead to the results presented in Theorem 1. Let g_t denote the cost-to-go function of period t, defined as

$$g_t\left(y_t,\vec{a}_t\right)=\begin{cases} C_t(y_t)+\alpha E_{d_t} f_{t+1}(y_t-d_t\,,a_{t+1},\vec{a}_{t+1}), & \text{if } T-n\le t\le T, \\ C_t(y_t)+\alpha E_{d_t,a_{t+n+1}} f_{t+1}(y_t-d_t\,,a_{t+1},\vec{a}_{t+1}), & \text{if } 1\le t\le T-n-1, \end{cases} \tag{6}$$

and we rewrite the minimal discounted expected cost function $f_t\left(x_t,a_t,\vec{a}_t\right)$ as

$$f_t\left(x_t,a_t,\vec{a}_t\right)=\min_{y_t\ge x_t} g_t\left(y_t,\vec{a}_t\right),\quad \text{if } 1\le t\le T. \tag{7}$$

We first show the essential convexity results that will allow us to establish the optimal policy.

Lemma 1: For any arbitrary value of information horizon n and value of the ACI vector $\vec{a}_t$, the following holds for all t:

1. $g_t\left(y,\vec{a}\right)$ is convex in y,
2. $f_t\left(x,\vec{a}\right)$ is convex in x.

Proof : The proof starts by backward induction in time period T.
$t=T$: From equation. , we have $g_T(y_T)=C_T(y_T)$ by taking $f_{T+1}(\cdot)\equiv 0$ into account. Since the reassigned single-period cost function $C_T(y_T)$ is assumed to be convex, function $g_T(y_T)$ is also convex. For $f_T(x_T,a_T)=\min_{y_T\ge x_T} g_T(y_T)$ we apply Lemma 1 and show that function $f_T(x_T,a_T)$ is convex.
$t=T-1$: $f_{T-1}(x_{T-1},a_{T-1},\vec{a}_{T-1})=\min_{y_{T-1}\ge x_{T-1}}\{C_{T-1}(y_{T-1})+\alpha E_{d_{T-1}} f_T(y_{T-1}-d_{T-1},a_T)$. We have shown that $f_T(x_T,a_T)$ is convex, thus using an affine mapping property (Hiriart-Urruty and Lamaréchal, 1996) we show that function $\tilde{f}_T(y_{T-1},d_{T-1},a_T):=f_T(y_{T-1}-d_{T-1},a_T)$ is also convex (the update of the inventory position is linear; thus a linear translation with $b=d_{T-1}$). $\alpha E_{d_{T-1}} f_T$ is convex since expectation preserves convexity (Heyman and Sobel, 1984) and by adding cost function $C_{T-1}(y_{T-1})$ and using a weighted sum property (Hiriart-Urruty and Lamaréchal, 1996) we show that $g_{T-1}(y_{T-1},a_T)$ is convex. g_{T-1} is then minimised and through Lemma 1 we conclude that $f_{T-1}(x_{T-1},a_{T-1},\vec{a}_{T-1})$ is also convex.
$t=T-2,\ldots,1$: The proof follows the same line as the previous step using backward induction on t, and thus proving the convexity of $f_t(x_t,a_t,\vec{a}_t)$. □
Lemma 2: If $g(y,e)$ is convex then $f(x,c,e)=\min_{x\le y\le x+c} g(y,e)$ is also convex for any $c\ge 0$.

Proof: Let $h(b,e):=\min_{Ay\le b} g(y,e)$ where $A=\left[1,1\right]$ and $b=\left[-x,x+c\right]$. By minimisation on the polyhedral property (Porteus, 2002, Mincsovics et al., 2009), we conclude that $h(b,e)$ is convex. Since $h(b,e)=f(x,c,e)$, f is also convex. □

Proof of Theorem 1: The convexity results of Theorem 1 directly imply the proposed structure of the optimal policy.

RECEIVED: APRIL 2009

REFERENCES

Bush, Chandler M. and Douglas W. Cooper (1988), "Inventory Level Decision Support", *Production and Inventory Management Journal* 29(1), 16–20.

Ciarallo, Frank W., Ramakrishna Akella and Thomas E. Morton (1994), "A Periodic Review, Production Planning Model with Uncertain Capacity and Uncertain Demand – Optimality of Extended Myopic Policies", *Management Science* 40, 320–332.

Güllü, Refik, Ebru Önol and Nesim Erkip (1997), "Analysis of a Deterministic Demand Production/Inventory System Under Nonstationary Supply Uncertainty", *IIE Transactions* 29, 703–709.

Güllü, Refik, Ebru Önol and Nesim Erkip (1999), "Analysis of an Inventory System Under Supply Uncertainty", *Int. J. Production Economics* 59, 377–385.

Henig, Mordechai and Yigal Gerchak (1990), "The Structure of Periodic Review Policies in the Presence of Random Yield", *Operations Research* 38 (4), 634–654.

Heyman, Daniel P. and Matthew J. Sobel (1984), *Stochastic Models in Operations Research Vol. II*, New York: McGraw-Hill.

Hiriart-Urruty, Jean-Batiste and Claude Lemaréchal (1996), *Convex Analysis and Minimization Algorithms II*, Berlin: Springer-Verlag.

Jacobs, Robert F., Richard B. Chase and Nicholas J. Aquilano (2008), *Operations and Supply Management*, 12th Edition, New York: McGraw-Hill/Irwin.

Jakšič, Marko, Jan C. Fransoo, Tarkan Tan, Ton de Kok and Borut Rusjan (2008), "Inventory Management with Advance Capacity Information", Beta publicatie, wp 249, Beta Research School for Operations Management and Logistics, Eindhoven University of Technology, The Netherlands.

Metters, Richard (1997), "Production Planning with Stochastic Seasonal Demand and Capacitated Production", *IIE Transactions* 29, 1017–1029.

Mincsovics, Gergely, Tarkan Tan and Osman Alp (2009), "Integrated Capacity and Inventory Management with Capacity Acquisition Lead Times", *European Journal of Operational Research* 196, 949–958.

Monczka, Robert M., Robert B. Handfield, Larry C. Giunipero and James L. Patterson (2009), *Purchasing and Supply Chain Management*, 4th Edition, Mason: South-Western Cangage Learning.

Özer, Özalp and Wei Wei (2004), "Inventory Control with Limited Capacity and Advance Demand Information", *Operations Research* 52, 988–1000.

Porteus, Evan L. (2002), *Foundations of Stochastic Inventory Theory*, Stanford: Stanford University Press.

Schwarz, Leroy B. (2005), "The State of Practice in Supply-Chain Management: A Research Perspective", In *Applications of Supply Chain Management and E-Commerce Research*: Springer.

Scott, Bjork R. (2000), *Inventory Management Systems - Statistical Data Included*, Internal Auditor, Institute of Internal Auditors, Inc., US.

Tan, Tarkan, Refik Güllü and Nesim Erkip (2007), "Modelling Imperfect Advance Demand Information and Analysis of Optimal Inventory Policies", *European Journal of Operational Research* 177, 897–923.

Wang, Yunzeng and Yigal Gerchak (1996), "Periodic Review Production Models with Variable Capacity, Random Yield, and Uncertain Demand", *Management Science* 42, 130–137.