INTERACTION BETWEEN TOTAL COST AND FILL RATE: A CASE STUDY

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ABSTRACT: *Forecasting plays a central role in the efficient operation of a supply chain – i.e., the total costs and fill rate. As forecasts of demand are required on a regular basis for a very large number of products, the methods developed should be fast, flexible, user-friendly, and able to produce results that are reliable and easy to interpret by a manager. In this paper we show that the supply chain costs cannot be optimal if the forecasting method is treated separately from the inventory model. We analyse the performance of the joint optimization of the modified Holt-Winters forecasting method and a stock control policy and investigate the effect of different penalties for unsatisfied demand on the total cost and fill rate of the supply chain. From the results obtained with 1,428 real time series from M3-Competition we show that an essential reduction of supply chain costs and an increase of fill rate can be achieved if we use the joint model with the modified Holt-Winters method.*

Keywords: *forecasting, inventory, fill rate, Holt-Winters method, optimization, M3-Competition*

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INTRODUCTION

The management of the global supply chain and its performance appear to be possible to boost considerably by striving for forecasting accuracy and the information sharing in order to harmonize different activities in the supply chain. As a result, costs may be lowered and customer services enhanced. So, determining the best inventory control policies is heavily dependent on the following three factors: the customers' demand pattern, the lead times and the information sharing (De Sensi et al., 2008; Wadhwa et al., 2009; Jakšič and Rusjan, 2009; Escuin et al., 2017). As demand rates are changing with time due to seasonal variations, business cycle and irregular fluctuations, effectively managing the supply chain with time-varying demand is an important issue (Zhao et al., 2016). Several authors (see, e.g., Hayya et al., 2006; Tiacci and Saetta, 2009; Syntetos et al., 2010; Liao and Chang, 2010; Danese and Kalchschmidt, 2011; Acar and Gardner, 2012) have performed research on the importance of forecasting in a supply chain. Authors investigated the impact of how forecasting is conducted on forecast accuracy and operational performances (i.e. cost and delivery performances).

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Forecasts of demand are required on a regular basis for a very large number of products so that inventory levels can be planned in order to provide an acceptable level of service to customers (Hyndman et al., 2002). The developed forecasting methods should therefore be fast, flexible, user-friendly, and able to produce results that are reliable and easy to interpret by a manager. Exponential smoothing methods are a class of methods that produce forecasts, taking into account trend and seasonal effects of data (more details can be found in Gardner (2006)). These procedures are widely used as forecasting techniques in inventory management and sales forecasting. Distinguished by their simplicity, their forecasts are comparable to those of more complex statistical time series models (Makridakis and Hibon, 2000).

Although demand data result from a demand forecasting system, they are regarded as an independent input to the stock control model in most studies. Usually, they include two steps: 1. calculate forecasts (for instance, minimising the mean square error) and 2. use obtained forecasts as an input to the inventory/production model and optimise the stock control policy (minimise the total cost). Even though this weakness has been highlighted in the academic literature, little empirical work has been conducted to develop understanding of the interaction between forecasting and stock control (Ferbar Tratar 2010; Strijbosch et al. 2011; Ma et al. 2013).

Regarding the above mentioned facts we were interested if the total cost of the supply chain and fill rate are optimal if we use the best model for forecasting demand and for inventory control policy for supply chain with centralised demand information. In the first case we treat these two models separately and calculate the total costs and fill rate (for different penalties) for forecasts obtained with different methods regarding minimising MSE. In the second case we inspect the performance of the joint model, where we determine the parameters of forecasting method to minimise the total cost of the supply chain. We use 1,428 real time series from M3-Competition to evaluate the performance of the modified Holt-Winters method. We will show that forecasts interact with the inventory model and consequently result in lower inventory costs as well as higher fill rate. We do not prescribe the required fill rate but rather analyse how the joint model with the proposed modified HW method, where the inventory costs are minimised, effects the fill rate.

The remainder of the paper is organized as follows. In Section 2 we describe the classical Holt-Winters forecasting procedure, a modified Holt-Winters procedure, our model of the supply chain, and present the proposed joint model. After the description and classification of the real time series from M3-Competition (Section 3), in Section 4, a performance of the modified HW method is demonstrated and the main findings of the paper are described.

2 METHODOLOGY

2.1 The Holt-Winters forecasting procedures and a modified HW method

Exponential smoothing methods are a class of methods that produce forecasts with simple formulae, taking into account trend and seasonal effects of the data. The HW method estimates three smoothing parameters associated with level, trend and seasonal factors. We estimated smoothing and initial parameters in HW methods by minimising the mean square error (MSE).

In the multiplicative seasonal form of HW method (MHW) fundamental equations for level (L_t), trend (b_t), seasonal factors (S_t) and forecast (F_{t+m}) are (Makridakis et al. 1998):

$$L_t = \alpha\left(Y_t / S_{t-s}\right) + \left(1-\alpha\right)\left(L_{t-1} + b_{t-1}\right) \tag{1}$$

$$b_t = \beta\left(L_t - L_{t-1}\right) + \left(1-\beta\right)b_{t-1} \tag{2}$$

$$S_t = \gamma\left(Y_t / L_t\right) + \left(1-\gamma\right)S_{t-s} \tag{3}$$

$$F_{t+m} = \left(L_t + b_t m\right)S_{t-s+m} \tag{4}$$

where m is the number of forecasts ahead, s is the length of seasonality (e.g., number of months or quarters in a year) and Y_t is the observed data at time point t. There have been many suggestions regarding restricting the parameter space for smoothing parameters α , β and γ (Hyndman and Khandakar 2008). In this paper, we follow the traditional approach, requiring that all parameters lie in the interval $\left[0,1\right]$. These estimates are set to minimize the discrepancies between the in-sample one-step-ahead predictions F_{t+1} and the observed values Y_{t+1}.

Empirical study (see Bermudez et al. 2006) illustrates that the method used to designate the initial vector has very little effect on the accuracy of the predictions obtained when smoothing and the initial parameters of the forecasting method are determined to minimise the forecast error measure. So, to initialize the level, we set $L_s = \left(Y_1 + Y_2 + \cdots + Y_n\right)/s$; to initialize the trend, we use $b_s = \left(Y_{s+1} - Y_1 + Y_{s+2} - Y_2 + \cdots + Y_{2s} - Y_s\right)/s^2$; and for initial seasonal indices we calculate $S_p = Y_p / L_s, p = 1, 2, \cdots, s$.

The additive seasonal form of HW method (AHW) works with the following equations:

$$L_t = \alpha\left(Y_t - S_{t-s}\right) + \left(1-\alpha\right)\left(L_{t-1} + b_{t-1}\right) \tag{5}$$

$$b_t = \beta\left(L_t - L_{t-1}\right) + \left(1-\beta\right)b_{t-1} \tag{6}$$

$$S_t = \gamma\left(Y_t - L_t\right) + \left(1-\gamma\right)S_{t-s} \tag{7}$$

$$F_{t+m} = L_t + b_t m + S_{t-s+m} \tag{8}$$

The equation (6) is identical to equation (2). The only differences in the other equations are that the seasonal indices are now added and subtracted instead of relying on products and ratios. The initial values for level and trend are identical to those for the multiplicative method. To initialize the seasonal indices we use $S_p = Y_p - L_s, p = 1,2,\cdots,s$.

The modified HW method (MoHW) contains the following equations (see Ferbar Tratar (2015a)):

$$L_t = \alpha Y_t - S_{t-s} + (1-\alpha)(L_{t-1} + b_{t-1}) \tag{9}$$

$$b_t = \beta(L_t - L_{t-1}) + (1-\beta) b_{t-1} \tag{10}$$

$$S_t = \gamma(Y_t - L_t) + (1-\gamma) S_{t-s} \tag{11}$$

$$F_{t+m} = L_t + b_t m + S_{t-s+m} \tag{12}$$

The only difference between the additive and modified HW method is in the equation (9). For the modified HW method in contrast to the additive HW method the smoothing parameter α occurs only at observed data Y_t and not at seasonal factor S_{t-s}. If we consider equation (9) and replace $S_t = \alpha S_t^*$, the equation (11) becomes:

$$S_t^* = \gamma^* (Y_t - L_t) + (1-\alpha\gamma^*) S_{t-s}^*, \quad \gamma^* = \gamma / \alpha \tag{13}$$

The other equations for the MoHW now conform to the AHW format. Thus, when we minimize forecast error with respect to the smoothing parameters, the new effect is to smooth the seasonal factors by changing them less. The initial values for level, trend and seasonal indices are identical to those for the additive method.

2.2 ETS method

The analyses were carried out also in the program R (R Core Team, 2014). The function sbplx from the nonlinear optimization package nloptr (Ypma and Borchers, 2014; Johnson, 2013) was used to estimate the smoothing parameters. For each of the series we used ets function to obtain the MSE, where we set opt.crit='mse', ic='aic', bounds='usual', so that the MSE was minimized to estimate the parameters of each model. AIC was used to select the best model (the best exponential smoothing method according the minimised MSE and the number of the smoothing parameters) and the standard parameter restrictions were applied (smoothing parameters lie in the interval $[0,1]$). We use notation ETS method. It is a state space model that includes some transition equations that describe how the unobserved components or states (level, trend, seasonal) change over time. The classical decomposition method splits a time series into a trend and a seasonal component and projects them in the forecast horizon (Escuin et al., 2017).

2.3 Symmetric relative efficiency measure

The efficiency of the MoHW method was measured in terms of the mean squared error (MSE) of the in-sample one-step-ahead forecasts and compared to that of AHW and MHW methods. Because the first two complete seasons were used to initialize the methods, these observations were excluded from the reported MSE:

$$\mathrm{MSE} = \frac{1}{T-2s}\sum_{t=2s+1}^{T}\left(Y_t - F_t\right)^2 \tag{14}$$

where Y_t is the observed data and F_t the forecast at time point *t*.

To compare the MoHW method with the other method, we first find their mean squared errors $\mathrm{MSE}_{\mathrm{MoHW}}$ and $\mathrm{MSE}_{\mathrm{method}}$ as defined above. We define the symmetric relative efficiency measure as

$$\mathrm{SREM}_{\mathrm{MoHW/method}} = \begin{cases} 1-\dfrac{\mathrm{MSE}_{\mathrm{MoHW}}}{\mathrm{MSE}_{\mathrm{method}}}; & \mathrm{MSE}_{\mathrm{MoHW}} < \mathrm{MSE}_{\mathrm{method}} \\ \dfrac{\mathrm{MSE}_{\mathrm{method}}}{\mathrm{MSE}_{\mathrm{MoHW}}}-1; & \mathrm{MSE}_{\mathrm{MoHW}} \geq \mathrm{MSE}_{\mathrm{method}} \end{cases} \tag{15}$$

The value of SREM is bounded by the interval $\left[-1,1\right]$, which mitigates the possibility of an individual time series to substantially over-weigh other series in the group. This is especially important in the study, where some methods on some series give MSE close to or equal to 0. If the average of SREM values over a group of time series is positive, this indicates that the MoHW method outperforms the other method for this particular group of series. The interpretation does not depend on the number of series in the group, so SREM can easily be applied to the M3-Competition data where different disciplines (or types) have different numbers of time series.

2.4 The supply chain model and joint optimisation

Consider a single-stage supply chain (with centralized demand information) consisting of one retailer (the most downstream unit of the supply chain) and one distributor (Ferbar Tratar et al., 2009; Ferbar Tratar, 2010). The retailer holds inventory in order to meet an external demand and places inventory replenishment orders to the distributor. Orders are placed at every time period. At time *t*, the last known value of the external demand is D_{t-1}. The retailer places order Q_t to the distributor, taking into account the demand forecast for period $t+1$ (using eq. (4), (8) or (12) for $F_{(t-1)+2} = F_{t+1}$). We assume that the order placed one period ago is received (lead time is one period). After the order placement, the external demand D_t is observed and filled. At the end of each period, the inventory cost are evaluated. If the retailer has on-hand inventory, the holding cost appears. The unsatisfied demand is backlogged and causes backordering cost for the retailer. The distributor is able to supply any requested quantity. The order placed at time *t* is received at time $t+1$ and is available to the retailer to fulfil external demand D_{t+1}.

Assuming that the retailer follows an order-up-to inventory policy, the order Q_t placed by the retailer to the distributor can be expressed as $Q_t = F_{t+1} - FS_{t-1}$, where F_{t+1} is the forecasted demand for the period $t+1$ (taking into account that the last known value of the external demand is D_{t-1}) and FS_{t-1} is the final stock for the period $t-1$ (if $FS_{t-1} > 0$ the retailer has on-hand inventory, if $FS_{t-1} < 0$ the unsatisfied demand occurs). When it is $Q_t < 0$, an order is not placed. The final stock is calculated as $FS_t = IS_t - D_t$, where the initial stock IS_t is obtained as $IS_t = Q_{t-1} + FS_{t-1}$. As the distributor has information about the external demand (centralized supply chain), it places the order, which is equal to the forecasted demand (less FS_{t-1}, if $FS_{t-1} > 0$). The missing amount of products supplied from the marketplace (assuming that a perfect substitute for the product exists) causes backordering cost for the distributor.

The costs of the supply chain are the sum of the holding and the backordering costs for all links in the supply chain. We assume the backordering cost to be higher than the holding cost, which is expressed by introducing a weight, *penalty* (= backordering cost / holding cost), that is greater than 1. In our analysis, for all calculations of total costs (average costs and minimised average costs) we assume that penalty is equal to 3 or 5.

In other words, using the common notation $X^+ = \max(X,0)$, the supply chain costs in time period *t* are expressed as (*n=2* – total number of links in the supply chain):

$$C_t = \sum_{l=1}^{n} C_t^l = \sum_{l=1}^{n} \left(\left(IS_t^l - D_t^l \right)^+ + penalty \times \left(D_t^l - IS_t^l \right)^+ \right) \qquad (16)$$

where the initial stock can be expressed with forecast and final stock as:

$$IS_t^l = \left(F_t^l - FS_{t-2}^l \right)^+ + FS_{t-1}^l \qquad (17)$$

Because the first two seasons were used to initialize the methods, the average costs (AC) are calculated as:

$$AC = \frac{1}{T-2s} \sum_{t=2s+1}^{T} C_t \qquad (18)$$

where the supply chain costs C_t in time period *t* are defined with eq. (16).

We use definition of SREM to compare the MoHW method with others methods regarding average costs. In these cases, SREM1 measures the percentage increase or decrease of the average costs:

$$\mathrm{SREM1}_{\mathrm{MoHW/method}} = \begin{cases} 1-\dfrac{\mathrm{AC}_{\mathrm{MoHW}}}{\mathrm{AC}_{\mathrm{method}}}; & \mathrm{AC}_{\mathrm{MoHW}} < \mathrm{AC}_{\mathrm{method}} \\ \dfrac{\mathrm{AC}_{\mathrm{method}}}{\mathrm{AC}_{\mathrm{MoHW}}}-1; & \mathrm{AC}_{\mathrm{MoHW}} \geq \mathrm{AC}_{\mathrm{method}} \end{cases} \quad (19)$$

Since forecast is usually considered as input to the model in stock control studies the average costs for forecasts obtained with different forecasting methods regarding minimising MSE were calculated and the SREM1 of MoHW with respect to the AHW, MHW and ETS were computed. After that the smoothing and initial parameters of the forecasting method in the joint model are estimated by minimising the average costs and the SREM1 of JMoHW with respect to the AHW, JAHW, MHW, JMHW and ETS were computed (where letter 'J' means usage of the joint model).

2.5 Fill rate

A fill rate is a service metric and measures the number of units filled as a percentage of the total ordered (Guijarro et al., 2012). If customer orders total 1000 units and we can only meet 900 units of that order, the fill rate is 90%. We calculated fill rate for the retailer for every period

$$FR_t = 1 - \frac{\left(D_t - IS_t\right)^+}{D_t} \quad (20)$$

and presented the average fill rate:

$$AFR = \frac{1}{T-2s}\sum_{t=2s+1}^{T} FR_t \quad (21)$$

3 DATA

The Makridakis Competitions, known in the literature as the M-Competitions, are empirical studies that have compared the performance of a large number of major time series methods using recognized experts who provide forecasts for their method of expertise (Makridakis & Hibon, 2000). The first M-Competition (1982) used 1001 time series and 15 forecasting methods. The second M2-Competition (1993) used only 29 time series. The third M3-Competition (2000) was intended to both replicate and extend the features of the first two competitions. A total of 3003 time series was used.

The real time series from the M3-Competition are still widely used for testing new and evaluating old forecasting methods and models (Gorr and Schneider 2013; Petropoulos et al. 2014). The data sets used refer mainly to business and economic time series, although the conclusions are relevant to other disciplines as well. The original time series data can be found in R package Mcomp (Hyndman et al. 2013).

We used real seasonal time series from the M3-Competition to evaluate the performance of the modified Holt-Winters method. The analyses were carried out in Solver (Microsoft Excel 2010) and the program R (R Core Team 2014). The starting values in the minimization step were set to $\alpha_0 = \beta_0 = \gamma_0 = 0.5$ and the maximum number of iterations was set to 25,000.

In our study, we analysed 1428 monthly series. They refer to six different disciplines, as shown in Table 1. First we used ets function from R package forecast (Hyndman et al. 2014; Hynmdan and Khandakar 2008) to classify the series by the form of their trend, seasonality and noise. Table 1 also shows this classification. Here 'A' stands for 'additive', 'M' for 'multiplicative', and 'N' for 'none'.

Table 1: *Classification of monthly time series from M3-Competition*

Discipline	***Number***	***Noise***	***Trend***	***Season***	***Number***
DEMOGRAPHIC	111	A	N	N	123
FINANCE	145	A	N	A	115
INDUSTRY	334	A	A	N	167
MACRO	312	A	A	A	97
MICRO	474	M	N	N	124
OTHER	52	M	N	A	95
TOTAL	1428	M	N	M	124
		M	A	N	179
		M	A	A	56
		M	A	M	99
		M	M	N	159
		M	M	M	90
				TOTAL	1428

We applied AHW, MHW and MoHW methods on each of the series independently of its discipline and ets classification. The estimated smoothing and initial parameters and in-sample MSE values were saved and the SREM of MoHW with respect to the AHW, MHW and ETS were computed.

4 RESULTS OF THE STUDY AND DISCUSSION

For each method and series, the symmetric relative efficiency measures (SREM and SREM1) of MoHW with respect to AHW, MHW and ETS were computed. Table 2 shows averages of SREM for monthly time series. We can observe that with the MoHW method the MSE can be reduced on average by more than 4% (6%) in comparison with the AHW (MHW) method. Also, the MoHW method outperforms ETS in 77% of cases, on average by almost 16%.

The MoHW method is particularly good in capturing the behavior of microeconomic time series, where the MoHW method performs better than the ETS method on average by 26%. The MoHW method substantially outperforms other methods for classes with no seasonal component (xNN, xAN and xMN), irrespective of noise. Surprisingly, the fit of the MoHW method is better even in xAA and xAM classes, where AHW and MHW methods are theoretically the correct methods. This indicates the universality of the MoHW method regarding ETS which tries to select the most appropriate method.

Since demand data is usually considered as input to the model in stock control studies, the average costs (for the time interval $t = 2s,\ldots,T$) for forecasts obtained with different forecasting methods were calculated. Table 2 also shows the averages of SREM1 (percentage of improvement of the average costs) of MoHW with respect to AHW, MHW and ETS. We can observe that averages of the SREM1 are more than 2%, 4% and 10% (for penalty = 3 and penalty = 5) with respect to AHW, MHW and ETS. Almost the same as we observe for SREM holds for SREM1. If the MoHW substantially outperforms classical methods in some classes regarding MSE, the MoHW substantially outperforms them in the same classes regarding the average costs (as in this case the costs are calculated for forecasts considered as an input to the stock control model). We can also observe that the improvement of MoHW in comparison with other methods increases as penalty increases.

Table 2: *Averages of the SREM and SREM1*

	MSE → COST	SREM			SREM1					
					penalty = 3			penalty = 5		
		MoHW/ AHW	MoHW/ MHW	MoHW/ ETS	MoHW/ AHW	MoHW/ MHW	MoHW/ ETS	MoHW/ AHW	MoHW/ MHW	MoHW/ ETS
Discipline	DEMOGRAPHIC	3.3%	8.3%	13.7%	2.2%	6.4%	9.0%	2.5%	6.7%	10.1%
	FINANCE	4.1%	8.7%	18.0%	2.1%	3.5%	9.8%	2.2%	3.7%	10.5%
	INDUSTRY	2.5%	7.0%	6.7%	1.7%	4.4%	6.4%	1.8%	4.4%	7.4%
	MACRO	6.4%	4.3%	8.0%	4.1%	3.2%	6.9%	4.2%	3.5%	8.0%
	MICRO	5.3%	7.2%	26.0%	3.2%	5.0%	14.9%	3.5%	5.4%	15.5%
	OTHER	1.5%	6.6%	22.7%	1.0%	5.1%	9.6%	1.2%	5.6%	11.1%

	MSE → COST	SREM			SREM1					
					penalty = 3			penalty = 5		
		MoHW/ AHW	MoHW/ MHW	MoHW/ ETS	MoHW/ AHW	MoHW/ MHW	MoHW/ ETS	MoHW/ AHW	MoHW/ MHW	MoHW/ ETS
Type	ANN	4.4%	5.3%	26.5%	1.9%	3.2%	14.3%	2.0%	3.3%	14.6%
	ANA	2.2%	7.1%	5.5%	1.5%	3.9%	6.1%	1.6%	3.9%	7.1%
	AAN	2.9%	7.1%	20.2%	1.7%	5.4%	12.4%	1.8%	5.8%	12.6%
	AAA	4.1%	6.7%	7.0%	2.0%	4.5%	5.6%	2.1%	4.8%	6.4%
	MNN	2.9%	7.8%	26.5%	1.8%	3.7%	14.4%	2.0%	4.2%	15.4%
	MNA	3.5%	8.3%	8.5%	2.2%	7.2%	6.8%	2.4%	7.4%	8.0%
	MNM	3.9%	5.2%	9.3%	3.3%	5.3%	8.3%	3.6%	5.7%	10.3%
	MAN	3.9%	6.1%	17.5%	2.2%	2.9%	10.1%	2.4%	3.1%	10.8%
	MAA	3.3%	6.2%	8.9%	1.9%	3.9%	6.5%	2.1%	4.1%	7.8%
	MAM	8.2%	5.1%	8.2%	4.6%	4.0%	6.9%	4.8%	4.5%	7.9%
	MMN	3.5%	6.6%	20.3%	2.5%	5.0%	11.9%	3.0%	5.2%	12.3%
	MMM	9.8%	7.7%	7.7%	6.5%	5.7%	5.9%	7.1%	5.9%	6.7%
	Total	**4.5%**	**6.7%**	**15.9%**	**2.7%**	**4.5%**	**10.1%**	**2.9%**	**4.7%**	**10.9%**

In Table 3 we present the results of fill rate for the models in which forecasting and an inventory model were treated separately. The ETS method on average slightly outperforms other methods. ETS outperforms AHW and MHW for demographic and microeconomic series and for series with multiplicative noise. ETS outperforms MoHW for all dicsiplines except for industry and macroeconomics series and for all types except for ANA, AAN.

From the joint optimisation of supply chain (costs) model for $1,428$ monthly series (see Table 4), we observe the following: on average JMoHW can reduce the average costs by 5.9% (7.2%) in comparison with JAHW (JMHW) for penalty = 3 and by 9.2% (11.8%) for penalty = 5. We can see that the averages of the SREM1 increase as penalty increases. The JMoHW method outperforms the JAHW and JMHW methods for all disciplines and it is particularly good for microeconomic and demographic time series. Also, the JMoHW method outperforms the other two methods for all types and it is particularly good in MNA, MAM and MMx (multiplicative noise and trend) classes.

Table 3: *Fill rate results obtained from the supply chain model with the forecasts obtained regarding minimising MSE*

	MSE → COST	FILL RATE			
		ETS	AHW	MHW	MoHW
Discipline	DEMOGRAPHIC	98.90%	98.71%	98.80%	98.47%
	FINANCE	97.54%	97.61%	97.55%	97.48%
	INDUSTRY	96.94%	96.95%	96.97%	96.96%
	MACRO	98.83%	98.89%	98.87%	98.87%
	MICRO	93.70%	93.29%	93.35%	93.00%
	OTHER	97.39%	97.47%	97.47%	97.35%
Type	ANN	94.29%	94.53%	94.37%	94.12%
	ANA	96.31%	96.49%	96.51%	96.38%
	AAN	98.09%	98.28%	98.19%	98.24%
	AAA	98.03%	98.08%	98.05%	98.00%
	MNN	94.50%	94.33%	94.32%	94.17%
	MNA	96.54%	96.51%	96.26%	96.14%
	MNM	94.77%	93.60%	94.15%	93.48%
	MAN	97.53%	97.58%	97.58%	97.36%
	MAA	97.49%	97.48%	97.52%	97.48%
	MAM	96.84%	96.51%	96.61%	96.46%
	MMN	96.56%	96.52%	96.35%	96.43%
	MMM	97.22%	96.44%	96.90%	96.65%
	Total	**96.50%**	**96.38%**	**96.40%**	**96.25%**

Table 4: *Averages of the SREM1 obtained with the joint optimisation*

	JOINT	SREM1			
		penalty = 3		penalty = 5	
		JMOHW/ JAHW	JMOHW/ JMHW	JMOHW/ JAHW	JMOHW/ JMHW
Discipline	DEMOGRAPHIC	**7.0%**	**8.9%**	**12.5%**	**16.1%**
	FINANCE	4.0%	6.8%	7.1%	11.6%
	INDUSTRY	2.9%	5.1%	4.7%	7.1%
	MACRO	6.3%	5.5%	8.9%	8.3%
	MICRO	**7.8%**	**9.5%**	**12.6%**	**16.9%**
	OTHER	7.8%	7.3%	10.0%	11.1%

	JOINT	SREM1			
		penalty = 3		penalty = 5	
		JMOHW/ JAHW	JMOHW/ JMHW	JMOHW/ JAHW	JMOHW/ JMHW
Type	ANN	5.8%	6.6%	7.2%	10.6%
	ANA	3.1%	4.9%	5.2%	7.1%
	AAN	5.7%	6.6%	9.1%	10.6%
	AAA	5.4%	6.7%	8.9%	11.8%
	MNN	5.3%	7.3%	8.0%	11.7%
	MNA	**5.3%**	**10.0%**	**8.8%**	**15.6%**
	MNM	4.7%	8.3%	8.5%	12.3%
	MAN	4.3%	5.6%	6.9%	7.9%
	MAA	6.1%	6.5%	9.9%	9.7%
	MAM	**8.5%**	**8.7%**	**12.0%**	**14.9%**
	MMN	**7.1%**	**8.7%**	**13.1%**	**14.8%**
	MMM	**8.6%**	**8.4%**	**12.5%**	**16.6%**
	Total	**5.9%**	**7.2%**	**9.2%**	**11.8%**

In Table 5 we present the results of fill rate for the joint model. Fill rate increases as penalty increases. The MoHW methods outperforms all other methods for all types and all disciplines, except AHW for demographic time series (penalty = 3) and MHW for other time series (penalty = 5). The fill rate of the MoHW method reaches the highest value for macroeconomics time series and for AAN type series.

If we compare these results with those in Table 3, we can observe that the use of the joint model increases the fill rate of the MoHW method in comparison with the earlier superior ETS method by more than 2.5 percentage points.

The result for SREM and SREM1 (Table 2) confirms that the ETS (AHW, MHW) method more tends to over or under forecasts than the MoHW method. If we consider also the result for fill rate (Table 3), we can see that on average the ETS method gives more "positive inventory", so the ETS method over forecasts in comparison with the MoHW method. When the joint model is applied (Table 5), the MoHW method increases forecasts and higher orders are placed. Consequently, the fill rate of the MoHW method increases.

So, if the joint model is used the adapted forecasts cause more efficient ordering which provides the appropriate order-up-to level and consequently lowers the total costs and improves the fill rate.

Table 5: *Fill rate results obtained with the joint optimisation*

	JOINT	FILL RATE					
		penalty = 3			penalty = 5		
		AHW	MHW	MoHW	AHW	MHW	MoHW
Discipline	DEMOGRAPHIC	***99.37%***	99.22%	99.34%	99.52%	99.32%	99.61%
	FINANCE	99.10%	98.91%	99.23%	99.43%	99.31%	99.55%
	INDUSTRY	98.84%	98.77%	98.93%	99.36%	99.31%	99.44%
	MACRO	99.53%	99.46%	**99.64%**	99.71%	99.66%	**99.80%**
	MICRO	98.12%	97.71%	98.62%	98.83%	98.45%	99.43%
	OTHER	99.12%	99.19%	99.22%	99.58%	***99.71%***	99.68%
	JOINT	FILL RATE					
		penalty = 3			penalty = 5		
		AHW	MHW	MoHW	AHW	MHW	MoHW
Type	ANN	98.74%	98.22%	98.75%	99.30%	98.93%	99.56%
	ANA	98.83%	98.81%	98.88%	99.40%	99.39%	99.48%
	AAN	99.56%	99.27%	**99.63%**	99.70%	99.49%	**99.84%**
	AAA	99.23%	99.15%	99.38%	99.51%	99.46%	99.68%
	MNN	98.50%	98.06%	98.81%	99.16%	98.95%	99.49%
	MNA	98.76%	98.45%	98.99%	99.23%	98.82%	99.50%
	MNM	97.82%	97.49%	98.20%	98.75%	98.45%	99.05%
	MAN	99.34%	99.09%	99.36%	99.57%	99.44%	99.66%
	MAA	99.05%	99.07%	99.27%	99.44%	99.51%	99.64%
	MAM	98.47%	98.55%	98.85%	99.02%	98.99%	99.39%
	MMN	98.93%	98.85%	99.23%	99.29%	99.11%	99.67%
	MMM	98.20%	98.30%	99.01%	98.72%	98.70%	99.45%
	Total	**98.83%**	**98.63%**	**99.05%**	**99.29%**	**99.12%**	**99.55%**

Finally, if we use joint optimisation with the MoHW method (JMoHW) instead of the models where forecasts are calculated with the AHW, MHW or ETS method regarding minimising MSE, we can observe the following (see Table 6): on average JMoHW can reduce the average costs by more than 24% (23% and 28%) in comparison with the AHW (MHW and ETS) method for penalty = 3 and by more than 41% (40% and 43%) for penalty = 5.

The averages of the SREM1 within different disciplines vary between 18.9% and 33.1% for penalty = 3 and between 33.5% and 48.9% for penalty = 5. The JMoHW substantially outperforms other methods for microeconomic time series. The averages of the SREM1 within different classes vary between 18.3% and 36.2% for penalty = 3 and between 35.5% and 51.9% for penalty = 5. Also, the JMoHW method substantially outperforms the classical methods if a time series does not have a trend and a seasonal component. For these two classes, ANN and MNN, the averages of the SREM1 vary between 25.9% and 36.2% for penalty = 3 and between 44.3% and 51.9% for penalty = 5.

Table 6: *Averages of the SREM1 (comparison of the joint model with the MoHW method and models in which forecasting and an inventory model were treated separately)*

	JOINT/MSE	**SREM1**					
		penalty = 3			**penalty = 5**		
		JMOHW/ AHW	JMOHW/ MHW	JMOHW/ ETS	JMOHW/ AHW	JMOHW/ MHW	JMOHW/ ETS
Discipline	DEMOGRAPHIC	18.9%	19.0%	22.6%	33.5%	33.7%	36.7%
	FINANCE	21.5%	20.8%	27.0%	36.8%	36.6%	41.6%
	INDUSTRY	23.1%	23.2%	24.8%	39.6%	39.7%	41.1%
	MACRO	23.1%	22.3%	26.1%	39.2%	38.6%	42.1%
	MICRO	**27.2%**	**25.9%**	**33.1%**	**46.2%**	**45.3%**	**48.9%**
	OTHER	27.7%	28.6%	31.7%	46.2%	46.9%	48.3%
Type	ANN	**27.3%**	**27.1%**	**36.2%**	**45.7%**	**45.7%**	**51.9%**
	ANA	23.9%	24.5%	26.9%	41.0%	41.4%	43.5%
	AAN	20.1%	21.8%	27.9%	35.5%	37.1%	42.0%
	AAA	23.1%	23.4%	24.8%	40.7%	41.1%	42.2%
	MNN	**27.1%**	**25.9%**	**35.2%**	**45.2%**	**44.3%**	**50.6%**
	MNA	24.4%	26.6%	26.5%	42.3%	44.1%	42.9%
	MNM	22.6%	18.3%	21.0%	40.0%	36.7%	37.0%
	MAN	23.7%	23.1%	29.0%	40.2%	39.8%	44.4%
	MAA	23.3%	23.8%	25.3%	40.4%	40.9%	41.8%
	MAM	26.3%	23.4%	25.1%	42.8%	40.6%	41.6%
	MMN	24.6%	24.3%	30.5%	41.8%	41.7%	46.4%
	MMM	24.6%	20.6%	23.1%	40.6%	37.6%	38.7%
	Total	**24.1%**	**23.5%**	**28.1%**	**41.2%**	**40.8%**	**43.9%**

As we can see, the JMoHW method outperforms all three methods and it does not perform generally worse in any of the classes, which indicates the universality of the JMoHW method. The JMoHW method is general enough to be used as the encompassing method when the same method is applied to all time series.

5 CONCLUSION

Demand forecasting is used throughout the world more often because of proper source management and the rising need to plan. One of the most commonly used forecasting techniques is exponential smoothing, which is relatively inexpensive, fast and simple.

In this paper we presented the modified Holt-Winters method and the problem of the local optimisation of forecasting methods when the calculated forecasts are used in the inventory model. We therefore proposed the MoHW method for a simultaneous optimisation of demand forecasting and a stock control policy. The method is computationally stable, requires little storage and produces results that are easy to interpret.

This paper differs from the study of Ferbar Tratar (2015b) in adding interaction between the total cost and fill rate of the supply chain. In Ferbar Tratar (2015a), the simulation (not real data) study showed that the modified HW method can reduce the forecast error (MSE) in comparison with the other classical methods (AHW, MHW). The analysis is focused on the influence of parameters (slope, seasonality and noise) that they have on MSE obtained with the modified method. In Ferbar Tratar (2010) for the first time the problem of "the local optimisation" of the forecasting methods (AHW, MHW and improved multiplicative (not additive) HW method) was exposed. The added value to Ferbar Tratar (2010) is a case study of 1,428 real time series (and detailed inspection within different classes of discipline and type), validation of functionality of the joint model with the modified HW method and investigation into how the minimisation of the total cost influences the fill rate.

We tested the method on 1,428 real series from M3-Competition. We developed the symmetric relative efficiency measure to compare the performance of different methods. Taking averages of these measures across several time series allowed us to indicate which method is preferable in general. We showed that forecasts interact with the inventory model and consequently result in lower inventory costs as well as higher fill rates when the joint model is used. The average total costs can be reduced on average by more than 25% for penalty = 3 and by more than 41% for penalty = 5 in comparison with the models where forecasts are calculated with the AHW, MHW or ETS method regarding minimising MSE and treated separately from the inventory model. At the same time, the joint model with the MoHW method improves fill rate on average by 2.5 percentage points for penalty = 3 and by 3 percentage points for penalty = 5.

Based on the M3-Competition monthly time series we showed that the MoHW method is particularly good for microeconomic time series and for time series with multiplicative noise, trend and seasonal component. We showed that the MoHW method is general enough to be used as the encompassing method when the single method is applied to all time series.

As the method can be easily implemented in an Excel spreadsheet, we suggest that the managers and supply chain decision-makers use the JMoHW method to make better predictions and reduce costs.

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