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SIMULATION OF FLUID SLOSHING IN A TANK USING THE SPH AND PENDULUM METHODS

SIMULACIJA PLJUSKANJA KAPLJEVINE V CISTERNI PO METODI HIDRODINAMIKE ZGLAJENIH DELCEV (SPH) IN PO METODAH NIHALA

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Abstract

We simulated liquid sloshing in a circular road tanker during two typical manoeuvres, namely steady-turn and lane change. A quasi-static pendulum, a modified dynamic pendulum with adjustable rod length, and the SPH (Smooth Particle Hydrodynamics) model Tis Isat were applied to simulate liquid oscillations. A simplified vehicle-tanker overturning model was developed and applied in order to determine the overturning threshold for the first manoeuvre. The agreement of liquid oscillations between the applied methods was better in the second manoeuvre, while the maximum inclination of the liquid gravity centre was successfully simulated in both cases. Both dynamic methods, the SPH and the dynamic pendulum, show a significantly lower overturning threshold, while all methods show similar overturning behaviour for a vehicle with liquid cargo. The threshold computed using the SPH model is slightly higher than with the dynamic pendulum due to wall-particle and particle-particle interactions. The results and comparisons confirm the suitability of the SPH method for simulating sloshing in road tankers and also show the method's advantages: realistic description of the non-linear free surface in real-time, along with a consideration of mixing and friction processes within the fluid.

Keywords: SPH method, pendulum, fluid oscillations, closed domain, liquid cargo, overturning threshold.

Izvleček

V prispevku obravnavamo pljuskanja kapljevine v cisterni krožnega prečnega prereza med dvema tipičnima manevroma: vstopom in vožnjo skozi krožišče ter menjavo voznega pasu. Za simulacije nihanja kapljevine smo uporabili tri metode: modela kvazistatičnega nihala in modificiranega dinamičnega nihala s spremenljivo ročico ter model Tis-Isat, ki deluje po metodi hidromehanike zglajenih delcev (SPH – Smooth Particle Hydrodynamics). Za določitev praga prevrnitve pri prvem manevru smo izdelali in uporabili model poenostavljenega vozila s cisterno. Ujemanje nihanja kapljevine je bilo boljše pri drugem manevru, maksimalni odmik težišča kapljevine pa smo uspešno simulirali v obeh obravnavanih primerih. Vse tri metode pokažejo podobno obnašanje vozila s tekočim tovorom ob prevrnitvi, obe dinamični metodi, SPH in dinamično

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nihalo, pa dajeta znatno nižji prag prevrnitve. Pri SPH je prag prevrnitve nekoliko višji kot pri dinamičnem nihalu zaradi interakcije med delci kapljevine ter kapljevino in ostenjem. Primerjava rezultatov dinamičnih metod potrjuje uporabnost metode SPH za simulacije pljuskanja v tovornih cisternah, prav tako pa tudi prednosti metode SPH: opis nelinearne proste gladine v realnem času ter upoštevanje viskoznih procesov in mešanja v kapljevini.

Ključne besede: metoda SPH, nihalo, nihanje kapljevine, zaprto območje, tekoči tovor, prag prevrnitve.

1. Introduction

Traffic accidents with a rollover of liquid cargo are relatively frequent and the environmental effect of spilled fluids can be severe, as the fluids being transported often contain toxic and environmentally hazardous substances. The exact mechanism of overturning is not yet well understood. Various methods have been applied to describe the forces of sloshing liquid on the tank and the vehicle. Since the 1960s various pendulum-based methods have been used for calculations with several tank shapes (Rattaya, 1965) and in developing simplified models of liquid sloshing (Abramson, 1966; Rattaya, 1965). Computations are usually based on the analogy between the movement of the liquid gravity centre and the oscillations of a pendulum (Aliabadi et al., 2003; Casasanta, 2010; Salem, 2000). The pendulum's parameters are determined by the shape of the tank and the type of liquid (Rattaya, 1965). Several studies on two- and three-dimensional sloshing were also carried out using finite element methods (Aliabadi et al., 2003; Behr and Tezduyar, 1994; Okamoto and Kawahara, 1990; Ramaswamy et al., 1986; Wu et al., 1998). Furthermore, analytical solutions were explored (Kolaei et al., 2014) as well as other numerical methods, such as boundary element methods (Kolaei et al., 2015) and the particle based MSP (moving particle semi-implicit) method (Jena and Biswal, 2017). Recently the SPH (smoothed particle hydrodynamics) method has been applied for research studies on similar phenomena due to its ability to simulating the exact free surface, the interactions between the particles, and the interactions between the liquid and the solid boundaries. The SPH method's appropriateness for simulating sloshing in tankers and tanks has been examined and verified against laboratory experiments and comparisons. Various SPH models have been used recently to perform such simulations (Cao et al., 2014; Shao et al., 2012; Vorobyev et al., 2011). In this study we applied the Tis Isat model (Džebo et al., 2014, 2013; Petkovšek et al., 2010) developed at the Faculty of Civil and Geodetic Engineering of the University of Ljubljana.

Assessing the forces of sloshing liquid facilitates computation of tanker's (vehicle's) overturning. The threshold of vehicle overturning was studied extensively by Issermann (1976), who also proposed an analytical model. Another analytical model was proposed by Kolaei, Rakheja, and Richard (Kolaei et al., 2014). Strandberg (1978) compared the overturning of vehicles carrying solid and liquid cargo based on laboratory experiments and examined the dynamic forces caused by splashing during the overturning of a vehicle. Rakheja and Ranganathan (1993) proposed a simple methodology to estimate the rollover threshold of partially filled liquid cargo vehicles. Salem (2000) developed 2D and 3D models using the pendulum method. Kang et al. (2001) analysed partially filled tanks of various cross sections (circular, modified oval and two optimized cross sections). They also performed a sensitivity analysis for several parameters describing the roll stability characteristics of vehicles carrying partially filled tanks. Modaressi-Tehrani et al. (2006) analysed the overturning moment caused by transient liquid slosh inside a partially-filled moving tank with different overturning models.

In this study we present a modification of the method described in Aliabadi et al. (2003) by using a pendulum rod of adjustable length. We furthermore present a comparison of forces during a typical manoeuvre computed with the SPH model Tis-Isat and the modified pendulum model. The forces and the moments due to sloshing are applied in a simplified overturning model. The physical principles behind the oscillations of a solid body (pendulum) and particle-described liquid (SPH) differ significantly; we therefore expect significant differences in forces, moments, and overturning threshold between the two methods. We also expect to obtain a more detailed description of liquid behaviour using the SPH method.

The main aim of the performed study was to estimate the rollover threshold of the vehicle and to compare the differences between the two applied methods with regard to the computation of forces and moments in partially filled tanks with increasing free surface levels.

2. Methods and materials

2.1 SPH model Tis-Isat

Tis-Isat is a mesh-free, particle-based numerical model based on the SPH method (Gingold and Monaghan, 1977; Lucy, 1977). It solves the mass conservation equation, momentum equation, and the equation of state for weakly compressible fluids, and also introduces a non-discrete boundary condition with friction into the SPH method (Petkovšek et al., 2010). The derivation of basic equations is described in numerous sources (Delorme, 2009; Gomez-Gesteira et al., 2010; Jones and Belton, 2006; Liu and Liu, 2003; Monaghan, 1992). In the Tis Isat model we used the cubic spline kernel function proposed by Monaghan and Lattanzio (1985), the Verlet algorithm for time stepping (Verlet, 1967), and additional density flux between particles (Molteni and Colagrossi, 2008) for preventing oscillations of the pressure field. Re-initialisation was performed every 10 time steps in order to prevent the simulations from instability.

The Tis-isat model was verified against two benchmark experiments. The modelling of two-dimensional water column collapse (Martin and Moyce, 1952) and the propagation of dam break wave (Rajar, 1972) are described in Petkovšek et al. (2010) and Džebo et al. (2014), respectively.

2.2 Pendulum model

Several pendulum methods are available for simulating fluid sloshing in circular tanks (Abramson, 1966; Aliabadi et al., 2003; Casasanta, 2010; Ranganathan et al., 1993; Rattaya, 1965; Salem, 2000). We modified the pendulum method for circular tanks described in Aliabadi et al. (2003) by using a pendulum rod of adjustable length. The following assumptions are valid for the pendulum (Figure 1): (i) the mass of liquid is concentrated in the gravitational centre of the fluid at the end point of the rod; (ii) the rod is rotating around a frictionless pivot in the fixed centre point of the tank S; (iii) a constant inertial force is impacting the pendulum due to centrifugal acceleration. The force in the rod F_v is computed from the oscillation equations and depends on the parameters of the model: arm length l and fluid mass m_L. These depend on the level of fluid in the tank and the fluid density.

Following Aliabadi et al. (2003), the governing equation of pendulum motion due to gravitational (g) and radial ($\underline{a_Y}$) accelerations is:

$$\ddot{\theta} = \frac{1}{l}(a_Y \cdot \cos\theta - g \cdot \sin\theta) \tag{1}$$

$$\ddot{\theta} = \frac{\mathrm{d}\dot{\theta}}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} \tag{2}$$

where θ denotes the angle between the rod and the vertical axis, while the two derivatives over time ($\dot{\theta}$ and $\ddot{\theta}$) denote angular velocity and angular acceleration. $\dot{\theta}$ and θ can be computed from the acceleration using trapezoidal numerical integration in each time step with the following initial conditions: $\theta|_{\mathrm{t}=0} = 0, \dot{\theta}\big|_{\mathrm{t}=0} = 0$.

The forces F_v computed using the modified model were compared to the results of the original model (Aliabadi et al., 2003). We obtained excellent agreement for four levels of liquid using a standard calibration manoeuvre, where the horizontal accelerations a_Y arise due to the tanker traveling at a constant speed of 10 m·s^{-1} through a curve with a constant radius 250 m (Vidmar, 2012), Figures 6 and 7 therein).

2.3 Overturning models

2.3.1 Quasi-static pendulum overturning model

In the first approximation of rollover threshold computations we used a two-dimensional simplified overturning model presented by Rakheja and Ranganathan (1993). We assume that the forces acting on vehicle do not change temporally and refer to the model as quasi-static. If we assume (i) a small roll angle; (ii) a low unsprung weight; (iii) that the roll centre is in the ground plane, then we can determine the point of roll instability. Vehicle rollover occurs when the overturning moment exceeds the restoring moment; then, the horizontal acceleration is calculated as (Rakheja and Ranganathan, 1993):

$$a_{Yn} = \frac{(W_T + W_L)}{W_T \cdot h_T + W_L \cdot (h_L + R - Z_{tt})} \cdot T \tag{3}$$

where a_{Yn} denotes normalized lateral acceleration with respect to g, W_T is the vehicle weight without cargo, W_L is the weight of liquid cargo, h_T and h_L denote the central gravity heights of the vehicle and the cargo, respectively, R is the diameter of the tank, T is the vehicle's half-width, and Z_{tt} is the initial distance between the bottom of the tank and the gravity centre of the fluid, depending on the quantity (level) of fluid in the tank (Figures 1 and 2).

2.3.2 Dynamic pendulum overturning moment

For the final computations of rollover threshold and comparison to the SPH model, we used dynamic equations in the pendulum model. The acceleration of the pendulum a_{Yp} due to the radial acceleration (a_Y) and the gravity (g) is:

$$a_{Yp} = -g \cdot \sin\theta + a_Y \cdot \cos\theta\ , \tag{4}$$

while the force in the rod F_v and the force of liquid on the tank F_C can be written as:

$$F_V = \left(-g \cdot \cos\theta + a_{Yp} \cdot \sin\theta\right) \cdot m_L \tag{5}$$

$$F_C = -F_V\ , \tag{6}$$

where m_L denotes the mass of the liquid.

The two terms in the overturning moment (M_O) equation

$$M_O = M_T + M_L \tag{7}$$

denote the contributions of the vehicle (M_T) and the oscillating liquid (M_L). These are further calculated as:

$$M_T = F_T \cdot h_T \tag{8}$$

$$F_T = -m_T \cdot a_Y\ , \tag{9}$$

where F_T denotes the force of the vehicle, with the vehicle's mass denoted by m_T; and

$$M_L = F_C \cdot H_S\ , \tag{10}$$

where H_s is the lever in the oblique plane:

$$H_S = h_S \cdot \sin\theta \tag{11}$$

and h_s is the distance between the centre of the tank and ground.

2.3.3 SPH method overturning moment

In the SPH model (Figure 3), the equations for calculating the overturning moment are as follows:

$$\mathbf{M_{Lj}} = \textstyle\sum_i^n \mathbf{M_{Cij}} \tag{12}$$

which is an equivalent to Eq. (10) in the j-th time step, where

$$\mathbf{M_{Cij}} = \mathbf{F_{Cij}} \times \mathbf{d_{ij}} \tag{13}$$

and $\mathbf{F_{Cij}}$ is the force of the i-th particle on the tank in j-th time step. Here

$$\mathbf{F_{Cij}} = -\,\mathbf{F_{Dij}} \tag{14}$$

and $\mathbf{F_{Dij}}$ can be written as

$$\mathbf{F_{Dij}} = \left[F_{YDij}\ F_{ZDij}\right] \tag{15}$$

where F_{YDij} and F_{ZDij} are represented by:

$$F_{YDij} = m_i \cdot a_{Yij} \tag{16}$$

$$F_{ZDij} = m_i \cdot a_{Zij} \tag{17}$$

and m_i and d_{ij} are the mass and the position vector of the i-th particle, respectively. The last vector in Eq. (13) is:

$$\mathbf{d_{ij}} = \left[d_{Yij}\ d_{Zij}\right] \tag{18}$$

Acceleration of the i-th particle in the j-th time step is further calculated from particle velocities as:

$$a_{Yij} = a_Y + \frac{v_{Yij(t+dt)} - v_{Yij(t)}}{dt} \tag{19}$$

$$a_{Zij} = -g + \frac{v_{Zij(t+dt)} - v_{Zij(t)}}{dt} \tag{20}$$

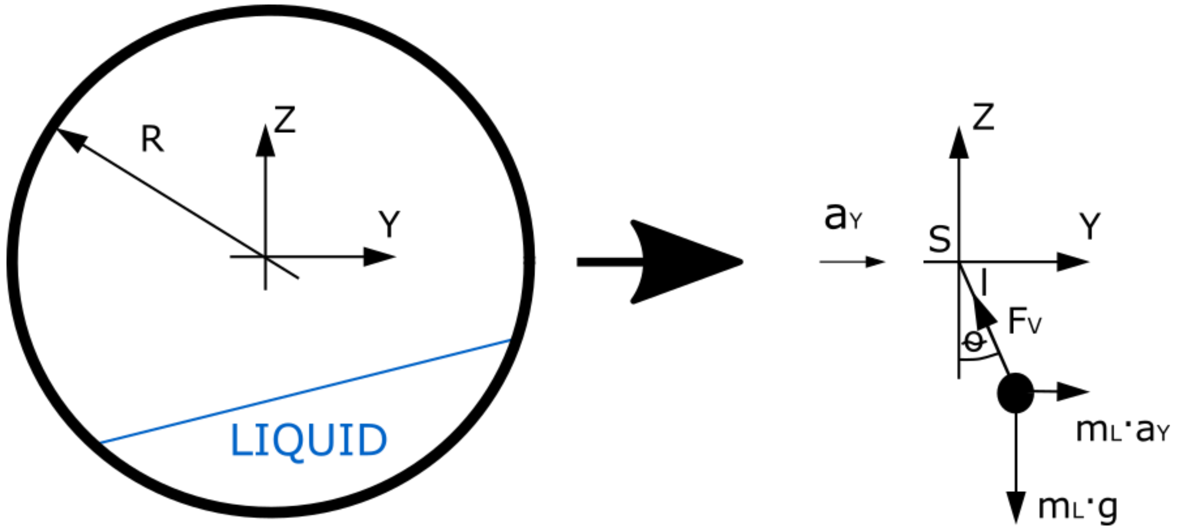

Figure 1: *Pendulum model.*

Slika 1: *Model nihala.*

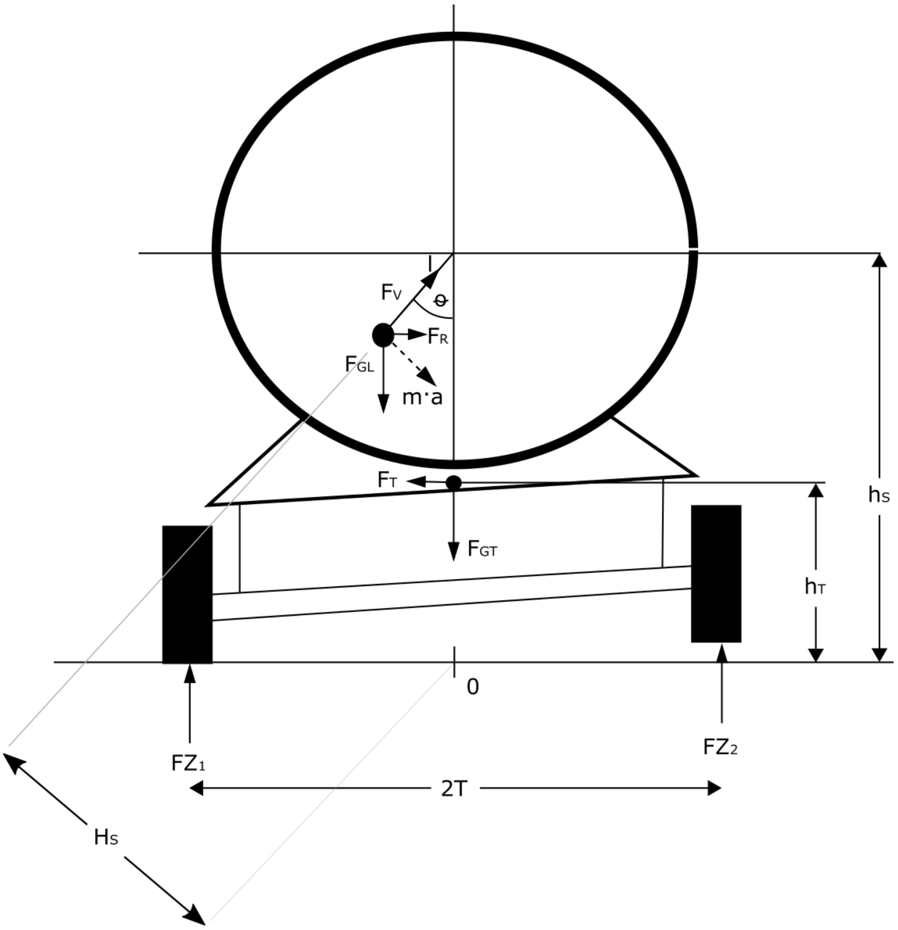

Figure 2: *Overturning mechanism in the pendulum method.*

Slika 2: *Prevrnitveni mehanizem pri metodi nihala.*

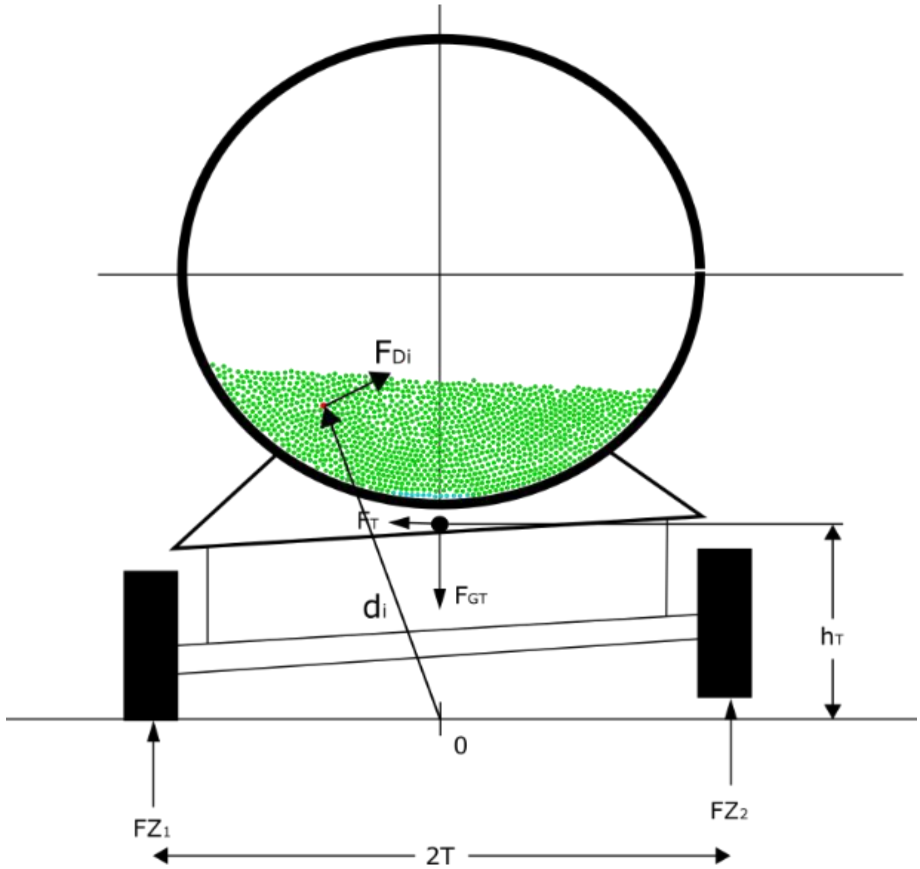

Figure 3: *Overturning mechanism in the SPH method.*

Slika 3: *Prevrnitveni mehanizem pri metodi SPH.*

Table 1: *Vehicle and cargo parameters.*

Preglednica 1: *Parametri vozila in tovora.*

Truck		
Weight	46882	N
Trailer and cargo		
Weight (empty)	87448	N
Diameter	2.00	m
Centre height of trailer and tank	2.29	m
Center height of tank	3.29	m

Table 2: *Pendulum and SPH parameters.*

Preglednica 2: *Parametri nihala in metode SPH.*

Filling [%]	W_L [N]	Particles
10	20511	640
20	56113	1752
30	99442	3104
40	147214	4595
50	197058	6151
60	246902	7707
70	294675	9198
80	338003	10551
90	373606	11662

Several time-step lengths were tested in order to achieve stable simulations and a smooth graphic representation of results. We adopted $dt = 10^{-4}$ s in all SPH simulations.

The restoring moment of the observed system depends on the gravity centre position of the sloshing liquid; when depicted by a pendulum, the gravity centre can be determined from the pendulum's inclination, while when using the SPH method it can be calculated from the particles' position vectors. In order to be consistent with the quasi-static pendulum we adopted another simplification suggested by Rakheja and Ranganathan (1993), and Ranganathan et al. (1993). A small inclination was adopted for the pendulum in Eq. (3), inducing the maximum restoring moment (of static cargo) to resist to the overturning. Even though the inclination can be significant, particularly with lower liquid levels, we adopted the small angle principle in order to enable a comparison of the three applied methods.

The resisting moment is therefore independent of the applied method:

$$M_R = (m_T + m_L) \cdot g \cdot T \tag{21}$$

where m_T and m_L are the mass of the vehicle and the liquid cargo, respectively; g is the gravitational constant and T is the half width of the vehicle (Figure 2). In all performed computations we used a vehicle described by the parameters in Table 1, while the pendulum and SPH parameters are described in Table 2.

2.4 Manoeuvres

We present two characteristic manoeuvres: (i) lateral liquid cargo shifting during a steady turn – driving through a curve with constant radius (Rakheja and Ranganathan, 1993); (ii) sloshing due to a lane change manoeuvre – avoiding an obstacle by changing the lane, a standard NATO AVTP 03-160W test (Casasanta, 2010). The input data for each manoeuvre are presented below.

2.4.1 Steady turn

In this manoeuvre the vehicle turns instantaneously from a straight section into a constant radius curve at time t_s. In both pendulum methods the radial acceleration applied as the input data was:

$$Acc_R = \begin{cases} 0, 0 < t < t_s \\ \frac{v_t^2}{r}, t_s < t < t_{end} \end{cases} \tag{22}$$

where Acc_R is radial acceleration, v_t denotes tangential velocity, r is the curve radius, while t_s and t_{end} mark the manoeuvre's beginning and end, respectively.

In the SPH method we adjusted the input data; instead of acceleration, we applied the acceleration integral (VEL) with the input data given in Table 3:

$$VEL = \begin{cases} 0, 0 < t < t_s \\ \int \frac{v_t^2}{r} dt,\ t_s < t < t_{end} \end{cases} \tag{23}$$

2.4.2 Lane change manoeuvre

Similarly, the lane change manoeuvre requires an adjustment of the input data in the SPH model. In the pendulum method we applied radial acceleration, approximated by two smooth symmetrical sinusoidal curves, following the principle described in Casasanta (2010):

$$Acc_R = \begin{cases} \frac{v_t^2}{r} \cdot \sin\omega(t), 0 < t < T \\ -\frac{v_t^2}{r} \cdot \sin\omega(t-T), T < t < 2T \\ 0, 2T < t < t_{end} \end{cases} \tag{24}$$

Where

$$T = \frac{L}{v_t} \tag{25}$$

and

$$\omega = \frac{2\pi}{T} \tag{26}$$

In Eq. (24–27), ω denotes the angular frequency and T is the oscillation period.

For the SPH method we again used the integral of acceleration defined as:

$$VEL = \begin{cases} \int \frac{v_t^2}{r} \cdot \sin\omega(t), 0 < t < T \\ \int -\frac{v_t^2}{r} \cdot \sin\omega(t-T), T < t < 2T \\ const = 0, 2T < t < t_{end} \end{cases} \tag{27}$$

where v_t denotes tangential velocity and t_{end} represents the manoeuvre end (Table 4).

Table 3: *Parameters of the steady turn manoeuvre.*

Preglednica 3: *Parametri manevra ob vožnji v križišče.*

Manoeuvre 1		
t_s	0	s
t_{end}	5	s
dt	10^{-4}	s
r	250	m
v_t	10	$m \cdot s^{-1}$

Table 4: *Parameters of the lane change manoeuvre.*

Preglednica 4: *Parametri manevra ob menjavi voznega pasu.*

Manoeuvre 2		
T	9.29	s
t_{end}	20.00	s
dt	10^{-4}	s
L	103.9	m
r	12.5	m
v_t	11.18	$m \cdot s^{-1}$

3. Results and discussion

3.1 Manoeuvre 1: Constant radius

Dynamic pendulum and SPH simulations of manoeuvre 1 were in relatively good agreement by low fluid levels (Figure 4). The differences between the applied methods, in both phase and amplitude, increase proportionally with the liquid height (Figure 5). Such a disagreement was expected, as the applied methods simulate fluid oscillations on a different basic principle. All pendulum methods are approximate by considering the oscillations of a non-deformable body; the accuracy of these methods accordingly decreases with the filling height when the dynamic phenomena within the fluid become more significant. On the contrary, the SPH method simulates direct dynamic interactions between the neighbouring moving fluid particles. The gravity centre therefore oscillates differently when using the SPH method, and the oscillations are severely damped due to viscous friction between particles. Casasanta (2010) demonstrated that damping only had a minor influence on pendulum oscillations. Furthermore the damping coefficients would be extremely difficult to determine. Therefore, we omitted damping in all simulations performed with pendulums. The comparison of maximum gravity centre inclination (Figure 6) reveals approximately same level of agreement between the methods (within 20%, decreasing with fluid level).

3.2 Manoeuvre 2: Lane change

Although the lane change manoeuvre is longer and more complicated due to time-variable radial acceleration, we achieved good visual agreement between the results of the applied methods (Figures 7 and 8).

The agreement in both amplitude and phase is better when fluid levels are higher, with lower secondary oscillations of the gravity centre achieved by using the SPH method (Figure 8). The most probable reason for smoother curve using the SPH method is "noise"-damping due to mixing and friction between particles, a process typical for particle-based methods.

Pendulum methods are well established for simulations of liquid oscillation in closed domains, being used for more than 50 years (e.g. Abramson, 1966). We achieved relatively good agreement in gravity centre oscillations using the SPH method. Based on the simulations performed we can conclude that the SPH method is suitable for simulations in similar case studies. Moreover, the significant advantages of this method lie in the real-time graphic representation of the liquid (particles) oscillation, the sloshing of the liquid in a closed domain, and the description of the non-linear free surface (Figure 9).

We also compared maximum inclination of the gravity centre between the methods (Figure 10). The difference between the methods is mostly within 10º, exceeding 10% relative discrepancy only at low liquid levels, and generally decreasing with the filling height.

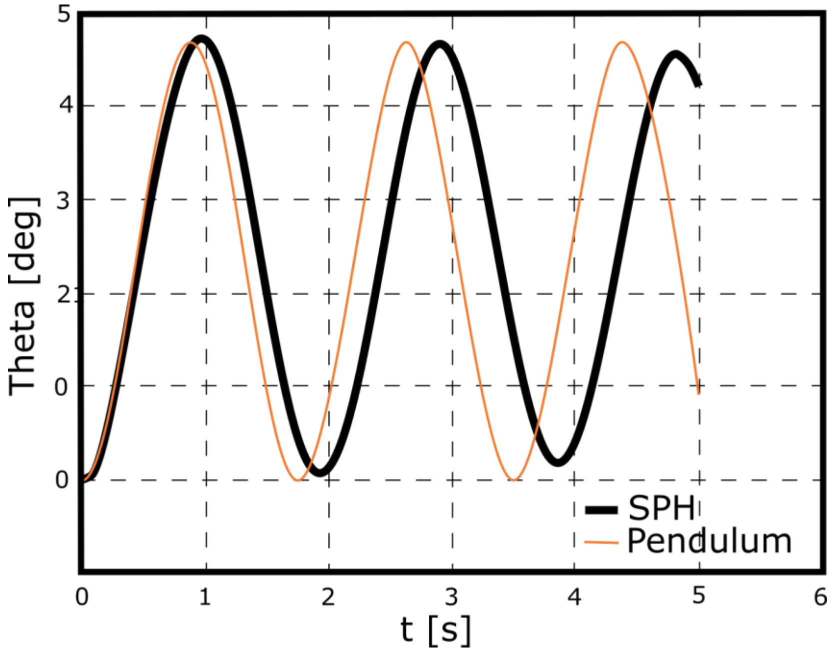

Figure 4: *Manoeuvre 1: comparison of gravity centre oscillations between the SPH and dynamic pendulum; 20% filling height.*

Slika 4: *Manever 1: primerjava nihanja težišča med rezultati metode SPH in dinamičnim nihalom; 20 % polnitev.*

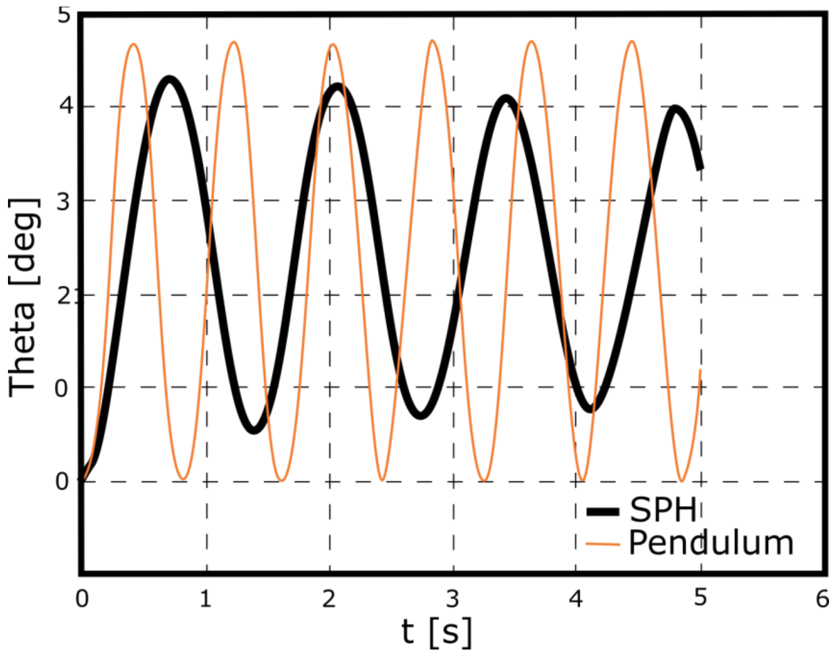

Figure 5: *Manoeuvre 1: comparison of gravity centre oscillations between the SPH and dynamic pendulum; 80% filling height.*

Slika 5: *Manever 1: primerjava nihanja težišča med rezultati metode SPH in dinamičnim nihalom; 80 % polnitev.*

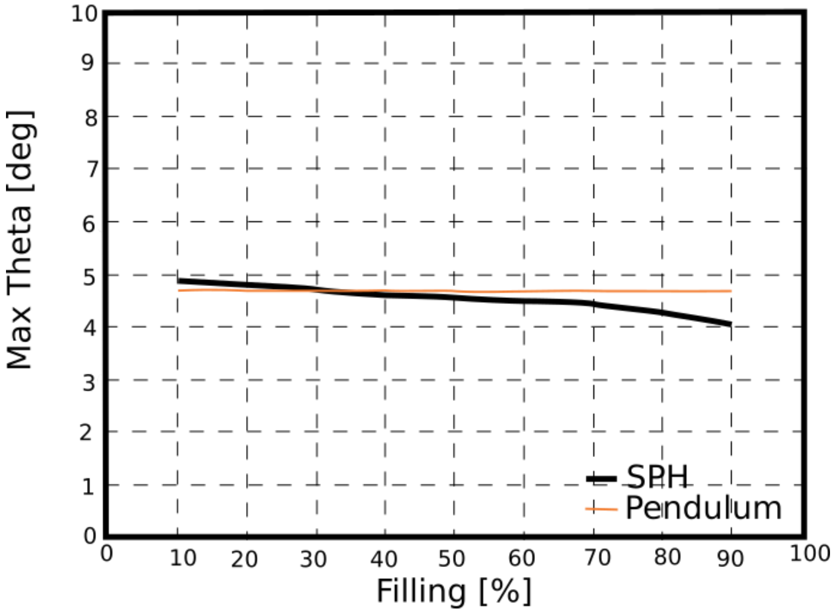

Figure 6: *Manoeuvre 1: maximum gravity centre inclination as a function of filling height.*

Slika 6: *Manever 1: maksimalni odmik težišča v odvisnosti od višine polnitve.*

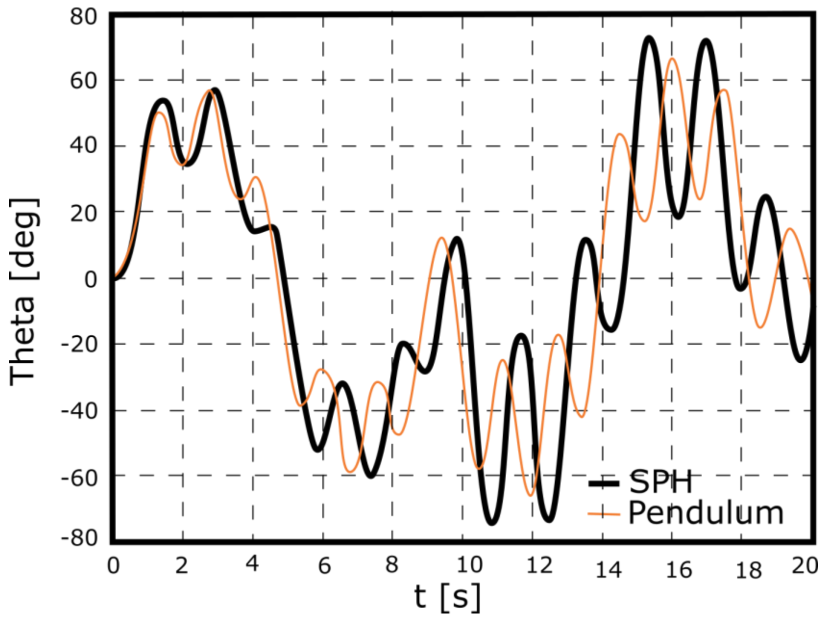

Figure 7: *Manoeuvre 2: comparison of gravity centre oscillations between the SPH and dynamic pendulum; 20% filling height.*

Slika 7: *Manever 2: primerjava nihanja težišča med rezultati metode SPH in dinamičnim nihalom; 20 % polnitev.*

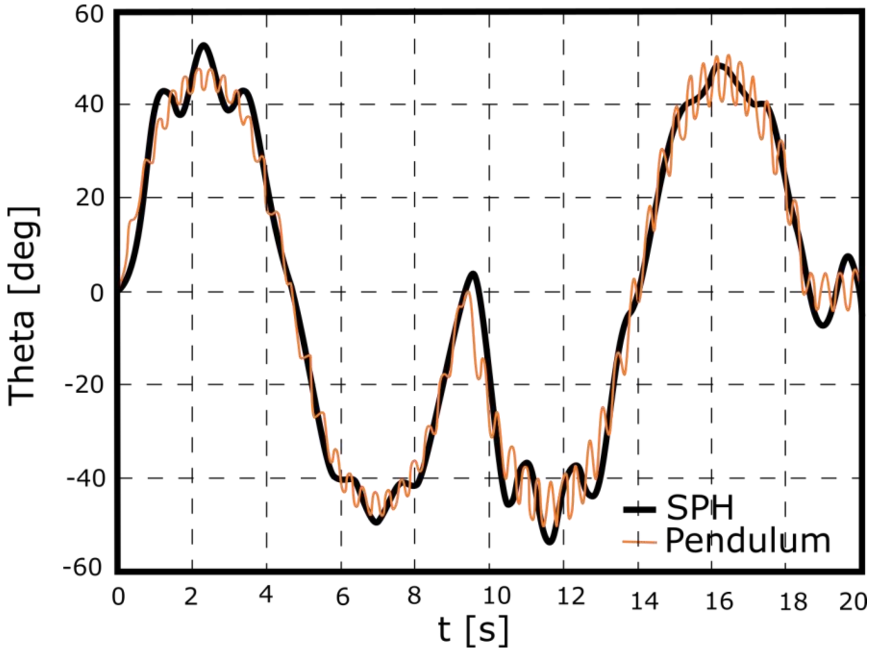

Figure 8: *Manoeuvre 2: comparison of gravity centre oscillations between the SPH and dynamic pendulum; 80% filling height.*

Slika 8: *Manever 2: primerjava nihanja težišča med rezultati metode SPH in dinamičnim nihalom; 80 % polnitev.*

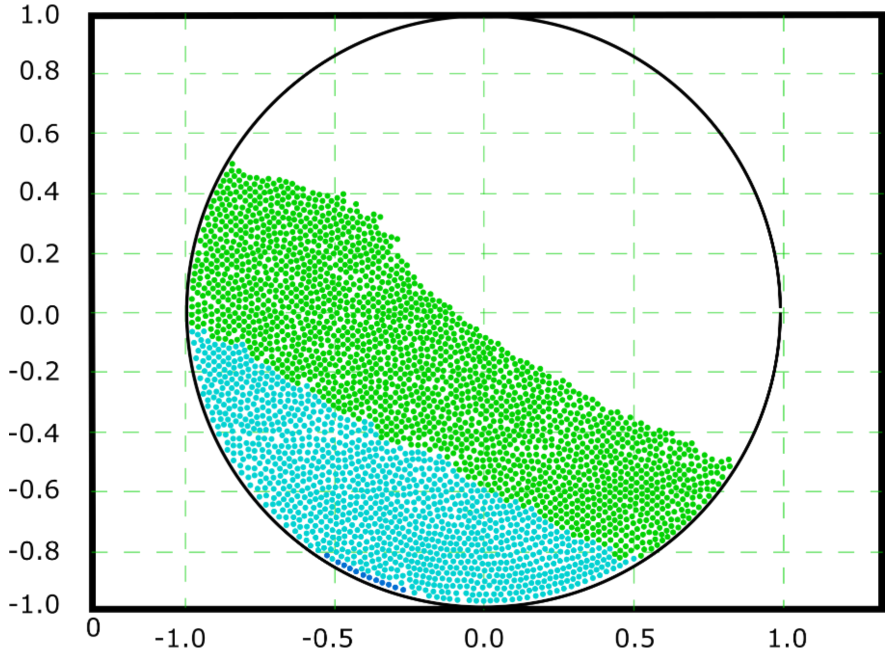

Figure 9: *Manoeuvre 1 (overturning threshold determination): position of particles in time t = 3.8 s; 50% filling height.*

Slika 9: *Manever 1 (določanje praga prevrnitve): položaj delcev v času t = 3,8 s; 50 % polnitev.*

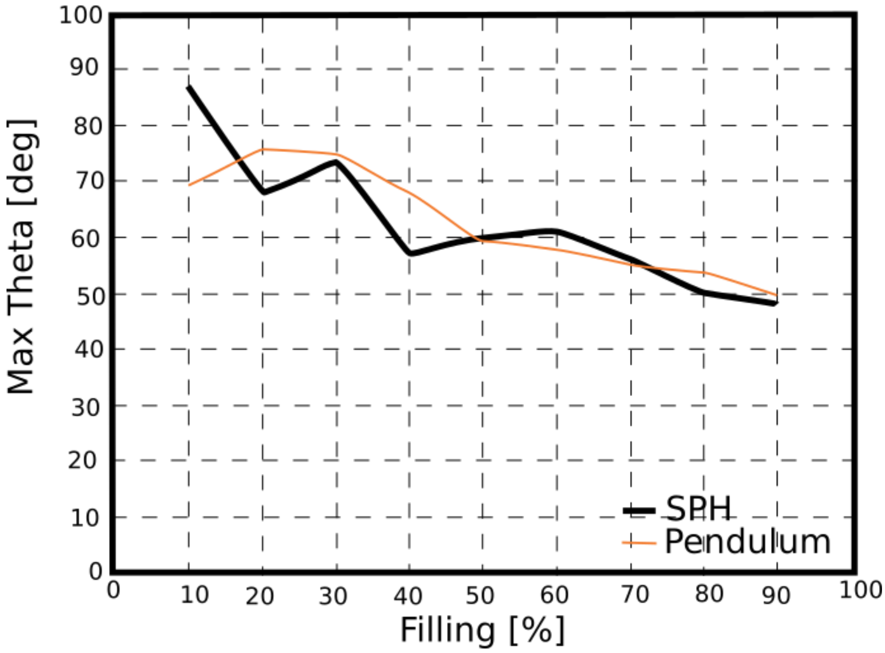

Figure 10: *Manoeuvre 2: maximum gravity centre inclination as a function of filling height.*

Slika 10: *Manever 2: maksimalni odmik težišča v odvisnosti od višine polnitve.*

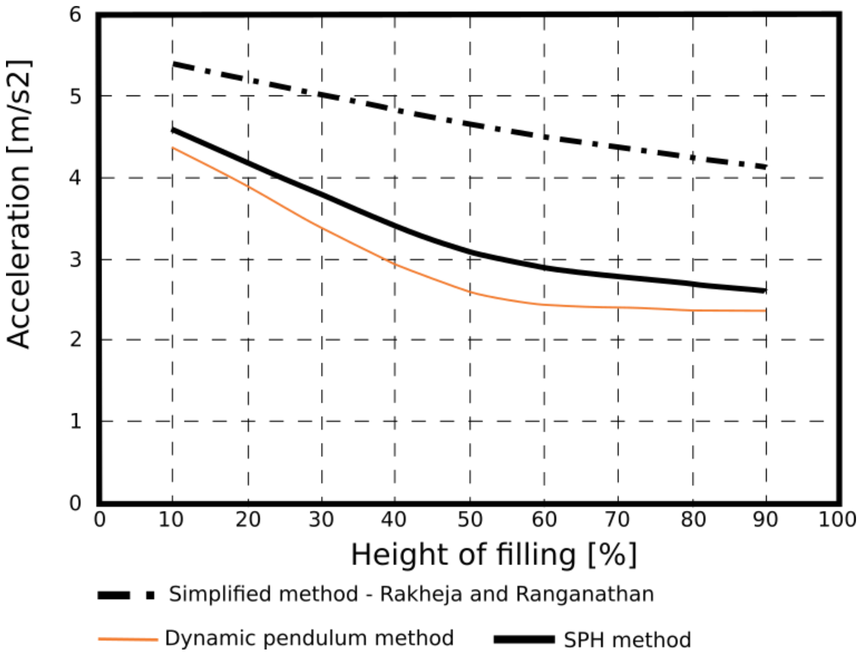

Figure 11: *Overturning threshold – acceleration vs. filling height; comparison between quasi-static and dynamic pendulum, and the SPH methods.*

Slika 11: *Prag prevrnitve – pospešek v odvisnosti od višine polnitve; primerjava kvazistatičnega in dinamičnega nihala ter metode SPH.*

3.3 Manoeuvre 1: Overturning threshold

We determined the overturning threshold for manoeuvre 1 using all described methods. The overturning model suitable for cylindrical tankers performing steady-turn manoeuvre, described by Rakheja and Ranganathan (1993) was applied. In the simulated cases we varied the tangential velocity and the corresponding radial acceleration until the overturning moment M_O reached the resisting moment M_R.

The radial acceleration causing overturning using quasi-static pendulum was computed by Eq. (3). In the dynamic pendulum case we applied Eq. (4–11), and in the SPH model Eqs. (12–20) were used for computing M_O. The resisting moment M_R was computed following Eq. (21).

The results of both the pendulum (quasi-static and dynamic) and the SPH models were compared. Radial accelerations causing overturning by different liquid levels are depicted in Figure 11.

The simplified model is basically a quasi-static pendulum overturning model; therefore the overturning threshold in this method is, expectedly, significantly higher than with the modified pendulum, which takes into account the dynamics of the oscillations during the manoeuvre. We achieved relatively good agreement between the dynamic pendulum and the SPH results, the overturning threshold being slightly higher when using the SPH method. This method makes it possible to exactly describe each particle's position and velocity during sloshing and mixing, and accounts for friction between the moving particles and the wall, as well as for friction between the particles themselves. Qualitatively speaking, all methods reveal a decreasing radial acceleration with increasing liquid height. The inclination of both dynamic methods is, however, more similar: they decrease more steeply in the lower half of the tank and stabilise towards the higher liquid heights. Although the gravity centre of the vehicle and the fluid increases with liquid height, there is nonetheless less space for sloshing. Such conditions lead to increased relative stability, as computed with dynamic models.

4. Conclusions

We compared fluid movement in a closed domain for two manoeuvres: steady-turn and lane change. Three different methods were applied. Two pendulum-based methods (quasi-static and dynamic – modified by means of adjustable rod length) were compared to the SPH. The overturning threshold of a vehicle with liquid cargo was computed for the steady-turn manoeuvre.

The SPH method is known for its suitability in simulations with rapid free-surface changes. Although the method has been applied for similar case studies (sloshing of liquid cargo on vessels, filling of ships in rough sea conditions), to the best of our knowledge it has never been used for simulating sloshing in road tankers. We achieved relatively good agreement of oscillations for the second manoeuvre, while maximum inclination of the gravity centre was simulated well in both cases.

The simple overturning model was applied for the first manoeuvre. While both dynamic methods reveal a similar, but significantly lower overturning threshold compared to the quasi-static pendulum, the threshold is higher using the SPH method.

The described results and comparisons confirm the suitability of the SPH method for simulating sloshing in road tankers. Furthermore, the SPH method has significant advantages over the pendulum methods: neither real-time, non-linear, free-surface depiction nor the sloshing of a liquid within a closed domain can be accounted for by applying any kind of pendulum. Furthermore, friction at the wall-particle interface as well as between the particles is difficult to describe by applying any pendulum method, as the damping coefficients in such cases are difficult to determine.

Validation of the applied models is, however, necessary in order to quantify the agreement of both dynamic models with measurements. Although the suitability of the SPH method is not in question, a rigorous calibration of the SPH model based on physical model results is required before the model can be used for determining the overturning threshold in realistic conditions.

Several improvements to the simulations of sloshing in road tankers are further possible. A more realistic description of vehicle can be applied: vehicle suspension, tyre flexibility, and slipping/friction conditions between tyres and pavement can be taken into account. Furthermore a comparison of three-dimensional pendula connected with springs to an adequately improved three-dimensional SPH model would allow for simulations of sloshing due to tangential acceleration and simulations in chamber-divided tankers with differing liquid height in each chamber.

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