RELAXATION MESH DYNAMICS IN THE METHOD OF FINITE DIFFERENCES

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UDK: 519.852

The number of relaxation mesh points in the finite difference method is defined and essentially restrained with the fast memory size of the computer in use. The boundary conditions for the second order differential equation of the first degree include in many technical applications certain fine structure in their geometry. With an uniform relaxation mesh a given finite number of mesh points that fine structure can not be taken properly into consideration. The relaxation mesh dynamics represents a procedure which gives some possibilities for the improvements. The applicability and the constraints of the relaxation mesh dynamics are quoted. The uniqueness of the solution to the second order differential equation is mentioned with regard to the application of the relaxation mesh dynamics.

The method of finite differences in its computational version yields a digital solution to the second order differential equation of the first degree. It provides in a certain way the uniqueness of that solution. The solution uniqueness could be achieved with an algorithm, which should exactly define all the steps in the numerical approach to the solution of the differential equation.

The finite fast memory size in the computer defines and restrains the number of mesh points in the relaxation mesh.

There is a possibility to increase the number of mesh points in a given small part of the region, and simultaneously decrease the number of mesh points in the rest of the whole region given within the extensions of the independent variables. Those extensions are defined with the boundary conditions to the differential equation.

The uniqueness of the solution cancels, when the mesh thinning and/or the mesh thickening is performed optionally and independently from the form of the differential equation solution. A complete uniqueness of the solution should be achieved, and including the relaxation mesh dynamics, when the mesh dynamics could follow the equipotential curves of the differential equation expected solution.

The basic suggestion how to approach the relaxation mesh to the equipotential curves of the expected solution for a given differential equation and the boundary conditions is described by A.M. WINSLOW in the year 1966. A successful application of such a relaxation mesh dynamics is usually connected to the interactive graphics at the computer in use. That application is introduced by J.S. COLONIAS in the year 1967.

When designing the relaxation mesh dynamics in the procedure, where the boundary conditions should be satisfied, the logical lay-out has to be strictly distinguished from the lay-out of the real geometry, given with the boundary conditions of the problem under consideration. With another words, one has to distinguish the boundary conditions input data as given into the programme from the data about the boundary conditions, accepted within the programme performance. The relaxation mesh dynamics has to be adjusted to the lay-out of the real geometry of the problem under consideration.

Therefore the relaxation mesh generator should provide the solution uniqueness in the differential equation as well as the mesh dynamics.

It should be done in such a way, that the solution accuracy is evenly distributed over the whole region, given within the extensions of the independent variables and defined with the boundary conditions.

In this report, we do intend to describe some experiences with the relaxation mesh dynamics in the method of finite differences, in the computational work, done during the past several years.

The simplest approach of the relaxation mesh dynamics represents the partitioning and the compounding of the solution matrix, respectively. Usually the matrix partitioning and the matrix compounding are performed simultaneously, although not necessarily, with the thinning and/or the thickening of the relaxation mesh, respectively. That approach includes the solution value linear interpolation at the transition from the thicker mesh to the thinner one. The matrix partitioning and/or compounding involves very large data handling.

The next approach in the relaxation mesh dynamics is the programmed mesh thinning. The interpolation conditions require, the region of the thinner mesh has to be surrounded and located inside the preceding thicker one. In this report some of that approach significances with the programmed mesh thinning are described.

It is possible to programme as many mesh thinning as they could be accepted in the finite fast memory size of the computer central processor. At the RRC CYBER 172 computer the procedure with five sequential mesh thinning is implemented. The basic mesh extension could be that way decreased 2^4 times. That appears as a satisfactory solution to the most of the problems under consideration.

At each of the mesh thinning, it is firstly necessary to determine the interpolated solution values of the differential equation at the thinner mesh region boundary. With the defined solution values at the region boundary for each of the relaxation mesh thinning, an interation procedure for a given kind of relaxation is performed. The kind of relaxation, i.e. the over-relaxation and/or the under-relaxation depends on the linearity and the non-linearity of the problem, respectively. The actual matrix of the differential equation solutions for a given relaxation mesh thinning is saved on the magnetic tape for the later final consideration.

It is necessary to remark, the mesh thinning could be simply developed only for the method of finite differences at the accuracy of second order. At the accuracy of the fourth order, as introduced by P. BONJOUR and S. NATALIS in the year 1972, the relaxation mesh thinning would require an extremely involved procedure.

The linear interpolation of the values for the differential equation solution at the mesh thinning does not appear always as a justified one. An interpolation with higher order splines should yield a more applicable results.

The relaxation mesh dynamics provides a better interpretation of the boundary condition fine structure and a more uniform distribution of the solution accuracy over the whole region under consideration. A better accuracy of the unique solution in the whole region could be accomplished with the method of finite differences of the fourth order and applying an exact consideration of accurately defined boundary conditions.

The suitable and an applicable method of finite differences using the accuracy of the second order should include several ten thousand of mesh points in the relaxation mesh. The solution accuracy is directly proportional to the inverse value of the mesh points number in the relaxation mesh.

For the purpose of completeness and briefness this report is supported with only two references. The report [1] from the Dubna Institute is quoted for the early publications and historical background. The work by P. BONJOUR [2] includes more recent achievements, particularly the intentions how to approach the relaxation mesh dynamics to the form of the expected solution of the second order differential equation.

REFERENCES

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