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THE APPLICATION OF NANOMECHANICS IN ENERGY TECHNOLOGIES

UPORABA NANOMEHANIKE V ENERGESKIH TEHNOLOGIJAH

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Keywords: nanomechanics, nanotechnology, nanomaterials, nanoenergetics

Abstract

Nanotechnologies may be very efficient for improving and optimizing energy technologies. The following will be shown in this paper: nanomaterials are very important in energy technology. Nanofluids and nanostructured materials may be applied in refrigerators, heaters, and similar equipment. In the micro- and nano-regions, the classic laws of mechanics are insufficiently accurate for the calculation of characteristics and properties. In this article, the application of microfluidics and vibration theory in the micro and nano-regions will be shown.

Povzetek

Nanotehnologije so lahko zelo učinkovite za izboljšanje in optimizacijo energetskih procesov. Razvoj nanomaterialov je izjemnega pomena za energetiko. Nanotekočine in nano nanostrukturirani materiali se lahko uporabljajo v hladilnih strojih, grelnikih.... V mikro in nano-področju zakoni klasične termomehanike v mnogih primerih niso dovolj natančni. V ta namen je tudi prikazana problematika v mikrofluidni tehniki in mehanskih nihanjih.

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1 INTRODUCTION

Along with the development of increasingly larger devices, many inventors and scientists have attempted to penetrate the secrets of the nanoworld. For centuries, watchmakers were miniaturizing mechanical devices. The discovery of the microscope in the 17th century enabled people to observe microbial, plant, and animal cells. Only in the late 20th century, however, were the technologies of microdevices developed. The size of transistors in integrated circuits is currently 0.18 micrometres, and laboratories are already developing transistors of 10 nanometres. One of the great scientific and technical advancements at the turn of the 21st century is the creation of nanomaterials and nanotechnology. The area that covers all significant problems from that field is called "mechanics" in the broadest sense, which is divided into:

- macromechanics 10^{-4}-10^{-5} m,
- mesomechanics 10^{-5}-10^{-7} m,
- micromechanics 10^{-7}-10^{-8} m,
- nanomechanics: 10^{-8}-10^{-9} m.

Since the atomic level (interatomic distance in a crystal lattice) has an order of one to several Å (10^{-10} m), the nanolevel is restricted to 10^{-9} m.

Nanotechnology currently allows the successful manufacture of micro-electro-mechanical systems (MEMS), nano-electro-mechanical systems (NEMS) and biological micro-electro-mechanical systems (BIO MEMS). Significant among these are microchips, micro sensors, micro mirrors biochips, and nanomaterials.

Micro-electro-mechanical systems are devices with a characteristic length ranging between 1 mm and 1 micrometre and consisting of electric and mechanical components. Microdevices, for example, are smaller than the diameter of a human hair. Nanodevices (NEMS) are devices of even smaller dimensions, containing nanocomponents or devices based on special materials called nanomaterials.

Nanomechanics and micromechanics are becoming even more important in modern industry. Ideas about extremely small aircraft, pumps and other technical devices have become reality today, *[1-9]*. At the same time, unforeseen problems have emerged. If a fluid flows through a pipe with the diameter of a few nanometres or micrometres, thermodynamic and transport properties of the fluid are modelled completely differently due to the high impact of the surface effects. Similarly, the equations of conventional hydromechanics do not apply in the entire area. In addition to temperature and pressure, the significance of the Knudsen number (Kn) is also growing. This number is the ratio of the mean free path length of the molecules to a characteristic length L, *[2-5].*

The Euler equation for the calculation of flow gives poor results in almost the entire range, from the Navier-Stokes equation at a Knudsen number of 0.1 to the Burnett equation at a Knudsen number of 10. To analyse the molecular free path in micro- and nano-channels, non-equilibrium mechanics and the original Boltzmann equation should be used to be able to calculate hydro-mechanical problems in the entire range of Knudsen numbers.

Nanofluids of exceptional properties are increasingly economically important. In addition to nanofluids, the application of nanopipes with even better properties is also increasing in engineering practice.

Nanotechnology will expand into many areas in the future. Digital data storage with much higher recording densities will be possible as well as analyses of individual cells in cases of serious illnesses as well as the manufacture of ultra-light materials with exceptional properties. The size of nanoparticles and nanowires is nearly the same as the size of biomolecules, such as DNA molecules and proteins. In the future, it will be possible to use nanoparticles in various areas of medicines. Consideration is being given to applying nanoparticles as a suitable probe for small DNA samples or proteins.

2 NANOMATERIALS

Nanomaterials are classified in the next divisions, *[1-3]*:

1) Carbon-based nanomaterials

2) Metallic nano-materials

2) Nanocomposites

3) Metals & alloys

4) Biological nanomaterials

5) Nano-polymers

6) Nano-glasses

7) Nano-ceramics

8) Natural nanoparticles

The term 'nanofluid' is envisioned as describing a solid-liquid mixture that consists of nanoparticles and a base liquid; this is one of the new challenges of nano-technology for thermo-sciences. The possible application area of nanofluids is in advanced cooling systems, and in micro/nano electromechanical systems. The investigation of the effective thermal conductivity of liquid with nanoparticles attract much more interest experimentally and theoretically. The effective thermal conductivity of nanoparticle suspension can be much higher than for the fluid without nanoparticles.

The calculation of properties for nanofluids for real substances is possible with the use of classical and statistical mechanics. Classical mechanics has no insight into the microstructure of the substance. Statistical mechanics, in contrast, calculates the properties of a state on the basis of the molecular motions in a space, and on the basis of the intermolecular interactions. The equations obtained by means of classical thermodynamics are empirical and apply only in the region under observation. The main drawback of classical thermodynamics is that it lacks the insight into the substance of microstructures. In contrast to classical mechanics, statistical mechanics calculates the thermomechanical properties of state on the basis of intermolecular and intramolecular interactions between particles in the same system of molecules. It deals with the systems composed of a vast number of particles. Figure 1 and Figure 2 shows the thermal conductivity and viscosity for nanofluids obtained by statistical thermomechanics.

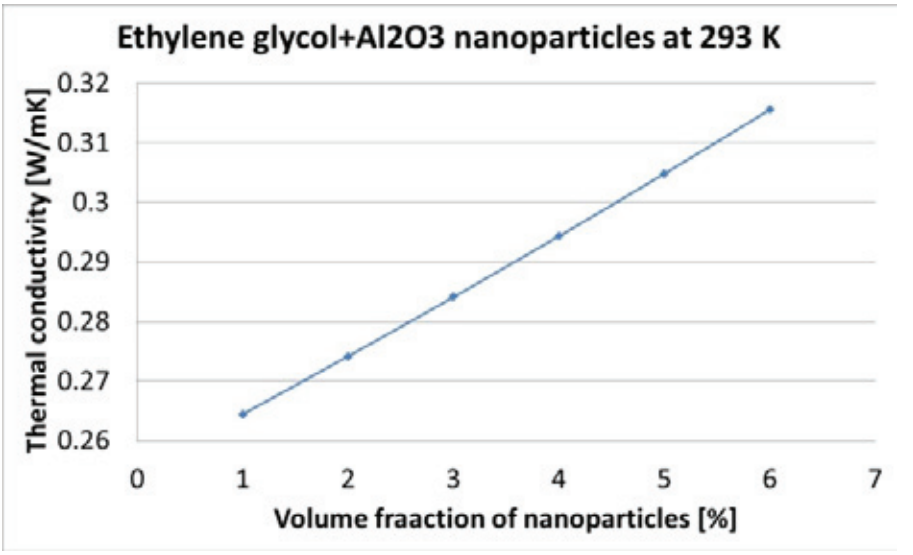


Figure 1: *Thermal conductivity of mixture between ethylene glycol and Al_2O_3 nanoparticles*

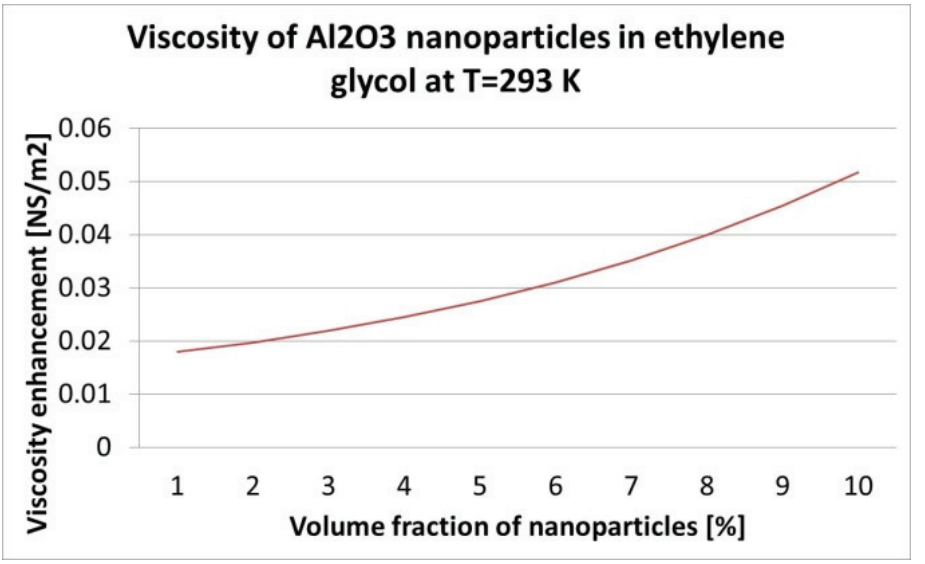


Figure 2: *Thermal conductivity of mixture between ethylene glycol and Al2O3 nanoparticles*

3 MICROFLUIDICS

Fluid flow in channels, minichannels, and microchannels is driven due to the presence of electric fields, magnetic fields or pressure-driven flows and some other effects, *[1]*.

Electrohydrodynamics (EHD), known as electrokinetics, is the theory of the flow of electrically charged fluids. It is the study of the motions of ionised particles or molecules and their interactions with electric fields and the surrounding fluid. For EHD flow, low electrical conducting fluids, such as organic fluids or alcohols, are required, *[2-4].* The electrokinetic flow can be classified into the following types: electrophoresis, electroosmosis, streaming potential and sedimental potential, *[2-5]*.

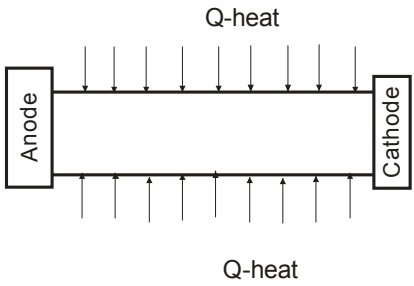
The fundamental concept for magnetohydrodynamics (MHD) is that magnetic fields can induce currents in a moving conductive fluid, which in turn creates forces on the fluid and also changes the magnetic field itself, *[2].* For MHD flows, highly conductive fluids, such as plasmas, liquid metals, electrolytes and salt water, are needed.

f the flow is transported by pressure differential, the phenomenon is called pressure hydrodynamics (PHD).

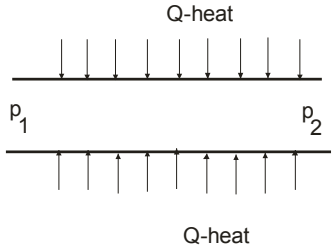
Ferrohydrodynamics (FHD) is the theory of magnetic fluid flow.

The term ‘nanofluid’ is envisioned as describing a solid-liquid mixture that consists of nanoparticles and a base liquid; this is one of the new challenges for thermo-sciences provided by nano-technology. The possible application area of nanofluids is in advanced cooling systems, and in micro/nano-electro-mechanical systems. The investigation of the effective thermal conductivity of liquid with nanoparticles has attracted much more interest experimentally and theoretically. The effective thermal conductivity of nanoparticle suspensions can be much higher than for the fluid without nanoparticles.

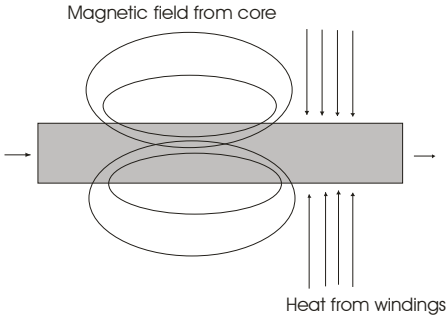
The calculation of properties for nanofluids for real substances is done using classical and statistical mechanics. Classical mechanics has no insight into the microstructure of the substance. Statistical mechanics, in contrast, calculates the properties of the state on the basis of molecular motions in a space, and on the basis of the intermolecular interactions. The equations obtained by means of classical thermodynamics are empirical and apply only in the region under observation. The main drawback of classical thermodynamics is that it lacks insight into the substance of the microstructure. In contrast to classical mechanics, statistical mechanics calculates the thermomechanical properties of state on the basis of intermolecular and intramolecular interactions between particles in the same system of molecules. It deals with systems composed of a very large number of particles.



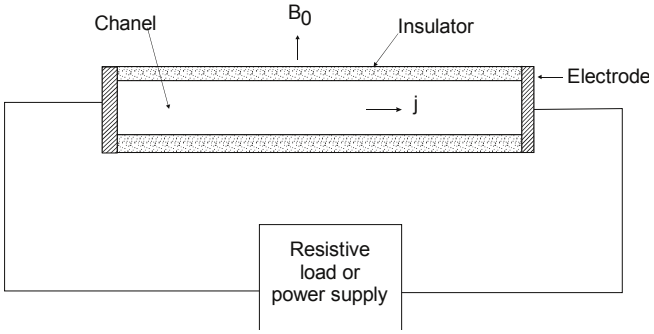
a) Electrokinetic flow



b) Pressure-driven flow



c) Ferrofluid flow



d) MHD flow

Figure 3: *Types of fluid flow*

Consider the MHD flow in rectangular and circular microchannels (Figure 3). The charged surface of a microchannel wall may attract ions of the opposite charge in the surrounding fluid. The general form of the momentum equation for MHD flow is:

$$\rho\frac{\partial\vec{v}}{\partial t}+\rho\vec{v}\nabla\vec{v}=-\nabla p+\nabla\left(\mu\nabla\vec{v}\right)+\vec{i}\times\vec{B}\,, \tag{3.1}$$

where the last term represents the electromagnetic force, and i and B refer to the current density and magnetic field strength, respectively. For steady-state flow in a microchannel at small Reynolds numbers, the transient and inertia terms can be neglected, so Eq. (3.1) is simplified in the next equation:

$$0=-\nabla p+\nabla\left(\mu\nabla\vec{v}\right)+\vec{i}\times\vec{B}\,, \tag{3.2}$$

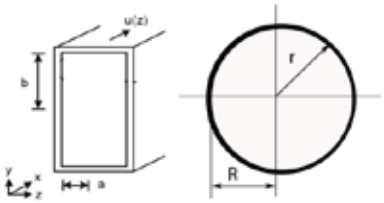


Figure 4: *Rectangular and circular microchannels*

Assuming the fluid velocity, magnetic field and current density are orthogonal, the reduced momentum equation becomes:

$$0=-\frac{dp}{dx}+\mu\frac{d^2u}{dz^2}+i_yB_z \tag{3.3}$$

The terms represent pressure, viscous and electromagnetic forces in the liquid. Using Ohm's Law to express the current density in terms of fluid velocity:

$$\mu\frac{d^2u}{dz^2}+\sigma_eB_z^2u=\frac{dp}{dx}\,, \tag{3.4}$$

where σ_e and B_z refer to the electrical conductivity and magnetic field strength. For fully developed flow in a microchannel, the pressure gradient becomes constant and independent of the magnetic field strength. In terms of the Hartman number, M_H ($M_H=aB_z\sqrt{\sigma_e/\mu}$),

$$\mu\frac{d^2u}{dz^2}-\left(\frac{M_H{}^2\eta}{a^2}\right)u=\frac{dp}{dx} \tag{3.5}$$

Applying the no-slip boundary conditions at z=0 and z=-2a, the analytical solution of Eq. (3.5) becomes:

$$u = -\frac{a^2\left(\frac{dp}{dx}\right)}{M_H{}^2\mu} + \frac{a^2\left(\frac{dp}{dx}\right)}{(1+2e^{2M_H})M_H{}^2\mu} e^{\frac{z\cdot M_H}{a}} + \frac{a^2\left(\frac{dp}{dx}\right)e^{2M_H}}{(1+2e^{2M_H})M_H{}^2\mu} e^{\frac{-z\cdot M_H}{a}} \tag{3.6}$$

The mean velocity within the microchannel becomes:

$$u_b = \frac{1}{2a}\int_0^{2a} u(z)dz \qquad u_b = \frac{a^2\left(\frac{dp}{dx}\right)(-M_H+Tanh(M_H))}{M_H{}^3\mu}$$

Non-dimensionalizing this result (z^*=z/a, u^*=u/u_b), we obtain the next equation:

$$u^* = \frac{M_H\left(-1+\frac{1}{(1+2e^{2M_H})}e^{z^*\cdot M_H}+\frac{e^{2M_H}}{(1+2e^{2M_H})}e^{-z^*\cdot M_H}\right)}{(-M_H+Tanh(M_H))} \tag{3.7}$$

Without electromagnetic effects, Equations (3.5-3.7) transform into the following expressions:

$$u(z) = \frac{\left(\frac{dp}{dx}\right)}{2\mu}\left(-2az+z^2\right), u_b = \frac{1}{2a}\int_{-a}^{a} u(z)dz = \frac{a^2\left(\frac{dp}{dx}\right)}{3\mu}, u^* = \frac{3}{2}\left(2-z^*\right)z^* \tag{3.8}$$

For the circular microchannel without electromagnetic forces, the governing equation is

$$\frac{dp}{dx} = \mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{3.9}$$

Solving the differential equation subject to boundary conditions,

$$u = \frac{\left(\frac{dp}{dx}\right)}{4\mu}(r^2-R^2),\ u_b = \frac{1}{\pi R^2}\int_0^R u2\pi\, r\cdot dr = -\frac{\left(\frac{dp}{dx}\right)R^2}{8\mu}\ u^* = 2-2r^{*2} \tag{3.10}$$

If we wish to calculate the velocity profile for MHD flow in a circular channel, we have to solve the following differential equation:

$$\frac{dp}{dx} = \mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) - \mu\frac{M_H{}^2}{R^2}u \tag{3.11}$$

The analytical solution of equation (3.11) is slightly more complicated. We have obtained the next solution of the differential equation with the boundary conditions (u(R)=0, u'(0)=0):

$$u[r] = \frac{R^2\left(\frac{dp}{dx}\right)(-\mathrm{BesselI}[0,M]+\mathrm{BesselI}[0,\frac{Mr}{R}])}{M^2\mu\,\mathrm{BesselI}[0,M]} \tag{3.12}$$

$$u_b = \frac{1}{\pi R^2}\int_0^R u2\pi\, r\cdot dr = \frac{R^2\left(\frac{dp}{dx}\right)\mathrm{BesselI}[2,M]}{M^2\mu\,\mathrm{BesselI}[0,M]} \tag{3.13}$$

Previous investigations of the pressure gradient for electro-osmotic liquid flow in microchannels have generally used no-slip conditions, while for gas flow in microchannels, the slip boundary condition is taken into account. Some previous experimental investigations have demonstrated the existence of liquid slip on a microchannel wall, *[2-5]*. In addition, some previous studies were numerically performed considering a slip velocity for a liquid flow in a microchannel made from hydrophobic surfaces, taking into account the electric field and pressure gradient, *[3-5]*. Furthermore, some previous studies numerically predicted a slip velocity for liquid flow in a microchannel made from hydrophobic surfaces, taking into account an imposed electric field and

a pressure gradient without heat transfer. On the basis of a slip velocity profile and boundary conditions in liquid microchannel flow, the velocity profile can be determined by

$$u(0) = u_{s1}, u(2a) = u_{s2} \tag{3.14}$$

$$u_{s1} = \beta \frac{\partial u}{\partial z}\Big|_{z=0}, \; u_{s2} = \beta \frac{\partial u}{\partial z}\Big|_{z=2a} \tag{3.15}$$

Figures 5-9 show the analytical results for the velocity profile in fluids in dependance of slip conditions, type of microchannel and type of fluid flow.

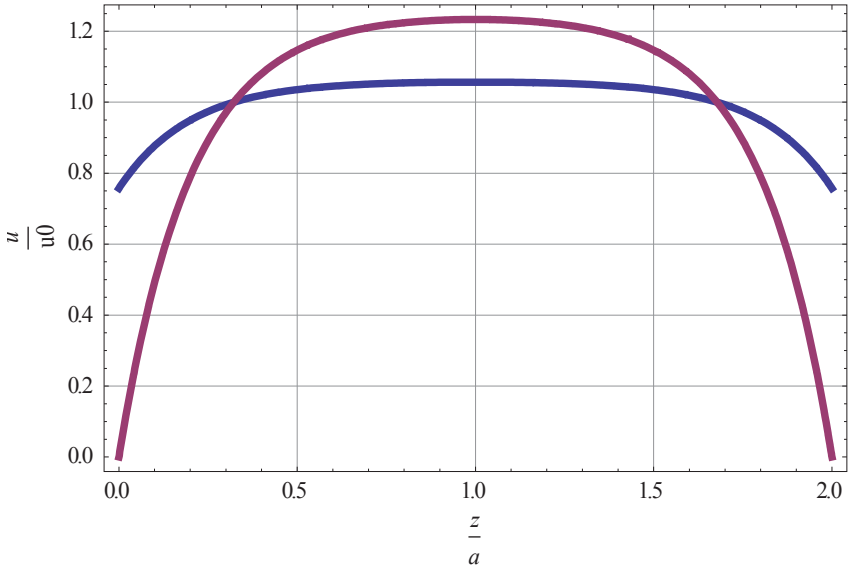


Figure 5: *Velocity profile (a) without slip (red line) and with slip β*=0.5 (blue line) at MH=5 in rectangular minichannel*

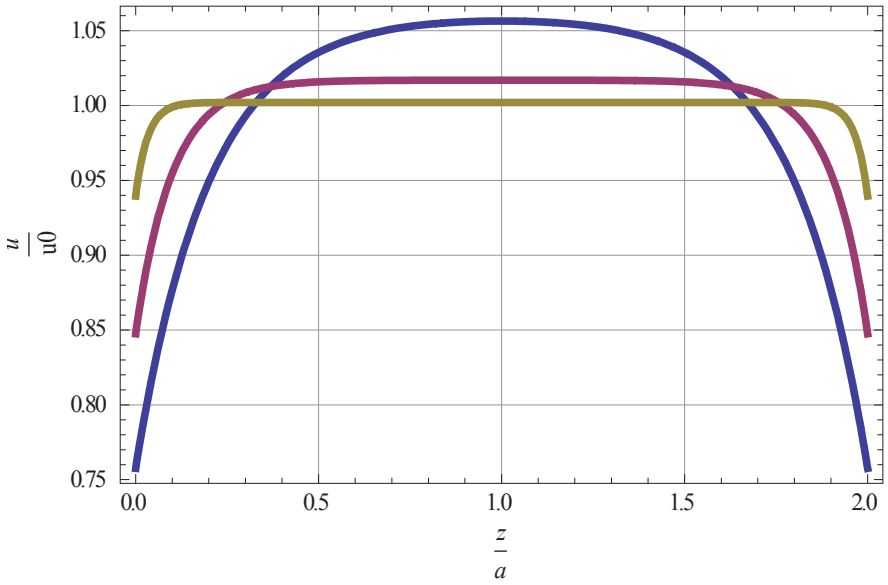


Figure 6: *Velocity profile with slip (β*=0.5) at MH=5 (blue line), MH=10 (red line) and MH=30 (green line) in rectangular minichannel with slip β*=0.5*

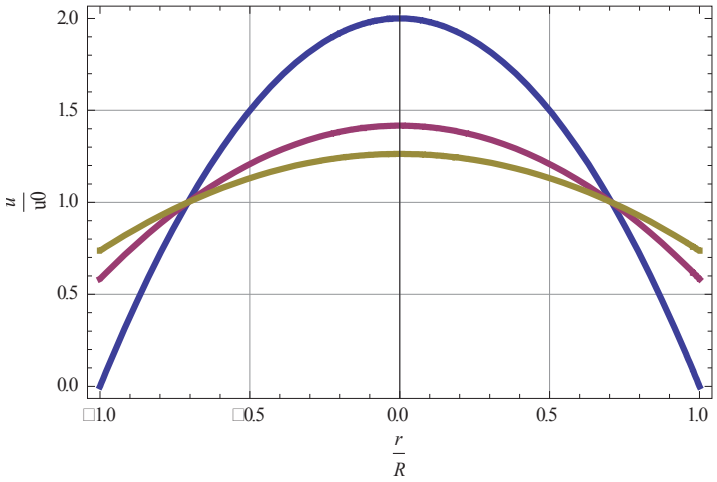


Figure 7: *Velocity profile in a circular microchannel without electromagnetic effects (blue line), with a slip with slip β*=0.35 (red line) and with slip β*=0.7 (green line)*

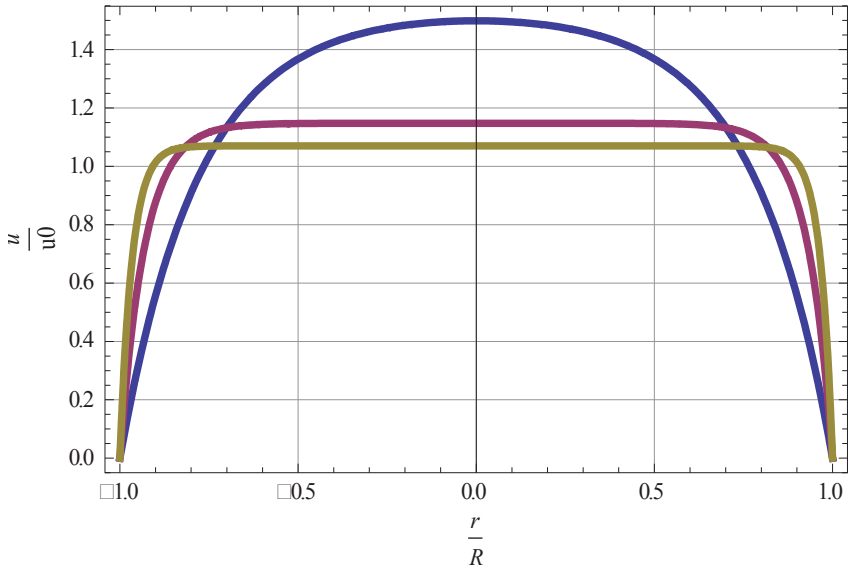


Figure 8: *Velocity profile with slip (β*=0.5) at MH=5 (blue line), MH=10 (red line) and MH=30 (green line) in circular microchannel without slip*

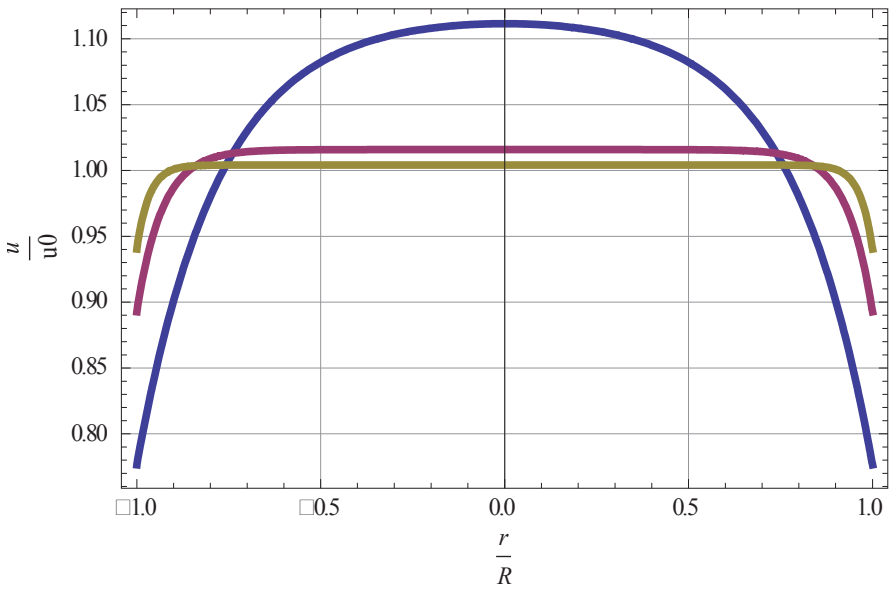


Figure 9: *Velocity profile with slip β*=0.5 at MH=5 (blue line), MH=10 (red line) and MH=30 (green line) in circular microchannel*

4 THE VIBRATIONS OF MICROBEAMS AND NANOBEAMS

The vibrations of beams, microbeams and nanobeams are of vital importance in mechanical engineering. Mechanical machines very frequently operate under diverse temperature conditions. In internal combustion engines, rocket systems, satellites, as well as MEMS and NEMS, conditions are particularly temperature-sensitive. Thermodynamic effects are frequently ignored in research, which may yield completely incorrect results. Literature, *[3]*, shows that even the slightest temperature change leads to significant alteration of the clamped-beam vibration properties. In the present paper the impact of a change in thermodynamic properties, which have to be taken into consideration with major temperature changes, is not neglected. Carbon nanotubes in dependence of the chiral angle can be classified into three types: armchair, zigzag, and chiral. Numerous studies are available on the physical properties of armchair and zigzag carbon nanotubes [14-15]. However, only a limited portion of the literature studied nanotubes in dependence of temperature field. This article develops a model that analyses the frequency of the chiral single-walled carbon nanotubes (SWCNTs) subjected to thermal vibrations by using the Timoshenko beam model, including the effect of rotary inertia and shear deformation. The Timoshenko model has been compared with the Euler model.

Carbon nanotubes can be classified into single wall nanotubes (SWNT) and multi-wall nanotubes (MWNT). On the basis of molecular simulation, many researchers have found that the modulus of elasticity is no longer constant, and is dependent on the diameter of nanotube and thickness of nanotube *[6-16]*. On the basis of molecular dynamics calculation, equations for surface Young modulus and Poisson number for armchair SWNT can be expressed:

$$Y_s = \frac{4\mu K_p}{\sqrt{3}(\lambda + 3\mu)} \tag{4.1}$$

where:

$$\mathrm{v} = \frac{\lambda - \zeta\mu}{\lambda + \zeta\mu}, \lambda = \frac{7 - \cos(\pi/n)}{34 + 2\cos(\pi/n)}, \mu = \frac{K_\theta^2}{K_\rho r_0^2} \tag{4.2}$$

The above equations are obtained on the basis of continuous mechanics and molecular simulation, *[6-8]*, where Y_s means surface Young modulus, and v the Poisson number. From Fig. 3.10, we can see that material properties are both temperature and size dependent.

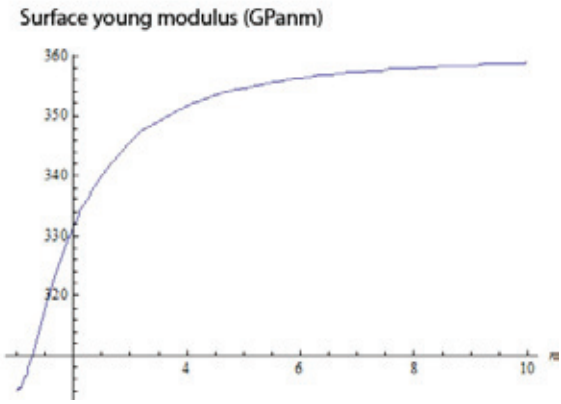


Figure 10: *Young modulus of armchair nanotubes*

The Young modulus of carbon nanotubes and the linear expansion coefficient are also dependent on the temperature field. On the basis of Prakash, *[16]*, a molecular dynamics simulation is obtained and the next relation for modulus of elasticity and linear expansion coefficient α for SWNT:

$$Y_s = Y_s(1 - 0.000075T) \tag{4.3}$$

$$\alpha = \frac{1}{l}\left(\frac{dl}{dT}\right) = \frac{10^{-18}T^2 - 2*10^{-15}*T + 10^{-13}}{10^{-18}T^3 - 10^{-15}T^2 + 10^{-13}T + 3*10^{-8}} \tag{4.4}$$

4.1 Local Euler-Bernoulli beam model under thermal stresses

Let us assume that the support is homogenous, having the same temperature over its entire length. As a result of thermal expansion, an additional axial force F_T occurs:

$$F_T = \alpha\theta EA \tag{4.5}$$

In equation (3.7), α is the linear thermal extension coefficient, and θ is the temperature difference between the actual and initial or reference temperature. The equation, by means of which we can resolve the problem using the axial force, is as follows according to Wear, Timoshenko and Young, *[4]*:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + F_T\frac{\partial^2 w(x,t)}{\partial x^2} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0\,, \tag{4.6}$$

where E means Young modulus, I area moment of inertia, A area, ρ density of material, t time and w the displacement. Using the method of separation of variables $w(x,t)=X(x)\Omega(t)$ and introducing the new functions, Equation (4.6) can be written down in a slightly less complicated way:

$$c^2\frac{X''''(x)}{X(x)}+2\gamma\frac{X''(x)}{X(x)}-\frac{\ddot{\Omega}(t)}{\Omega}=\omega^2, \tag{4.7}$$

where the partial derivatives have been replaced with total derivatives.

$$\ddot{\Omega}(t)+\omega^2\Omega(t)=0 \tag{4.8}$$

$$X''''(x)+2\gamma X''(x)-\beta^4X(x)=0 \tag{4.9}$$

In Equation (4.7), the new symbols represent the following functional relations:

$$\beta^2=\frac{\omega}{c},c^2=\frac{EI}{\rho A},\ \gamma=\frac{F_T}{2EI} \tag{4.10}$$

Thus, a general solution to Equations (4.8) and (4.9) is ($\lambda=\sqrt{\beta^4+\gamma^2}$):

$$X(x)=C_1\cos\left(\sqrt{\lambda+\gamma}x\right)+C_2\cosh\left(\sqrt{\lambda-\gamma}x\right)+C_3\sin\left(\sqrt{\lambda+\gamma}x\right)+C_4\sinh\left(\sqrt{\lambda-\gamma}x\right) \tag{4.11}$$

$$\Omega(t)=A\sin(\omega t)+B\cos(\omega t) \tag{4.12}$$

In the equation (4.11), the value of λ (where the influence of angular frequency ω is hidden) and three of four constants of integration C_1, C2, C_3, and C_4 are determined from the boundary conditions. The fourth constant is possible to find in the combination with the constants A and B in equation (4.11). For a given beam at a defined temperature, the values by λ depend upon the boundary conditions, *[5-9]*. Using boundary conditions, the following solutions can be analytically computed ($\Gamma=L^2\gamma,\Lambda=L^2\lambda$):

With the known angular frequencies ω_n of individual modes of vibration, it is possible to calculate X_n and Ω_n of individual modes of vibration. To determine the solution for the displacement, we have to solve the equation, *[6-15]*:

$$w(x,t)=\sum_{i=1}^{\infty}\left(A_n \sin(\omega_n t)+B_n \cos(\omega_n t)\right)X_n(x), \tag{4.13}$$

where the modal shapes can be shown to be orthogonal:

$$\int_0^l X_n(x)X_m(x)dx=0 \quad for\ n\neq m \tag{4.14}$$

The model presented in our paper is fully analytical, but when compared with the measured results it points to a large deviation from reality, *[6-15]*. The biggest problem with this model is that the clamped wall can fully withstand the beam for the beam to have a constant length all the time in the mathematical model in question. The above assumption is not realistic. As a result, a new model was designed to reduce to at least to some extent the huge differences between the analytical results and the measured values.

4.2 Local Timoshenko beam model under thermal stresses

The Timoshenko beam model, *[6-10]*, includes the effect of rotary inertia and shear deformation. The Timoshenko vibrational beam model gives the next expression:

$$\frac{EI}{\rho A}\frac{\partial^4 Y}{\partial x^4}+\frac{F_T}{\rho A}\frac{\partial^4 Y}{\partial x^4}+\frac{\partial^2}{\partial t^2}-\frac{I}{A}\left(1+\frac{E}{KG}\right)\frac{\partial^4 Y}{\partial x^2 \partial t^2}+\frac{I}{A}\frac{\rho}{KG}\frac{\partial^4 Y}{\partial t^4} \tag{4.15}$$

$$K=\frac{2(1+\mu)}{4+3\mu} \tag{4.16}$$

In Eq. (4.15), I is the dynamic moment of inertia of the beam, nanotube, K is the shear coefficient of the nanotube, and μ is the Poisson's ratio. F_t presents additional thermal force:

$$F_t=\alpha TEA \tag{4.17}$$

The solution of Eq. (4.15) could be expressed as:

$$Y(x,t)=y(x)e^{-i\omega t} \tag{4.18}$$

In Eq. (4.18), ω is the angular frequency. In the above approximations, the following dimensionless forms can be expressed as:

$$\frac{d^4\eta}{d\zeta^4}+\left[(\alpha+\beta)\Gamma^2+\delta\right]\frac{d^2\eta}{d\zeta^4}-\left(1-\Gamma^{2\alpha}\alpha\beta\right)\Gamma^2\eta \tag{4.19}$$

$$\eta=\frac{y}{L};\zeta=\frac{x}{L},\alpha=\frac{l}{AL^2},\beta=\frac{El}{KGAL^2} \tag{4.20}$$

$$\Gamma=\frac{\rho A\omega^2L^4}{El},\delta=\frac{F_TL^2}{El} \tag{4.21}$$

Where δ represents the effect of thermal vibration of the frequency of SWCNT. The general solution of Eq. (4.19) could be expressed as

$$\eta(\xi)=C_1\cos\left(\sqrt{\lambda-\gamma}\varepsilon\right)+C_2\cosh\left(\sqrt{\lambda+\gamma}\varepsilon\right)+C_3\sin\left(\sqrt{\lambda-\gamma}\varepsilon\right)+C_4\sinh\left(\sqrt{\lambda+\gamma}\varepsilon\right) \tag{4.22}$$

When the solution integrates with the boundary condition for the support-simply support nanotube model we obtain the solution:

$$\Gamma^4-\frac{1+(n\pi)^{2^{(\alpha+\beta)}}}{\alpha\beta}\Gamma^2+\frac{n^2\pi^2}{\alpha\beta}\left(-n^2\pi^2+\delta\right)=0 \tag{4.23}$$

For the case of Euler-Bernouli beam ($\alpha=\beta=0$), we obtain the following equation:

$$\Gamma=n\pi\sqrt{-\delta+(n\pi)^2} \tag{4.24}$$

The presented mathematical model was used to calculate the thermodynamic properties of the state of a pure aluminium microbeam. Table 1 contains the main important data of the beam. The aluminium beam is particularly interesting due to its relatively high expansion coefficients. In the presented section, we have calculated the vibrational characteristics for the supported-simply supported systems. For carbon nanotubes, we have used data for Young modulus and linear expansion coefficient shown in the Prakash Thesis, [16]. Figures 11 and 12 show angular frequency for nanotubes for the first, second and third orders. From both figures, we see that when we have relatively long nanotubes (L/D>10), the results for Timoshenko and Euler-Bernouli model give similar results; in contrast, when we have short nanotubes the Timoshenko model gives much better results for first and higher orders.

Table 1: *Fundamental constants for aluminium beam*

	Beam
Length (m)	$6.35 \cdot 10^{-2}$
Width (m)	$2.04 \cdot 10^{-2}$
Thickness (m)	$1.62 \cdot 10^{-3}$
Young modulus (N/m^2)	$6.9 \cdot 10^{10}$
Volume expansion coefficient (1/K)	$24 \cdot 10^{-6}\ K^{-1}$
Spring constant (N/m)	$1.553 \cdot 10^{5}$
Density (kg/m^3)	2780

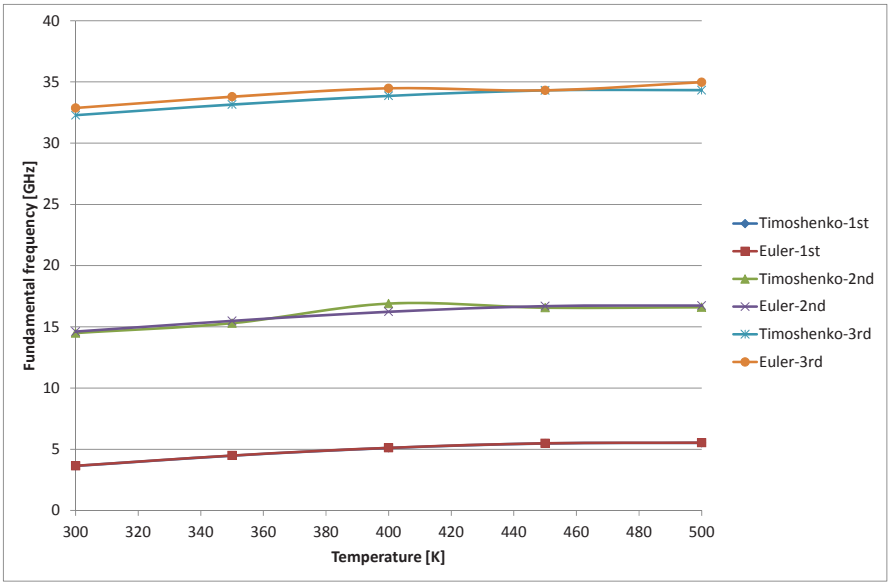


Figure 11: *Fundamental frequency for nanotube with L/D=40 with Timoshenko and Euler-Bernouli model*

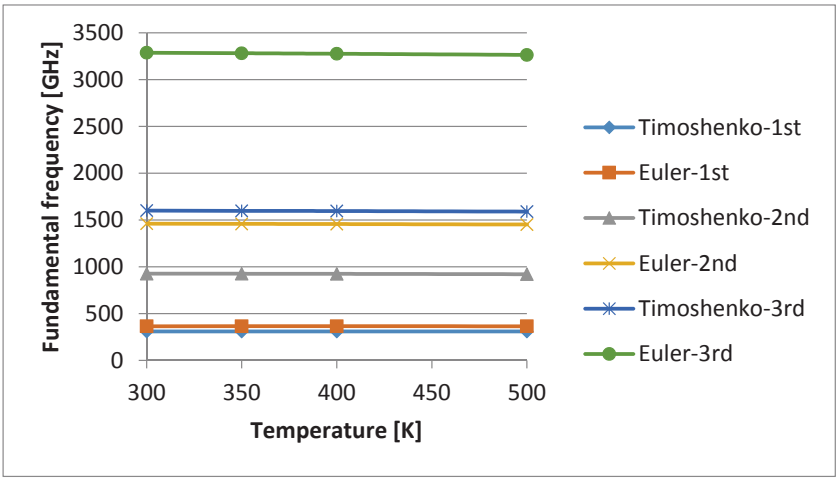


Figure 12: *Fundamental frequency for nanotube with L/D=4 with Timoshenko and Euler-Bernouli model*

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