

1. Matematično modeliranje hidravličnega ovna  
Mathematical Modelling of a Hydraulic Ram Pump System
2. Obratovanje hidravličnega turbostroja med prehodnimi pojavi  
The Behaviour of a Hydraulic Turbomachine during Transients
3. Dvodimenzijsko modeliranje gibanja droberskega toka v Logu pod Mangartom kot primer nenewtonske tekočine  
Two-Dimensional Modelling of Debris-Flow Movement in Log pod Mangartom as an Example of a Non-Newtonian Fluid
4. Dvodimenzijski matematični model transporta lebdečih plavin  
A Two-Dimensional Mathematical Model of Suspended-Sediment Transport
5. Optimizacija aksialnih vodnih turbin  
The Optimization of Axial Turbines



## Vsebina

### Contents

Strojniški vestnik - Journal of Mechanical Engineering  
letnik - volume 49, (2003), številka - number 3

#### Uvodnik

Bergant, A., Četina, M.

#### Razprave

Filipan, V., Virag, Z., Bergant, A.: Matematično modeliranje hidravličnega ovna

Bergant, A.: Obratovanje hidravličnega turbostroja med prehodnimi pojavi

Četina, M., Krzyk, M.: Dvodimenzijsko modeliranje gibanja drobirskega toka v Logu pod Mangartom kot primer nenenewtonske tekočine

Krzyk, M., Četina, M.: Dvodimenzijski matematični model transporta lebdečih plavin

Lipej, A.: Optimizacija aksialnih vodnih turbin

#### Osebne vesti

#### Navodila avtorjem

#### Editorial

136 Bergant, A., Četina, M.

#### Papers

137 Filipan, V., Virag, Z., Bergant, A.: Mathematical Modelling of a Hydraulic Ram Pump System

150 Bergant, A.: The Behaviour of a Hydraulic Turbomachine during Transients

161 Četina, M., Krzyk, M.: Two-Dimensional modelling of Debris-Flow Movement in Log pod Mangartom as an Example of a Non-Newtonian Fluid

173 Krzyk, M., Četina, M.: A Two-Dimensional Mathematical Model of Suspended-Sediment Transport

185 Lipej, A.: The Optimization of Axial Turbines

#### 196 Personal Events

#### 197 Instructions for Authors

## Uvodnik

### Editorial

Slovensko društvo za hidravlične raziskave (SDHR) je prostovoljno združenje, ki spremlja in vzpodbuja strokovno in znanstveno-raziskovalno delo hidravličnih raziskav na področju strojništva, gradbeništva in okoljskega inženirstva. Ustanovljeno je bilo leta 1994 in ima danes 53 dejavnih članov. Zastavljene cilje dosega z uresničevanjem naslednjih nalog: izobraževanje članov društva na področju hidravličnih raziskav in s tem povezanih znanstvenih področij, sodelovanje pri pripravi in dajanje pobud za spremembo predpisov s področja hidrotehnike, sodelovanje pri pripravi ustreznih standardov na področju hidravlike ter organizacija strokovnih srečanj in ekskurzij. Delo društva je javno in odprto za koristne pobude, zato ima razvejano sodelovanje z drugimi sorodnimi domačimi in mednarodnimi organizacijami. Omenimo samo nekatere izmed njih: Slovensko društvo za velike pregrade – SLOCOLD, Društvo vodarjev Slovenije in Mednarodno združenje za hidravlične raziskave – IAHR.

Ena od pomembnih nalog društva je tudi objavljane strokovnih dosežkov, s čimer želimo prek strokovnih in javnih glasil svoj delo in novosti v stroki predstaviti tako domačim in tujim strokovnjakom kakor tudi širši javnosti. Zato smo se odločili, da naredimo izbor strokovnih predavanj, organiziranih od SDHR, od leta 2000 naprej. Izbrali smo Strojniški vestnik, ki je dvojezičen, ima razvejano mednarodno izmenjavo in je indeksiran v številnih bazah podatkov.

Med osmimi predavanji je bilo za objavo izbranih pet predavanj. Vsi članki so bili rigorozno recenzirani, po vnesenih končnih popravkih so bila slovenska in angleška besedila še lektorirana. Pri recenziranju objavljenih prispevkov so sodelovali prof.dr. Roman Klasinc, TU Gradec; prof.dr. Matjaž Mikoš, Univerza v Ljubljani; prof. Predrag Popovski, Univerza Sv. Cirila in Metoda, Skopje; doc.dr. Andrej Predin, Univerza v Mariboru; prof.dr. Rudi Rajar, Univerza v Ljubljani; dr. Arris Tijsseling, TU Eindhoven; prof.dr. Zvonimir Vukelić, Univerza Sv. Cirila in Metoda, Skopje, za njihov trud pa se jim gostujoča urednika iskreno zahvaljujemo. Za skrbno in natančno delo se zahvaljujemo tudi obema lektorjema, gospe Anji Baras za slovenščino in dr. Paulu McGuinnessu za angleščino ter tehnični urednici gospe Suzani Domjan.

Gostujoča urednika  
doc.dr. Anton Bergant  
prof.dr. Matjaž Četina

The Slovenian Association for Hydraulic Research (SDHR) is a voluntary association that promotes and encourages professional and scientific hydraulic research work in the field of mechanical, civil and environmental engineering. It was founded in 1994 and has 53 active members. Its main aims are to educate the association's members in the field of hydraulic research and associated research areas, to prepare and suggest changes to hydraulic engineering rules and standards, and to organise professional meetings and visits. As the association is public and open to useful suggestions, it has good and fruitful collaborations with other professional organisations in Slovenia and abroad. The most important of these collaborations are the Slovenian Committee on Large Dams (SLOCOLD), the Water Management Association of Slovenia and the International Association of Hydraulic Research (IAHR).

One of the most important tasks of the association is to publish its professional achievements and news in domestic and international journals in order to present our work to other professionals and the interested public. For this reason it was decided to select some of the lectures that were presented at the SDHR's meetings since 2000 and publish them in the Journal of Mechanical Engineering. The JME is an international journal that is indexed in a number of databases.

Five lectures were selected for publication. All the papers were peer reviewed and for the final versions of the Slovene and English texts careful proofreading was provided for both languages. By reviewing the published papers, the following colleagues provided much help: Assoc.Prof. Roman Klasinc, TU Graz; Assoc.Prof. Matjaž Mikoš, University of Ljubljana; Prof. Predrag Popovski, St. Cyril and Methodius University, Skopje; Assist.Prof. Andrej Predin, University of Maribor; Prof. Rudi Rajar, University of Ljubljana; Dr. Arris Tijsseling, TU Eindhoven; Prof. Zvonimir Vukelić, St. Cyril and Methodius University, Skopje. The guest editors would like to thank them all. We would also like to thank Mrs. Anja Baras and Dr. Paul McGuinness for their proofreading, and the technical editor, Mrs. Suzana Domjan, for her careful work in preparing the journal for publication.

Guest Editors  
Assist.Prof.Dr. Anton Bergant  
Prof.Dr. Matjaž Četina

## Matematično modeliranje hidravličnega ovna Mathematical Modelling of a Hydraulic Ram Pump System

Veljko Filipan, Zdravko Virag, Anton Bergant

*V prispevku podajamo matematični model hidravličnega ovna (HO), vgrajenega v cevni sistem. Obravnavamo poenostavljeni delovni postopek. Pretok v sistemu z ovnom je neustaljen, zato podajamo enačbe neustaljenega toka v ceveh in metodo karakteristik (MK). Podan je podrobni oris matematičnega modeliranja elementov sistema HO. Pogonsko in dobavno cev modeliramo s pomočjo pravokotne mreže karakteristik, pogonski in dobavni zbiralnik pa sta robna pogoja z nespremenljivo gladino vode. HO modeliramo kot robni pogoj, ki popisuje delovanje ovna v področjih delovnega postopka. Ta robni pogoj je opisan z 11 enačbami in 11 odvisnimi spremenljivkami, rešujemo ga z metodo iteracij v časovnih korakih računskega postopka. Postavljen model je programiran na računalniku. V podani računski simulaciji smo uporabili podatke iz literature, časovni potek tlaka in pretočne hitrosti so prikazani v grafični obliki.*

© 2003 Strojniški vestnik. Vse pravice pridržane.

**(Ključne besede: ovni hidravlični, sistemi črpalni, udari tlačni, metode karakteristik)**

*In this paper we present the mathematical modelling of a hydraulic ram pump (HRP) system and an explanation of the simplified working cycle of its action. The flow in the HRP system is unsteady; therefore, unsteady pipe-flow equations and the method of characteristics (MOC) are given. The mathematical modelling of particular components of the HRP system is explained in detail. The drive and delivery pipes are modelled by a fixed grid MOC, and the supply and delivery tanks are the boundary conditions – open reservoirs with a constant water level. The HRP is modelled as a unit boundary condition describing the physics of its action. This boundary condition consists of 11 equations with 11 dependent variables and is solved by an iterative procedure for each time step of the computational run. The derived model is programmed for a digital computer. A computer simulation using input data from the literature is made and the results are presented in the form of pressure and velocity vs. time graphs.*

© 2003 Journal of Mechanical Engineering. All rights reserved.

**(Keywords: hydraulic rams, pumping systems, water hammer, method of characteristics - MOC)**

### 0 UUVOD

Sistem s hidravličnim ovnom (HO) je preprost in obsega dve cevi, dva ventila in tlačni kotel. Hidravlični oven izkorišča energijo vodnega toka pri relativno majhni pogonski višini za črpanje ustrezne pretočne količine na višjo dobavno raven. HO obratuje brez dodatnih zunanjih pogonskih naprav (električnih ali mehaničnih), prav tako ni potreben dodatni vir energije. Preprosta izvedba in zanemarljivi obratovalni stroški sta dva ključna elementa za uporabo sistema s hidravličnim ovnom v moderni inženirski praksi, več ko 200 let po njegovem izumu.

V začetnem obdobju razvoja od leta 1797 (leto, ko je J. M. Montgolfier izumil HO) do konca 19. stoletja so številni raziskovalci neuspešno poskušali postaviti teoretični model delovanja HO. Žukovskij je leta 1898 [1] postavil teoretični model vodnega udara, po

### 0 INTRODUCTION

The hydraulic ram pump (HRP) system is a very simple device that consists of two pipes, two valves and a pressure vessel. The momentum of the water flow under a relatively low drive head is used to pump a small portion of the flow to a considerably higher delivery head. The HRP operates without any other external (electrical or mechanical) driving device, and with no other external energy input. The simplicity of its structure and the negligible operating costs are the two main reasons why the HRP is currently very interesting, although it was invented more than 200 years ago.

There have been many unsuccessful theoretical attempts to explain the HRP's action, from its invention in 1797 (by J. M. Montgolfier) until the end of the 19<sup>th</sup> century, when the water hammer – the main use of the HRP's action – was first explained in detail

katerem deluje hidravlični oven. Delovanje hidravličnega ovna je bilo zadovoljivo popisano na podlagi sklopljene teoretične in eksperimentalne analize HO v prvi polovici 20. stoletja. Najpomembnejši izvajalci sklopljenih raziskav v tem obdobju so bili: O'Brien in Gosline [2], Lansford in Dugan [3] ter Krol [4]. Omeniti moramo tudi Eytelweina [5] in Calverta [6], ki sta izvedla obširne eksperimentalne raziskave HO.

Na podlagi analize časovnega poteka tlaka sta O'Brien in Gosline [2] leta 1933 prva uspešno popisala načelo črpanja z uporabo HO. Kakorkoli že, raziskovalca sta v svojem modelu uporabila teorijo elastičnega vodnega udara samo v črpalnem taktu. Preostale takte delovnega postopka HO sta popisala s teorijo togega vodnega udara. Podoben model z malenkostnimi izboljšavami sta razvila Lansford in Dugan [3]. Krol [4] je uporabil teorijo togega udara za popis celotnega delovnega postopka HO. Obravnavani avtorji [2] do [4] so v matematičnih modelih uporabili statične karakteristike ventilov in cevi. Poenostavljen model togega vodnega udara je razvil tudi Iversen [7]. Basfeld in Müller [8] sta predstavila podoben model ob uporabi izmerjenih koeficientov pretočne hitrosti. Chiang [9] je uporabil impedančno metodo za popis delovanja in optimizacijo HO.

Rennie in Bunt [10] sta razvila grafoanalitični model, ki sloni na grafoanalitični rešitvi enačb neustaljenega toka v cevi za praktično celoten delovni postopek HO. Rezultati izračuna so se dobro ujemali z rezultati meritev. Sam postopek grafičnega reševanja v ravnini piezometrična višina ( $h$ ) – pretočna hitrost ( $u$ ), ki je fizikalno jasen, pa je težaven in zamuden. Avorja sta objavila tudi rezultate svojih obsežnih eksperimentalnih raziskav [11].

Razvoj na koncu 20. stoletja je bil usmerjen v izdelavo preprostih izvedb HO, namenjenih za proizvodnjo v skromno opremljenih delovnih obratih dežel v razvoju. Levji delež raziskav so opravili na Univerzi v Warwicku v obdobju med 1985 in 1997. Na tej univerzi so razvili niz izvedb hidravličnih ovnov in postavili poenostavljeni algebraični model [12], da bi čim bolj razširili uporabo HO v nerazvitih državah. V letih 1982 do 1984 so tudi na Tehnični univerzi v Delftu potekale obsežne laboratorijske raziskave, ki so obsegale preizkus 12 hidravličnih ovnov iz industrije. Rezultate laboratorijskih raziskav je objavil Tacke [13]. V tem poročilu Tacke podaja matematični model, ki sloni na teoriji togega vodnega udara z izjemo črpalnega takta. Črpalni takt je opisan s teorijo elastičnega udara.

Leta 1999 so Najm, Azoury & Piasecki [14] razvili numerični model HO, ki sloni na metodi karakteristik. Model zajema vse sestavne dele naprave, razviti program pa omogoča modeliranje vseh pretočnih režimov HO. V

by Zhukovsky in 1898 [1]. Combined theoretical and experimental investigations in the first half of the 20<sup>th</sup> century led to an adequate explanation of the HRP's action. In a combined approach O'Brien & Gosline [2], Lansford & Dugan [3] and Krol [4] made the most important contributions. Eytelwein [5] and Calvert [6] should also be mentioned because of their exhaustive and extensive experimental investigations of the HRP.

In 1933 using experimental data of pressure vs. time O'Brien & Gosline [2] were the first to successfully explain the HRP's pumping action. However, in their model they used the elastic water hammer theory only for the pumping period. For other parts of the HRP's working cycle they considered using the rigid column theory. A similar approach, with some improvements, was reported by Lansford & Dugan [3]. Krol [4] used the rigid column theory for the whole cycle of the HRP's action. All these authors [2] to [4] used the measured static characteristics of valves and pipes in their mathematical models. A simplified rigid column theory model was also derived by Iversen [7]. Basfeld & Müller [8] introduced a similar model using experimentally obtained velocity coefficients. Chiang [9] used a model for the simulation and optimisation of a HRP based on an impedance method.

Rennie & Bunt [10] derived a graphic-analytical model and they used unsteady pipe-flow equations and their graphic-analytical solution for almost the whole of the HRP cycle. The results of the model agreed well with their experimental results, but the graphical procedure in the plane of the piezometric head ( $h$ ) and the velocity ( $u$ ), although physically clear, is very cumbersome. The authors presented comprehensive experimental results with all the necessary details in [11].

Developments at the end of the 20<sup>th</sup> century were directed towards simplified solutions of the HRP so that they could be manufactured in the workshops of developing countries. Great efforts were made at the University of Warwick, where they ran a project from 1985 to 1997. They developed several designs of HRP and introduced a simplified algebraic model [12], all with the aim of enabling a wider application of the HRP in developing countries. An extensive investigation with a large number of laboratory tests on twelve commercially available hydraulic rams was also carried out at the Delft University of Technology in the period 1982 to 1984. The results of the laboratory investigations were reported by Tacke in 1988 [13]. In this report Tacke developed a mathematical model based on rigid column theory, except for the pumping period. This period was considered using the pressure-wave theory.

In 1999 Najm, Azoury & Piasecki [14] developed a numerical model of the HRP based on the method of characteristics. The model encompasses all the parts of the device and the derived program accounts for all possible flow situations of the HRP. This is very similar to the model of the HRP system developed in a doctoral

pričujočem prispevku podajamo podoben model, ki ga je v doktorskem delu razvil prvi avtor [15]. Model sloni na teoriji elastičnega vodnega udara in zajema vse elemente HO. Obravnavani model je programiran na računalniku, v numerični simulaciji pa smo uporabili podatke Rennieja in Bunta [11]. Dobljeni rezultati izračuna so primerjani z rezultati meritev iz literature.

## 1 DELOVNI POSTOPEK HIDRAVLIČNEGA OVNA

Slika 1 prikazuje značilni sistem z vgrajenim HO. Sistem sestavljajo: HO oziroma črpalka (1), pogonska cev (2), dobavna cev (3), pogonski zbiralnik (4) in dobavni zbiralnik (5). Črpalko (1) sestavljajo: okrov (1a), vzbujevalni ventil (1b), dobavni ventil (1c) in tlačni kotel (1d) z zračno blazino. Oven izrablja spremembo kinetične energije ustrezno velike količine vode (pogonski pretok,  $Q$ ), ki se pretaka iz pogonskega zbiralnika (pogonska višina,  $H$ ), pri hitri zapori vzbujevalnega ventila. Narastek tlaka omogoči črpanje vode (dobavni pretok,  $q$ ) v dobavni zbiralnik (dobavna višina,  $h$ ). Obratovanje HO sloni na izmeničnem delovanju obeh ventilov. Vzbujevalni ventil se odpre z uporabo stalno delujoče sile uteži ali vzmeti. Obravnavani ventil avtomatično zapre hidrodinamična sila delujoča na ventil. Dobavni ventil je preprosta izvedba povratnega ventila, ki odpira in zapira na podlagi razlike tlakov.

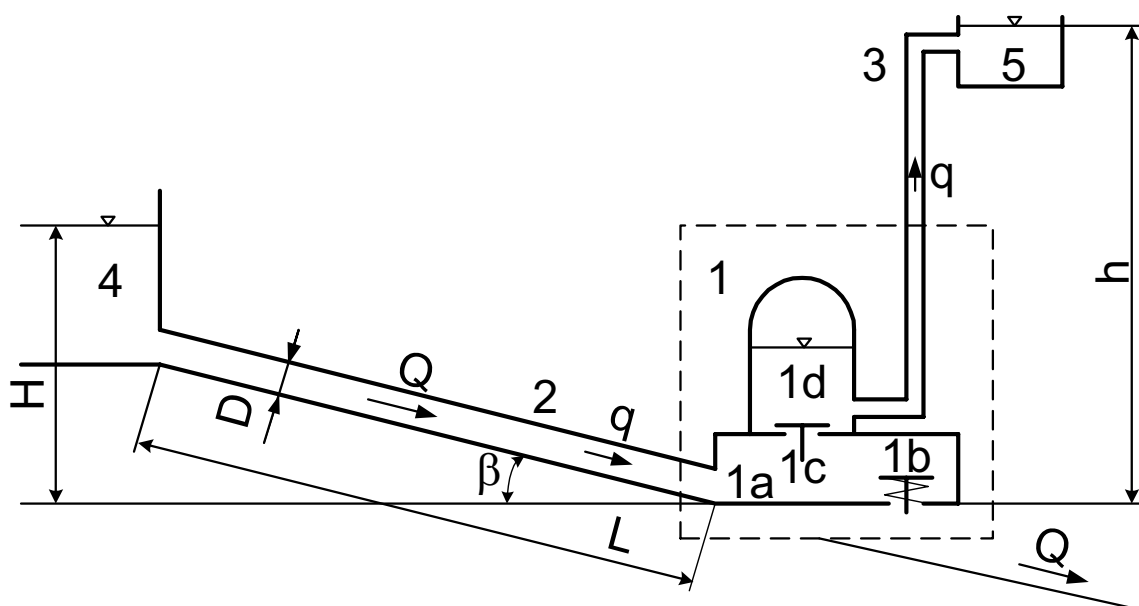
V prvem približku lahko delovni postopek HO razdelimo na tri značilne delovne takte, kakor je prikazano na sliki 2.

thesis by the first author [15] and outlined in this paper. The model is based on elastic water-hammer theory and takes into account all the parts of the HRP system. The HRP unit is modelled as a complex boundary condition and a significant extension is made in the impulse valve modelling. The model is programmed for a digital computer and a simulation using the experimental data of Rennie and Bunt [11] is made. The obtained results are compared with experimental data from the literature.

## 1 THE WORKING CYCLE OF THE HRP

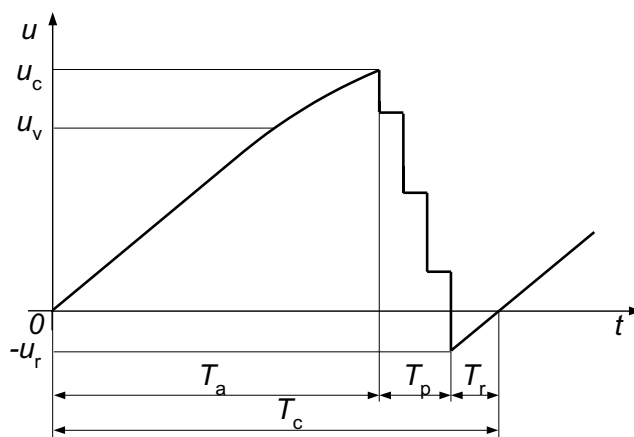
A typical HRP system is shown in Fig 1. It consists of a HRP unit or pump (1), drive pipe (2), delivery pipe (3), supply tank (4) and delivery tank (5). The pump (1) consists of a pump body or casing (1a), impulse valve (1b), delivery valve (1c) and a pressure vessel (1d) partially filled with air. The HRP operates by extracting the energy from a large quantity of water (drive flow,  $Q$ ), which flows from a supply tank (driving head,  $H$ ), by rapid closure of the impulse valve. A small quantity of water (delivery flow,  $q$ ) is then pumped to a delivery tank (delivery head,  $h$ ). The operation is due to the automatic action of two valves that work intermittently. The impulse valve is open due to a continuously acting force (dead weight or spring). The valve is closed automatically under the dynamic force of flowing water. The delivery valve is a simple non-return valve that is opened and closed under a pressure difference.

In a simplified manner the HRP working cycle can be divided into three different periods of action, as depicted in Fig. 2.



Sl 1. Značilni sistem s HO (1-črpalka: 1a-okrov, 1b-vzbujevalni ventil, 1c-dobavni ventil, 1d-tlačni kotel; 2-pogonska cev; 3-dobavna cev; 4-pogonski zbiralnik; 5-dobavni zbiralnik)

Fig 1. A typical HRP system (1-pump: 1a-casing, 1b-impulse valve, 1c-delivery valve, 1d-pressure vessel; 2-drive pipe; 3-delivery pipe; 4-supply tank; 5-delivery tank.)



Sl. 2. Poenostavljen delovni postopek HO  
Fig. 2. The simplified working cycle of an HRP

V pospeševalnem taktu ( $T_a$ ) teče voda skozi odprt vzbujevalni ventil, povratni ventil je zaprt. Voda v pogonski cevi je pospešena od  $u=0$  do  $u=u_v$  (faza 1) in nato na  $u=u_c$  (faza 2). V prvi fazi je vzbujevalni ventil polno odprt, v drugi fazi pa se zapira zaradi hidrodinamične sile delujoče na ventil. V sklepnem delu pospeševalnega takta se vzbujevalni ventil hitro zapre, pretok vode se zaustavi, zaustavitev pa povzroči vodni udar z visokim tlakom. Tlak vodnega udara v okrovu črpalke je večji od tlaka v tlačnem kotlu. Posledično se nato odpre dobavni ventil, vodni tok se usmeri proti tlačnemu kotlu in naprej v dobavni cevovod, kjer se na koncu izteka v dobavni zbiralnik. V drugem taktu postopka, imenovanem črpalni takt ( $T_p$ ), voda teče skozi odprt dobavni ventil, medtem ko je vzbujevalni ventil zaprt. Pretočna hitrost v sistemu se korakoma zmanjšuje glede na dinamiko potovanja udarnih valov v pogonski cevi. Tlak v okrovu črpalke se postopoma zmanjša na kritični tlak, pri katerem se zapre dobavni ventil. Temu sledi odprtje vzbujevalnega ventila z uporabo sile uteži ali vzmeti in pojav povratnega toka v pogonski cevi (voda se pretaka iz okrova črpalke v pogonsko cev proti pogonskemu zbiralniku). Pretok v pogonski cevi postopoma pojema od  $u=-u_r$  na  $u=0$ , obravnavani tretji takt imenujemo sunek ( $T_r$ ). Sunek konča delovni proces hidravličnega ovna, ki se nato samodejno ponavlja s periodo  $T_c$ .

In the acceleration period ( $T_a$ ) water discharges through the opened impulse valve while the delivery valve is closed. Water in the drive pipe is accelerated from  $u=0$  to  $u=u_v$  (phase 1) and after that to  $u=u_c$  (phase 2). In phase 1 the impulse valve is fully opened and in phase 2 it is closing due to the dynamic force of the water. At the end of the acceleration period the impulse valve is closed rapidly, the flow is stopped, and a water hammer with high pressure is created as a consequence. The induced pressure in the pump body is higher than the pressure in the air chamber. As a consequence, the delivery valve opens, enabling water to flow to the pressure vessel and further through the delivery pipe to the delivery tank. In this second part of the cycle, called the pumping period ( $T_p$ ), water continues to flow through the opened delivery valve while the impulse valve is closed. The flow velocity diminishes stepwise after each returning of the subsequent pressure waves that are travelling along the drive pipe. Gradually the pressure in the pump body drops below the critical pressure, causing the delivery valve to close. After this the impulse valve opens due to active force and negative flow in the drive pipe (from the pump body to the supply tank) occurs. The flow is gradually decelerated from  $u=-u_r$  to  $u=0$  in this part of action, called the recoil period ( $T_r$ ). The end of the recoil period is the beginning of the second cycle and the action of the HRP continues automatically with a period of  $T_c$ .

## 2 ENAČBE NEUSTALJENEGA TOKA V CEVI IN METODA KARAKTERISTIK

Tok v sistemu s HO lahko popišemo z enačbami neustaljenega toka v cevi [16]. To sta kontinuitetna enačba:

$$\partial h / \partial t + u \partial h / \partial x + u \sin \beta + c^2 / g \partial u / \partial x = 0 \quad (1)$$

in gibalna enačba:

## 2 THE UNSTEADY-PIPE-FLOW EQUATIONS AND THE METHOD OF CHARACTERISTICS

The flow in a HRP system can be described using unsteady-pipe-flow equations [16]. These are the continuity equation:

and the momentum equation:

$$\partial u / \partial t + u \partial u / \partial x + g \partial h / \partial x + \lambda |u| u / (2D) = 0 \quad (2),$$

kjer so:  $h$  – piezometrična višina,  $t$  – čas,  $u$  – pretočna hitrost,  $x$  – koordinata,  $\beta$  – strmina cevovoda,  $c$  – hitrost vala,  $g$  – zemeljski pospešek,  $\lambda$  – Darcy-Weisbachov koeficient trenja in  $D$  – premer cevi. Pri izpeljavi enačb (1) in (2) smo prevzeli naslednje postavke: enakomerna porazdelitev tlaka, hitrost je povprečena po prerezu cevi, tlačni tok v cevi, cev je okroglega prereza, elastične napetosti v steni cevi, Newtonska tekočina.

Par kvazilinearnih parcialnih diferencialnih enačb (1) in (2) nima analitične rešitve, za reševanje obravnavanih enačb so razvite številne metode ([16] in [17]). V prispevku bomo uporabili metodo karakteristik (MK). Sprememba enačb (1) in (2) po metodi karakteristik da štiri navadne diferencialne enačbe, dve združljivostni enačbi:

$$\pm g/c \, dh/dt + du/dt \pm g/c \, u \sin\beta + \lambda |u| u/(2D) = 0 \tag{3}$$

in dve enačbi karakteristik:

$$dx/dt = u \pm c \tag{4}$$

Vsaka združljivostna enačba (3) je veljavna samo vzdolž ustrezajoče karakteristične enačbe (4). Člen  $g/c \, u \sin\beta$  lahko zanemarimo, ker je hitrost vala  $c$  mnogo večja od pretočne hitrosti  $u$ . Enačbi integriramo z metodo končnih razlik ([16] in [17]). Rezultat integracije je sistem algebraičnih enačb. Te enačbe podajamo v poglavju 3 skupaj z enačbami robnih pogojev v sistemu s hidravličnim ovnom.

### 3 MATEMATIČNO MODELIRANJE SISTEMA S HIDRAVLIČNIM OVONOM

Elementi sistema s HO z uporabljenimi označbami so prikazani na sliki 3.

in which  $h$  is the piezometric head,  $t$  is the time,  $u$  is the flow velocity,  $x$  is the distance,  $\beta$  is the pipe slope,  $c$  is the wave speed,  $g$  is the gravitational acceleration,  $\lambda$  is the Darcy-Weisbach friction factor and  $D$  is the pipe diameter. Equations (1) and (2) are derived using the following assumptions: the distribution of pressure is uniform, the velocity is averaged over the pipe's cross section, the pipe is full with fluid, the pipe is of circular cross section, the pipe strains are in the elastic domain, the fluid is Newtonian.

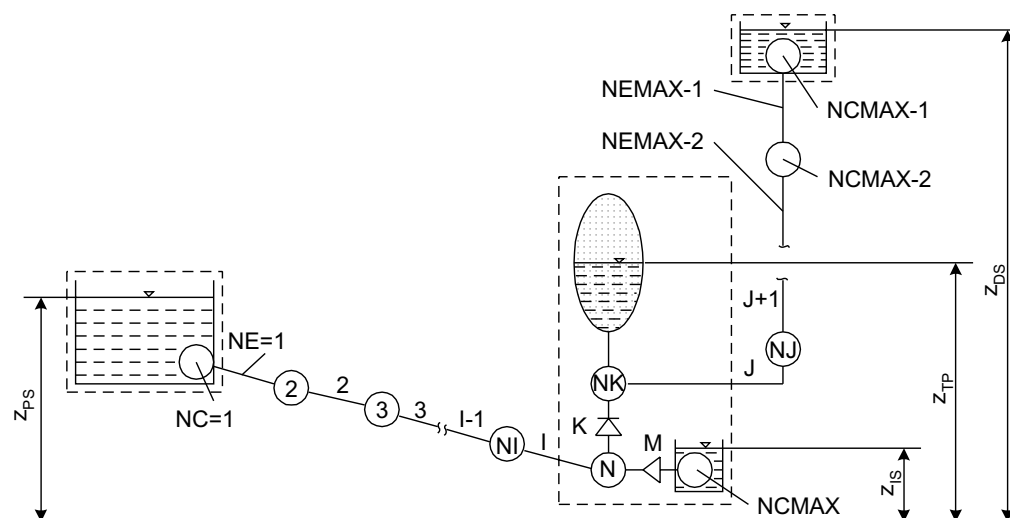
The pair of quasi-linear partial differential Equations (1) and (2) has no analytical solution and a number of methods for their solution have been developed ([16] and [17]). The method of characteristics (MOC) is used in this paper. The MOC transformation of equations (1) and (2) results in four ordinary differential equations, two compatibility equations:

and two characteristic equations:

Each compatibility Equation (3) is only valid along the corresponding characteristic Equation (4). The term  $g/c \, u \sin\beta$  in Eqn. (3) can be neglected because the wave speed  $c$  is much higher than the flow velocity  $u$ . The equations are integrated using a discrete finite-difference method ([16] and [17]). As a result a set of algebraic equations is obtained. These equations are presented in Chapter 3 together with the boundary conditions of the HRP model.

### 3 MATHEMATICAL MODEL OF THE HRP SYSTEM

The components of the HRP system with symbols used in model are shown in Fig 3.



Sl. 3. Elementi modela sistema s HO  
Fig. 3. The components of HRP system model



### Modeliranje pogonske in dobavne cevi

V numeričnem modelu [13] razdelimo pogonsko cev na izbrano število cevnih odsekov I (NE=1 do I) enake dolžine  $\Delta x$ . Dobavna cev je razdeljena na ustrezno število cevnih odsekov NEMAX-(I+1) (NE=J do NEMAX-1). Vsak cevni odsek je omejen z dvema vozliščema, ki ju oštevilčimo z NC=1 do NCMAX. V numeričnem izračunu uporabimo pravokotno mrežo MK. Časovni korak  $\Delta t$  izberemo na podlagi Courant-Friedrich-Lewyjevega kriterija stabilnosti:

$$\Delta t \leq \Delta x / (|u| + c) \quad (5).$$

Neznanki za tlak (piezometrična višina,  $h$ ) in pretočno hitrost ( $u$ ) v računski točki P v času  $t_0 + \Delta t$  (sl. 4) lahko dobimo iz naslednjih enačb, zapisanih v splošni obliki [15]:

$$h_p = (g(A_R h_R / c_R / (F_R + 1) + A_S h_S / c_S / (F_S + 1)) - (A_R u_R (F_R - 1) / (F_R + 1) - A_S u_S (F_S - 1) / (F_S + 1))) / (g(A_R / c_R / (F_R + 1) + A_S / c_S / (F_S + 1))) \quad (6)$$

$$u_{p1} = u_R (1 - F_R) / (F_R + 1) - g(h_p - h_R) / c_R / (F_R + 1) \quad (7)$$

$$u_{p2} = u_S (1 - F_S) / (F_S + 1) - g(h_p - h_S) / c_S / (F_S + 1) \quad (8),$$

kjer so:  $A$  – prečni prerez,  $F_R$  in  $F_S$  – znani stalnici, določeni kakor sledi:

$$F_R = (u_{O1} + u_R) \lambda_R \Delta t / (8D_R) \quad (9)$$

$$F_S = (u_{O2} + u_S) \lambda_S \Delta t / (8D_S) \quad (10).$$

Piezometrični višini v točkah  $P_1$  in  $P_2$  sta enaki ( $h_{p1} = h_{p2} = h_p$ ), pretočni hitrosti  $u_{p1}$  in  $u_{p2}$  sta različni v primeru nezveznosti (notranji robni pogoj).

### Modelling the Drive and Delivery Pipes

In the numerical model [15], the drive pipe is divided into a selected number of pipe reaches I (NE=1 to I) of equal length  $\Delta x$ . The delivery pipe is divided into a corresponding number of pipe reaches NEMAX-(I+1) (NE=J to NEMAX-1). Each pipe reach is bounded by two nodes numbered NC=1 to NCMAX. A fixed grid of the MOC is used for the numerical computation. The time step  $\Delta t$  is selected according to the stability criterion of Courant-Friedrich-Lewy:

The unknown pressure (piezometric head,  $h$ ) and velocity ( $u$ ) at a computational section P for time  $t_0 + \Delta t$  (Fig. 4) can be obtained from the following equations written in a generalized form [15]:

in which  $A$  is the pipe area,  $F_R$  and  $F_S$  are known constants defined as follows:

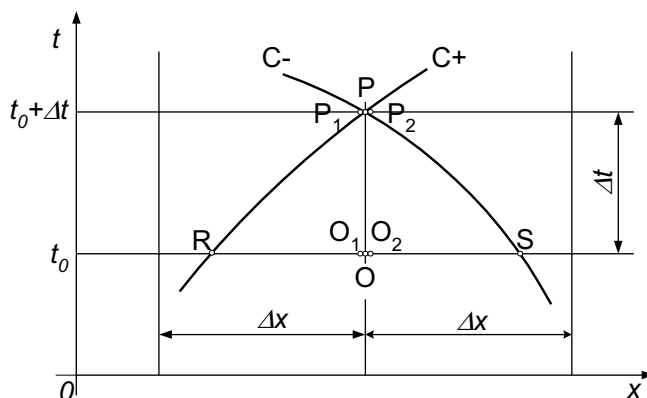
The piezometric heads at sections  $P_1$  and  $P_2$  are equal ( $h_{p1} = h_{p2} = h_p$ ) and the velocities  $u_{p1}$  and  $u_{p2}$  are different only in the case of a discontinuity (internal boundary condition).

### Modeliranje pogonskega in dobavnega zbiralnika

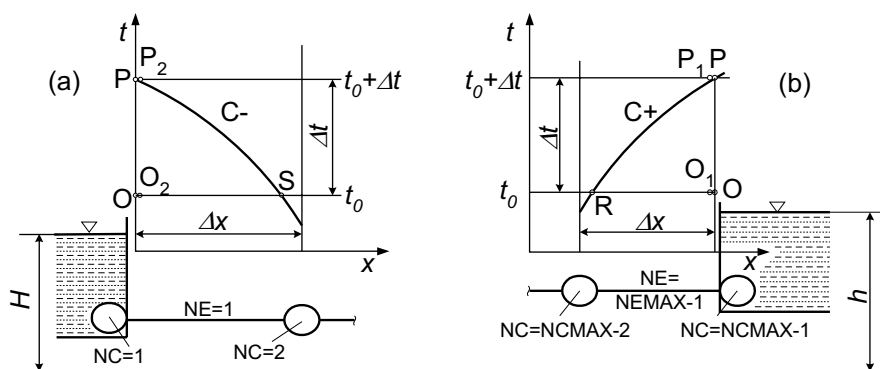
Pogonski in dobavni zbiralnik modeliramo kot robni pogoj. Postavimo nespreneljivo gladino vode

### Modelling the Supply and Delivery Tanks

The supply and delivery tanks are modelled as a boundary condition assuming that they are air-open



Sl. 4. Ravnina  $x-t$   
Fig. 4. The  $x-t$  plane



Sl. 5. Pogonski (a) in dobavni (b) zbiralnik  
Fig. 5. Supply (a) and delivery (b) tanks

v odprtem zbiralniku ( $H=\text{konst.}$  in  $h=\text{konst.}$ , sl. 5). Pretočni hitrosti  $u_{p2}$  in  $u_{p1}$  izračunamo iz ustreznih enačb (8) in (7).

### Enotni robni pogoj za HO

Hidravlični oven (črpalka) (sl. 1), sestavljajo ga okrov, vzbujevalni in dobavni ventil ter tlačni kotel, modeliramo kot enotni robni pogoj. Nespremenljivi tlak (višino) na izstopnem delu vzbujevalnega ventila v računski točki  $NC=NCMAX$  modeliramo kot zbiralnik s stalno gladino vode ( $z_{is}=\text{konst.}$ , sl. 3). Enačbe, ki popišejo druge dele obravnavanega robnega pogoja, so podane kakor sledi.

(1) Na navzdolnjem koncu pogonske cevi, ki je pritrjena na okrov ovna ( $NC=N$ ), lahko izrazimo združljivost enačbo  $C^+$  karakteristike v naslednji obliki:

$$g/c_r (h_N - h_r) + u_{+1} - u_r + F_r (u_{+1} + u_r) = 0 \quad (11)$$

ob upoštevanju  $R=NI$  po sliki 3.

Neznanki v enačbi (11) sta višina v okrovu ovna  $N$  ( $h_N$ ) in pretočna hitrost na navzdolnjem koncu pogonske cevi ( $u_{+1}$ ) v računskem času  $t_0 + \Delta t$ .

(2) Na navzgorjem koncu dobavne cevi, ki je pritrjena na tlačni kotel ( $NC=NK$ ), lahko izrazimo združljivost enačbo  $C^-$  karakteristike v naslednji obliki:

$$-g/c_s (h_{NK} - h_s) + u_{-j} - u_s + F_s (u_{-j} + u_s) = 0 \quad (12)$$

ob upoštevanju  $S=NJ$  po sliki 3.

Neznanki v enačbi (12) sta višina na priključku v tlačni kotel ( $h_{NK}$ ) in pretočna hitrost na navzgorjem koncu dobavne cevi ( $u_{-j}$ ) v računskem času  $t_0 + \Delta t$ .

(3) Enačbo politrope za zračno blazino v kotlu izrazimo v obliki:

$$p_p V_p^n = p_{p0} V_{p0}^n = \text{konst.} \quad (13),$$

reservoirs with a constant water level ( $H=\text{konst.}$  and  $h=\text{konst.}$ , see Fig. 5). The velocities  $u_{p2}$  and  $u_{p1}$  are calculated from the corresponding Equations (8) and (7).

### The HRP Unit Boundary Condition

The HRP unit (pump) (see Fig. 1) that consists of a pump body, impulse and delivery valves, and a pressure vessel, is modelled as a unit boundary condition. Constant pressure (head) at the outlet of the impulse valve at the computational node  $NC=NCMAX$  is modelled as an air-open outlet reservoir with a constant water level ( $z_{is}=\text{konst.}$ , see Fig. 3). The equations that describe the other parts of the HRP boundary condition are explained below.

(1) At the downstream-end section of the drive pipe coupled to the HRP pump body ( $NC=N$ ) the compatibility equation of the  $C^+$  characteristic can be expressed as:

with  $R=NI$  according to Fig. 3.

The unknowns in Eq. (11) are the head in the pump body ( $h_N$ ) and the velocity at the downstream end of the drive pipe ( $u_{+1}$ ) for the new time step  $t_0 + \Delta t$ .

(2) At the upstream-end section of the delivery pipe coupled to the pressure vessel ( $NC=NK$ ) the compatibility equation of the  $C^-$  characteristic can be expressed as:

with  $S=NJ$  according to Fig. 3.

The unknowns in Eq. (12) are the head just below the pressure vessel ( $h_{NK}$ ) and the velocity at the upstream-end section of the delivery pipe ( $u_{-j}$ ) for the new time step  $t_0 + \Delta t$ .

(3) The polytropic equation for air in the pressure vessel can be expressed as:

kjer so:  $n$  eksponent politrope,  $p_p$  in  $V_p$  sta neznan absolutni tlak in prostornina zraka v tlačnem kotlu v računskem času  $t_0 + \Delta t$ , absolutni tlak  $p_{p0}$  in prostornina zraka  $V_{p0}$  sta izračunana v predhodnem računskem času  $t_0$ .

(4) Absolutno piezometrično višino v vozlišču NK izrazimo z enačbo :

$$h_{NK} = p_p + \rho g (z_{TP} - z_{IS}) \quad (14),$$

kjer so  $\rho$  – gostota tekočine,  $z_{TP}$  in  $z_{IS}$  – gladini vode (sl. 3).

(5) Model dobavnega ventila upošteva izmerjene koeficiente tlačnih izgub na ventilu  $\xi_{DV} = f(u_K)$ . Ventil je odprt, ko je razlika tlakov med vozliščema N in NK pozitivna. V nasprotnem primeru, tj. pri negativni razliki tlakov, je ventil zaprt. Pretočno hitrost skozi ventil opišemo takole:

$$u_K = \begin{cases} (2g(h_N - h_{NK})/\xi_{DV})^{0.5} & \text{za/for } h_N > h_{NK} \\ u_K = 0 & \text{za/for } h_N \leq h_{NK} \end{cases} \quad (15)$$

(6) Kontinuitetno enačbo za računsko vozlišče NK zapišemo v obliki:

$$V_p - V_{p0} = (u_J + u_{J0}) A_J - (u_K + u_{K0}) A_K \quad (16),$$

kjer indeks 0 definira stanje v računskem času  $t_0$ .

(7) Vzbujevalni ventil modeliramo z uporabo drugega Newtonovega zakona ob upoštevanju vseh sil, ki delujejo na ventil. Obravnavano enačbo gibanja vzbujevalnega ventila zapišemo v naslednji obliki:

$$a_{pV} = (F_h - F_v) / m_{pV} = (\text{sign}(u_M - v_{pV}) C_{pV} \rho A_{pV} (u_M - v_{pV})^2 - F_v) / m_{pV} \quad (17),$$

kjer so  $a_{pV}$  – pospešek ventila,  $F_h$  – hidravlična sila,  $F_v$  – znana vzbujevalna sila za odprtje ventila proti smeri vodnega toka,  $m_{pV}$  – vplivna masa ventila in  $v_{pV}$  – hitrost diska ventila v smeri vodnega toka  $u_M$  (sl. 6).

where  $n$  is a polytropic exponent,  $p_p$  and  $V_p$  are the unknown absolute pressure and the volume of air in the pressure vessel for the new time step  $t_0 + \Delta t$ , while  $p_{p0}$  and  $V_{p0}$  are the known absolute pressure and the volume of air for the previous time step  $t_0$ .

(4) The absolute piezometric head at the node NK is expressed as :

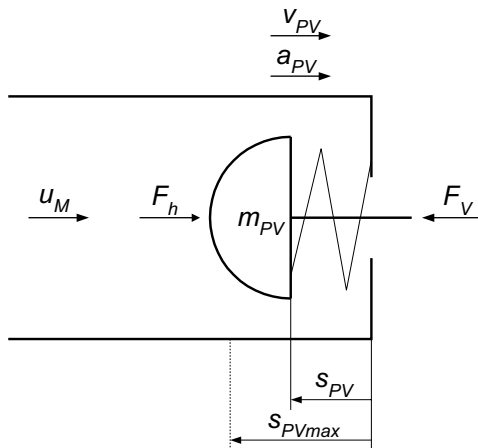
in which  $\rho$  is the fluid density, and  $z_{TP}$  and  $z_{IS}$  are the water-level elevations (see Fig. 3).

(5) The delivery valve is modelled using the measured valve head-loss coefficient  $\xi_{DV} = f(u_K)$ . If the pressure difference between points N and NK is positive, the valve is opened. On the other hand, if the pressure difference is negative, the valve is closed. The velocity through the valve is modelled as follows:

(6) The continuity equation for the computational node NK is written as follows:

where index 0 means the value for the previous time step  $t_0$ .

(7) The impulse valve's motion is modelled according to Newton's second law, taking into account all the forces acting on the valve. The equation of motion of the impulse valve can be expressed as:



Sl. 6. Model vzbujevalnega ventila  
Fig. 6. The impulse valve model

Izmerjena koeficient tlačnih izgub na vzbujevalnem ventilu  $\xi_{pv}$  in koeficient sile  $C_{pv}$  sta funkciji pomika ventila  $s_{pv}$  ([2] do [4], [10] in [11]).

Hitrost in pomik ventila dobimo z enačbo (17):

$$v_{pv} = v_{pv0} + a_{pv} \Delta t \quad (18)$$

$$s_{pv} = s_{pv0} - v_{pv} \Delta t - a_{pv} \Delta t^2 / 2 \quad (19)$$

Koeficient tlačnih izgub na vzbujevalnem ventilu  $\xi_{pv}$  je definiran za znani  $s_{pv}$ ; pretočno hitrost  $u_M$  izračunamo takole:

$$u_M = \begin{cases} \text{sign}(h_N - h_{NM}) (2g |h_N - h_{NM}| / \xi_{pv})^{0.5} & \text{za for } s_{pv} > 0 \\ u_M = 0 & \text{za for } s_{pv} = 0 \end{cases} \quad (20)$$

ob upoštevanju  $NM=NCMAX$  po sliki 3.

(8) Za rešitev sistema enačb (11) do (20) potrebujemo še kontinuitetno enačbo, zapisano za vozlišče N:

$$u_{+1} A_1 - u_M A_M - u_K A_K = 0 \quad (21)$$

Sistem 11 nelinearnih algebraičnih enačb (11) do (21) z 11 neznankami ( $h_N, h_{NK}, u_{+1}, u_{-1}, u_K, u_M, s_{pv}, v_{pv}, a_{pv}, p_p$  in  $V_p$ ) rešimo z iterativno metodo. Uporabimo prerezno metodo [18], ki ustreza fiziki delovanja HO. Obravnavana metoda je preprosta in robustna (stabilna in konvergentna) [15].

Pri zagonu HO v času  $t=0$ , s postavimo hidrostaticne tlačne razmere v sistemu. Dobavni ventil je zaprt, vzbujevalni ventil pa je polno odprt (nenadno odprtje). Temu sledi pospeševanje vodne mase v pogonski cevi po sliki 2.

#### 4 RAČUNALNIŠKI PROGRAM IN REZULTATI IZRAČUNA

Obravnavani matematični model programiramo v FORTRANu. Uporabimo standardno pravokotno mrežo MK za reševanje enačb neustaljenega toka v cevi. Razvili smo izvorni podprogram za HO, kjer v vsakem časovnem koraku izračunamo 11 neznank v enačbah (11) do (21) z uporabo iteracije. Za izračun tlakov (piezometričnih višin) in pretočnih hitrosti v računskih točkah uporabimo standardne podprograme.

Izvedli smo računalniško simulacijo delovanja HO ob uporabi eksperimentalnih podatkov, dobljenih v prispevku Rennieja in Bunta ([10] in [11]). Glavni podatki so naslednji: pogonska višina  $H=1,52$  m, dobavna višina  $h=32$  m (ta višina je bila krmiljena z ventilom, vgrajenim v dobavni cevi), dolžina pogonske cevi  $L=12,2$  m, premer pogonske cevi  $D=0,036$  m, dolžina dobavne cevi  $L=1,36$  m, premer

The measured impulse-valve head-loss coefficient  $\xi_{pv}$  and the force coefficient  $C_{pv}$  are functions of the valve displacement  $s_{pv}$  ([2] to [4], [10] and [11]).

The valve velocity and displacement are obtained with the aid of Eq. (17):

$$v_{pv} = v_{pv0} + a_{pv} \Delta t \quad (18)$$

$$s_{pv} = s_{pv0} - v_{pv} \Delta t - a_{pv} \Delta t^2 / 2 \quad (19)$$

For the known  $s_{pv}$  the value of the impulse-valve head-loss coefficient  $\xi_{pv}=f(s_{pv})$  is defined, and then the velocity  $u_M$  is calculated as follows:

$$u_M = \begin{cases} \text{sign}(h_N - h_{NM}) (2g |h_N - h_{NM}| / \xi_{pv})^{0.5} & \text{za for } s_{pv} > 0 \\ u_M = 0 & \text{za for } s_{pv} = 0 \end{cases} \quad (20)$$

with  $NM=NCMAX$  according to Fig. 3.

(8) One additional equation necessary for solving Eqs. (11) to (20) is the continuity equation for node N:

$$u_{+1} A_1 - u_M A_M - u_K A_K = 0 \quad (21)$$

The system of 11 non-linear algebraic Equations (11) to (21) with 11 unknowns ( $h_N, h_{NK}, u_{+1}, u_{-1}, u_K, u_M, s_{pv}, v_{pv}, a_{pv}, p_p$  and  $V_p$ ) is solved by an iterative method. The bisection method [18] based on the physics of the HRP action is found to be a simple and robust one (stable and converging) [15].

For the HRP start-up at time  $t=0$  it can be assumed that the water is under hydrostatic pressure in the system. The delivery valve is closed and the impulse valve is fully opened (it opens instantaneously). Then the water in the drive pipe starts to accelerate according to Fig. 2.

#### 4 THE COMPUTER PROGRAM AND THE SIMULATION RESULTS

The described mathematical model is programmed in the FORTRAN language. A standard fixed-grid MOC procedure for solving the unsteady pipe flow is used. An original subroutine for modelling the HRP is developed, in which an iterative procedure is used for each time step to predict 11 unknowns from Eqs. (11) to (21). Standard subroutines are used for the computation of pressures (piezometric heads) and the velocities at other computational sections.

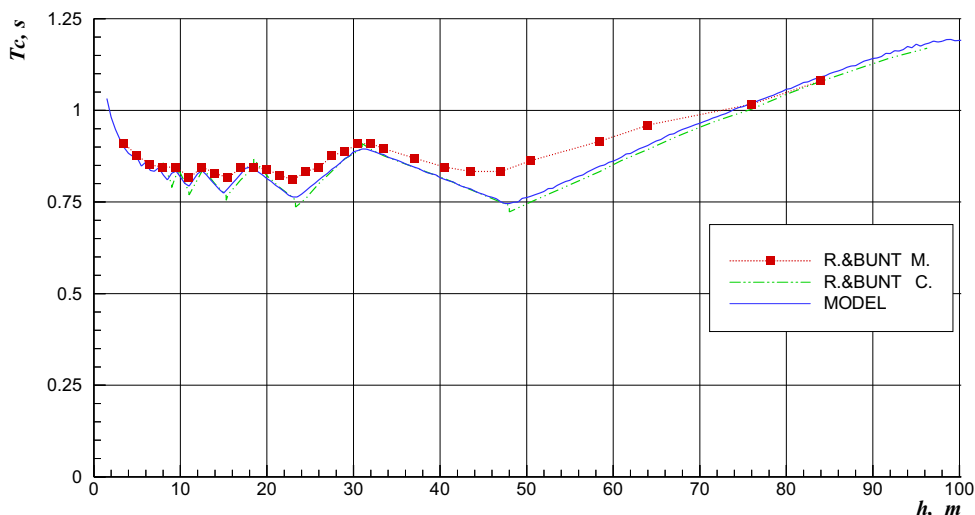
As an example, the computer simulation was carried out using the experimental pump system from the paper of Rennie & Bunt ([10] and [11]). The most important data of this HRP system are as follows: drive head  $H=1.52$  m, delivery head  $h=32$  m (the head was adjusted by a control valve in the delivery pipe), length of drive pipe  $L=12.2$  m, diameter of drive pipe  $D=0.036$  m, length of delivery pipe  $L=1.36$  m, diameter

dobavne cevi  $D=0,012$  m, največji pomik vzbujevalnega ventila  $s_{pv,max}=0,00381$  m, potrebna sila za odprtje vzbujevalnega ventila  $F_v=2,68$  N, prečni prerez vzbujevalnega ventila  $A_{pv}=0,001134$  m<sup>2</sup>, vplivna masa gibajočih delov vzbujevalnega ventila  $m_{pv}=0,273$  kg, prostornina tlačnega kotla  $V_{TP}=0,0083$  m<sup>3</sup>, stalnica tlačnega kotla  $p_{p0} V_{p0}^n=380$  Nm<sup>8/5</sup>, eksponent politrope  $n=1,2$ . Izmerjeni koeficienti dobavnega in vzbujevalnega ventila so približni, kakor sledi ([10], [11] in [15]):  $\xi_{pv}=0,00761$  s<sup>-1,38</sup>,  $C_{pv}=0,000216$  s<sup>-1,824</sup> in  $\xi_{dv}=200=konst.$  Ocenjena hitrost tlačnega vala v ceveh znaša  $c=1390$  m/s [10]. Pogonsko cev smo razdelili na 36 cevni odsekov dolžine  $\Delta x \approx 0,339$  m, dobavno cev pa na 4 odseke enake dolžine. Časovni korak v izračunu smo določili s pomočjo enačbe (5), dobimo vrednost  $\Delta t \approx 0,000242$  sekunde.

Rezultati izračuna postavljenega modela so primerjani z rezultati izračuna in meritev Rennie in Bunta ([10] in [11]). Rezultati se dobro ujemajo, kar

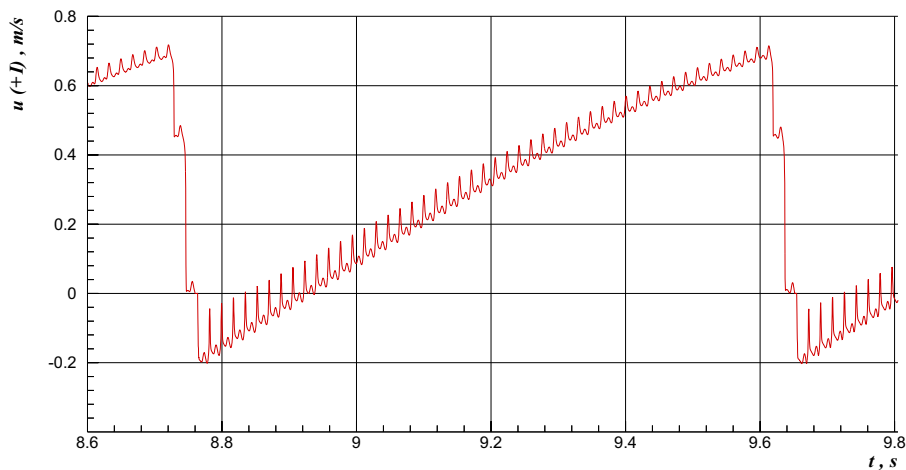
of delivery pipe  $D=0.012$  m, maximum displacement of impulse valve  $s_{pv,max}=0.00381$  m, force on impulse valve  $F_v=2.68$  N, impulse valve cross-sectional area  $A_{pv}=0.001134$  m<sup>2</sup>, effective mass of impulse-valve moving parts  $m_{pv}=0.273$  kg, the volume of the pressure vessel  $V_{TP}=0.0083$  m<sup>3</sup>, the pressure vessel constant  $p_{p0} V_{p0}^n=380$  Nm<sup>8/5</sup>, the polytropic exponent  $n=1.2$ . The measured coefficients of the impulse and delivery valves are approximated as follows ([10], [11] and [15]):  $\xi_{pv}=0.00761$  s<sup>-1.38</sup>,  $C_{pv}=0.000216$  s<sup>-1.824</sup> and  $\xi_{dv}=200=const.$  The wave speed in the pipes was estimated as  $c=1390$  m/s [10]. The drive pipe was divided into 36 pipe reaches of length  $\Delta x \approx 0.339$  m and the delivery pipe into 4 reaches of the same length. The time increment for the computational run was estimated according to Eq. (5) as  $\Delta t \approx 0.000242$  seconds.

The simulation results of the presented model are compared with the computational and experimental results of Rennie & Bunt ([10] and [11]), and good agreement, especially in the case of the HRP

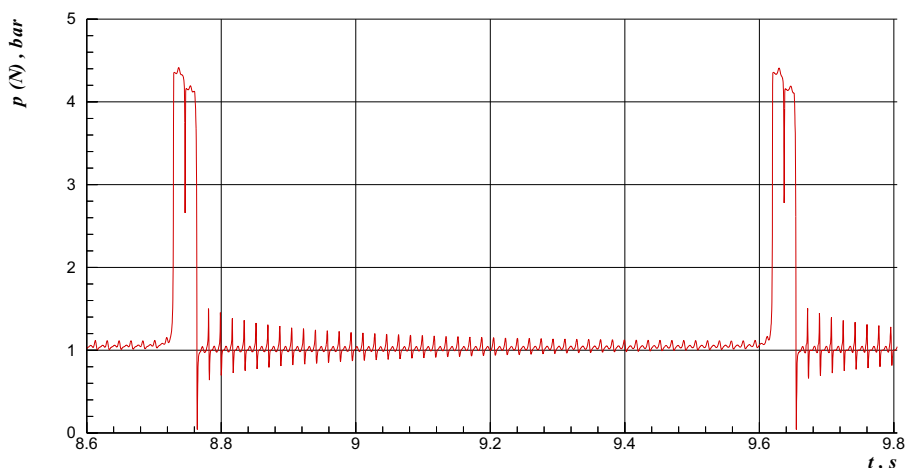


Sl. 7. Perioda  $T_c$  v odvisnosti od dobavne višine  $h$  (meritev - R.&BUNT M – in izračun - R.&BUNT C - Rennie in Bunta; izračun s postavljenim modelom – MODEL)

Fig. 7. The cycle period  $T_c$  vs. delivery head  $h$  (measured - R.&BUNT M – and computed - R.&BUNT C - by Rennie & Bunt; computed by described model – MODEL)



Sl. 8. Pretočna hitrost na navzdolnjem koncu pogonske cevi (točka +I)  
Fig. 8. Velocity at the downstream end of the drive pipe (section +I)



Sl. 9. Tlak na navzdolnjem koncu pogonske cevi, spojene z okrovom HO (vozlišče N)  
Fig. 9. Pressure at the downstream end of the drive pipe and pump body (node N)

posebno velja za potek karakteristik HO [15]. Slika 7 prikazuje primerjavo rezultatov izračuna periode ( $T_c$ ) v širokem pasu dobavnih višin ( $h$ ), dobljene z uporabo postavljenega modela (MODEL) ter Rennie in Buntovega modela (R.&BUNT C) z rezultati meritev Rennie in Bunta (R.&BUNT M).

Iz rezultatov izluščimo načelo delovanja in zapletene hidravlične interakcije v samem HO. Kot primer na slikah 8 in 9 podajamo rezultate izračuna časovnenega poteka pretočne hitrosti in tlaka na navzdolnjem koncu pogonske cevi za en delovni proces hidravličnega ovna (dobavna višina  $h=32$  m). S slik 8 in 9 razberemo, da tlačne in hitrostne motnje v črpalnem taktu ovna vplivajo na celotni delovni postopek.

Računalniški program računa v vsakem delovnem postopku naslednje karakteristike sistema s HO: pogonski pretok ( $Q$ ), dobavni pretok ( $q$ ), periodo postopka ( $T_c$ ) in izkoristek ( $\eta$ ). Izkoristek sistema s HO ( $\eta$ ) je definiran kot Rankinov izkoristek:

$$\eta = (h/H - 1) q/Q \quad (22),$$

kjer sta:  $H$  – pogonska višina in  $h$  – dobavna višina (sl. 1).

V občutljivostni analizi smo spremljali vpliv mase vzbujevalnega ventila (v obravnavanem primeru sile  $F_v$ , ki deluje na ventil z utežjo). Rezultati izračuna na sliki 10 prikazujejo odvisnost karakteristik hidravličnega ovna  $Q$ ,  $q$ ,  $T_c$  in  $\eta$  od sile  $F_v$ .

Ti rezultati so zelo uporabni v dejanskem primeru, ko moramo ustrezno nastaviti parametre sistema s HO. V splošnem zmanjšanje sile  $F_v$  vpliva na zmanjšanje pretočne količine ( $Q$  in  $q$ ) in periode delovnega postopka ( $T_c$ ), poveča pa se izkoristek ( $\eta$ ). V primeru, ko mora sistem s HO dobaviti večje količine vode, povečamo silo  $F_v$ , s tem pa zmanjšamo izkoristek. Povedati moramo, da je pas nastavitve sile  $F_v$  omejen za vsak posamezni sistem s HO.

characteristics, is obtained [15]. Fig. 7 shows a comparison between the computational results for the cycle period ( $T_c$ ) in a wide range of delivery heads ( $h$ ) obtained by the described model (MODEL) and the model of Rennie and Bunt (R.&BUNT C), and the results of the measurements (R.&BUNT M).

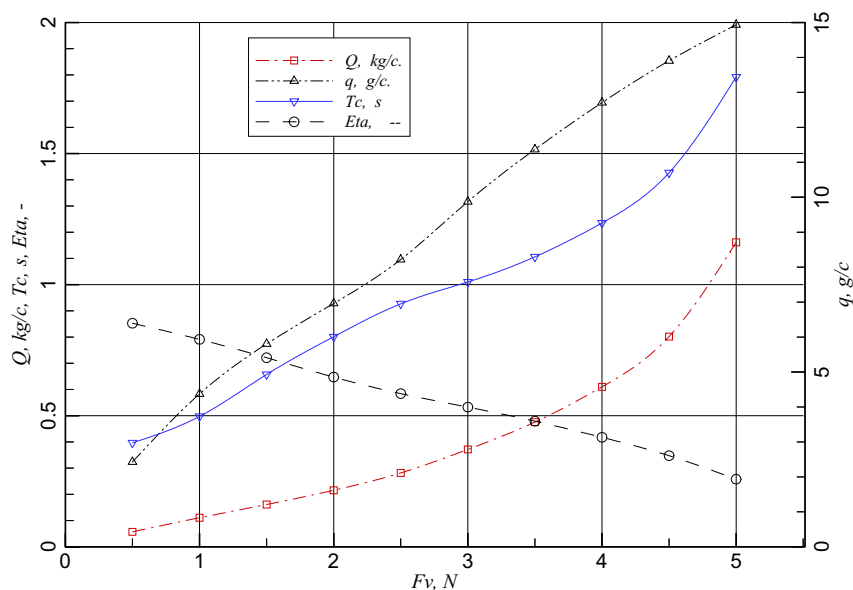
The results show the operational principle and the complex hydraulic interactions occurring within the HRP. As an example, the simulation results of velocity and pressure versus time traces at the downstream end of the drive pipe are depicted in Figs. 8 and 9, respectively, for one complete cycle of the HRP action (delivery head  $h=32$  m). Figs. 8 and 9 show that the pressure and velocity disturbances occurring in a pumping period influence the whole cycle of operation.

The computer program calculates the following HRP system characteristics: drive flow ( $Q$ ), delivery flow ( $q$ ), cycle period ( $T_c$ ) and efficiency ( $\eta$ ) for each successive cycle. The efficiency of the HRP system ( $\eta$ ) is defined as the Rankine efficiency:

where  $H$  is the driving head and  $h$  is the delivery head (see Fig. 1).

The sensitivity analysis with respect to the mass of the impulse valve (e.g. the force  $F_v$  in present case where the valve is loaded by a dead weight) was made. The simulation results of the analysis are shown in Fig. 10 as dependencies of the hydraulic ram pump characteristics  $Q$ ,  $q$ ,  $T_c$  and  $\eta$  vs. force  $F_v$ .

These results are very useful in a real case where the HRP system parameters should be adequately adjusted. Generally speaking, lower  $F_v$  values result in lower flow rates ( $Q$  and  $q$ ) and a lower cycle period ( $T_c$ ), whereas the efficiency ( $\eta$ ) is higher. In the case when the HRP system has to deliver more water, the adjustment must be made for higher forces  $F_v$ , but then the efficiency is lower. It must be pointed out that there is only a limited range of  $F_v$  values for each particular HRP system.



Sl. 10. Vpliv  $F_v$  na karakteristike hidravličnega ovna  $Q$ ,  $q$ ,  $T_c$  in  $\eta$   
 Fig. 10. The influence of  $F_v$  on hydraulic ram pump characteristics  $Q$ ,  $q$ ,  $T_c$  and  $\eta$

### 5 SKLEPI

V prispevku podajamo matematični model hidravličnega ovna (HO), vgrajenega v cevni sistem. Obravnavamo poenostavljeni delovni postopek. Pretok v sistemu z ovnom je neustaljen, zato podajamo enačbe neustaljenega toka v ceveh in metodo karakteristik (MK). Podan je podrobni oris matematičnega modeliranja elementov sistema HO. Pogonsko in dobavno cev modeliramo s pravokotno mrežo MK, pogonski in dobavni zbiralnik pa sta robna pogoja s stalno gladino vode. HO modeliramo kot robni pogoj, ki popisuje delovanje ovna v področjih delovnega postopka. Ta robni pogoj je opisan z 11 enačbami in odvisnimi spremenljivkami, rešujemo ga s pomočjo metode iteracij v časovnih korakih računskega postopka. Postavljen model je programiran na računalniku. V podani računski simulaciji smo uporabili podatke iz literature, časovni potek tlaka in pretočne hitrosti so prikazani v grafični obliki. Postavljeni model omogoča bolj natančno analizo delovanja hidravličnega ovna in izluščitev zapletenih hidravličnih interakcij med elementi, ki so vgrajeni v ovnu. V občutljivostni analizi smo spremljali vpliv sile, delujoče na vzbujevalni ventil.

### 5 CONCLUSIONS

Mathematical modelling of the HRP system is presented and the physics of a simplified working cycle of its action is explained. The flow in the HRP system is unsteady, therefore, unsteady-pipe-flow equations and the method of characteristics (MOC) are presented. The mathematical modelling of particular components of the HRP system is explained in detail. The drive and delivery pipes are modelled by the fixed grid MOC, and the supply and delivery tanks are the boundary conditions – open reservoirs with constant water levels. The HRP is modelled as a unit boundary condition describing the physics of its action. This boundary condition consists of 11 non-linear algebraic equations with 11 unknowns and is solved by an iterative procedure for each time step of the computational run. The derived model is programmed for a digital computer. A computer simulation using a pump system from the literature was made and the results are presented in the form of pressure and velocity vs. time graphs. The presented model enables a more detailed analysis of the operational principle of the HRP and gives insight into the complex hydraulic interactions occurring within the HRP. The sensitivity analysis presents the influence of the force acting on the impulse valve.

### 6 LITERATURA

### 6 REFERENCES

- [1] Joukowsky, N. (1900) Über den hydraulischen Stoss in Wasserleitungsröhren. Mémoires de l'Académie Impériale des Sciences de St.-Petersbourg, Series 8, Classe Physico-Mathématique, Vol. 9, No. 5.
- [2] O'Brien, M. P., J. E. Gosline (1933) The hydraulic ram. University of California Publication in Engineering, Vol. 3, No. 1, 1-58.
- [3] Lansford, W. M., W. G. Dugan (1941) An analytical and experimental study of the hydraulic ram. *University of Illinois Bulletin*, Vol 38, No. 22, 1-70.
- [4] Krol, J. (1951) The automatic hydraulic ram. *Proc. Inst. Mech. Eng.*, Vol. 165, 53-65.

- [5] Eytelwein, J.A. (1805) Bemerkungen über die Wirkung und vorteilhafte Anwendung des Stoßhebers (Bélier hydrauliques). *Realschulbuchhandlung*, Berlin.
- [6] Calvert, N. G. (1957) The hydraulic ram. *The Engineer*, Vol. 203, April 19, 597-600.
- [7] Iversen, H. W. (1975) An analysis of the hydraulic ram. *Trans. ASME, Journal of Fluids Engineering*, Vol. 97, No. 2, 191-196.
- [8] Basfeld, M., E.-A. Müller (1984) The hydraulic ram. *Forsch. Ing.-Wes.*, Band 50, Nr. 5, 141-147.
- [9] Chiang, Y.-C. (1984) Simulation and optimisation of transient oscillations, flow, and sound in complex piping system. Dissertation, *University of Wisconsin*, Madison.
- [10] Rennie, L. C., E. A., Bunt (1981) The automatic hydraulic ram. *The South African Mech. Eng.*, Vol. 31, No. 10, 258-273, No. 11, 286-311.
- [11] Rennie, L. C., E. A. Bunt (1990) The automatic hydraulic ram - experimental results. *Proc. Inst. Mech. Eng.*, Vol. 204, Part A: *Journal of Power and Energy*, 23-31.
- [12] Thomas, T. H. (1994) Algebraic modelling of the behaviour of hydraulic ram pump, Working Paper WP41, *University of Warwick*, Coventry.
- [13] Tacke, J.H.P.M. (1988) Hydraulic rams – a comparative investigation. *Communications on Hydraulic and Geotechnical Engineering*, Delft University of Technology, Faculty of Civil Engineering, Report 88-1.
- [14] Najm, H.N., P.H. Azoury, M. Piasecki (1999) Hydraulic ram analysis: a new look at an old problem. *Proc. Inst. Mech. Eng., Part A: Journal of Power and Energy*, Vol. 213, Nr. 2, 127-141.
- [15] Filipan, V. (1998) Hydrodynamic modelling of power hydraulic ram. Dissertation, *University of Zagreb*, Zagreb, (in Croatian).
- [16] Fox, J. A. (1989) Transient flow in pipes, open channels and sewers. *Ellis Horwood Ltd.*, Chichester.
- [17] Wylie, E. B., V.L. Streeter (1993) Fluid transients in systems. *Prentice Hall*, Englewood Cliffs.
- [18] Press, W. H., S.A. Teukolsky, W.T. Vetterling, B.P. Flannery (1994) Numerical recipes in Fortran, The art of scientific computing. 2nd Ed., *Cambridge University Press*, Cambridge.

Naslovi avtorjev: dr. Veljko Filipan  
Fakulteta za kemijsko inženirstvo  
in tehnologijo  
Univerza v Zagrebu  
Savska 16  
HR-10000 Zagreb, Hrvatska  
veljko.filipan@fkit.hr

prof. dr. Zdravko Virag  
Fakulteta za strojništvo in  
ladjedelništvo  
Univerza v Zagrebu  
Ivana Lučića 5  
HR-10000 Zagreb, Hrvatska  
zdravko.virag@fsb.hr

dr. Anton Bergant  
Litostroj E.I. d.o.o.  
Litostrojska 40  
SI-1000 Ljubljana  
anton.bergant@litostroj-ei.si

Authors' Addresses: Dr. Veljko Filipan  
Faculty of Chemical Eng.  
and Technology  
University of Zagreb  
Savska 16  
HR-10000 Zagreb, Croatia  
veljko.filipan@fkit.hr

Prof. Dr. Zdravko Virag  
Faculty of Mechanical Eng.  
and Naval Architecture  
University of Zagreb  
Ivana Lučića 5  
HR-10000 Zagreb, Croatia  
zdravko.virag@fsb.hr

Dr. Anton Bergant  
Litostroj E.I. Ltd.  
Litostrojska 40  
SI-1000 Ljubljana, Slovenia  
anton.bergant@litostroj-ei.si

Prejeto: 14.1.2003  
Received:

Sprejeto: 29.5.2003  
Accepted:

Odprt za diskusijo: 1 leto  
Open for discussion: 1 year



## Obratovanje hidravličnega turbostroja med prehodnimi pojavi

### The Behaviour of a Hydraulic Turbomachine during Transients

Anton Bergant

*Prispevek obravnava obratovanje hidravličnega turbostroja med prehodnimi pojavi (vodni udar) v cevnih sistemih. Podane so osnove metode karakteristik in prehodnega kavitacijskega toka v ceveh. Hidravlični turbostroj je popisan kot robni pogoj v deltoidni mreži metode karakteristik. Prikazana sta dva industrijska primera: trenutna razbremenitev dveh 34 MW francisovih turbin in izklop centrifugalne črpalke. Rezultati izračunov in meritev na terenu se dobro ujemajo v obeh primerih.*

© 2003 Strojniški vestnik. Vse pravice pridržane.

**(Ključne besede: stroji turbinski, udar tlačni, hidroelektrarne, sistemi črpalni)**

*This paper deals with the behaviour of a hydraulic turbomachine during transients (water hammer) in piping systems. A brief description of the method of characteristics and the fundamentals of transient cavitating pipe flow are given. The hydraulic turbomachine is treated as a boundary condition within the staggered grid of the method of characteristics. Case studies for a sudden load rejection of two 34-MW Francis turbines and a centrifugal pump rundown are presented. There is good agreement between the computational and the field-test results for both cases.*

© 2003 Journal of Mechanical Engineering. All rights reserved.

**(Keywords: turbomachines, water hammer, hydroelectric power plant, pumping systems)**

#### 0 UVOD

V fazi izbire in načrtovanja hidravličnega sistema moramo izdelati analizo prehodnih pojavov, da zagotovimo varno obratovanje izbranega sistema. Glavni cilj tega prispevka je izluščitev vpliva hidravličnega turbostroja na odziv pretočnega sistema med prehodnimi režimi. Med prehodnimi režimi se lahko pojavita ekstremni vodni udar in prekinitve kapljevinskega stebra v sistemu. V pretočnem sistemu hidroelektrarne se pojavijo naslednji režimi: obremenitev turbine, zmanjšanje obremenitve in trenutna razbremenitev, pobeg turbine, zaprtje varnostnih zapiral ter kombinirano obratovanje turbine in zapirala. V črpalnih sistemih se vodni udar pojavi pri startu, izklopu črpalke ter pri odpiranju in zapiranju ventilov. Poleg tega lahko hidravlični turbostroj obratuje kot črpalka – turbina. V tem primeru je turbostroj v turbinskih in črpalnih področjih obratovanja. Vodni udar lahko povzroči motnje v obratovanju hidravličnega sistema in poškoduje elemente sistema (zlom cevododa). Ekstremne tlačne utripe v pretočnem sistemu in vrtilno frekvenco hidravličnega stroja običajno krmilimo z ustreznimi obratovalnimi manevri (zapiranje in odpiranje

#### 0 INTRODUCTION

Feasibility and design studies of hydraulic systems should include a water-hammer analysis in order to ensure safe operation of the system. The main objective of this paper is to identify the influence of hydraulic turbomachine on system behaviour during transient regimes. Transient regimes may cause excessive water hammer and possible column separation in the system. These include the turbine-load acceptance, load reduction or sudden load rejection, turbine runaway, shutoff valve closure, and a combined operation of the turbine and valve in hydroelectric power plants. In pumping systems, water hammer may be induced by the pump start-up, the pump rundown, and the opening and closing of valves. In addition, the hydraulic turbomachine can operate as a pump-turbine. In this case the turbomachine undergoes transient regimes in the turbine- and pump-operating modes. Water hammer may disturb the operation of the hydraulic system and damage the system components (pipe rupture). High-pressure fluctuations in the flow-passage system and the hydromachine speed are traditionally controlled by appropriate operational means (closing and opening wicket gates, preventing flow

vodilnika turbine, preprečitev povratnega toka skozi črpalko). Vodni udar lahko blažimo tudi z vgradnjo zaščitne opreme proti vodnemu udaru (vztrajnik, tlačni varnostni ventil, izravnalnik, zračni kotel, prezračevalna cev, zračni ventil) in tehnično prerazporeditvijo cevnih elementov ([1] do [4]). Na izbiro metod za blažitev vodnega udara vplivajo obratovalni, varnostni in gospodarni kriteriji.

Vodni udar popišemo s sistemom hiperboličnih parcialnih diferencialnih enačb, kontinuitetne in gibalne enačbe. Obravnavani sistem enačb rešujemo z uporabo metode karakteristik. Hidravlični turbo stroj popišemo kot robni pogoj v deltoidni mreži metode karakteristik. Podan je kratek opis prekinitev kapljevinskega stebra in neprekinjenega kavitacijskega toka. Kavitacijski tok se pojavi, ko se tlak kapljevine v cevi zniža na parni tlak kapljevine.

V sklepnem delu prispevka preverimo teoretični model z dvema primeroma iz industrije. Prvi obravnavani sistem, hidroelektrarna Toro II (Kostarika), ima vgrajeni dve 34 MW francisovi turbini (nazivni neto padec 376 m, pretok 10 m<sup>3</sup>/s). Podani so rezultati trenutne razbremenitve obeh turbin s polne moči. Vodni udar blažimo z ustrezno nastavitvijo časa zapiranja vodilnika turbine. Drugi primer obravnava izklop centrifugalne črpalke v drenažnem črpalnem sistemu rudnika v Velenju. V obeh obravnavanih primerih se rezultati izračuna in meritev dobro ujemajo.

## 1 MODEL VODNEGA UDARA

Vodni udar popisuje potovanje tlačnih valov vzdolž cevovoda. Tlačni valovi so vzbujeni s spremembo pretočne hitrosti. Neustaljeni tok v zaprtih ceveh popišemo z dvema enačbama v eni razsežnosti, kontinuitetno enačbo in gibalno enačbo ([1], [3] in [5]):

$$\frac{\partial H}{\partial t} + v \frac{\partial H}{\partial x} - v \sin \theta_p + \frac{a^2}{g} \frac{\partial v}{\partial x} = 0 \quad (1)$$

in

and

$$g \frac{\partial H}{\partial x} + \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{\lambda v |v|}{2D} = 0 \quad (2),$$

kjer so  $H$  – piezometrična višina,  $t$  – čas,  $v$  – pretočna hitrost v cevi,  $x$  – koordinata,  $\theta_p$  – strmina cevovoda,  $a$  – hitrost vala,  $g$  – zemeljski pospešek,  $\lambda$  – Darcy-Weisbachov koeficient trenja in  $D$  – premer cevi. V večini inženirskih uporab so člen strmine cevovoda in konvekcijski členi v enačbah (1) in (2) majhni in jih zanemarimo ([1], [3] in [5]). V obravnavanih enačbah običajno vpeljemo pretok  $Q = vA$  namesto pretočne hitrosti  $v$ ;  $A$  – prečni prerez. Enačbi (1) in (2) sestavljata sistem navidezlinearnih hiperboličnih parcialnih diferencialnih enačb. Splošna rešitev za obravnavane enačbe ne obstaja. Običajna rešitev

reversal of the pump). Additional protective measures against unacceptable water hammer include installation of surge-control devices (flywheel, pressure-relief valve, surge tank, air-cushion surge chamber, aeration pipe, air valve) and redesign of the pipeline layout ([1] to [4]). Operational, safety and economic factors are decisive for the optimum selection of the method for controlling transients.

Water hammer is described by a set of hyperbolic partial differential equations, the continuity equation and the equation of motion. The method of characteristics is used for solving the water-hammer equations. The hydraulic turbomachine is treated as a boundary condition within the staggered grid of the method of characteristics. Liquid-column separation and distributed cavitation are briefly introduced. Cavitating flow occurs when the pressure in the pipe drops to the liquid vapour pressure.

The paper concludes with two case studies validating the theoretical model. The first system under consideration, the Toro II hydro-electric powerplant (Costarica), is fitted with two 34-MW Francis turbines (rated net head 376 m, discharge 10 m<sup>3</sup>/s). Results from the sudden load rejection of the two units are presented. Water hammer is controlled by appropriate adjustment of the wicket-gate closing manoeuvre. The second case study considers a centrifugal-pump rundown in the pumping system of a mine in Velenje (Slovenia). The computational results match reasonably well with the field test results in both systems.

## 1 WATER-HAMMER MODEL

Water hammer is the transmission of pressure waves along the pipeline resulting from a change in flow velocity. Unsteady flow in closed conduits is described by two one-dimensional equations: the continuity equation and the equation of motion ([1], [3] and [5]):

in which  $H$  – the piezometric head,  $t$  – the time,  $v$  – the pipe flow velocity,  $x$  – the distance,  $\theta_p$  – the pipe slope,  $a$  – the wave speed,  $g$  – the gravitational acceleration,  $\lambda$  – the Darcy-Weisbach friction factor, and  $D$  – the pipe diameter. For most engineering applications, the pipe slope and convective acceleration terms in Equations (1) and (2) are small and can be neglected ([1], [3] and [5]). Usually, the discharge  $Q = vA$  replaces the flow velocity  $v$ ;  $A$  – pipe area. Equations (1) and (2) are a set of quasi-linear hyperbolic partial differential equations. A general solution to these equations is not available. The common method of solving equations

enačb (1) in (2) je metoda karakteristik ([1] in [3]). Sprememba po metodi karakteristik da združljivostne enačbe vodnega udara, ki veljajo vzdolž karakterističnih krivulj. Numerično stabilne združljivostne enačbe vodnega udara, zapisane v obliki končnih razlik, se glasijo (majhne člene zanemarimo) ([3], [6] in [7]):

- vzdolž C<sup>+</sup> karakteristike ( $\Delta x/\Delta t = a$ ):

$$H_{j,t} - H_{j-1,t-\Delta t} + \frac{a}{gA} [(Q_u)_{j,t} - (Q_d)_{j-1,t-\Delta t}] + \frac{\lambda \Delta x}{2gDA^2} (Q_u)_{j,t} |(Q_d)_{j-1,t-\Delta t}| = 0 \quad (3)$$

- vzdolž C<sup>-</sup> karakteristike ( $\Delta x/\Delta t = -a$ ):

$$H_{j,t} - H_{j+1,t-\Delta t} - \frac{a}{gA} [(Q_d)_{j,t} - (Q_u)_{j+1,t-\Delta t}] - \frac{\lambda \Delta x}{2gDA^2} (Q_d)_{j,t} |(Q_u)_{j+1,t-\Delta t}| = 0 \quad (4),$$

kjer so  $j$  – indeks računске točke,  $Q_u$  – pretok na navzgorjem koncu računске točke,  $Q_d$  – pretok na navzdolnjem koncu računске točke,  $\Delta x$  – dolžina cevne odseka in  $\Delta t$  – časovni korak. V enačbah (3) in (4) uporabimo nespremenljivo vrednost Darcy-Weisbachovega koeficienta trenja  $\lambda$ . V primeru hitrih prehodnih pojavov to postavko popravimo z vpeljavo neustaljenega člena trenja v zgornjih enačbah ([7] do [9]). V primeru vodnega udara sta pretok na navzgorjem koncu računске točke  $Q_u$  in pretok na navzdolnjem koncu računске točke  $Q_d$  enaka ( $Q_u \equiv Q_d$ ), tlak v računski točki je večji od parnega tlaka kapljevine. Na robu enačba robnega pogoja nadomesti eno od združljivostnih enačb vodnega udara.

### Prehodni kavitacijski tok

Prehodni kavitacijski tok se pojavi, ko se tlak kapljevine zniža na parni tlak kapljevine. Kavitacija se lahko pojavi v dveh oblikah ([10] do [12]). Prva oblika je krajevna diskretna kavitacija s paro z velikim kavitacijskim razmernikom (pretrganje stebra). Druga oblika kavitacije je neprekinjen kavitacijski tok pri parnem tlaku kapljevine, ki se ustvari na daljši dolžini cevovoda (majhen kavitacijski razmernik). Za prehodni kavitacijski tok običajna metoda reševanja vodnega udara ne velja. V prispevku obravnavamo diskretni parni kavitacijski model ([3] in [13]).

Diskretni kavitacijski model dovoljuje stvaritev kavitacije s paro v vseh tistih računskih točkah numerične mreže metode karakteristik, kjer se tlak zniža na parni tlak kapljevine. V cevnih odsekih med računskimi točkami postavimo kapljevinsko fazo s stalno hitrostjo širjenja udarnega vala  $a$ . Dinamiko diskretne kavitacije s paro v poljubni računski točki  $j$  vzdolž cevovoda v celoti popišemo z dvema združljivostnima enačbama vodnega udara (3) in (4), kjer višini  $H$  priredimo vrednost  $z+h_v$  ( $z$  – geodetska višina,  $h_v$  – parna tlačna višina), in s kontinuitetno enačbo prostornine diskretne kavitacije s paro:

$$V_v = \int_{t_m}^t (Q - Q_u) dt \quad (5),$$

(1) and (2) is by the method-of-characteristics transformation ([1] and [3]). The transformation by the method of characteristics gives the water-hammer compatibility equations, which are valid along the characteristic curves. The numerically stable water-hammer compatibility equations, written in a finite-difference form, are (small terms are neglected) ([3], [6] and [7]):

- along the C<sup>+</sup> characteristic line ( $\Delta x/\Delta t = a$ ):

- along the C<sup>-</sup> characteristic line ( $\Delta x/\Delta t = -a$ ):

in which  $j$  – the computational section index,  $Q_u$  – the discharge at the upstream side of the computational section,  $Q_d$  – the discharge at the downstream side of the computational section,  $\Delta x$  – the reach length, and  $\Delta t$  – the time step. A constant value of the Darcy-Weisbach friction factor  $\lambda$  is used in Equations (3) and (4). This assumption may be corrected for the case of rapid transients by introducing an unsteady friction term in the above equations ([7] to [9]). Discharge at the upstream side of the computational section  $Q_u$  and the discharge at the downstream side of the section  $Q_d$  are identical for the water-hammer case ( $Q_u \equiv Q_d$ ), i.e. the pressure at a section is greater than the liquid vapour pressure. At a boundary, the boundary equation replaces one of the water-hammer compatibility equations.

### Transient Cavitating Flow

Transient cavitating flow in a pipeline system occurs when the pressure drops to the liquid vapour pressure. Two basic flow situations may occur ([10] to [12]). The first type is a localised, discrete vapour cavity with a large void fraction (column separation). The second type is a distributed, vaporous cavitation, which may extend over long sections of the pipe (small void fraction). The standard water-hammer solution is no longer valid. This paper deals with a discrete vapour-cavity model ([3] and [13]).

The discrete vapour-cavity model allows vapour cavities to form at all computing sections in the method-of-characteristics numerical model when the pressure drops to the liquid vapour pressure. A liquid phase with a constant wave speed  $a$  is assumed to occupy the full reach length between the computational sections. The behaviour of the discrete vapour cavity at an arbitrary computational section  $j$  along the pipeline is fully described by the two water-hammer compatibility Equations (3) and (4) with  $H$  set to  $z+h_v$  ( $z$  – the elevation above datum,  $h_v$  – the vapour pressure head), and the continuity equation for the discrete vapour-cavity volume:

kjer sta  $V_v$  – prostornina diskretne kavitacije s paro in  $t_{in}$  – čas nastanka kavitacije.

Numerična rešitev enačbe (5), zapisane v deltoidni mreži metode karakteristik, se glasi [14]:

$$(V_v)_{j,t} = (V_v)_{j,t-2\Delta t} + (\psi((Q_d)_{j,t} - (Q_u)_{j,t}) + (1-\psi)((Q_d)_{j,t-2\Delta t} - (Q_u)_{j,t-2\Delta t}))2\Delta t \quad (6)$$

kjer je  $\psi$  – utežni koeficient ( $\psi = 0,5$  do 1). Kavitacija se zruši, ko je zbirna prostornina kavitacije manjša od nič. Ponovno se vzpostavi kapljevinski tok in s tem tudi veljavnost rešitve enačb vodnega udara (3) in (4). Priporoča se, da največja prostornina diskretne kavitacije ne preseže 10 % prostornine cevnega odseka [13].

## 2 MODELIRANJE HIDRAVLIČNEGA TURBOSTROJA

Hidravlični turbostroj lahko preide turbinsko, črpalno ali črpalno-turbinsko področje obratovanja. Dinamični odziv krmiljene črpalke – turbine popišemo z enačbami črpalke – turbine (energijska enačba, enačba vrtenja turboagregata), krmilnika (enačba za popis spremembe vrtilne frekvence črpalke – turbine v odvisnosti od giba krmilnega mehanizma (ov)) in cevovoda (enačbe vodnega udara in prehodnega kavitacijskega toka). Razmerje med vplivnimi veličinami turbostroja upodobimo v obliki eksperimentalno določenih karakteristik črpalke – turbine (višina, moment, vzdolžna osna sila). V literaturi so na voljo številne metode reševanja zgoraj navedenih enačb ([1], [3] in [15]). V primeru spremembe obremenitve, ko je vrtilna frekvenca črpalke – turbine krmiljena, moramo v teoretičnem modelu upoštevati enačbe črpalke – turbine, krmilnika in cevovoda. Enačbe krmilnika ne upoštevamo v primeru analize trenutne razbremenitve turbine ali izklopa črpalke, pri kateri je sprememba vrtilne frekvence agregata odvisna od čistega hidravličnega momenta turbostroja. Robna pogoja za popis trenutne razbremenitve francisove turbine in izklopa centrifugalne črpalke sta definirana, kakor sledi.

### (1) Robni pogoj za trenutno razbremenitev francisove turbine

Robni pogoj za trenutno razbremenitev francisove turbine, vgrajene v cevovodu, zapisan v deltoidni mreži metode karakteristik, definiramo z naslednjimi enačbami (prehodna kavitacija na vstopnem in izstopnem robu francisove turbine ni dovoljena):

- združljivostni enačbi vodnega udara (3) in (4)
- energijska enačba:

$$H_u - H_r \left( \left( \frac{n}{n_r} \right)^2 + \left( \frac{Q}{Q_r} \right)^2 \right) W_H(y(t), x) - H_d = 0 \quad (7)$$

- enačba vrtenja sklopa turbine in generatorja:

in which  $V_v$  – discrete vapour-cavity volume, and  $t_{in}$  – the time of cavitation inception.

The numerical solution of equation (5) within the staggered grid of the method of characteristics is [14]:

in which  $\psi$  = the weighting factor ( $\psi = 0.5$  to 1). The cavity collapses when the cumulative cavity volume becomes less than zero. The liquid phase is re-established and the water-hammer solution using equations (3) and (4) is valid. It is recommended that the maximum size of the discrete vapour cavity at a section is less than 10 % of the reach volume [13].

## 2 MODELLING A HYDRAULIC TURBOMACHINE

The hydraulic turbomachine may undergo turbine, pump or pump-turbine operating modes. The dynamic behaviour of a governed pump-turbine is described by the pump-turbine (head balance equation, dynamic equation of rotating masses), the governor (dynamic equation that relates the pump-turbine rotational speed change to the position of the regulating mechanism(s)) and the pipeline equations (water-hammer and column-separation equations). The relationship among the influential turbomachine variables is presented in the form of the experimentally predicted pump-turbine characteristics (head, torque, axial force). Different methods for handling and solving the system of dynamic equations are available in the literature ([1], [3] and [15]). The complete set of hydraulic turbomachine-governor-pipeline equations should be used for the case of load reduction in which the pump-turbine speed is controlled by the governor. The governor equations are omitted in the analysis for the case of a turbine sudden load rejection or pump rundown in which the unit-speed change depends only on the net torque. Boundary conditions defining the sudden load rejection of the Francis turbine and the rundown of the centrifugal pump are as follows.

### (1) Boundary Condition for a Sudden Load Rejection of the Francis Turbine

The Francis turbine inline boundary condition for the case of sudden load rejection, which is incorporated into the staggered grid of the method of characteristics, is described by the following equations (no column separation is allowed at the Francis turbine inlet and outlet):

- water-hammer compatibility equations (3) and (4)
- head-balance equation:

- dynamic equation of the turbine-unit rotating masses:

$$\left( \left( \frac{n}{n_r} \right)^2 + \left( \frac{Q}{Q_r} \right)^2 \right) W_T(y(t), x) + \left( \frac{T}{T_r} \right)_{t-2\Delta t} - I \frac{\pi n_r}{30 T_r} \frac{1}{\Delta t} \left( \left( \frac{n}{n_r} \right) - \left( \frac{n}{n_r} \right)_{t-2\Delta t} \right) = 0 \quad (8),$$

kjer so  $H_u$  – višina na navzgorjem koncu turbine,  $H_r$  – imenski padec turbine,  $n$  – vrtilna frekvenca turbine (pozitivna v turbinski smeri vrtenja),  $r$  – imenski pogoji,  $W_H(y(t), x)$  – brezrazsežna turbinska tlačna karakteristika,  $y(t)$  – brezrazsežno odprtje vodilnika,  $x = \text{tg}^{-1}((Q/Q_r)/(n/n_r))$  – turbina v polarnem karakterističnem digramu, definiranem za turbinsko in disipacijsko področje (samo v enem kvadrantu),  $H_d$  – višina na navzdolnjem koncu turbine,  $W_T(y(t), x)$  – brezrazsežna turbinska momentna karakteristika,  $T$  – moment in  $I$  – polarni vztrajnostni moment vrtečih se delov.

## (2) Robni pogoj za izklop centrifugalne črpalke

Robni pogoj za izklop centrifugalne črpalke, vgrajene v cevovodu, zapisan v deltoidni mreži metode karakteristik, določimo z naslednjimi enačbami (prehodna kavitacija na vstopnem in izstopnem robu centrifugalne črpalke ni dovoljena):

- združljivostni enačbi vodnega udara (3) in (4)
- energijska enačba:

$$H_u + H_r \left( \left( \frac{n}{n_r} \right)^2 + \left( \frac{Q}{Q_r} \right)^2 \right) W_H(x) - H_d = 0 \quad (9)$$

- enačba vrtenja sklopa črpalke in elektromotorja:

$$\left( \left( \frac{n}{n_r} \right)^2 + \left( \frac{Q}{Q_r} \right)^2 \right) W_T(x) + \left( \frac{T}{T_r} \right)_{t-2\Delta t} + I \frac{\pi n_r}{30 T_r} \frac{1}{\Delta t} \left( \left( \frac{n}{n_r} \right) - \left( \frac{n}{n_r} \right)_{t-2\Delta t} \right) = 0 \quad (10),$$

kjer so  $H_u$  – višina na navzgorjem koncu črpalke,  $H_r$  – imenska črpalna višina,  $n$  – vrtilna frekvenca črpalke (pozitivna v črpalni smeri vrtenja),  $W_H(x)$  – brezrazsežna črpalna tlačna karakteristika,  $x = \pi + \text{tg}^{-1}((Q/Q_r)/(n/n_r))$  – črpalka v polarnem karakterističnem diagramu, določenem za vse štiri kvadrante,  $H_d$  – višina na navzdolnjem koncu črpalke in  $W_T(x)$  – brezrazsežna črpalna momentna karakteristika.

Neznanke v zgornjem sistemu enačb za francisovo turbino oziroma za centrifugalno črpalčko so: višini  $H_u$  in  $H_d$ , pretok  $Q$  ( $Q_d \equiv Q_u$ ) in vrtilna frekvenca turbostroja  $n$ . Sistem nelinearnih enačb (3), (4), ter (7) in (8) za turbino oziroma (9) in (10) za črpalčko rešimo s Newton-Raphsonovo metodo [16]. V primeru prehodnega kavitacijskega toka dodamo zgornjemu sistemu enačb spremenjeno enačbo (6), zapisano za navzgornji ali navzdolnji rob turbostroja.

## 3 PREVERITEV TEORETIČNEGA MODELA

Preveritev teoretičnega modela je prikazana za dva primera iz industrije. Prvi primer obravnava hidroelektrarno Toro II (Kostarika), ki ima vgrajeni dve 34 MW francisovi turbini [17]. Podajamo

in which  $H_u$  – the head at the upstream side of the turbine,  $H_r$  – the rated turbine head,  $n$  – the turbine rotational speed (positive in turbine direction),  $r$  – the rated conditions,  $W_H(y(t), x)$  – the dimensionless turbine head characteristic,  $y(t)$  – the dimensionless wicket-gates position,  $x = \text{tg}^{-1}((Q/Q_r)/(n/n_r))$  – the angular position of the turbine characteristic curve in generating and dissipating mode (only in one quadrant),  $H_d$  – the head at the downstream side of the turbine,  $W_T(y(t), x)$  – the dimensionless turbine torque characteristic,  $T$  – the torque, and  $I$  – the polar moment of inertia of rotating parts.

## (2) Boundary Condition for a Rundown of the Centrifugal Pump

The centrifugal pump inline boundary condition for the case of rundown, which is incorporated into the staggered grid of the method of characteristics, is described by the following equations (no column separation is allowed at the centrifugal pump inlet and outlet):

- water-hammer compatibility equations (3) and (4)
- head-balance equation:

$$H_u + H_r \left( \left( \frac{n}{n_r} \right)^2 + \left( \frac{Q}{Q_r} \right)^2 \right) W_H(x) - H_d = 0 \quad (9)$$

- dynamic equation of the pump-unit rotating masses:

$$\left( \left( \frac{n}{n_r} \right)^2 + \left( \frac{Q}{Q_r} \right)^2 \right) W_T(x) + \left( \frac{T}{T_r} \right)_{t-2\Delta t} + I \frac{\pi n_r}{30 T_r} \frac{1}{\Delta t} \left( \left( \frac{n}{n_r} \right) - \left( \frac{n}{n_r} \right)_{t-2\Delta t} \right) = 0 \quad (10),$$

in which  $H_u$  – the head at the upstream side of the pump,  $H_r$  – the rated pump head,  $n$  – the pump rotational speed (positive in pump direction),  $W_H(x)$  – the dimensionless pump head characteristic,  $x = \pi + \text{tg}^{-1}((Q/Q_r)/(n/n_r))$  – the angular position of the pump four-quadrant characteristic curve,  $H_d$  – the head at the downstream side of the pump,  $W_T(x)$  – the dimensionless pump torque characteristic.

The unknowns in the above system of equations for the Francis turbine and the centrifugal pump respectively, are the heads  $H_u$  and  $H_d$ , discharge  $Q$  ( $Q_d \equiv Q_u$ ) and turbomachine rotational speed  $n$ . The system of non-linear equations (3), (4), and (7) and (8) for the turbine, and (9) and (10) for the pump, respectively, is solved by the Newton-Raphson method [16]. The modified equation (6) is added to the above system of equations for the column-separation case, either at the upstream or the downstream side of the turbomachine.

## 3 VALIDATION OF THE THEORETICAL MODEL

Two case studies validating the theoretical model are presented. The first system under consideration, the Toro II hydro-electric powerplant (Costarica), is fitted with two 34-MW Francis turbines

rezultate za trenutno razbremenitev obeh turbin. Drugi primer obravnava izklop centrifugalne črpalke v drenažnem črpalnem sistemu v rudniku Velenje [2]. Izračuni so bili izdelani z uporabo računalniških programov za analizo prehodnih pojavov v Litostroju ([18] do [20]). V teh programih so zajeti elementi pretočnega sistema hidroelektrarn in črpalnih postaj (hidravlični turbostroj, ventil, izravnalnik, tlačni kotel itn.).

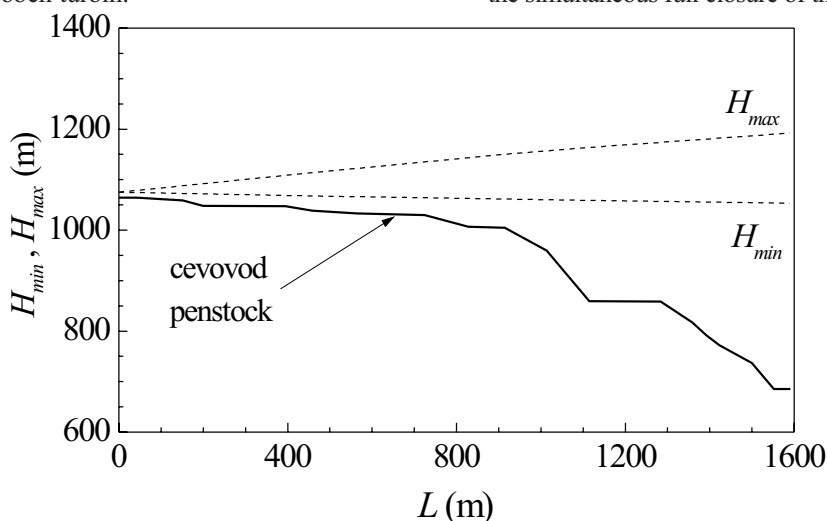
### Primer 1 – HE Toro II

Hidroelektrarno Toro II (Kostarika) sestavljajo navzgornji zbiralnik, cevovod z ustreznim premerom  $D = 2,23$  m in skupno dolžino  $L = 1577,3$  m (sl. 1), dve 34 MW francisovi turbini z navpično gredjo (imenski padec  $H_r = 376$  m, pretok  $Q_r = 10$  m<sup>3</sup>/s) priključeni na odvodni predor s premerom  $D = 2,5$  m in dolžino  $L = 22,8$  m ter navzdolnji zbiralnik. Gladina vode v navzgornjem zbiralniku  $z_u$  se giblje od 1069,5 m do 1075,0 m, gladina vode v navzdolnjem zbiralniku  $z_d$  pa od 689,7 m do 690,5 m. Imenska vrtilna frekvenca turbine je  $n_r = 720,0$  min<sup>-1</sup>, polarni vztrajnostni moment vrtečih se delov stroja pa  $I = 47,2 \times 10^3$  kgm<sup>2</sup>.

Prezemne meritve na terenu so zajemale naslednje preizkuse: start, obremenjevanje, zmanjšanje obremenitve ter trenutno razbremenitev ene ali dveh turbin. Nastali vodni udar blažimo z ustrežno nastavitvijo časov zapiranja in odpiranja vodilnika.

#### Trenutna razbremenitev dveh turbin

Trenutna razbremenitev dveh turbin je najbolj nevaren prehodni režim med normalnimi obratovalnimi razmerami [1]. Turbinska agregata izklopimo iz električnega omrežja, temu sledi hkratno polno zaprtje vodilnikov obeh turbin.



Sl. 1. Izračunane ovojnice največjih in najmanjših višin vzdolž cevovoda za primer trenutne razbremenitve dveh turbin ( $H_{max}$  – največja višina,  $H_{min}$  – najmanjša višina,  $L$  – dolžina cevovoda)  
Fig. 1. Computational envelopes of maximum and minimum heads along the penstock after the sudden load rejection of two turbines ( $H_{max}$  = maximum head,  $H_{min}$  = minimum head,  $L$  = penstock length)

[17]. Results from the sudden load rejection of the two units are presented. The second case study considers a centrifugal-pump rundown in Velenje (Slovenia) in the pumping system of a mine [2]. Calculations were performed with the aid of computer programs for hydraulic transient analysis in Litostroj ([18] to [20]). The hydropower plant and the pumping-station elements are included in these programs (hydraulic turbomachine, valve, surge tank, air receiver, etc.).

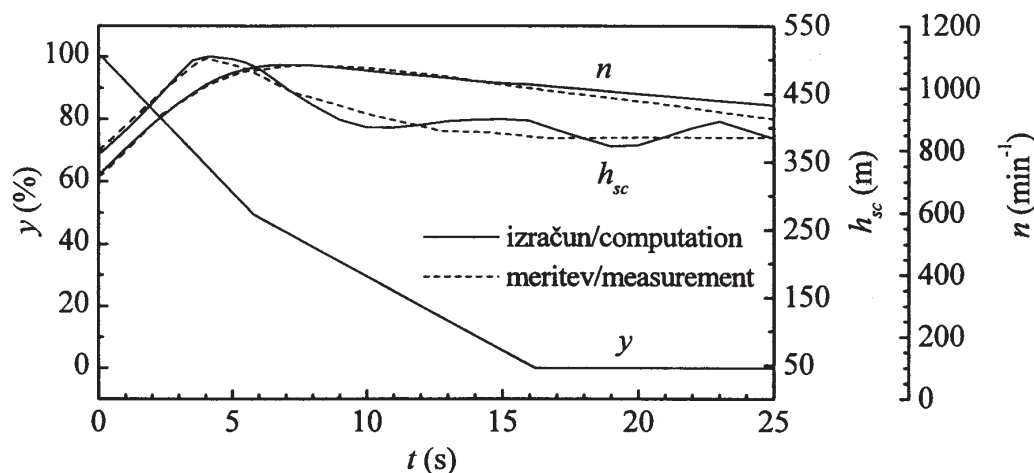
### Case Study 1 – Toro II HPP

The Toro II hydro-electric powerplant (Costarica) is comprised of an upstream reservoir; a penstock of equivalent diameter  $D = 2.23$  m and total length  $L = 1577.3$  m (see Fig. 1); two vertical-shaft 34-MW Francis turbines of rated head  $H_r = 376$  m and discharge  $Q_r = 10$  m<sup>3</sup>/s, which are connected to the outlet tunnel of diameter  $D = 2.5$  m and length  $L = 22.8$  m; and a downstream reservoir. The water level in the upstream reservoir  $z_u$  is in the range from 1069.5 m to 1075.0 m; the level in the downstream reservoir  $z_d$  is in the range from 689.7 m to 690.5 m. The rated speed of the turbine is  $n_r = 720.0$  min<sup>-1</sup> and the polar moment of inertia of the unit rotating parts  $I = 47.2 \times 10^3$  kgm<sup>2</sup>.

Various operating regimes were performed during the commissioning tests, including turbine start-up, load acceptance, load reduction and sudden load rejection of one or two turbines. The resulting water hammer was controlled by appropriate adjustment of wicket-gates closing and opening manoeuvres.

#### Sudden Load Rejection of Two Turbines

The sudden load rejection of two turbines is the most severe transient regime that occurs during normal operating conditions [1]. The turbines are simultaneously disconnected from the electrical grid followed by the simultaneous full closure of the wicket gates.



Sl. 2. Primerjava izračunanih in izmerjenih rezultatov za primer trenutne razbremenitve obeh turbin ( $y$  - brezrazsežno odprtje vodilnika,  $h_{sc}$  - tlačna višina v spirali,  $n$  - vrtilna frekvenca turbine,  $t$  - čas)  
 Fig. 2. Comparison of computational and experimental results after the sudden load rejection of two turbines ( $y$  - dimensionless wicket gates position,  $h_{sc}$  - scroll-case pressure head,  $n$  - turbine rotational speed,  $t$  - time)

Slika 1 prikazuje izračunane ovojnice največjih in najmanjših piezometričnih višin vzdolž profila cevovoda. Ta diagram omogoča inženirju načrtovanje varnega in gospodarnega cevne sistema. Iz ovojnice najmanjše višine ( $H_{min}$ ) izluščimo nevarnost pretrganja kapljevinskega stebra, ko se tlak zniža pod koto cevovoda. V našem primeru je izračunana višina zadosti nad vzdolžnim profilom cevovoda.

Največji tlak v spirali in največja vrtilna frekvenca turbine pomembno vplivata na izmere elementov turbine. Slika 2 prikazuje tlačno višino v spirali na vstopu v turbino  $h_{sc}$  (geodetska višina  $z = 685,0$  m) in vrtilno frekvenco turbine  $n$ . Rezultati izračuna in meritev se dobro ujemajo. Izračunana največja višina  $h_{sc,max,c} = 504,2$  m je rahlo večja od izmerjene višine  $h_{sc,max,m} = 501,0$  m. Izračunana največja vrtilna frekvenca turbine  $n_{max,c} = 1075$  min<sup>-1</sup> je rahlo manjša od izmerjene hitrosti  $n_{max,m} = 1082$  min<sup>-1</sup>. Odstopanja med rezultati se povečajo pri majhnih odprtjih.

### Primer 2 – Rudnik Velenje

Črpalni sistem v rudniku Velenje je visokotlačni sistem, kjer vodoravna večstopenjska centrifugalna črpalna potiska vodo v skoraj vertikalni cevovod s premerom  $D = 0,205$  m in skupno dolžino  $L = 441,5$  m (glej sl. 3). Voda prosto izteka v okolico. Koncentracijski razmernik trdnin v vodi je zanemarljiv. Na navzdolnjem robu črpalke je vgrajena nedušna povratna loputa, ki prepreči nasprotno vrtenje črpalke. Črpalka obratuje na imenski višini  $H_r = 382$  m, pretoku  $Q_r = 0,05$  m<sup>3</sup>/s in vrtilni frekvenci črpalke  $n_r = 720$  min<sup>-1</sup>.

Na terenu smo preizkusili zagon in izklop črpalke. Vodni udar v cevovodu je v veliki meri vplivan z dinamičnim odzivom povratne lopute [21].

Computed envelopes of the maximum and minimum piezometric heads along the penstock profile are shown in Fig. 1. This diagram is important for design engineers to help them design a safe and economic pipeline system. The envelope of the minimum head ( $H_{min}$ ) indicates the danger of liquid column separation when the pressure drops below the penstock profile. The computed minimum head is well above the penstock profile.

The maximum pressure in the scroll case and the maximum turbine rotational speed are two important parameters in turbine design. The pressure head in the scroll-case at the turbine inlet  $h_{sc}$  (datum level  $z = 685.0$  m) and the turbine rotational speed  $n$  are depicted in Fig. 2. There is a reasonable agreement between the results of the computation and the measurement. The computed maximum head  $h_{sc,max,c} = 504.2$  m is slightly higher than the measured one  $h_{sc,max,m} = 501.0$  m. The computed maximum turbine rotational speed  $n_{max,c} = 1075$  min<sup>-1</sup> is slightly lower than the measured speed  $n_{max,m} = 1082$  min<sup>-1</sup>. The discrepancies between the results increase at small wicket-gates openings.

### Case Study 2 – Velenje Mine

The Velenje mine pumping system (Slovenia) is a high head-system with a horizontal multistage centrifugal pump forcing water into a nearly vertical pipeline of diameter  $D = 0.205$  m and total length  $L = 441.5$  m (see Fig. 3). The water discharges freely into the atmosphere. The concentration ratio of solids in the water is negligible. An undamped swing-type check valve is installed at the downstream side of the pump to prevent pump-flow reversal. The pump operates at rated head  $H_r = 382$  m, discharge  $Q_r = 0.05$  m<sup>3</sup>/s and pump rotational speed  $n_r = 720$  min<sup>-1</sup>.

Pump start-up and rundown tests were performed in-situ. The water hammer in the pipeline was controlled by the dynamic action of the check valve [21].

### Izklop centrifugalne črpalke

Izklop centrifugalne črpalke je najbolj nevaren prehodni režim v obravnavanem črpalnem sistemu. Izklopimo elektromotor črpalke, povratna loputa se zapre v času  $t_{cv} = 1,1$  s, črpalka pa se zaustavi v času  $t_{ps} = 40$  s po izklopu.

Izračunane ovojnice največjih in najmanjših piezometričnih višin vzdolž profila cevovoda so prikazane na sliki 3. Iz slike 3 razberemo stvaritev področja neprekinjenega kavitacijskega toka na navzdolnjem koncu cevovoda, ki pa ima zanemarljiv vpliv na obliko obeh ovojnic. Obravnavani cevovod je projektiran tudi za podtlak.

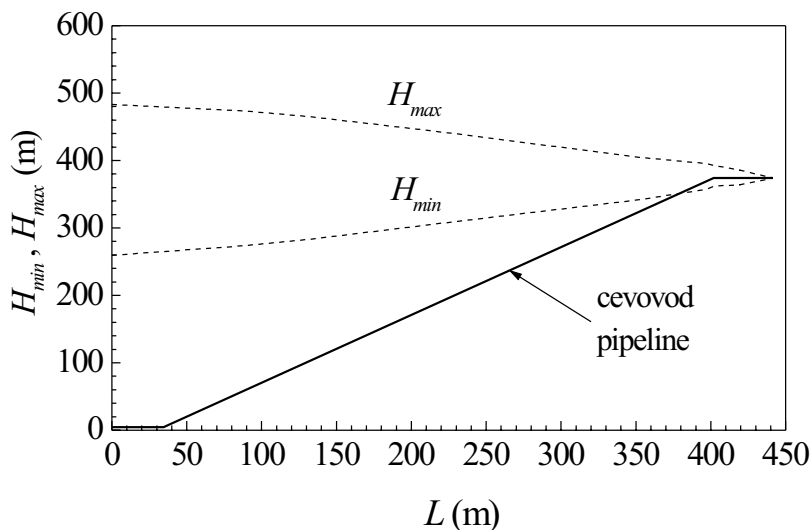
Na sliki 4 primerjamo izračunane in izmerjene tlačne višine  $h_{cv,d}$  na navzdolnjem koncu povratne lopute, ki je priključena k črpalci. Izračunana največja višina  $h_{cv,d,max,c} = 481,4$  m se dobro ujema z izmerjeno

### Centrifugal Pump Rundown

Pump rundown is the most severe transient regime in the considered pumping system. The pump-electromotor was switched off, the check valve shut in time  $t_{cv} = 1.1$  s and the pump stoppage time after power loss was  $t_{ps} = 40$  s.

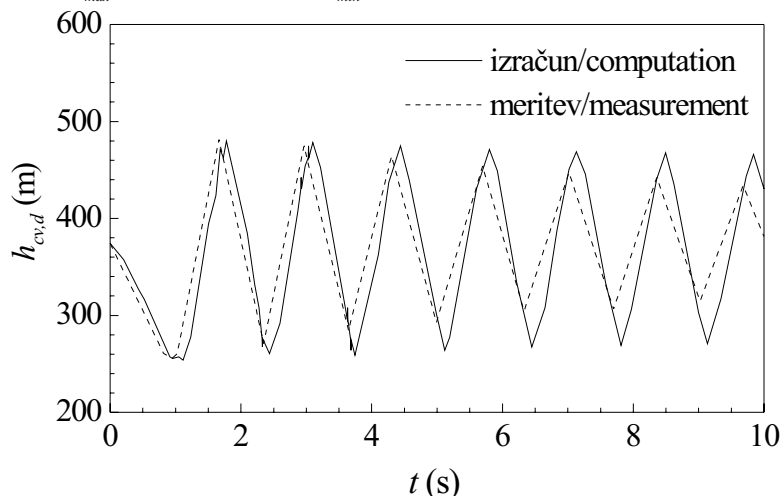
Computed envelopes of the maximum and minimum piezometric heads along the pipeline profile are shown in Fig. 3. As can be seen from Fig. 3, there is a distributed vaporous cavitation at the upper part of the pipeline, which does not significantly affect the shape of both envelopes. The pipeline is designed to withstand the underpressure.

Fig. 4 shows the comparison between computed and measured pressure heads  $h_{cv,d}$  at the downstream end of the check valve connected to the pump. The computed maximum head  $h_{cv,d,max,c} = 481.4$  m reasonably



Sl. 3. Izračunane ovojnice največjih in najmanjših višin vzdolž cevovoda za primer izklopa črpalke ( $H_{max}$  = največja višina,  $H_{min}$  = najmanjša višina,  $L$  = dolžina cevovoda)

Fig. 3. Computational envelopes of maximum and minimum heads along the pipeline after pump rundown ( $H_{max}$  = maximum head,  $H_{min}$  = minimum head,  $L$  = pipeline length)



Sl. 4. Primerjava izračunanih in izmerjenih rezultatov za primer izklopa črpalke ( $h_{cv,d}$  – tlačna višina na dolvodnem koncu povratne lopute,  $t$  – čas)

Fig.4. Comparison of computational and experimental results after pump rundown ( $h_{cv,d}$  – pressure head at the downstream end of the check valve,  $t$  – time)



višino  $h_{cv,d,max,m} = 483,3$  m. Časovni odmik med izračunanim in izmerjenim potekom tlačne višine izvira iz zapletenega modeliranja dinamičnega odziva povratne lopute. Na prvem in drugem računskem tlačnem nihanju razberemo šibke utripe. Ti utripi so inducirani z zrušitvijo šibkih diskretnih kavitacij na navzdolnjem delu cevovoda. Iz izmerjene tlačne višine ne razberemo obravnavanih kavitacijskih učinkov. Kondenzacija izračunanih diskretnih kavitacij generira večje tlake kakor pa kondenzacija dejanskega področja neprekinjenega kavitacijskega toka vzdolž cevovoda [12].

#### 4 SKLEP

Analiza prehodnih pojavov v pretočnih sistemih hidroelektrarn in črpalnih sistemih mora zajeti kritične obratovalne dogodke, da obremenitve, vzbujene z vodnim udarom, ne presežejo dopustnih vrednosti. Trenutna razbremenitev dveh francisovih turbin je najbolj nevaren prehodni režim v visokotlačni hidroelektrarni Toro II (Kostarika). Podobno vzbudi največje obremenitve izklop centrifugalne črpalke v visokotlačnem cevnom sistemu v rudniku Velenje. Rezultati izračuna, dobljeni z metodo karakteristik, se dobro ujemajo z rezultati meritev na terenu. Metodo karakteristik priporočamo za analizo prehodnih pojavov v hidravličnih sistemih, v katerih je dolžina cevovoda mnogo večja od premera cevi.

matches the measured one  $h_{cv,d,max,m} = 483,3$  m. The time shift between the calculated and measured head trace is mainly due to difficulties in the modelling of the dynamic behaviour of the check valve. The computed head exhibits low-amplitude pressure spikes superimposed on the first and the second pressure pulse. These spikes are due to discrete multi-cavity collapse at the upper part of the pipeline. The measured head does not exhibit such cavitating effects. Condensation of the computed discrete vapour cavities produces larger pressures than the condensation of the actual distributed vaporous cavitation zone along the pipeline [12].

#### 4 CONCLUSION

Transient analysis in hydro-electric power plants and pumping systems should include critical operating conditions such that the loads induced by water hammer are kept within the prescribed limits. Sudden load rejection of two Francis turbines is the most severe transient regime in the Toro II (Costarica) high-head hydro-electric powerplant. Similarly, centrifugal pump rundown is the most severe transient regime in the Velenje (Slovenia) high-head mine pumping system. The method-of-characteristics computational results agree reasonably with the results of the measurements for both cases. The method is recommended for the transient analysis in hydraulic systems where the pipeline length is much larger than the pipe diameter.

#### 5 OZNAČBE

#### 5 SYMBOLS

|  |            |  |
|--|------------|--|
| prečni prerez                                      | $A$        | pipe area  |
| hitrost vala                                       | $a$        | wave speed   |
| premer cevi  | $D$        | pipe diameter  |
| zemeljski pospešek                                 | $g$        | gravitational acceleration                             |
| piezometrična višina                               | $H$        | piezometric head                                       |
| tlačna višina na navzdolnjem koncu povratne lopute | $h_{cv}$   | pressure head at the downstream end of the check valve |
| tlačna višina v spirali                            | $h_{sc}$   | scroll-case pressure head                              |
| parna tlačna višina                                | $h_v$      | vapour pressure head                                   |
| polarni vztrajnostni moment vrtilnih delov         | $I$        | polar moment of inertia of rotating parts              |
| dolžina cevi                                       | $L$        | pipe length  |
| vrtilna frekvenca turbostroja                      | $n$        | turbomachine rotational speed                          |
| pretok   | $Q$        | discharge  |
| moment   | $T$        | torque   |
| čas  | $t$        | time   |
| čas zapiranja povratne lopute                      | $t_{cv}$   | check-valve closure time                               |
| čas nastanka kavitacije                            | $t_{in}$   | time of cavitation inception                           |
| čas zaustavitve črpalke                            | $t_{ps}$   | pump stoppage time                                     |
| prostornina diskretne kavitacije pare              | $V_v^{ps}$ | discrete vapour-cavity volume                          |
| pretočna hitrost                                   | $v$        | pipe flow velocity                                     |
| brezrazsežna tlačna karakteristika turbostroja     | $W_H$      | dimensionless turbomachine head characteristic         |
| brezrazsežna momentna karakteristika turbostroja   | $W_T$      | dimensionless turbomachine torque characteristic       |

|   |            |   |
|---|------------|---|
| koordinata, turbostroj v polarni karakteristiki | $x$        | distance, angular position in turbomachine characteristic curve |
| brezrazsežno odprtje vodilnika                  | $y$        | dimensionless wicket-gates position                             |
| geodetska višina                                | $z$        | elevation above datum   |
| časovni korak                                   | $\Delta t$ | time step   |
| dolžina cevnega odseka                          | $\Delta x$ | reach length  |
| Darcy-Weisbachov koeficient trenja              | $\lambda$  | Darcy-Weisbach friction factor                                  |
| strmina cevovoda                                | $\theta_p$ | pipe slope  |
| utežni koeficient                               | $\psi$     | weighting factor  |

*Indeksi*

|                       |
|-----------------------|
| izračun               |
| navzdolnje            |
| indeks računske točke |
| meritev               |
| največje              |
| najmanjše             |
| imensko               |
| čas                   |
| navzgoranje           |

*Subscripts*

|       |                             |
|-------|-----------------------------|
| $c$   | computation                 |
| $d$   | downstream                  |
| $j$   | computational section index |
| $m$   | measurement                 |
| $max$ | maximum                     |
| $min$ | minimum                     |
| $r$   | rated                       |
| $t$   | time                        |
| $u$   | upstream                    |

## 6 LITERATURA

## 6 REFERENCES

- [1] Chaudhry, M.H. (1987) Applied hydraulic transients. *Van Nostrand Reinhold Company*, New York, USA.
- [2] Bergant, A., A.R. Simpson, E. Sijamhodžić (1991) Water hammer analysis of pumping systems for control of water in underground mines. *Proceedings of the 4<sup>th</sup> International Mine Congress*, Ljubljana, Slovenia & Pörschach, Austria, Vol. 2, 9 - 20.
- [3] Wylie, E.B., V.L. Streeter (1993) Fluid transients in systems. *Prentice Hall*, Englewood Cliffs, USA.
- [4] Bergant, A., E. Sijamhodžić (1997) Water hammer problems related to refurbishment and upgrading of hydraulic machinery. *Hydropower into the Next Century*, Portorož, Slovenia, 611 - 622.
- [5] Bergant, A., A.R. Simpson (1997) Development of a generalised set of pipeline water hammer and column separation equations. *Report n. R149*, Dept. of Civil and Envir. Engrg., University of Adelaide, Adelaide, Australia.
- [6] Wylie, E.B. (1983) Advances in the use of MOC in unsteady pipeline flow. *BHRA Conference on Pressure Surges*, Bath, England, 349 - 356.
- [7] Anderson, A., M. Arfaie, R. Sandoval-Pena, K. Suwan (1991) Pipe-friction in water hammer. *XXIV IAHR Congress*, Madrid, Spain, D23 - D30.
- [8] Bergant, A., A.R. Simpson (1994) Estimating unsteady friction in transient cavitating pipe flow. *BHR Group Conference on Water Pipeline Systems*, Edinburgh, Scotland, 3 - 16.
- [9] Bergant, A., A.R. Simpson, V. Vítkovský (2001) Developments in unsteady pipe flow friction modeling. *IAHR Journal of Hydraulic Research*, 39(3), 249 - 257.
- [10] Simpson, A.R., E.B. Wylie (1991) Large water hammer pressures for column separation in pipelines. *ASCE Journal of Hydraulic Engineering*, 117(10), 1310 - 1316.
- [11] Bergant, A., A.R. Simpson (1992) Interface model for transient cavitating flow in pipelines. *Conference on Unsteady Flow and Fluid Transients*, Durham, England, 333 - 342.
- [12] Bergant, A., A.R. Simpson (1999) Pipeline column separation flow regimes. *ASCE Journal of Hydraulic Engineering*, 125(8), 835 - 848.
- [13] Simpson, A.R., A. Bergant (1994) Numerical comparison of pipe-column-separation models. *ASCE Journal of Hydraulic Engineering*, 120(3), 361 - 377.
- [14] Wylie, E.B. (1984). Simulation of vaporous and gaseous cavitation. *ASME Journal of Fluids Engineering*, 106(3), 307 - 311.
- [15] Krivčenko, G.I., N.N. Aršenevskij, E.V. Kvjatovskaja, V.M. Klabukov (1975) Hydromechanical transient processes in hydro power plants (in Russian). *Energija*, Moscow, Russia.
- [16] Carnahan, B., H.A. Luther, J.O. Wilks (1969). Applied numerical methods. *John Wiley & Sons*, New York, USA.
- [17] Bergant, A., E. Sijamhodžić (1998) Water hammer flow regimes in a high-head Francis turbine hydro powerplant. *Hydroturbo '98*, Loučná nad Desnou, Czech Republic, 237 - 245.

- [18] Fašalek, J. (1985) Unsteady phenomena at complete rundown of the pump turbine with particular reference to the pumping mode of operation (in Slovene). *Master of Science Thesis*, Dept. of Mech. Engrg., University of Ljubljana, Ljubljana, Slovenia.
- [19] Bergant, A. (1986) Review of modern theoretical methods for hydraulic transient analysis applied in Litostrój (in Serbian). *Transient Processes in Hydrotechnical Systems*, Belgrade, Serbia and Montenegro, Vol. 2, 8–20.
- [20] Bergant, A. (1992) Transient cavitating flow in piping systems (in Slovene). *Dissertation*, Dept. of Mech. Engrg., University of Ljubljana, Ljubljana, Slovenia.
- [21] Kruisbrink, A.C.H. (1988) Check valve closure behaviour. Experimental investigation and simulation in water hammer computer programs. *BHR Group Conference on Developments in Valves and Actuators*, Manchester, England, 21 pp.

Avtorjev naslov: doc.dr. Anton Bergant  
Litostrój E.I. d.o.o.  
Litostrójska 40  
1000 Ljubljana  
anton.bergant@litostrój-ei.si

Author's Address: Doc.Dr. Anton Bergant  
Litostrój E.I. Ltd.  
Litostrójska 40  
1000 Ljubljana, Slovenia  
anton.bergant@litostrój-ei.si

Prejeto: 13.12.2002  
Received:

Sprejeto: 29.5.2003  
Accepted:

Odprt za diskusijo: 1 leto  
Open for discussion: 1 year

## Dvodimenzijsko modeliranje gibanja drobirskega toka v Logu pod Mangartom kot primer nenewtonske tekočine

### Two-Dimensional Modelling of Debris-Flow Movement in Log pod Mangartom as an Example of a Non-Newtonian Fluid

Matjaž Četina - Mario Krzyk

Z matematičnimi modeli je bilo simulirano gibanje drobirskega toka, ki je nastal po sprožitvi plazu Stože, novembra 2000, in razdejal del vasi Log pod Mangartom. Na zgornjem delu vplivnega področja, kjer se je drobirski tok v glavnem gibal po ozkem kanjonu, je bil uporabljen enodimenzijski model DEBRIFID. Za izbrane primere mogočih prihodnjih plazov so bili tako določeni hidrogrami pretokov drobirskega toka na zgornjem koncu ogrožene vasi Log pod Mangartom. Dvodimenzijske simulacije drobirskega toka v vasi so bile narejene z dvema modeloma: s PCFLOW2D, ki je bil razvit na FGG Univerze v Ljubljani in s tržnim modelom FLO-2D. Vsi trije modeli so bili uspešno umerjeni s terenskimi meritvami dosega plazov, pri čemer so bili upoštevani tudi rezultati geomehanskih laboratorijskih raziskav. Narejena je bila analiza občutljivosti dvodimenzijskega modela PCFLOW2D na gostoto numerične mreže in različna robna pogoja na iztoku iz področja. Veljavnost kvadratne enačbe, ki izraža trenje pri Binghamovem plastičnem modelu nenewtonske tekočine, je bila potrjena tudi pri spreminjajočem se režimu toka. Modeli so bili uspešno uporabljeni za določitev ukrepov za zaščito vasi Log pod Mangartom. Najbolj učinkovit ukrep je odstranitev drobirskih mas, ki so se odložile po zadnjem plazov v strugi in ob njej vzdolž vasi, tako da bi se morebitni novi drobirski tokovi lahko gibali mimo vasi ali imeli prostor za odlaganje.

© 2003 Strojniški vestnik. Vse pravice pridržane.

**(Ključne besede: tokovi drobirski, modeli matematični, tekočine nenewtonske, urejanje hudournikov)**

Mathematical models have been used for simulations of the landslide (debris flow) below Stože that occurred in November 2000, and destroyed part of the village of Log pod Mangartom. The one-dimensional model, DEBRIFID, was first used for simulations along the upper part of the affected region, where the debris flow was mainly in a narrow canyon. Hydrographs of the debris-flow discharge at the upstream end of the affected village of Log pod Mangartom were thus determined for possible future landslides. Two-dimensional simulations of the debris flow in the village were carried out using two models: PCFLOW2D, developed at the University of Ljubljana; and a commercial model, FLO-2D. The three models were successfully calibrated using field measurements of the debris-flow inundation limits, taking into account the results of geo-mechanical laboratory experiments. The sensitivity analysis of the two-dimensional model PCFLOW2D with different grid sizes and two different outflow boundary conditions was also made. The validity of the quadratic equation expressing the resistance of the non-Newtonian Bingham plastic fluid model was roughly confirmed, even for changing flow regimes. The models were used successfully to determine the measures for the future protection of the village of Log pod Mangartom. The most effective measure is the removal of debris mass, which was deposited after the last landslide in the stream along side the village, so that the future debris flows will move through or will be deposited.

© 2003 Journal of Mechanical Engineering. All rights reserved.

**(Keywords: debris flow, mathematical models, non-Newtonian fluids, torrent control)**

#### 0 UVOD

Blatni in drobirski tokovi, ki lahko nastanejo po sprožitvi plazov v hribovitih področjih, često povzročijo veliko materialno škodo in celo žrtve. V Sloveniji je približno tretjina ozemlja pokrita s strmimi

#### 0 INTRODUCTION

Landslide-induced mud/debris flows are often the cause of a significant loss of property and casualties in mountainous regions. In Slovenia, approximately one third of the country is covered by

pobočji, ki so samo pogojno stabilna. Zaradi neugodne geološke sestave na njih pogosto prihaja do plazov, ki jih lahko povzročijo potresi ali intenzivne padavine [12]. Znanstveniki si prizadevajo razviti zanesljive numerične modele, s katerimi bi bilo mogoče napovedati doseg možnih plazov in določiti učinke različnih varovalnih ukrepov.

Pojav blatnih in drobirskih tokov je podoben nestalnemu toku vode, posebej valovom zaradi porušitev pregrad, in do neke mere tudi dinamiki snežnih plazov [17]. V vseh primerih se tok ravna po fizikalnih zakonih ohranitve mase in gibalne količine. Znano pa je, da je trenjski upor pri gibanju drobirskega toka in snežnega plazu bolj zapleten kakor pri vodnem toku [13]. Reološko obnašanje toka je v veliki meri odvisno od koncentracije sedimentov v vodi. Glede na to razmerje lahko prihaja do skoraj suhih plazov, drobirskih tokov, blatnih tokov in toka čiste vode [1]. Koncentracija se vzdolž poti drobirskega toka zaradi vnosa dodatnih sedimentov in/ali povečanega dotoka vode s pritoki pogosto spreminja, kar simulacije toka še otežuje.

V prispevku je prikazano eno- in dvodimenzijsko modeliranje drobirskega toka, do katerega je prišlo novembra 2000 v vasi Log pod Mangartom. Tragični dogodek razmeroma velikega obsega z začetno maso plazu 1.200.000 m<sup>3</sup> je zahteval sedem življenj in povzročil veliko materialno škodo v vasi Log ([6], [9], [10] in [11]). Podani so opis tega dogodka ter razvoj, umerjanje in preverjanje uporabljenih matematičnih modelov. Prikazani in ocenjeni so rezultati simulacij možnih prihodnjih drobirskih tokov, v sklepih pa so predlagani nekateri najbolj učinkoviti ukrepi za zaščito vasi Log pod Mangartom.

#### 1 OPIS DOGODKA POJAVA DROBIRSKEGA TOKA

Dne 15. novembra leta 2000 se je na pobočju pod Stožami utrgal zemeljski plaz, ki je zdrsil do mostu v Mlinču (most I), ga le delno prelil, večina mase pa se je ustavila nad mostom (odsek A, sl. 1). Po dvodnevnem močnem deževju in zaradi vtoka Mangartskega potoka se je plazovina zelo razmočila. Nekaj po polnoči 17. novembra je na pobočju pod Stožami očitno zdrsil dodatni manjši plaz. Drsel je do razmočenega prvega, tega ponovno sprožil v gibanje, nato pa je skupna masa drobirskega toka prelila in odnesla most I ter zdrvela prek ozkega kanjona Mangartskega potoka in naprej po soteski Predelice do Gorenjega Loga (območje B, sl. 1). Po približno štirih minutah (hitrost čela je bila okrog 15 m/s) je drobirski tok dosegel vas Gorenji Log (odsek C, sl. 1), kjer je v svojih domovih umrlo 7 ljudi, uničenih je bilo 6 hiš, resno poškodovanih pa 23 stanovanjskih ali gospodarskih poslopij ([6], [9] in [10]). Slika 2 prikazuje zračni posnetek opustošenega odseka C, iz katerega je razviden tudi največji doseg opisanega pojava drobirskega toka.

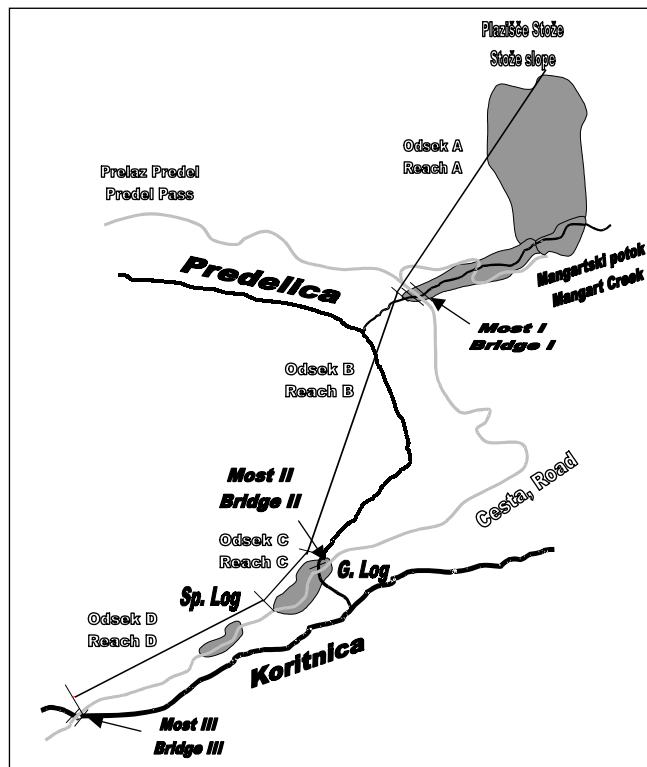
slopes, which are conditionally stable due to their unfavourable geological composition, and are often threatened by landslides caused, e.g. by earthquakes or heavy rainfalls [12]. Scientists are trying to develop reliable numerical models that are able to predict inundation limits for possible future landslides and to determine the effect of different protective measures.

The phenomenon of mud/debris flow is similar to the unsteady flow of water, in particular to dam-break flow, and also to snow avalanche dynamics [17]. In all these cases the flow is governed by the physical laws of conservation of mass and of momentum. However, it is well known that the resistance to flow movement is more complex in debris flows and snow avalanches than in pure water flows [13]. The rheological behaviour of the flow depends to a large extent on the debris-flow sediment/water concentration. This can range from nearly dry landslides to debris flows, mud flood and pure water flows [1]. The concentration often changes along the debris path, as additional sediment is scavenged into the flow and/or additional water streams join the debris flow, which makes the simulations even more difficult.

One- and two-dimensional mathematical modelling of the debris flow that occurred in November 2000 at the village of Log pod Mangartom is presented. This tragic event involved an initial landslide mass of 1.200.000 m<sup>3</sup> and resulted in the loss of seven lives and a lot of property ([6], [9], [10] and [11]). The paper includes a description of this event as well as the development, the calibration and the validation of the mathematical models used. The results of possible future debris-flow simulations are discussed and in the conclusions some of the most effective protective measures for the village of Log pod Mangartom are proposed.

#### 1 DESCRIPTION OF THE DEBRIS-FLOW EVENT

On November 15<sup>th</sup>, 2000, a landslide glided down the slope of Stože (Fig. 1). The mass mostly stopped at the Mlinč Bridge (Bridge I), just a few percent of the mass came over the bridge (Reach A, Fig. 1). During the next two days the mass was moistened by heavy rain and by the inflow of the Mangart Creek. Just after midnight on November 17<sup>th</sup> a smaller, additional landslide glided down the slope and triggered the very wet mass of the first landslide into a debris flow. The combined mass swept away the bridge, completely destroying it, and flowed down the narrow canyon of the Predelica Torrent (Reach B, Fig. 1). In about 4 minutes (front velocity about 15 m/s) it reached the village of Gorenji Log (Reach C, Fig. 1), where it killed 7 people in their homes, destroyed 6 and severely damaged 23 residential or farm buildings ([6], [9] and [10]). Fig. 2 shows the bottom view of the devastated Reach C, together with the envelope of the debris-flow event.



Sl. 1. Situacija področja in računski odseki

Fig. 1. Situation of the region and the computational reaches



Sl. 2. Fotografija območja drobirskega toka v Gorenjem Logu (foto: B. Vlaj, 22. 11. 2000)

Fig. 2. Photograph of the debris-flow area in Gorenji Log (photo: B. Vlaj, 22 Nov. 2000)

Na odseku C pri vasi Gorenji Log je postal tok zaradi dodatnih voda reke Koritnice redkejši in material še bolj razmočen. Po dolini Koritnice je tekel počasneje in deloma odlagal material vzdolž svoje poti. Večina materiala se je odložila navzgor od ožine reke Koritnica na

In Reach C, nearby the village of Gorenji Log, the debris flow received an additional water discharge from the Koritnica River, and it continued its movement more slowly, partly depositing the mass along its path. Most of the debris mass was deposited upstream of the very narrow section of the Koritnica

koncu odseka D, manjši del pa je nadaljeval pot proti reki Soči.

Skupaj se je s plazišča Stože premaknilo 1,200.000 m<sup>3</sup> materiala (sl. 1). Prostornina prvega plazu (15. novembra), ki se je ustavil na območju A, je bila ocenjena na približno 600.000 m<sup>3</sup>. 17. novembra pa je šlo skozi profil mostu I in naprej po reki navzdol okrog 950.000 m<sup>3</sup> materiala. Od tega ga je približno 800.000 m<sup>3</sup> doseglo vas Gorenji Log, ker se je 150.000 m<sup>3</sup> materiala odložilo vzdolž poti ([6], [9] in [10]).

## 2 TEORETIČNE OSNOVE MATEMATIČNIH MODELOV

### 2.1 Izbira modelov

Tok vzdolž odseka A je bil simuliran z modeloma DEBRIF1D in FLO-2D, tok vzdolž odseka B pa je bil tipično enodimenzijski in zato simuliran samo z modelom DEBRIF1D. Na odseku C pri vasi Gorenji Log je bil tok tipično dvodimenzijski in simuliran z obema modeloma, PCFLOW2D in FLO-2D, saj gre za pomembno območje, kjer lahko pride do žrtev in materialne škode. Odsek D je bil simuliran z enodimenzijskim modelom in v zgornjem delu (vas Spodnji Log, sl. 1) tudi z modelom FLO-2D. Za povezavo med zaporednimi odseki so bili uporabljeni izračunani hidrogrami na koncu vzdolžnih odsekov.

### 2.2 Enodimenzijski model DEBRIF1D

Že leta 1972 je bil na Fakulteti za gradbeništvo in geodezijo Univerze v Ljubljani razvit enorazsežen model LAXDAM [18] za simulacijo valov zaradi porušitev pregrad. Kasneje je bil model uporabljen kot osnova za razvoj numeričnega modela SNOWDYN za simulacijo dinamike snežnih plazov ([17] in [16]). Leta 2001 pa je bil model razširjen še za simulacijo drobirskih tokov, DEBRIF1D. Vsi trije modeli rešujejo enodimenzijske St. Venantove enačbe v t.i. "konservativni" obliki.

Glavna razlika med opisanimi pojavi je v formulaciji upora toku ter v začetnih in robnih pogojih. Model DEBRIF1D uporablja izraz za trenjski nagib  $S_f$ , ki je vzet iz [14]:

$$S_f = \frac{\tau_y}{\gamma_m h} + \frac{K V \eta}{8 \gamma_m h^2} + \frac{n_g^2 V^2}{h^{4/3}} + S_e + S_b \quad (1),$$

kjer so:  $\tau_y$  – mejna strižna trdnost mešanice vode in usedline;  $\gamma_m$  – specifična teža mešanice vode in usedline;  $h$  – globina toka;  $K$  je brezdimenzijski parameter upora; uporabili smo vrednost  $K=2285$ , ki jo priporoča O'Brien [7] za naravne vodotoke z zelo nepravilno razgibanostjo terena;  $\eta$  – dinamična viskoznost mešanice;  $V$  – hitrost toka;  $n_g$  – Manningov koeficient hrapavosti;  $S_e$  – nagib

Creek, at the end of Reach D, and a smaller part of it was transported further down to the Soča River.

In total, a volume of 1.200.000 m<sup>3</sup> was displaced from its original location on the Stože slope (Fig.1). The volume of the first debris-landslide (on November 15<sup>th</sup>), deposited along Reach A was estimated to be about 600,000 m<sup>3</sup>. On November 17<sup>th</sup> about 950,000 m<sup>3</sup> moved across the section of Bridge I downstream. A volume of about 800,000 m<sup>3</sup> reached the village of Gorenji Log, as 150,000 m<sup>3</sup> were deposited along the path ([6], [9] and [10]).

## 2 THEORETICAL BACKGROUND OF THE MATHEMATICAL MODELS

### 2.1 The choice of models

The flow along Reach A was simulated using the DEBRIF1D and FLO-2D models, the flow in Reach B was typically one-dimensional and thus simulated only with the DEBRIF1D model. Along Reach C (village of Gorenji Log) the flow was two-dimensional and, as this is the most important region (safety of inhabitants, loss of property), we simulated this reach with both (PCFLOW2D and FLO-2D) models. Reach D was simulated using the one-dimensional model, and along the upper part (village of Spodnji Log, Fig. 1) also with the FLO-2D model. The connections between the subsequent reaches were made with calculated hydrographs at the end of each upstream reach.

### 2.2 One-dimensional model DEBRIF1D

In 1972 a one-dimensional model, LAXDAM [18], for the simulation of a dam-break flow was developed at the Faculty of Civil and Geodetic Engineering of the University of Ljubljana. Later, this model was used as a basis for developing a numerical model for the simulation of snow avalanche dynamics, SNOWDYN ([17] and [16]). In 2001 this model was extended to a debris-flow model, DEBRIF1D. All three models solve one-dimensional St. Venant equations in what is called the »conservation« form.

The main difference between the three phenomena is in the formulation of the flow resistance and in the formulation of initial and boundary conditions. The DEBRIF1D model uses the formulation for resistance slope  $S_f$ , adapted from [14]:

where:  $\tau_y$  – the yield shear stress of the sediment-water mixture;  $\gamma_m$  – the specific weight of the water-sediment mixture;  $h$  – the flow depth;  $K$  is the non-dimensional friction parameter, we used the value of  $K=2285$  that was proposed by O'Brien [7] for natural streams with a highly irregular terrain configuration;  $\eta$  – the dynamic viscosity of the mixture;  $V$  – the flow velocity;  $n_g$  – the Manning roughness coefficient;  $S_e$

energijskih izgub zaradi vrtincev v hitrih razširitvah, ki so približno enake razliki kinetičnih členov v dveh zaporednih prečnih prerezih [18];  $S_b$  – energijski nagib zaradi ovinkov; uporabljena je bila preprosta enačba, ki je znana iz hidravlike ustaljenega toka [18]:

$$S_b = 2 \frac{B_b}{r_b} \frac{V^2}{\Delta x} \frac{1}{2g} \quad (2),$$

kjer pomenijo:  $B_b$  – širino proste gladine v ovinku;  $r_b$  – polmer ovinka;  $\Delta x$  – vzdolžno razdaljo med prečnimi prerezi.

Enačba (1) je bila izpeljana iz enačbe za strižno napetost, ki je znana kot kvadratni reološki model:

$$\tau = \tau_y + \eta \frac{dV}{dy} + C \left( \frac{dV}{dy} \right)^2 \quad (3).$$

V literaturi je enačba (3) znana kot model O'Brien in Juliena, kjer so  $\tau_y$  – vsota kohezivne strižne napetosti in Mohr-Coulombove napetosti;  $C$  – stalnica,  $y$  – smer, pravokotno na hitrost toka  $V$ .

Za reševanje je uporabljena eksplicitna numerična metoda Lax-Wendroff. Ker je momentna enačba upoštevana v konservativni obliki in rešena s primerno numerično metodo končnih razlik, lahko model delno zajame tudi izgube na čelu vala. To omogoča enoten račun čela s preostalim tokom, brez posebnih tehnik za napredovanje čela [18].

Pomembna lastnost modela DEBRIF1D je, da vključuje izračun začetnega hidrograma  $Q(t)$  na nizvodnem koncu začetne drobirske mase. Postopek je vzet iz modela za račun porušnih valov, kjer se v prvem trenutku po porušitvi ( $t = 0$ ) kota vode in hitrost na mestu pregrade izračuna z dinamično in kontinuitetno enačbo ter enačbo pozitivne karakteristike [18]. Znana mora biti geometrijska oblika plazju pred premikom. V času  $t > 0$  teče račun zvezno od vzvodnega konca plazeče se mase do čela drobirskega toka (premikajoči se rob), hidrogram na pregradnem mestu pa se izračuna posredno. Večina modelov drobirskega toka, med njimi tudi modela PCFLOW2D in FLO-2D, zahteva namreč določitev začetnega hidrograma z uporabo približnih metod in začetne drobirske mase.

### 2.3 Dvodimenzijski model PCFLOW2D

Kontinuitetna (4) in enačbi ohranitve gibalne količine (5) in (6), ki opisujejo dvodimenzijski, nestisljiv, globinsko povprečni drobirski tok, so uporabljene v naslednji obliki:

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = 0 \quad (4)$$

$$\frac{\partial(hu)}{\partial t} + \frac{\partial(hu^2)}{\partial x} + \frac{\partial(huv)}{\partial y} = -gh \frac{\partial h}{\partial x} - gh \frac{\partial z_b}{\partial x} - gh S_{fx} + \frac{\partial}{\partial x} (h v_{ef} \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y} (h v_{ef} \frac{\partial u}{\partial y}) \quad (5)$$

– the resistance slope due to eddy losses at sharp channel expansions, equal to the difference of the values of the kinetic terms in two subsequent cross-sections [18];  $S_b$  – the resistance slope due to bends; a simple equation, known from steady-state river hydraulics [18] was used:

in which  $B_b$  – the free surface width of the bend;  $r_b$  – the radius of the bend;  $\Delta x$  – the longitudinal distance between cross sections.

Equation (1) was derived from the equation for shear stress, which is known as the quadratic rheologic model:

In the literature Equation (3) is known as the O'Brien-Julien model, where in  $\tau_y$  the sum of cohesive yield stress and Mohr-Coulomb's stress is included;  $C$  is a constant and  $y$  is a direction normal to the flow velocity  $V$ .

The explicit, numerical Lax-Wendroff method is used for the solution. Since the momentum equation is used in conservative form and solved by the appropriate finite-difference numerical method, the model can partly take into account the energy losses at the wave front. This enables a direct simulation of the wave front without any special wave-front tracking technique [18].

One important feature of the DEBRIF1D model is that it includes the computation of the initial flow hydrograph  $Q(t)$  at the downstream end of the initial debris mass. The procedure is taken from the dam-break flow model, where during the first instant after the dam collapse ( $t = 0$ ) the water level and the velocity at the dam site are calculated using the momentum and continuity equations and the equation of the forward characteristic [18]. The geometry of the landslide mass before the displacement must be known. At  $t > 0$  the computation runs continuously from the upstream end of the moving landslide mass to the front of the debris flow (moving boundary), and the hydrograph at the "dam site" is calculated implicitly. Most debris-flow models, among them PCFLOW2D and FLO-2D, demand the determination of the initial hydrograph using approximate methods, based on the initial debris mass.

### 2.3 Two-dimensional model PCFLOW2D

The continuity (4) and momentum Equations (5) and (6), describing two-dimensional, incompressible, unsteady, depth-averaged debris flow, are used in the following form:



$$\frac{\partial(hv)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial(hv^2)}{\partial y} = -gh \frac{\partial h}{\partial y} - gh \frac{\partial z_b}{\partial y} - ghS_{fy} + \frac{\partial}{\partial x} (hv_{ef} \frac{\partial v}{\partial x}) + \frac{\partial}{\partial y} (hv_{ef} \frac{\partial v}{\partial y}) \quad (6),$$

kjer so:  $t$  – čas;  $h$  – globina toka;  $u$  in  $v$  – globinsko povprečni komponenti hitrosti v smereh  $x$  in  $y$ ;  $z_b$  – kota dna,  $g$  – gravitacijska stalnica in  $v_{ef}$  – koeficient efektivne viskoznosti. Člena  $S_{fx}$  in  $S_{fy}$  izražata nagib energijske črte v smereh  $x$  in  $y$  in se lahko analogno kakor pri formulaciji za enorazsežen tok (1) zapišeta v naslednji obliki:

$$S_{fx} = \frac{\tau_y}{\gamma_m h} + \frac{K u \eta}{8 \gamma_m h^2} + \frac{n_g^2 u \sqrt{u^2 + v^2}}{h^{4/3}} \quad (7)$$

$$S_{fy} = \frac{\tau_x}{\gamma_m h} + \frac{K v \eta}{8 \gamma_m h^2} + \frac{n_g^2 v \sqrt{u^2 + v^2}}{h^{4/3}} \quad (8).$$

Sistem povezanih parcialnih diferencialnih enačb (4) do (6) je rešen z numerično metodo končnih prostornin [15]. Glavne značilnosti metode so premaknjena numerična mreža, kombinacija vzvodne in centralno diferenčne sheme (t.i. hibridna shema) ter iterativni postopek popravkov globin (SIMPLE). Za integracijo po času je uporabljena polna implicitna shema, ki zagotavlja stabilno in dovolj točno rešitev tudi pri razmeroma visokih Courantovih številih (do 10). Simulirati je mogoče tako mirne kakor tudi deroče tokove [5].

Za račune drobirskega toka v Logu pod Mangartom je bila upoštevana enakomerna numerična mreža s korakoma  $\Delta x = \Delta y = 2$  m, zajeti pa so bili tudi vsi objekti na obravnavanem računskem področju.

## 2.4 Dvodimenzijski model FLO-2D

To je tržni dvodimenzijski model, ki temelji na metodi končnih razlik in je bil razvit v ZDA [7]. Model je bil narejen za račun razširjanja nenevtonskih poplavnih valov, blatnih in drobirskih tokov. V ZDA ga za uporabo priporoča Zvezna uprava nezgod (Federal Emergency Management Agency - FEMA), široko pa se uporablja tudi v Avstriji, Švici in Italiji. Dvodimenzijske kontinuitetna in dinamični enačbi so podobne enačbam (4) do (6), edina razlika je, da so uporabljene v nekonservativni obliki. Reološki členi so ovrednoteni na podlagi prvih treh členov v enačbi (1), po katerih dobimo dvodimenzijski enačbi (7) in (8). Na računskem področju Loga pod Mangartom je bila upoštevana enakomerna numerična mreža velikosti  $\Delta x = \Delta y = 4$  m.

Ker je bil model že uspešno uporabljen za simulacije različnih praktičnih primerov drobirskega toka [8], smo ga uporabili hkrati z našim modelom PCFLOW2D, da bi dobili bolj zanesljive rezultate. Kakor bo prikazano pri umerjanju, je bilo ujemanje med rezultati obeh modelov dovolj dobro.

where:  $t$  – the time;  $h$  – the flow depth;  $u$  and  $v$  – the depth-averaged velocity components in the  $x$  and  $y$  directions;  $z_b$  – the bottom level;  $g$  – the gravity constant and  $v_{ef}$  – the effective coefficient of viscosity. The terms  $S_{fx}$  and  $S_{fy}$  express energy-line slopes in the  $x$  and  $y$  directions and can be analogous to the one-dimensional flow formulation (Eq. 1) written as follows:

The set of coupled partial differential equations (4) to (6) is solved by the finite-volume numerical method [15]. The main characteristics of the method are a staggered numerical grid, the combination of an upwind and central-difference schemes (the so-called hybrid scheme) and an iterative procedure of depth corrections (SIMPLE). A fully implicit scheme is used for time integration providing a stable and accurate solution even at relatively high Courant numbers (up to about 10). It is possible to simulate both subcritical or supercritical flows [5].

For the computations of the debris flow at Log pod Mangartom, the uniform mesh  $\Delta x = \Delta y = 2$  m was used and all buildings in the computational domain were also taken into account.

## 2.4 Two-dimensional model FLO-2D

This is a commercial, two-dimensional, finite-difference model, developed in the USA [7]. The model was conceived for routing non-Newtonian flood flows, as well as mudflows and debris flows. In the USA it is recommended for use by the Federal Emergency Management Agency (FEMA), and it is also widely used in Austria, Switzerland and Italy. The two-dimensional continuity and momentum equations are similar to Eqs. (4) to (6), the only difference is that they are used in a non-conservative form. The rheological terms are evaluated as the first three terms in Eq. (1) resulting in the two-dimensional Eqs. (7) and (8). The uniform mesh  $\Delta x = \Delta y = 4$  m was taken into account for the computational domain at the village of Log pod Mangartom.

As the model has been successfully used for several practical cases of debris-flow simulations [8], we used this model in parallel with our PCFLOW2D model to get more reliable results. As we showed in the calibration process, we obtained a reasonable agreement between the results of both models.

### 3 UMERJANJE MATEMATIČNIH MODELOV

Pri umerjanju eno- in dvodimenzijskih modelov smo iskali ustrezne vrednosti treh reoloških parametrov v enačbah (1), (7) in (8):  $\tau_y$ ,  $\eta$  in  $n_g$ . Uporabili smo tri metode za določitev teh vrednosti: (a) razpoložljive vrednosti iz literature; (b) rezultate geomehanskih laboratorijskih preizkusov [9]; in (c) vrednosti, dobljene z umerjanjem modela na temelju primerjave s terenskimi meritvami.

Prvi plaz na odseku A (15. novembra 2000) je bil razmeroma suh in se je ustavil na pobočju z nagibom 16%. Enorazsežen model je ta pojav simuliral z vrednostmi  $\tau_y = 2000 \text{ N/m}^2$  in  $\eta = 156 \text{ Pa.s}$ . Zelo moker drobirski tok vzdolž odseka B (17. novembra 2000) je bil umerjen pri vrednostih  $\tau_y = 20 \text{ N/m}^2$  in  $\eta = 40 \text{ Pa.s}$ . Ker obsega odsek B zelo strm, ozek kanjon s povprečnim padcem 17% in največjim padcem 119%, se je Manningov koeficient spreminjal v zelo širokih mejah od 0,03 do  $0,35 \text{ sm}^{-1/3}$ . Največje vrednosti so bile umerjene na najstrmejših delih odseka B, kjer je moral biti tok podoben vodnemu slapu. Slika 3 prikazuje ovojnico simuliranih najvišjih kot in na podlagi največjega dosega zabeležene gladine drobirskega toka vzdolž zgornjega dela odseka B. Menimo, da je ujemanje glede na zapletenost pojava zadovoljivo.

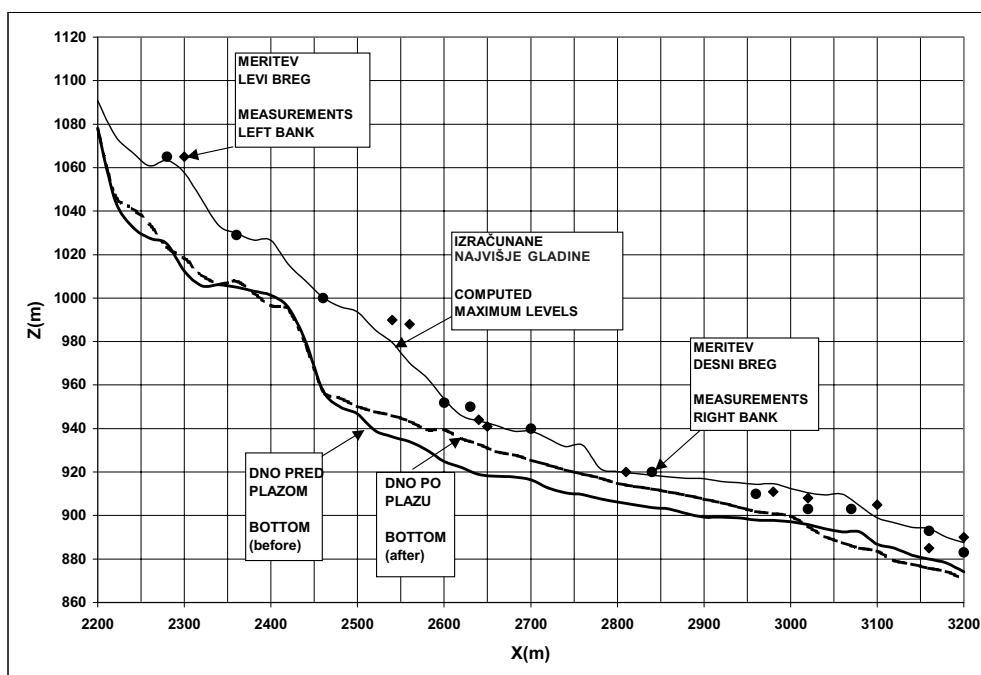
Podobna točnost je bila dobljena tudi na odsekih A, C in D. Za odsek C je primerjava merjenega in z dvodimenzijskima modeloma PCFLOW2D in FLO-2D simuliranega dosega drobirskega toka prikazana na sliki 4. Umerjene

### 3 CALIBRATION OF THE MATHEMATICAL MODELS

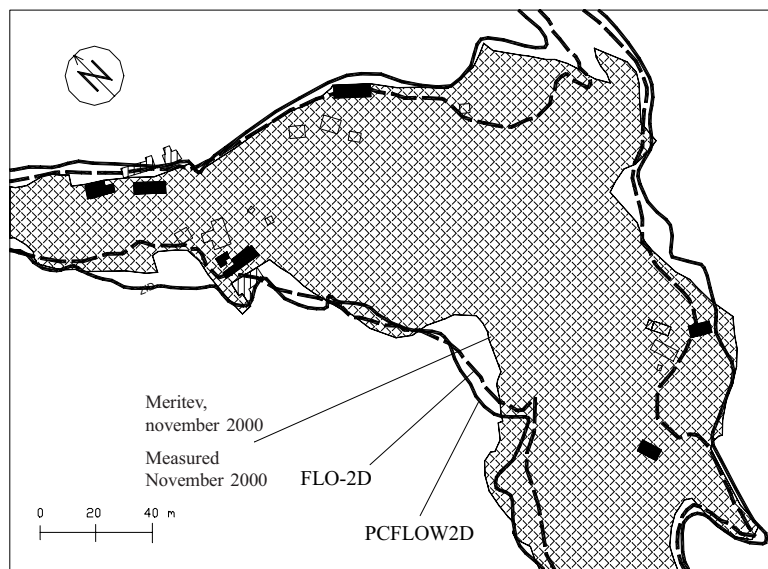
Calibration of both the one- and two-dimensional models was made by finding proper values of the three rheological parameters in Eqs. (1), (7) and (8):  $\tau_y$ ,  $\eta$ , and  $n_g$ . We used three possible methods for determining these values: (a) values available in the literature; (b) results of geomechanical laboratory measurements [9]; and (c) values obtained by the calibration of the model – comparison with the field measurements.

The first landslide along Reach A (November 15<sup>th</sup>, 2000) was relatively dry and it stopped on a slope of 16%. The one-dimensional model simulated this phenomenon with values of  $\tau_y = 2000 \text{ N/m}^2$ , and  $\eta = 156 \text{ Pa.s}$ . The very wet debris flow along Reach B (November 17<sup>th</sup>, 2000) was calibrated with the values  $\tau_y = 20 \text{ N/m}^2$  and  $\eta = 40 \text{ Pa.s}$ . Since Reach B is a very steep, narrow canyon, with an average slope of 17%, and a maximum slope of 119%, the Manning coefficient changed in broad limits from 0.03 to  $0.35 \text{ sm}^{-1/3}$ . The greatest values were found along the steepest parts of Reach B, where the flow must have been similar to a waterfall. Fig. 3 shows the longitudinal profile of the simulated and measured maximum debris-flow surface elevations along the upper part of Reach B. We can consider the agreement as satisfactory.

Similar accuracy was also obtained for Reaches A, C and D. Along Reach C, the comparison of the measured and simulated inundation limits obtained by the two-dimensional models PCFLOW2D and FLO-2D is shown in Fig. 4. The calibrated debris-



Sl. 3. Primerjava merjenih in izračunanih kot drobirskega toka vzdolž zgornjega dela odseka B  
 Fig. 3. Comparison of measured and computed debris-flow elevations along the upstream part of Reach B



Sl. 4. Primerjava merjenega in izračunane območja dosega drobirskega toka vzdolž odseka C  
 Fig. 4. Comparison of measured and computed inundation limits of the debris flow along Reach C

vrednosti drobirskega toka na odseku C so bile  $\tau_y = 20 \text{ N/m}^2$ ,  $\eta = 10 \text{ Pa}\cdot\text{s}$  in  $n_g = 0,05$  do  $0,065 \text{ sm}^{-1/3}$ . Medsebojna primerjava izračunanih najvišjih gladin modelov PCFLOW2D in FLO-2D ter primerjava z najvišjimi kotami drobirskega toka, zabeleženimi na podlagi sledov dosega drobirskega toka z dne 17. novembra 2000 (primer a, slika 5) je pokazala, da lahko točnost modelov na odseku C ocenimo na  $\pm 1,5 \text{ m}$  za najvišje kote v osi strug Predelice in Koritnice ([2] do [4]).

flow values along Reach C were  $\tau_y = 20 \text{ N/m}^2$ ,  $\eta = 10 \text{ Pa}\cdot\text{s}$  and  $n_g = 0.05$  to  $0.065 \text{ sm}^{-1/3}$ . The comparison of the maximum debris-flow elevations computed by the PCFLOW2D and FLO-2D models, as well as the comparison with the measured elevations obtained by the inundation limits on November 17<sup>th</sup>, 2000 (Case a, Fig. 5), has demonstrated that the accuracy of both models. On Reach C, it can be estimated to  $\pm 1.5 \text{ m}$  for the maximum elevations along the axes of the Predelica and Koritnica Creeks ([2] to [4]).

#### 4 RAČUNSKI REZULTATI

#### 4 COMPUTATIONAL RESULTS

##### 4.1 Določitev primerov za simulacijo

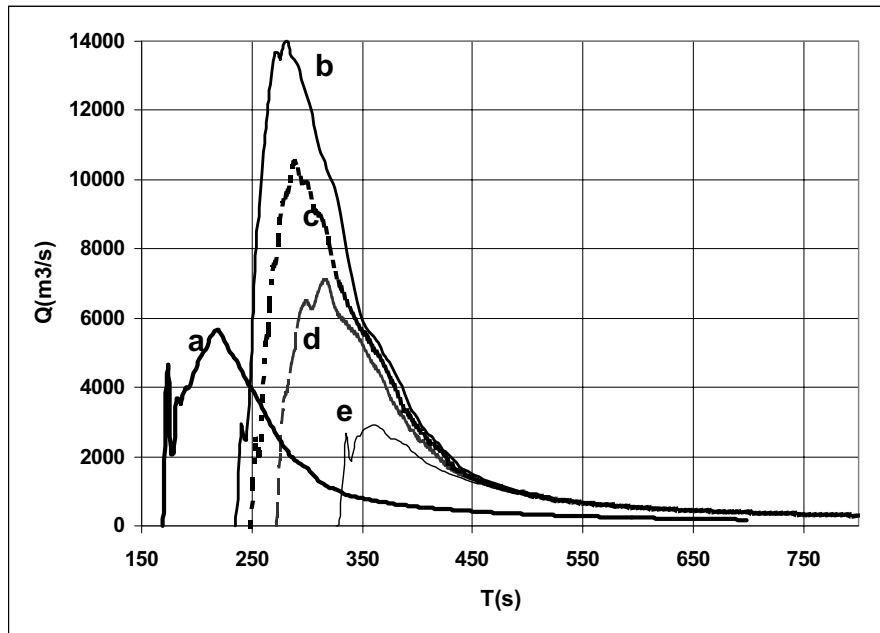
##### 4.1 Determination of cases for simulation

Na podlagi podrobnih geoloških, hidroloških in geomehanskih analiz je bilo ugotovljeno, da so nad odlomnim robom plazov v Stožah še vedno obstoječe nestabilne mase, tako da obstaja možna nevarnost novih plazov enakega ali celo večjega obsega kakor novembra 2000 ([6], [9] in [10]). Med strokovnjaki je prevladalo enotno mnenje, da bi morala biti vas varovana proti plazovom enakega reda velikosti kakor v novembru 2000, ko se je premaknilo  $1,200.000 \text{ m}^3$  materiala. Pri dejanskem plazju se je okrog  $400.000 \text{ m}^3$  odložilo vzdolž odsekov A in B, v prihodnje pa je moč pričakovati odlaganje samo  $200.000 \text{ m}^3$ . Dno struge je namreč deloma zapolnjeno z materialom prejšnjih plazov, tako da bi bila prostornina materiala, ki bi dosegel vas,  $1,000.000 \text{ m}^3$ . Za oceno možne nevarnosti so bile narejene še druge simulacije z naslednjimi možnimi začetnimi prostorninami materiala:  $2,000.000 \text{ m}^3$ ,  $1,600.000 \text{ m}^3$  in  $800.000 \text{ m}^3$ . Narejenih je bilo tudi več simulacij, pri katerih je bila začetna prostornina zmanjšana za  $600.000 \text{ m}^3$ . Toliko namreč približno znaša zadrževalna prostornina štirih pregrad. Slika 5 prikazuje hidrograme drobirskega toka pri mostu II v Gorenjem

Based on detailed geological, hydrological, and geo-mechanical analyses, it was determined that above the region of the last landslide on the Stože slope there are potentially unstable masses and that in the future there is a potential danger of further landslides of the same or even greater magnitude as that in November 2000 ([6], [9] and [10]). Among professionals it was agreed that the village should be protected against the same magnitude of landslide mass as that of November 2000, when the total displaced volume was  $1.200,000 \text{ m}^3$ . In the original landslide about  $400,000 \text{ m}^3$  were deposited along Reaches A and B, while in the future only a deposition of  $200,000 \text{ m}^3$  can be expected, as the bed is partially filled with the material of previous landslides, and thus a volume of  $1,000,000 \text{ m}^3$  would reach the village. To estimate the potential danger, other simulations were also made for possible initial volumes of  $2,000,000 \text{ m}^3$ ,  $1,600,000 \text{ m}^3$ , and  $800,000 \text{ m}^3$ . A series of simulations has also been made with the initial volumes reduced for  $600,000 \text{ m}^3$ . This would be approximately the retention volume of four dams. Fig. 5 shows debris-flow hydrographs at the site of Bridge II for the event of November 17<sup>th</sup>, 2000, and for

Logu za dogodek 17. novembra 2000 in štiri primere mogočih plazov oz. drobirskih tokov, ki so bili dobljeni z modelom DEBRIF1D. Čas  $T = 0$  s pomeni trenutek sprožitve drobirskega toka na plazišču v Stožah, razen v primeru ponovitve dogodka, dne 17. novembra 2000, ko  $T = 0$  pomeni čas sprožitve toka pri mostu I v Mlinč. Izračunani hidrogrami so bili temelj za simulacije drobirskega toka vzdolž odsekov C in D.

four cases of potential future landslide/debris flows, as simulated by the DEBRIF1D model. Time  $T = 0$  s is the moment when the debris flow starts to move from the Stože slope, except for the case of November 17<sup>th</sup>, 2000, where  $T = 0$  s means the time when the debris flow started to move at the Bridge I in Mlinč. Computed hydrographs were the basis for all the simulations of the debris flow along the downstream reaches C and D.



Sl. 5. Izračunani hidrogrami pretokov drobirskega toka pri mostu II v Gorenjem Logu. Krivulje ustrezajo naslednjim drobirskim prostorninam a:  $V = 0,8 \cdot 10^6 \text{ m}^3$  (dejanski dogodek 17. novembra 2000); b:  $V = 1,8 \cdot 10^6 \text{ m}^3$ ; c:  $V = 1,4 \cdot 10^6 \text{ m}^3$ ; d:  $V = 1,0 \cdot 10^6 \text{ m}^3$ ; e:  $V = 0,6 \cdot 10^6 \text{ m}^3$ .

Fig. 5. Computed hydrographs of the debris-flow discharge at Bridge II in Gorenji Log. Curves correspond to debris volumes: a:  $V = 0.8 \cdot 10^6 \text{ m}^3$  (real event on November 17<sup>th</sup>, 2000); b:  $V = 1.8 \cdot 10^6 \text{ m}^3$ ; c:  $V = 1.4 \cdot 10^6 \text{ m}^3$ ; d:  $V = 1.0 \cdot 10^6 \text{ m}^3$ ; e:  $V = 0.6 \cdot 10^6 \text{ m}^3$ .

#### 4.2 Rezultati in razprava

Za primer, ko bi vas doseglo  $1,000,000 \text{ m}^3$  drobirja, so na slikah 6 in 7 prikazani izračunani vektorji hitrosti in prosta gladina drobirskega toka vzdolž odseka C 70 sekund po prehodu čela vala skozi prečni profil mostu II v Gorenjem Logu. Oba rezultata sta bila dobljena z dvodimenzijskim modelom PCFLOW2D za primer, ko bi bil odložen material prejšnjih drobirskih tokov iz dna struge Predelice vzdolž odseka C v glavnem že odstranjen.

Za oceno obstoječe nevarnosti so bile narejene še simulacije z drugimi možnimi prostorninami, opisanimi v poglavju 4.1. Upoštevanih je bilo tudi več različic ureditve strug Predelice in Koritnice vzdolž odsekov C in D, da bi določili cenovno najugodnejšo možnost varovanja obeh vasi. Rezultati teh simulacij so podani v poročilih [2], [3] in [4], kjer so podrobno navedeni tudi končno predlagani ukrepi varovanja vasi Log pod Mangartom. Najbolj učinkoviti in cenovno sprejemljivi ukrepi so: odstranitev nasutega

#### 4.2 Results and discussion

For the case when  $1,000,000 \text{ m}^3$  of the debris volume would reach the village, Figs. 6 and 7 show the computed velocity vectors and the free surface of the debris flow along Reach C 70 seconds after the transition of the wave front through the cross-section of Bridge II. Both results were obtained by the two-dimensional model PCFLOW2D for the situation when the old debris material from the bed of Predelica along Reach C had been mainly removed.

To estimate the potential danger, other simulations were also made for the possible initial volumes described in section 4.1. Several variations of the riverbed training measures along the Predelica and Koritnica Creeks at Reaches C and D were taken into account in the simulations, to determine the most cost-effective solution to protect both villages. The results of these computations can be found in the reports [2], [3] and [4], where a detailed description of the measures to protect Log pod Mangartom village is also given. The most effective and economic measures are the

materiala prejšnjih drobirskih tokov iz struge Predelice vzdolž odseka C, razširitev ozkega dela doline pri mostu II v Gorenjem Logu ter izgradnja vzdolžnih zidov višine do 2 m ob naseljih Gorenji in Spodnji Log.

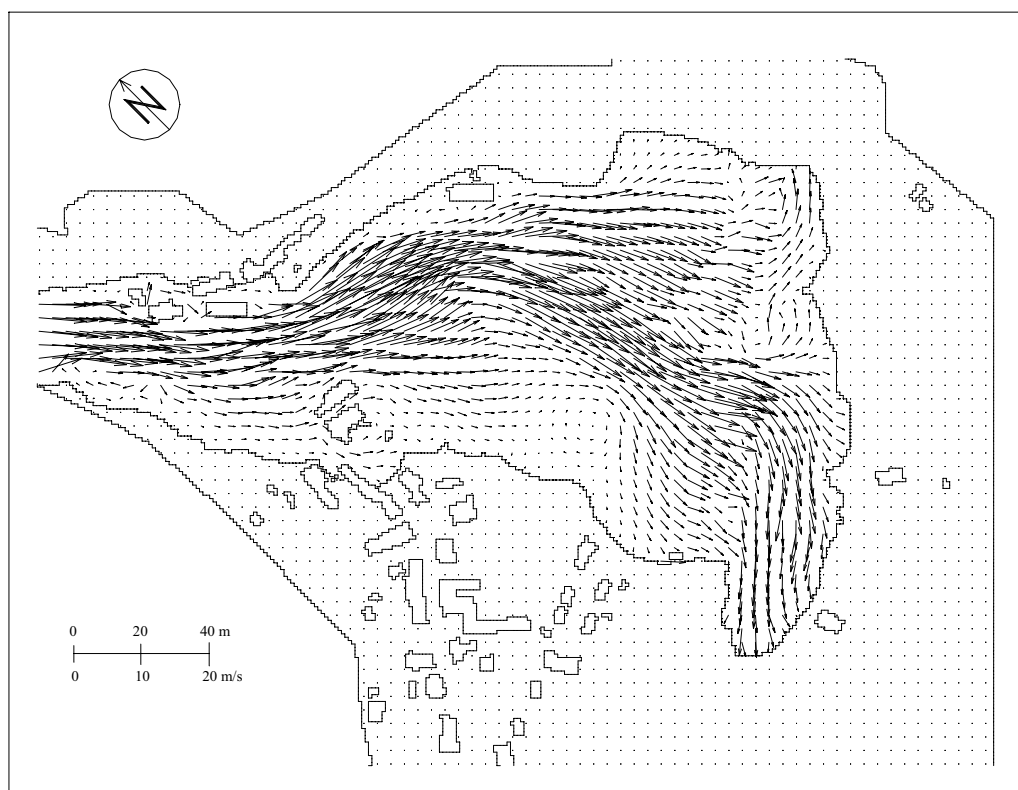
Za model PCFLOW2D je bila narejena tudi analiza občutljivosti rezultatov za spremembo gostote numerične mreže in iztočnega robnega pogoja [6]. Pri redkejši numerični mreži velikosti 4 x 4 m so se rezultati v primerjavi z mrežo 2 x 2 m, ki je bila uporabljena pri končnih izračunih, le malo spremenili. Različni robni pogoji na iztoku (kritični tok ali ničelni gradienti hitrosti in globin v smeri iztoka) pa je na izračunane gladine vplival krajevno, vendar ne dlje kakor okoli 80 m navzgor od iztoka.

Omenimo še, da v modelih erozija in odlaganje drobirskega materiala niso bili zajeti neposredno z enačbami, temveč posredno. Na mestih očitne erozije so bili tako pri umerjanju kakor pri končnih računih upoštevani profili, izmerjeni po prehodu drobirskega toka. Odlaganje na vzdolžnih odsekih je bilo navzdol po reki upoštevano tako, da je bil pri vtočnem hidrogramu  $Q - t$  odrezan zadnji, padajoči del hidrograma, katerega prostornina je ustrezala odloženemu materialu.

removal of the deposited debris material from the bed of the Predelica Creek along Reach C, the enlargement of the narrow part of the valley at Bridge II in Gorenji Log and the construction of longitudinal walls up to 2 m high along the villages of Gorenji and Spodnji Log.

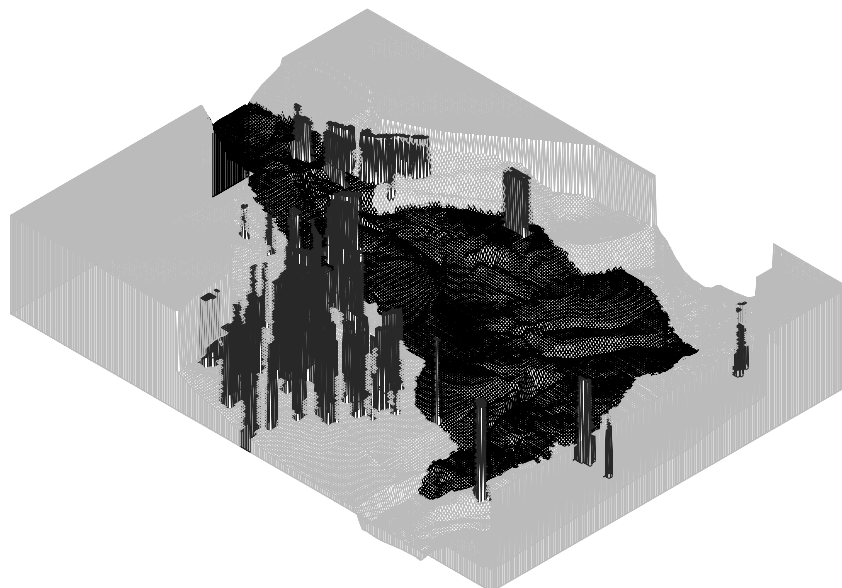
For the model PCFLOW2D, the sensitivity analysis for different mesh sizes and outflow boundary conditions was also made [6]. For the coarser grid of 4 x 4 m the results did not change significantly in comparison with the mesh of 2 x 2 m, which was used in the final computations. Different outflow boundary conditions (critical flow or zero velocity and depth gradients in the direction of the outflow) influenced debris-flow elevations locally up to about 80-m upstream from the outflow cross-section.

It is worth mentioning that the erosion and deposition of the debris material were not simulated directly by the equations, but they were taken into account indirectly. At the places where the erosion was obvious, the cross-sections that had been measured after the transition of the debris flow were considered during the calibration process and final computations. The deposition at upstream sections was taken into account downstream by cutting off the falling part of the inflow hydrographs that corresponded to the volume of the deposited material.



Sl. 6. Izračunani vektorji hitrosti vzdolž odseka C 70 sekund po prehodu čela vala skozi prečni profil mostu II v Gorenjem Logu

Fig. 6. Computed velocity vectors along Reach C 70 seconds after the transition of the wave front through the cross-section of Bridge II in Gorenji Log



Sl. 7. 3D pogled na izračunano gladino drobirskega toka na odseku C 70 sekund po prehodu čela vala skozi prečni profil mostu II v Gorenjem Logu

Fig. 7. 3D view of the computed debris-flow surface along Reach C 70 seconds after the transition of the wave front through the cross-section of Bridge II in Gorenji Log

## 5 SKLEPI

Končni cilji matematičnega modeliranja gibanja drobirskega toka so zapleteni: poleg varovanja vasi Log pod Mangartom pred mogočimi prihodnjimi drobirskimi tokovi mora biti zagotovljena tudi poplavna varnost. Končna ureditev strug Predelice in Koritnice ter okoliškega terena mora biti optimizirana še z ekonomskega in okoljskega vidika. Pomembni so tudi socialni vidiki, predvsem mora biti upoštevano mnenje vaščanov, ki jih pojav drobirskega toka ogroža.

V primeru drobirskega toka pod plaziščem Stože se je kombinacija enodimenzijskega modela DEBRIF1D in dveh dvodimenzijskih modelov PCFLOW2D in FLO-2D pokazala kot pravilna metodologija za simulacijo prihodnjih podobnih nesreč in načrtovanja ukrepov varovanja vasi Log pod Mangartom. Overitev vseh treh modelov je potrdila, da je zanesljivost modelnih rezultatov zadovoljiva. Kvadratni zakon upora drobirskega toka, ki ga je predlagal O'Brien [14], je dala sprejemljive rezultate tudi za primer daljših odsekov, kjer se režim toka spreminja. FLO-2D je pokazal določeno glajenje izračunanih kot v primerjavi z modelom PCFLOW2D, v povprečju pa je ujemanje rezultatov obeh modelov sprejemljivo, natančnost pa primerljiva.

## 5 CONCLUSIONS

The final goal of the mathematical modelling of debris flow is complex: besides protection against possible future debris flow in the village of Log pod Mangartom, protection against high water floods also has to be ensured. The final arrangement of the Predelica and Koritnica riverbeds and the nearby terrain has to be optimized from the economic and from the environmental points of view. Also the social aspects – the opinions of the local villagers, who are affected by the danger – have to be taken into account.

In the case of the debris flow below Stože, the combination of the one-dimensional model DEBRIF1D and the two two-dimensional models PCFLOW2D and FLO-2D proved to be the right methodology for the simulations of possible future disastrous events and to design measures to protect the village of Log pod Mangartom. The verification of the three models has proved the satisfactory reliability of the model results. The quadratic law of debris-flow resistance, proposed by O'Brien [14], gives acceptable results, even for flows along longer reaches, where the flow regimes change. FLO-2D shows a certain smoothing of the computed levels in comparison with PCFLOW2D, but on average the agreement of the results of both models is acceptable and the accuracy is comparable.

## 6 LITERATURA 6 REFERENCES

- [1] Coussot, P., M. Menuier (1996) Recognition, classification and mechanical description of debris flows. *Earth Science Review* 40, 209-227.

- [2] Četina, M., T. Hojnik, R. Rajar, M. Krzyk, M. Zakrajšek (2003) Dopolnilni računi z enodimenzijskim in dvodimenzijskima matematičnima modeloma murastega toka od plazišča Stože do Loga pod Mangartom in novelacija ocene ogroženosti Gorenjega in Spodnjega Loga. *FGG Ljubljana in VGB Maribor*, projekt d-57-1, d-56-2, 30.
- [3] Četina, M., R. Rajar, M. Krzyk, T. Hojnik (2001) Dvodimenzijski matematični model murastega toka od plazu Stože pod Mangartom do Loga pod Mangartom in ocena ogroženosti naselja Log pod Mangartom; dodatni 2D in 1D računi murastega toka zaradi optimizacije potrebnega izkopa v Logu pod Mangartom. *FGG Ljubljana in VGB Maribor*, projekt d-56, d-57, 12.
- [4] Četina, M., T. Hojnik, R. Rajar, M. Krzyk, M. Zakrajšek (2001) Enodimenzijski in dvodimenzijski matematični model murastega toka od plazu Stože pod Mangartom do Loga pod Mangartom in ocena ogroženosti naselja Log pod Mangartom, *FGG Ljubljana*, 38.
- [5] Četina, M., R. Rajar (1993) Mathematical simulation of flow in kayak racing channel, *Proc. of the 5<sup>th</sup> International Symposium on Refined Flow Modelling and Turbulence Measurements* (Paris, France, 7. - 10. 9.), ed. P.L. Violette, 637-644, ISBN 2-85978-202-8.
- [6] Fazarinc, R. (2002) Matematično modeliranje drobirskega toka v Logu pod Mangartom. Magistrska naloga, *FGG Ljubljana*.
- [7] FLO Engineering Inc. (1999) *FLO-2D, Users manual, Version 99.1*. Civil and Water Resources Engineering.
- [8] Julien, P.Y., J.S. O'Brien (1997) Selected notes on debris flow dynamics. Recent developments on debris flows, *Lecture notes in earth sciences*, Springer, 144-162.
- [9] Majes, B., A. Petkovsek, J. Logar (2002) Landslide Stože - consequences and feasibility of corrective measures. *Proceedings of the 12<sup>th</sup> Danube - European Conference*, Passau, May 27-28, 235 – 238.
- [10] Majes, B. (2001) Analiza plazu in možnosti njegove sanacije. *Ujma 14-15*, letnik 2000/2001, 80-91.
- [11] Mikoš, M., M. Četina, M. Brilly (2002) Debris flow disaster in the village of Log pod Mangartom, Slovenia: First analysis of the event. Submitted to *Engineering Geology*.
- [12] Mikoš, M., R. Fazarinc (2000) Earthquake-induced erosion processes in two alpine valleys in Slovenia. *Proc. of the Int. Symp. INTERPRAEVENT* (Villach, Austria), Vol. 2, eds. F. Zollinger & G. Fiebiger, 143-154.
- [13] Ming, J., D.L. Fread (1999) 1D modeling of mud/debris unsteady flows. *J. Hydraul. Eng.* 125(8), 827-834.
- [14] O'Brien, J.S., P.Y. Julien, W.T. Fullerton (1993) Two-dimensional water flood and mudflow simulation. *J. Hydraul. Eng.* 119(2), 244-261.
- [15] Patankar, S.V. (1980) Numerical heat transfer and fluid flow, *McGraw-Hill*, New York.
- [16] Rajar, R. (1982) Mathematical simulation of snow avalanche dynamics, *A. Kuhelj Memorial Volume*, Ed. by Slovene Academy of Science, Ljubljana, 303-320.
- [17] Rajar, R. (1980) Mathematical models for simulation of flood waves, dam-break waves and snow avalanches. *INTERPRAEVENT*, Bad Ischl, 232-134.
- [18] Rajar, R. (1978) Mathematical simulation of dam-break flow. *J. Hydraul. Eng.* 104(7), 1011-1026.

Naslov avtorjev: prof.dr. Matjaž Četina  
 mag. Mario Krzyk  
 Univerza v Ljubljani  
 Fakulteta za gradbeništvo in  
 geodezijo  
 1000 Ljubljana  
 mcetina@fgg.uni-lj.si

Authors' Address: Prof.Dr. Matjaž Četina  
 Mag. Mario Krzyk  
 University of Ljubljana  
 Faculty of Civil and Geodetic  
 Engineering  
 SI-1000, Ljubljana, Slovenia  
 mcetina@fgg.uni-lj.si

Prejeto:  
 Received: 30.1.2003

Sprejeto:  
 Accepted: 29.5.2003

Odprt za diskusijo: 1 leto  
 Open for discussion: 1 year

## Dvodimenzijski matematični model transporta lebdečih plavin

### A Two-Dimensional Mathematical Model of Suspended-Sediment Transport

Mario Krzyk - Matjaž Četina

Po kratki predstavitvi teoretičnih osnov hidrodinamičnih značilnosti tokov, ki jih obravnavamo kot dvorazsežne v vodoravni ravnini, z globinsko povprečnimi hitrostmi, je v prispevku predstavljena enačba transporta lebdečih plavin, ki je uporabljena v matematičnem modelu. To je adveksijsko-difuzijska enačba z dodatnim izvornim členom, ki opisuje spremembo koncentracije plavin, povzročeno z usedanjem ali dvigovanjem materiala z dna struge. Globinsko povprečno vrednost koncentracije lebdečih plavin smo dobili na podlagi analize transportne enačbe v navpični ravnini. Izračun izvornega člena sloni na transportni enačbi, definirani v navpični ravnini, ki daje značilen raspored koncentracije lebdečih plavin z najmanjšo koncentracijo na površini in največjo na dnu struge. Rezultati izračuna so odvisni od razlike med vtočno (izračunano) globinsko povprečno koncentracijo plavin in povprečno vrednostjo koncentracije plavin v ravnovesnem stanju, za določene hidrodinamične pogoje znotraj kontrolnega volumna. Kot primer uporabe matematičnega modela je obdelana problematika Ptujkega jezera. To je izpostavljeno vplivu usedanja lebdečih plavin, ki ga nosi reka Drava. Prikazani so rezultati opravljenih meritev, postopek umerjanja hidrodinamičnega dela matematičnega modela ter rezultati modela za račun prenosa lebdečih plavin.

© 2003 Strojniški vestnik. Vse pravice pridržane.

**(Ključne besede: modeli matematični, transport materiala, plavine lebdeče, Ptujsko jezero)**

After a brief review of the theoretical basis of the hydrodynamic characteristics of two-dimensional depth-averaged flow in a horizontal plane, in this paper we present an equation for suspended sediment transport. It is an advective-diffusion equation with an added source term that describes the concentration of a suspended sediment caused by sedimentation or erosion. The depth-averaged concentration of the suspended load is a result of an analysis of the transport equation in the vertical plane. The source-term definition is based on the transport equation in the vertical plane, which gives a characteristic concentration distribution of the suspended load with a minimum concentration at the surface and a maximum at the bottom of the bed. The calculation results depend on the difference between the inflow (calculated), depth-averaged concentration of the suspension and the averaged equilibrium suspension concentration in a numeric cell under certain hydrodynamic conditions. As an example of the application of the mathematical model, the problem of Ptuj lake (Slovenia) is presented. It is very exposed to the sedimentation of suspended sediment that is brought by the river Drava. The results of the measurements, the procedure of the hydrodynamic part of the mathematical model calibration and the results of the suspended-load module are presented.

© 2003 Journal of Mechanical Engineering. All rights reserved.

**(Keywords: mathematical model, materials transport, suspended sediment load, Ptuj lake)**

#### 0 UVOD

Eden izmed zelo pomembnih problemov, s katerim se srečujemo pri gospodarjenju z umetnimi akumulacijami, je problem gibanja plavin. To je pojav, ki je odvisen od hidrodinamičnih sil ter drugih pogojev, ki so definirani z značilnostmi plavin. Gibanje plavin povzroča dotok materiala iz zaledja porečja in njegovo usedanje, spremembo ravni podtalnice in večja stopnja ogroženosti zaradi morebitnega iztekanja vode iz struge in poplavljanja obrečnega območja.

#### 0 INTRODUCTION

The transport of suspended sediment is a very real problem in the management of artificial reservoirs. It depends on the kinds of hydrodynamic forces present and on other conditions that are defined by the characteristic of the sediment. Suspended-sediment transport takes into account the processes of erosion, the transport of materials and their settling. The major consequences of settling material are: the filling up of riverbeds, the lifting of the river bottom and, in some



Pojav usedanja plavin je posebej pomemben pri akumulacijah, zgrajenih na rekah z večjo transportno zmogljivostjo, ker povzroča zmanjšanje prostornine akumulacije in kakovosti vode, tako da postaja vprašljiva večnamenska uporabnost akumulacije.

Z nalogami, ki so s časom postajale vse bolj zapletene, se je razvijala tudi metodologija načina prenosa plavin. Od prvih, bolj grobih napovedi, zasnovanih na empiričnih in polempiričnih enačbah, je dandanes večji poudarek na matematičnih modelih, ki naj bi bili zmožni napovedati koncentracijo lebdečih plavin in bilanco proda v prostoru in času z uporabo diferencialnih enačb, ki opisujejo ta fizikalni pojav. Zaradi spremembe koncentracije plavin, povzročene z njihovim usedanjem ali erozijo, je treba prenos plavin obravnavati kot prenos nekonservativne snovi.

Za določanje bilance prenosa plavin obstajajo različne merske in računske metode. Ne glede na razpoložljivo mersko in računalniško opremo je še vedno težko napovedati bilanco plavin in spremembo konfiguracije dna. Za potrebe kalibracije bolj točnih in tudi bolj zahtevnih matematičnih modelov, je treba razpolagati z večjim številom podatkov, ki jih je mogoče dobiti le s celovitimi in dolgotrajnimi meritvami (v določenih primerih tudi do 10 let). Zato je treba, glede na kakovost in število razpoložljivih podatkov, racionalno izbrati kompleksnost matematičnega modela. Na ta način se lahko izognemo uporabi parametrov, za katere nimamo znanih vrednosti.

Problem prenosa plavin je v splošnem trodimenzijski problem. Glede na geometrijske značilnosti toka, je mogoče pri reševanju praktičnih problemov, uvesti ustrezne poenostavitve. Najbolj pogoste so:

- dvodimenzijski pristop (2D), kadar problematiko obravnavamo v navpični ali vodoravni ravnini (kot model enotne širine ali povprečen po globini) in
- enodimenzijski (1D) pristop.

Za analizo tokov na krajših odsekih v bližini izpustov tekočine, kjer je snov pod vplivom turbulence v navpični ravnini že enakomerno premešana ali pa v plitvih jezerih s prevladujočimi izmerami v vodoravni ravnini, se po navadi uporabljajo dvodimenzijski, po globini povprečni matematični modeli. Takšen model je uporabljen tudi pri analizi tokov in prenosa lebdečih plavin v Ptujskem jezeru.

## 1 HIDRODINAMIČNI MATEMATIČNI MODEL

### 1.1 Osnovne enačbe

Tok vode je temeljni parameter prenosa snovi v obliki raztopine ali delcev. Zato je natančno poznavanje hidrodinamičnih parametrov, kot sta

cases, an increased danger of floods, etc. This process has a special importance for retention basins that are located on gravel-bed rivers, because it causes a reduction in the basin's volume and water quality, so its multipurpose usage might become questionable.

As the practical problems have become more complex, the methodology of the approach to sediment-transport problems has developed. The first, more approximate, predictions were based on empirical and half-empirical equations; today, the main stress is on mathematical models, which should be able to predict the concentration of suspended load and the balance of the load in space and time by using differential equations for describing certain physical events. Sediment transport should be treated as the transport of non-conservative matter, as its concentration changes with settling and erosion.

Different measurement and calculation methods are present for the defining the balance of sediment transport. Regardless of the available measuring and computer equipment, it is still difficult to predict the balance of sediment transport and changes to the bottom configuration. If the aim is to calibrate a more accurate and more complex mathematical model, it is necessary to have more data available, which could be provided by integral and long-term measurements (sometimes for 10 years). So, depending on the quantity and quality of the available data, an appropriate mathematical model should be chosen. In this way it is possible to avoid the need for parameters for which there are no available data.

Matter-concentration transport is, in general, a three-dimensional problem. However, by taking into account the geometrical characteristics of water flow in practical work, the solving of hydrodynamic and concentration transport equations can be simplified. The most common simplifications are:

- a two-dimensional treatment of the problems - in a vertical or a horizontal plane (as a depth-averaged model),
- a one-dimensional treatment.

A two-dimensional, depth-averaged treatment is usually used for situations where problems appear in the region near the outflow into the recipient, where the matter is already well mixed in the vertical profile by the influence of turbulence and bottom friction, or in shallow lakes with the largest dimensions in the horizontal plain. Such a two-dimensional hydrodynamic mathematical model was applied to our sediment-transport analysis of the Ptuj lake.

## 1 HYDRODYNAMIC MATHEMATICAL MODEL

### 1.1 Basic equations

The basic parameter of the solution or parts transport is water flow. So, hydrodynamic parameters, such as velocity and flow depth, should be accurately

hitrost in globina toka, zelo pomembno. Izračun prenosa plavin je mogoče opraviti šele potem, ko imamo na voljo preverjeno hidrodinamično sliko toka.

Hidrodinamične značilnosti opišemo z dvodimenzijskimi, po globini povprečnimi enačbami. Uporabljene kontinuitetna in dinamični enačbi v konservativni obliki, ki opisujejo stalni tok v kartezičnem pravokotnem koordinatnem sistemu, so naslednje:

$$\frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = 0 \quad (1)$$

$$\frac{\partial(hu^2)}{\partial x} + \frac{\partial(huv)}{\partial y} = -gh \frac{\partial h}{\partial x} - gh \frac{\partial z_b}{\partial x} - ghn^2 \frac{u\sqrt{u^2+v^2}}{h^{3/2}} + \frac{\partial}{\partial x}(hv_{ef} \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(hv_{ef} \frac{\partial u}{\partial y}) \quad (2)$$

$$\frac{\partial(huv)}{\partial x} + \frac{\partial(hv^2)}{\partial y} = -gh \frac{\partial h}{\partial y} - gh \frac{\partial z_b}{\partial y} - ghn^2 \frac{v\sqrt{u^2+v^2}}{h^{3/2}} + \frac{\partial}{\partial x}(hv_{ef} \frac{\partial v}{\partial x}) + \frac{\partial}{\partial y}(hv_{ef} \frac{\partial v}{\partial y}) \quad (3)$$

Ker se lahko ponekod pojavljajo območja z bolj izraženim vrtničnim tokom ter v skladu z dosedanjimi izkušnjami pri matematičnem modeliranju tokov v rekah in jezerih z uporabo dvodimenzijskih modelov in z namenom, da bi dololočili vrednosti dodatnih turbulentnih napetosti, je uporabljena  $k - \varepsilon$  verzija hidrodinamičnega modela [2]. Dodatne transportne enačbe za turbulentno kinetično energijo  $k$  in njeno disipacijo  $\varepsilon$  so uporabljene v naslednji obliki:

$$\frac{\partial(huk)}{\partial x} + \frac{\partial(hvk)}{\partial y} = \frac{\partial}{\partial x}(h \frac{v_{ef}}{\sigma_k} \frac{\partial k}{\partial x}) + \frac{\partial}{\partial y}(h \frac{v_{ef}}{\sigma_k} \frac{\partial k}{\partial y}) + hG - c_D h \varepsilon + hP_{kv} \quad (4)$$

$$\frac{\partial(hu\varepsilon)}{\partial x} + \frac{\partial(hv\varepsilon)}{\partial y} = \frac{\partial}{\partial x}(h \frac{v_{ef}}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x}) + \frac{\partial}{\partial y}(h \frac{v_{ef}}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial y}) + c_1 \frac{\varepsilon}{k} hG - c_2 \frac{\varepsilon^2}{k} h + hP_{\varepsilon v} \quad (5)$$

kjer so:  $h$  – globina vode;  $u$  in  $v$  – komponente hitrosti v smereh  $x$  in  $y$ ;  $z_b$  – kota dna;  $n$  – Manningov koeficient hrupavosti;  $v_{ef}$  – kinematični koeficient efektivne viskoznosti;  $g$  – gravitacijski pospešek;  $k$  – turbulentna kinetična energija na enoto mase in  $\varepsilon$  – stopnja njene disipacije. Izrazi za  $G$  (produkcija  $k$  zaradi vodoravnih gradientov hitrosti) ter  $P_{kv}$  in  $P_{\varepsilon v}$  (izvorna člena zaradi trenja ob dno) so skupaj s stalnicami, ki se pojavljajo v modelu turbulence ( $c_D, c_{\mu}, c_1, c_2, c_{\varepsilon}, \sigma_k$  in  $\sigma_\varepsilon$ ), bolj natančno razloženi v literaturi [2].

## 1.2 Metoda reševanja in računalniški program

Sistem nelinearnih parcialnih diferencialnih enačb (1) do (5) rešujemo numerično z metodo kontrolnih volumnov, ki je natančno prikazana v literaturi [2] in [6]. Njene glavne značilnosti so premaknjena numerična mreža, hibridna shema (kombinacija centralnodiferenčne in sheme vzvodnih razlik) ter iteracijsko reševanje na podlagi popravkov globin. Izbrana shema je kompromis med točnostjo in ekonomičnostjo ter daje fizikalno realne rezultate pri vseh Pecletovih številih (razmerje med konvekcijskim in difuzijskim prenosom), razen v bližini vrednosti  $\pm 2$ .

known. The sediment-transport calculation can only be performed when the available hydrodynamic conditions are checked.

The hydrodynamic characteristics can be described by two-dimensional, depth-averaged equations. The continuity and dynamic equations for the cartesian orthogonal coordinate system are used in the following form:

As the appearance of regions with vortices in the flow is possible, and based on our experience of the mathematical modeling of river and lake flows using two-dimensional models, and in order to define additional turbulent stresses, the  $k - \varepsilon$  turbulent model seemed to be justified [2]. Additional transport equations for the turbulent kinetic energy  $k$  and its dissipation  $\varepsilon$  are used in the following form:

where is:  $h$  – water depth;  $u$  and  $v$  – velocity components in the  $x$  and  $y$  directions;  $z_b$  – bottom level;  $n$  – Manning’s roughness coefficient;  $v_{ef}$  – kinematic coefficient of effective viscosity;  $g$  – acceleration due to gravity;  $k$  – turbulent kinetic energy per mass unit; and  $\varepsilon$  – its level of dissipation. The term  $G$  (the production of  $k$  caused by horizontal velocity gradients),  $P_{kv}$  and  $P_{\varepsilon v}$  (the source terms caused by bottom friction), together with the constants that appear in the turbulence model ( $c_D, c_{\mu}, c_1, c_2, c_{\varepsilon}, \sigma_k$  and  $\sigma_\varepsilon$ ), are explained in the literature [2].

## 1.2 Solution method and computer program

The control-volume method, described in [2] and [6], was used for solving the partial differential equations (1) to (5). Its main characteristics are a staggered numeric grid, a hybrid scheme (a combination of the central-differences and upwind schemes) and an iterative solution based on water-depth corrections. The chosen scheme is a compromise between accuracy and economy. It gives physically “real” results across the whole range of Peclet numbers (the ratio between advective and diffusion transport) accept near the value of  $\pm 2$ .

Osnova hidrodinamičnega matematičnega modela je zelo razširjen računalniški program TEACH [4]. Program je dopolnjen z možnostjo računanja tokov s povprečno globino z uporabo  $k - \varepsilon$  modela turbulence ob upoštevanju spremenljive geometrijske oblike ([2] do [4]). Tako je nastala nova različica programa, ki smo jo poimenovali PCFLOW2D.

Enačbe od (1) do (5) so nelinearne parcialne diferencialne enačbe drugega reda, hiperboličnega tipa. Zato potrebujemo začetne in robne pogoje za hitrosti  $u$  in  $v$  ter globino  $h$  na vseh štirih robovih. Tudi za preračun kinetične energije in njene disipacije,  $k$  in  $\varepsilon$ , potrebujemo robne pogoje na vseh štirih robovih računskega področja. Podrobneje so ti pogoji navedeni v literaturi [3] in [4].

## 2 MATEMATIČNI MODEL PRENOSA LEBDEČIH PLAVIN

### 2.1 Osnovne enačbe

Adveksijsko-difuzijska enačba, ki opisuje prenos snovi v dvodimenzijskem prostoru vodoravne ravnine, se lahko napiše v naslednji obliki:

$$\frac{\partial}{\partial x}(h\bar{c}\bar{u}) + \frac{\partial}{\partial y}(h\bar{c}\bar{v}) = \frac{\partial}{\partial x}\left(h\frac{\nu_{ef}}{\sigma_{sm}}\frac{\partial \bar{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(h\frac{\nu_{ef}}{\sigma_{sm}}\frac{\partial \bar{c}}{\partial y}\right) \pm S_c \quad (6)$$

Oznake v enačbi (6) pomenijo:  $h$  - globino vode,  $\bar{u}$  in  $\bar{v}$  - globinsko povprečni komponenti hitrosti v smereh  $x$  in  $y$ ,  $\bar{c}$  - globinsko-povprečno koncentracijo lebdečih plavin,  $\nu_{ef}$  - kinematični koeficient efektivne viskoznosti,  $\sigma_{sm}$  - Schmidtovo število in  $S_c$  - izvorni člen, s katerim opišemo spremembo koncentracije plavin v računskem polju, odvisno od dejanske koncentracije in hidrodinamičnih pogojev.

Izvorni člen  $S_c$  smo definirali na podlagi dvodimenzijske analize toka in razporeda koncentracije lebdečih plavin v navpični ravnini kontrolnega volumna [1]. Dvodimenzijska oblika diferencialne enačbe, ki opisuje prenos lebdečih plavin v navpični ravnini, se glasi:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} - w_s\frac{\partial c}{\partial z} = \frac{\partial}{\partial z}\varepsilon_{s,z}\frac{\partial c}{\partial z} \quad (7)$$

Pomen členov v enačbi je:

- $u$  hitrost toka v vzdolžni smeri ( $x$ ) (prevladujoča smer toka);
- $w_s$  hitrost usedanja zrna plavine v navpični smeri (os  $z$ ), pozitivna smer hitrosti je usmerjena v negativni smeri osi  $z$ ;
- $t$  čas;
- $c$  koncentracija lebdečih plavin in
- $\varepsilon_{s,z}$  koeficient difuzije plavin.

Enačba (7) se po navadi rešuje tako, da se ob definiranih začetnih in robnih pogojih kot rešitev dobi razpored koncentracije lebdečih plavin  $c$  po globini

The basis of the hydrodynamic model is the well-known computer program called TEACH [4]. The computer program is also able to calculate the depth-averaged flows using the  $k - \varepsilon$  turbulence model with respect to variable geometry ([2] to [4]). This resulted in a new version of the computer program called PCFLOW2D.

Equations (1) to (5) are non-linear, partial differential equations of the second-order, hyperbolic type. The initial and boundary conditions for the velocities  $u$  and  $v$  and the water depth  $h$  should be defined on all four boundaries. Also, for the production and dissipation of the kinetic energy,  $k$  and  $\varepsilon$ , the boundary conditions should be known for all four boundaries of the calculation area. These conditions are described in detail in [3] and [4].

## 2 MATHEMATICAL MODEL FOR THE TRANSPORT OF SUSPENDED SEDIMENT

### 2.1 Basic equation

The advection-diffusion equation for matter transport in a two-dimensional space in a horizontal plane can be written in the following form:

using the following terms:  $h$  - water depth,  $\bar{u}$  and  $\bar{v}$  - depth-averaged components of the velocities in the  $x$  and  $y$  directions,  $\bar{c}$  - depth-averaged concentration of suspended sediments,  $\nu_{ef}$  - kinematic coefficient of effective viscosity,  $\sigma_{sm}$  - Schmidt number, and  $S_c$  - source term, describing the change of sediment concentration in a calculation cell, depending on the real concentration and the hydrodynamic conditions.

The source term  $S_c$  is determined on the basis of a two-dimensional analysis of flow and the distribution of the suspended load concentration in a vertical plane in a certain control volume [1]. The suspended sediment transport in a vertical plane is described by the following differential equation in a two-dimensional form:

Where:

- $u$  is the velocity in the longitudinal ( $x$ ) direction (dominant flow direction);
- $w_s$  is the settling velocity in the vertical direction ( $z$  axis), the positive direction is opposite to the positive direction of the  $z$  axis;
- $t$  is the time;
- $c$  is the suspended load concentration,
- $\varepsilon_{s,z}$  is the suspension diffusive coefficient.

Equation (7) is usually solved so that the solution is a suspension concentration distribution  $c$  along the water depth under defined initial and

toka. Pri tem se vpliv neenakomerne razporeditve velikosti zrna po globini po navadi ne upošteva. Obravnavano enačbo lahko rešujemo tudi neodvisno za vsako posamezno frakcijo plavin. Pomanjkljivost takega načina je v tem, da ni upoštevan medsebojni vpliv posameznih frakcij. Pri reševanju enačbe (7) smo uporabili prvi način, ob upoštevanju naslednjih robnih pogojev ([7] in [8]):

- na gladini, kjer je  $z = z_d + h = z_g$ :

$$\varepsilon_{s,z} \frac{\partial c}{\partial z} + w_s c = 0 \quad (8),$$

- na dnu robni pogoj izraža pretok mase plavin kot vsoto usedlega in erodiranega materiala:

$$-\left(\varepsilon_{s,z} \frac{\partial c}{\partial z} + w_s c\right)_{z_d} = -\alpha w_s (c - c_*)_{z_d} \quad (9).$$

V enačbi (9) je pomen oznak naslednji:

- $c_*$  – ravnotežna koncentracija, odvisna od hidrodinamičnih pogojev in značilnosti plavin,
- $\alpha$  – bredimenzijski koeficient, ki izraža medsebojni vpliv dna in plavin. Zmnožek  $\alpha w_s$  je znan kot dejanska hitrost usedanja. Vrednost  $\alpha = 0$  pomeni idealen odziv dna, dokler velike vrednosti tega koeficienta ustrezajo adsorbirajoči površini (idealni ponor).

Prvi člen na desni strani enačbe (9) ( $-\alpha w_s c$ ) pomeni pretok plavin v navpični smeri, ki se useda. Drugi člen desne strani,  $(+\alpha w_s c_*)$  pomeni pretok snovi, ki se erodira z dna in vključuje v tok. Ustrezno enačbi (9) je pretok plavin (usedanje ali erozija zrn) sorazmeren razliki med dejansko koncentracijo plavin in transportne zmogljivosti toka.

Z integriranjem pretoka plavin po globini, upoštevaje enačbo (7), dobimo vrednost globinsko povprečene koncentracije plavin  $\bar{c}$  na razdalji  $x = |\vec{V}|t$  od vtoka v kontrolni volumen, kjer je vtočna koncentracija  $\bar{c}_0$ . Pri tem je  $|\vec{V}|$  velikost rezultante vektorja hitrosti v kontrolnem volumnu in  $t$  čas zadrževanja delcev znotraj kontrolnega volumna [5]:

$$\bar{c} = (\bar{c}_0 - \bar{c}_*) \exp\left(-\frac{\alpha w_s}{h} t\right) + \bar{c}_* \quad (10),$$

kjer so:  $\bar{c}_*$  - ravnovesna globinsko-povprečna koncentracija lebdečih plavin,  $\alpha$  - brezdimenzijski koeficient, ki izraža medsebojni vpliv med dnom in plavinami in  $w_s$  - hitrost usedanja delcev plavin.

Izvorni člen iz konveksijsko-difuzijske enačbe transporta lebdečih plavin (6) pa je definiran kot razlika med vteklo koncentracijo in dejansko koncentracijo v kontrolnem volumnu:

$$S_c = \bar{c} - \bar{c}_0 \quad (11).$$

boundary conditions. The influence of a non-uniform distribution of the grain diameter along the water depth was not taken into account. It is possible to solve the treated equation for each grain fraction independently. The problem with this approach is that the mutual influence of each grain size is not considered. The first approach was used in the solving of Equation (7), and the following conditions were considered ([7] and [8]):

- on the free surface, where  $z = z_d + h = z_g$ :

- the bottom condition is explained as sediment discharge, as the sum of the settled and eroded material:

The meaning of the terms in equation (9) is as follows:

- $c_*$  is the equilibrium concentration, and it depends on the hydrodynamic conditions and the sediment characteristics,
- $\alpha$  is a non-dimensional coefficient that explains the mutual influence of the bottom and the sediments. The term  $\alpha w_s$  is known as the actual settled velocity. The value of  $\alpha = 0$  corresponds to an ideal bottom response, a larger value corresponds to an adsorption plain (ideal sink).

The first term on the right-hand side of equation (9),  $-\alpha w_s c$ , represents the sediment discharge in the vertical direction as settling. The second term,  $+\alpha w_s c_*$ , is the sediment discharge of the eroded material that entrainments in a flow. According to Equation (9) the sediment discharge (the settling or erosion of grains) is proportional to the difference between a certain sediment concentration and the flow transport capacity.

The depth-averaged sediment concentration  $\bar{c}$  is calculated by integrating the sediment discharge over the depth, by considering Equation (7), at a distance  $x = |\vec{V}|t$  from the inflow boundary of the control volume, at the point of the inflow concentration  $\bar{c}_0$ . There is a  $|\vec{V}|$  velocity vector magnitude in a control volume and a  $t$  dwell time of the particles inside the control volume [5]:

where the following terms are used:  $\bar{c}_*$  – equilibrium, depth-averaged concentration of the suspended load,  $\alpha$  – non-dimensional coefficient expressing the interacting influence between the bottom and the sediments, and  $w_s$  – the settling velocity of the particles.

The source term from the advective-diffusive equation of the suspended sediment (6) is defined as the difference between the inflow concentration and the actual concentration in the control volume:

## 2.2 Robni, začetni pogoji in računalniški program

Pri reševanju advekcijsko-difuzijske enačbe je bila podana znana koncentracija lebdečih plavin na vtočnem robu matematičnega modela in na iztočnem robu je bil robni pogoj definiran tako, da je gradient koncentracije v prevledujoči smeri toka enak nič. Na zaprtih (neprepustnih) robovih je postavljen pogoj neprepustne stene tako, da skozi takšen rob ni pretoka plavin.

Na začetku iterativnega postopka je treba podati vrednosti koncentracije lebdečih plavin v vseh kontrolnih volumnih. Pri razvoju računalniškega programa smo ugotovili, da se iteracijski postopek najhitreje konča, če je v vseh kontrolnih volumnih (razen v tistih z znano koncentracijo) podana vrednost enaka nič. Da bi se izognili težavam, ki bi lahko nastale z deljenjem z ničlo, podamo za začetno vrednost koncentracije kakšno majhno vrednost.

Za reševanje enačbe (6) je podobno, kakor že pri reševanju hidrodinamičnih enačb, uporabljena metoda kontrolnih volumnov s hibridno shemo. Sistem algebraskih enačb se v iteracijskem postopku rešuje z istim podprogramom, prevzetim iz hidrodinamičnega modela.

### 3 PRIMER UPORABE MATEMATIČNEGA MODELA NA PTUJSKEM JEZERU

#### 3.1 O Ptujskem jezeru

Ptujsko jezero je umetna akumulacija, zgrajena za potrebe proizvodnje električne energije na reki Dravi. To je zadnja akumulacija v slovenskem delu verige dravskih jezer. Od pregrade v Markovcih do strojnice HE Formin je zgrajen dovodni kanal.

## 2.2 Boundary, initial conditions and computer program

During the solving of the advective-diffusive equation, the known concentration of the suspended load was prescribed at the inflow boundary of the mathematical model, and the outflow boundary was defined such that the gradient of the concentration in the dominant direction of the flow is equal to zero; at the closed, non-permeable boundaries it was defined as the condition of a non-permeable wall, which means there is no sediment flow through these boundaries.

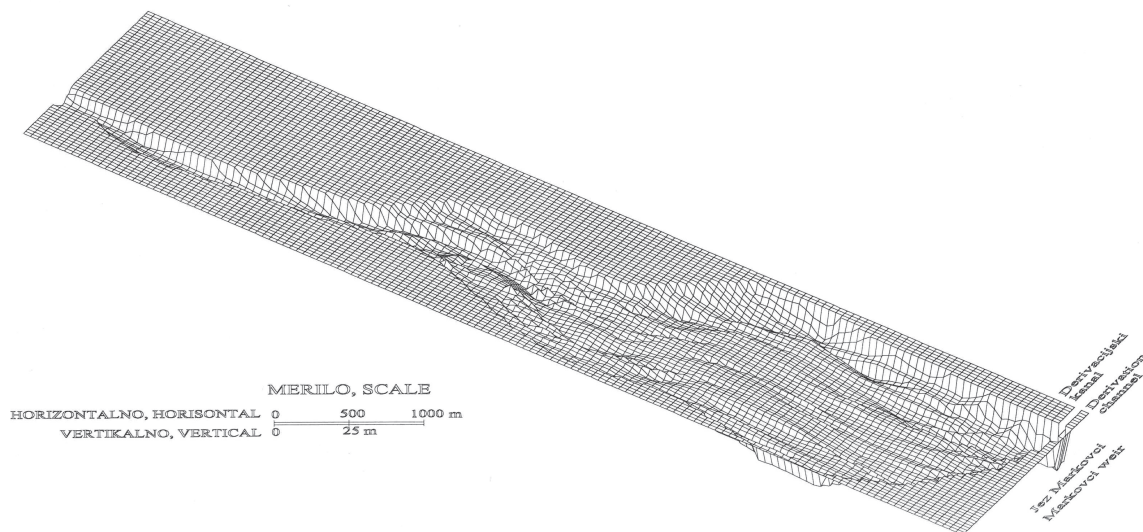
The concentration values should be defined on all the control volumes at the start of the iterative process. During the development of the computer program it was found that the iterative process takes the shortest time if all the control volumes (except those with a known concentration) had defined values of zero. To avoid the difficulties caused by dividing by zero, the starting concentration was chosen to be a small value.

Like during the solving of hydrodynamic equations, for the solving of Equation (6) the control volume method with a hybrid scheme was used. A system of algebraic equations was solved in the subprogram adopted from the hydrodynamic model.

### 3 AN EXAMPLE OF USING THE MATHEMATICAL MODEL FOR PTUJ LAKE

#### 3.1 About Ptuj lake

Ptuj lake is a reservoir on the Drava river; its main purpose is the production of electric power. It is the last reservoir in a chain of several hydro-electric power plants on the Drava river. From the weir in Markovci to HEPP Formin, there is a derivation



Sl. 1. Aksonometrična slika razporeda dna Ptujkega jezera  
 Fig. 1. An axonometric view of the bottom configuration of Ptuj lake

Prostornina akumulacije je 22 milijonov m<sup>3</sup>. Jezero je dolgo okoli 5,5 km, njegova povprečna širina je 1 km in povprečna globina 5 m. Največja globina je v bližini pregrade in znaša 12 m. Aksonometrični prikaz dna akumulacije je podan na sliki 1.

Ob reševanju elektroenergetskih problemov so bila z izgradnjo akumulacije zaščitena obrežna zemljišča reke Drave pred poplavljanjem. Izgradnja akumulacije je povzročila spremembo okolja in je imela tudi negativne posledice, predvsem na živalske vrste, značilne za to območje. Poleg tega se je zaradi pojavov evtrofikacije v usedlinah jezera poslabšala kakovost vode. Do te težave je prišlo zaradi porušitve ravnovesja med dotokom in prenosom plavin na obravnavanem odseku. Povečano usedanje materiala je povzročilo zmanjšanje globine (na nekaterih mestih se je znižala na 0,5 m) in prostornine zbiralnika (za 11% do leta 1993).

Da bi rešili omenjeno problematiko, so bile leta 1993 na Fakulteti za gradbeništvo in geodezijo, Univerze v Ljubljani, organizirane terenske meritve in opravljeni hidrodinamični računi toka v Ptujem jezera.

Te dejavnosti so obsegale:

1. Terenske meritve hitrosti toka na Ptujem jezera.
2. Zbiranje vzorcev vode in usedlega materiala z dna ter njihovo laboratorijsko analizo.
3. Pripravo topografskih, hidravličnih in hidroloških podatkov ter robnih pogojev za 2D hidrodinamični račun tokov.
4. Dvodimenzijske račune tokov za značilne pretoke in verifikacijo dobljenih rezultatov s pomočjo merjenih vrednosti.
5. Dvodimenzijske račune prenosa lebdečih plavin na podlagi izmerjenih značilnosti plavin in ustrezno izbranih koeficientov.

Opozoriti je treba, da so meritve temperature in drugih parametrov kakovosti vode po globini jezera pokazale, da stratifikacija ni opazna, kar upravičuje uporabo dvodimenzijskega matematičnega modela.

### 3.2 Terenske meritve

Glede na dejstvo, da se večina materiala prenaša v času večjih pretokov, je bilo treba terenske meritve časovno uskladiti s pojavom ustreznih pogojev. Opravljene so bile oktobra leta 1993 pri srednji vrednosti pretoka 964 m<sup>3</sup>/s, od tega je približno 460 m<sup>3</sup>/s teklo skozi turbine, preostalih 497 m<sup>3</sup>/s pa preko prelivnih polj. Merili smo naslednje parametre:

- hitrosti toka z uporabo hidrometričnih kril na območju dela zbiralnika z večjimi hitrostmi ter elektromagnetnih sond in plovcev na območju z manjšimi hitrostmi toka (pod 0,2 m/s). Gibanje plovcev smo časovno spremljali z geodetskim opazovanjem. Obdelani rezultati vseh meritev so podani na sliki 2;

channel. The volume of the reservoir is 22 million m<sup>3</sup>. The length of the lake is about 5.5 km, with a main width of 1 km and a main depth of 5 m. The maximum depth is near the weir, and it is 12 m. An axonometric view of the lake bottom is presented in Figure 1.

The building of the reservoir solved some electro-power problems and also protects the surrounding lands of the river Drava from flooding. At the same time, it has caused environmental changes and resulted in some negative consequences, especially to fauna that is characteristic of the area. The quality of the water was decreased by the eutrofication process. This was caused by upsetting the balance in the inflow and the available sediment transport along the reservoir. A decreasing in the depth of the water (at certain parts of the lake it is less than 0.5 m) and the volume (11% since 1993) was caused by the settling of sediments.

In an attempt to solve these problems, in 1993 at the Faculty of Civil and Geodetic Engineering of the University of Ljubljana, field measurements were arranged and hydrodynamic calculations of the flow in Ptuj lake were performed.

These activities were:

1. Field measurements of the velocities in Ptuj lake.
2. Water and bottom-sediment sampling and laboratory analyses.
3. Preparing topographic, hydraulic and hydrological data and boundary conditions for the 2D hydrodynamic flow calculation.
4. 2D calculations for the characteristic discharges and result verifications with the help of measured values.
5. 2D calculation of the suspended-sediment transport based on measured sediment characteristics and suitably chosen coefficients.

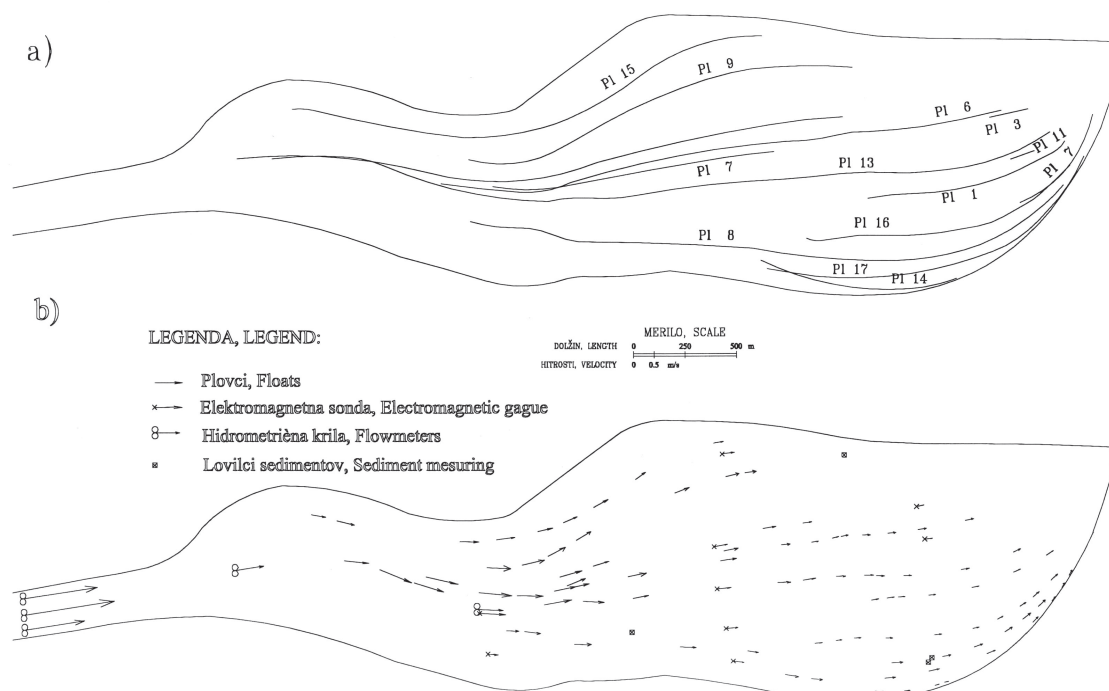
It should be noted that the measurement of temperature and other parameters of the water quality along the depth of reservoir showed that the lake was not stratified and that the use of the 2D model was justified.

### 3.2 Field measurements

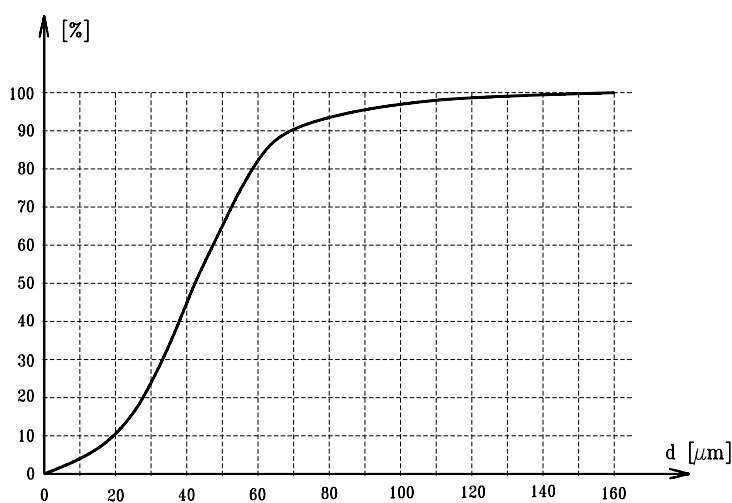
Because most of the material is transported during times of high discharges, measurements were adjusted appropriately with suitable conditions. The measurements were performed in October 1993 under the following conditions: discharge of 964 m<sup>3</sup>/s, 460 m<sup>3</sup>/s flew through turbines. The remaining discharge was overtopped on the weir. The following parameters were measured:

- velocities were measured with flowmeters in the region of the reservoir with higher velocities, and with electro-magnetic gauges and floats in the region with smaller flow velocities (smaller than 0.2 m/s). The movement of the floats was followed using geodetic observation. The results of all the measurements are presented in Fig. 2,

- hitrosti vetra, ki so bile izvedene na treh mestih na in ob jezeru z namenom, da bi ugotovili morebitni vpliv vetra na smer in hitrosti toka vode. Ugotovljeno je bilo, da je bila hitrost vetra v času meritev majhna in da je njegov vpliv na tok vode zanemarljiv;
- vzorčevanje vode smo opravili zato, da bi pridobili podatke o značilnostih lebdečih plavin, njihovi koncentraciji, gostoti in zrnavostni sestavi. Vzorčili smo na več globinah v jezeru, tako da smo lahko dobili ustrezno sliko razporeda koncentracij lebdečih plavin. Primer rezultatov analize zrnivosti je podan na sliki 3. Po globini povprečena koncentracija lebdečih plavin v korenu akumulacije je bila
- the wind velocity was measured at three locations on the lake surface and on the banks to ascertain if there was any wind influence on the water stream. We found that the wind velocity was so small that its influence on the water flow could be neglected,
- the water sampling was performed to obtain data on the suspended sediment's characteristics, its concentration, specific weight and grain size. The sampling was performed on several verticals in the lake, so the distribution of the suspended sediment's concentration was determined. The result of the grain size analysis is presented in Figure 3. The depth-averaged concentration in the



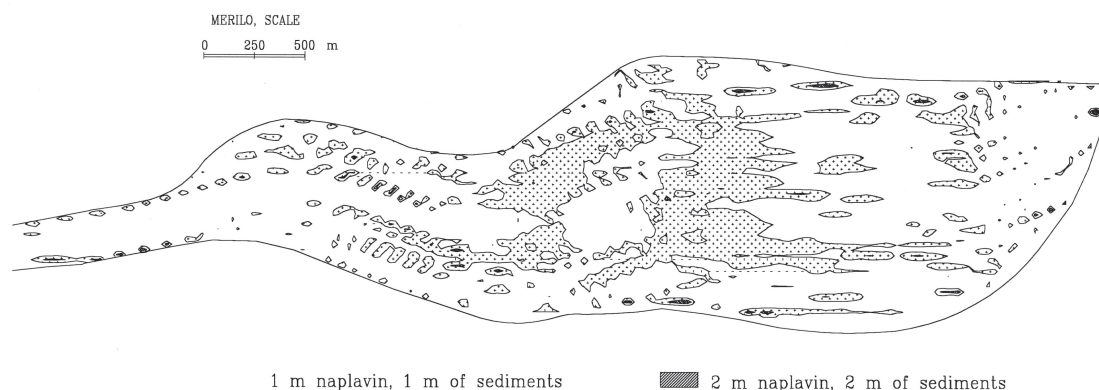
Sl. 2. Rezultati opravljenih meritev na Ptujem jezeru (26.10.1993), a) tokovnice, b) merjene hitrosti  
 Fig. 2. Results of hydrodynamic measurement on Ptuj lake (26.10.1993), a) streamlines, b) velocities



Sl. 3. Zrnavostna krivulja lebdečih plavin na vtoku v akumulacijo (26.10.1993)  
 Fig. 3. Grain size curve of the suspended load in the upper part of the lake (26.10.1993)

$\bar{c}_0 = 525 \text{ g/m}^3$ , medtem ko je bila zaradi prisotnosti organskih snovi gostota  $\rho_s = 2,2 \times 10^3 \text{ kg/m}^3$ .

Na podlagi občasnih meritev prečnih profilov, ki se opravljajo vsakih pet let, je izvedena analiza višine usedlih plavin. Rezultati dveh snemanj, ki sta bili opravljena v letih 1981 in 1991, sta matematično obdelani s programom QuickSurf (delujočega pod programom AutoCAD). Rezultat obdelave je prikazan na sliki 4, kjer so podane izočrte višine usedlega materiala. Opozoriti je treba, da je usedanje materiala opazno na vsem območju akumulacije, čeprav daje slika (zaradi precejšnjega razmika izočrt) nasproten vtis, saj je višina usedlega materiala na večjem delu zbiralnika manjša od 1 m. Rezultat, prikazan na sliki 4, je bil podlaga za umerjanje matematičnega modela za račun prenosa lebdečih plavin.



Sl. 4. Višina usedlih lebdečih plavin v Ptujem jezeru v obdobju od 1981 do 1991  
Fig. 4. Sediment height in Ptuj lake during the period from 1981 to 1991

### 3.3 Rezultati matematičnega modela

Pred simulacijo prenosa lebdečih plavin v Ptujem jezeru je bilo treba opraviti umerjanje hidrodinamičnega dela modela. To vključuje določanje ustreznih vrednosti in razporeda koeficienta hrapavosti dna. Rezultati opravljenih računov so pokazali največjo podobnost z izmerjenimi vrednostmi hitrosti ob upoštevanju enotnega koeficienta hrapavosti po vsem dnu jezera, čigar vrednost znaša po Manningu  $n = 0,026 \text{ sm}^{-1/3}$ . Na podlagi dosedanjih izkušenj ter opravljenih analiz zasipavanja akumulacije smo ugotovili, da večino materiala prinašajo pretoki enoletne povratne dobe ali večji. Ker je bil v prvi fazi pripravljen relativno preprost način računanja z uporabo modela za račun stalnega toka, je bil kot reprezentativen izbran pretok, ki ustreza petletni povratni dobi  $Q_5 = 1239 \text{ m}^3/\text{s}$ . Izračunano polje hitrosti in tokovnice za ta pretok so podane na sliki 5.

Na podlagi podatkov o značilnostih lebdečih plavin, višini usedlega materiala in hidrodinamičnih rezultatov, je bilo mogoče nadaljevati z modeliranjem prenosa lebdečih plavin. Določiti je

upper part of the lake was  $\bar{c}_0 = 525 \text{ g/m}^3$ , with specific weight  $\rho_s = 2.2 \times 10^3 \text{ kg/m}^3$ .

Based on periodic measurements of the cross-sections, which were made every five years, the height of the settled material was analysed. Two measurements, made in 1981 and 1991, were analyzed with the QuickSurf program (acting in AutoCAD). The result is presented in Figure 4 with isolines of the settled material depth. It should be pointed out that the sedimentation was present over the whole of the lake, even though Figure 4 gives (as the equidistance is relatively high) the opposite impression. The height of the sediments for the major region of the lake was less than 1 m. Figure 4 was the basis for the calibration of the mathematical model used to calculate the transport of the suspended sediment.

### 3.3 Results of the mathematical model

Before the transport of the suspended sediments in Ptuj lake was calculated the hydrodynamic model needed to be calibrated. It took into account the determination of a suitable value and the distribution of the roughness coefficient of the bottom of the lake. Several calculations showed that the best agreement with the measured velocities was reached with uniform distribution over the whole region of the lake, and with a value of Manning's coefficient  $n = 0.026 \text{ sm}^{-1/3}$ . Based on experience and the analysis of filling up the reservoir, we found that the largest amount of material brought discharges of a one year return period or higher. As for the first phase of the analysis, it was prepared with the relatively simple approach of a steady-state calculation. The representative discharge was from five-year return period,  $Q_5 = 1239 \text{ m}^3/\text{s}$ . The calculated velocity field with streamlines for that discharge is presented in Figure 5.

Based on the data of the suspended sediment's characteristics – the height of the sediments and the hydrodynamics results – it was possible to continue with the modeling of the suspended-sediment transport. The Schmidt number  $\sigma_{sm}$  and the non-



bilo treba vrednosti Schmidtovega števila  $\sigma_{sm}$  in bredimenzijskega koeficienta  $\alpha$  iz enačbe (9).

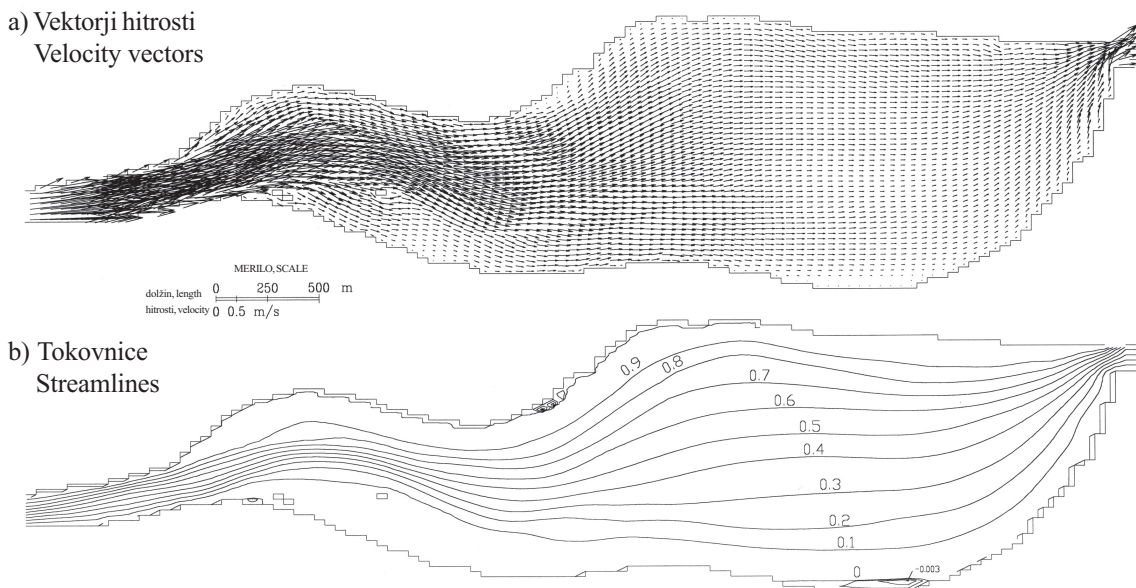
Po podatkih iz literature se vrednost Schmidtovega števila giblje med 0,5 in 1,0. Ker so analize pokazale precejšnjo neobčutljivost rezultatov matematičnega modela za račun prenosa lebdečih plavin na vrednost Schmidtovega števila v priporočenih mejah, tega problema v nadaljevanju nismo več obravnavali. V modelu smo upoštevali zgornjo priporočeno vrednost,  $\sigma_{sm} = 1$ .

Določiti smo morali še vrednost koeficienta  $\alpha$ . S testnimi računi smo ugotovili, da se doseže najboljše ujemanje z merjenimi vrednostmi z upoštevanjem  $\alpha = 0,1$ . Rezultati računov za obdobje od leta 1981 do leta 1991 so podani na sliki 6. Zaradi nepopolnih podatkov o značilnostih materiala v dnu akumulacije (poroznosti in podobno), prikaz rezultatov je podan samo primerjalno. Z znanimi manjkajočimi podatki, bi bilo

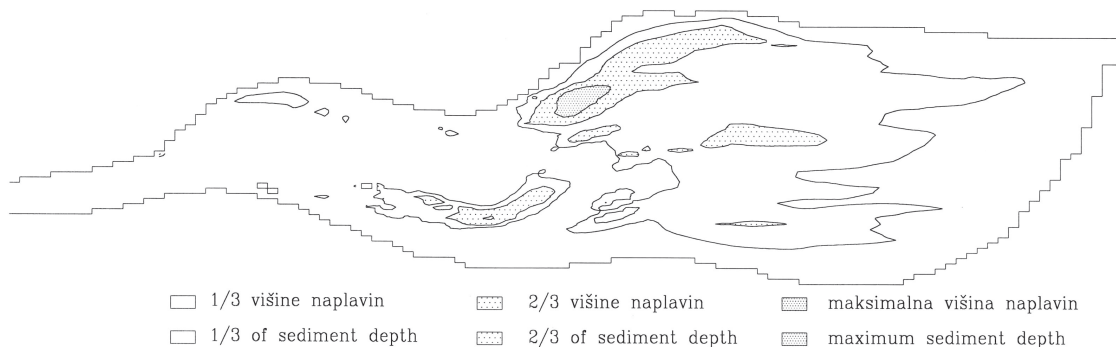
dimensional coefficient  $\alpha$  from the equation (9) needed to be determined.

Using data from the literature, the value of the Schmidt number was between 0.5 and 1.0. Since the analysis showed a rather low sensitivity of the mathematical model's results for the calculation of the suspended sediment transport of the chosen value of the Schmidt number in the recommended boundaries this problem was not treated further. The upper recommended value of  $\sigma_{sm} = 1$  was used in the model.

The value of coefficient  $\alpha$  should also be determined. Using test calculations we found that the best agreement with the measured results was obtained for  $\alpha = 0.1$ . The results for the period 1981 to 1991 are presented in Figure 6. As the data for the characteristics of the sediment were not full (porosity etc.), the result is presented only for a comparison. By including the missing data it would be possible to



Sl. 5. Hidrodinamični rezultati računov v Ptujem jezeru,  $Q=1239 \text{ m}^3/\text{s}$ ,  $n=0,026 \text{ sm}^{-1/3}$ , a) hitrostno polje, b) tokovnice  
 Fig. 5. Results of the hydrodynamic calculations in Ptuj lake  $Q=1239 \text{ m}^3/\text{s}$ ,  $n=0.026 \text{ sm}^{-1/3}$ , a) velocity field, b) streamlines



Sl. 6. Rezultati izračuna višine usedlih plavin v Ptujem jezeru v obdobju od 1981 do 1991  
 Fig. 6. Calculated sediment height in Ptuj lake for the period from 1981 to 1991

mogoče določiti dejansko višino usedlih plavin na posameznih delih dna zbiralnika.

determine the real sediment height and the distribution over the reservoir.

#### 4 SKLEPI

#### 4 CONCLUSIONS

Matematični model je omogočil naslednje:

- s hidrodinamičnim modelom je bila določena dvodimenzijska razporeditev hitrosti toka v jezeru pri različnih pretokih. Poznavanje hidrodinamične slike je pomembno zaradi več razlogov, predvsem pa za boljše razumevanje biokemičnih procesov in pravilno napovedovanje dogajanj, povezanih s prenosom plavin;
- model prenosa lebdečih plavin je omogočil kakovostno simulacijo usedanja lebdečih plavin v Ptujskem jezeru na podlagi razpoložljivih merskih podatkov. Ugotovimo lahko relativno dobro ujemanje rezultatov matematičnega modela in izmerjenimi vrednostmi (sl. 4, 6). Predvidevamo, da bi z ustrezno krivočrtno numerično mrežo dobili boljše ujemanje izračunane in izmerjene hidrodinamične slike. To bi prispevalo k še boljšim rezultatom računa prenosa lebdečih plavin ter s tem tudi k bolj natančni sliki višine usedlega materiala. Upoštevati je treba tudi to, da uporabljeni model zdaj obravnava problematiko prenosa plavin kot stalni pojav pri nekem referenčnem pretoku. To pomeni, da vpliva usedlega materiala in posledično s tem tudi spremembe konfiguracije dna na tokove ni bilo mogoče upoštevati. V naslednjem modelu nestalnega toka bo treba z modelom zajeti časovno spremembo konfiguracije dna in njen vpliv na hidrodinamično sliko ter s tem tudi na usedanje materiala.

The mathematical model made possible the following:

- by using the hydrodynamic model, a two-dimensional distribution of the velocities in the lake was determined in several discharges. Knowledge of the hydrodynamic characteristics of the lake is important for regularly predicting the sediment transport;
- a qualitative simulation of the settling suspended sediments in Ptuj lake, based on the available measured data. The agreement between the results of the mathematical model and the measured values is relatively good (Figures 4 and 6); however, a calculation with a curvilinear numeric mesh should offer better agreement between the calculated and measured results. This will contribute to better calculation results of the suspended sediments and a higher accuracy for the sediment height. It should be considered that the mathematical model used treats the sediment transport as a semi-steady-state process with a certain reference discharge. This means that the influence of settled material and consequently the changes of the bottom configuration on the water flow were not taken into account. The next mathematical model of the unsteady-state flow should take into account changes of the bottom configuration and its influence on the hydrodynamic picture as well as on the sedimentation of the load.

#### 5 LITERATURA

#### 5 REFERENCES

- [1] Cheng, K. J. (1985) An integrated suspended load equation for non-equilibrium transport of non-uniform Sediment, *Journal of Hydrology*, No. 79, 359-364.
- [2] Četina, M. (1988) Matematično modeliranje dvodimenzionalnih turbulentnih tokov, Magistrsko delo, FAGG, Ljubljana.
- [3] Četina, M. (1989) Uporaba  $k - e$  modela turbulence pri računu toka vode s prosto gladino, Kuhljevi dnevi '89, Rogla, 19.-20.10.1989, *Zbornik del*, 253-262.
- [4] Gosman, A. D., F.J.K. Ideriah (1976) TEACH-T: A general computer program for two-dimensional, turbulent, recirculating flows, *Dept. of Mechanical Engineering*, Imperial College, London.
- [5] Krzyk, M. (1996) Dvodimenzijski matematični model konvekcijsko-difuzijskega prenosa snovi in lebdečih plavin v površinskih vodah, Magistrsko delo, FGG, Ljubljana.
- [6] Patankar, S.V. (1980) Numerical heat transfer and fluid flow, *McGraw-Hill Book Company*.
- [7] Rijn, L. C. van (1984) Sediment transport, Part II, Suspended load transport, *Journal of Hydraulic Engineering*, Vol. 110, No. 11, November, 1613-1641.
- [8] Rijn, L. C. van (1993) Principles of sediment transport in rivers, Estuaries and Coastal Seas, *AQUA Publications*.

Naslov avtorjev: Mario Krzyk  
Matjaž Četina  
Univerza v Ljubljani  
Fakulteta za gradbeništvo in  
geodezijo  
Jamova 2  
SI - 1000 Ljubljana  
mkrzyk@fgg.uni-lj.si

Authors' Address: Mario Krzyk  
Matjaž Četina  
University of Ljubljana  
Faculty of Civil and Geodetic  
Engineering  
Jamova 2  
SI - 1000 Ljubljana, Slovenia  
mkrzyk@fgg.uni-lj.si

Prejeto: 30.1.2003  
Received:

Sprejeto: 29.5.2003  
Accepted:

Odprt za diskusijo: 1 leto  
Open for discussion: 1 year

# Optimizacija aksialnih vodnih turbin

## The Optimization of Axial Turbines

Andrej Lipej

*Delo obravnava numerično analizo toka v aksialnih turbinah in oblikovanje optimalnih hidravličnih oblik z uporabo genetičnih algoritmov.*

© 2003 Strojniški vestnik. Vse pravice pridržane.

**(Ključne besede: turbine vodne, optimiranje, metode numerične, oblikovanje gonilnikov)**

*This paper deals with numerical flow analysis and the design of optimum hydraulic shapes for axial water turbines using genetic algorithms.*

© 2003 Journal of Mechanical Engineering. All rights reserved.

**(Keywords: water turbines, optimizations, numerical methods, axial runner design)**

### 0 UVOD

Razvoj vodnih turbin se je v zadnjem desetletju preusmeril iz pretežno eksperimentalne analize v numerično preučevanje tokovnih razmer v različnih delih turbinskih strojev. Pri razvoju turbin je predvsem pomembno nadomestiti drage in zamudne eksperimentalne raziskave z izračuni. Da bi omogočili tekočini optimalno spremembo energije v rotorju, je treba poiskati najboljšo obliko lopatic.

V prispevku so predstavljene naslednje teme:

- oblikovanje aksialnih rotorjev,
- analiza toka v aksialnih rotorjih
  - eksperimentalna metoda (meritev hitrostnega polja)
  - numerična metoda
  - analiza energetske in kavitacijske lastnosti,
- optimizacijska metoda
  - genetična metoda,

ki združene v celoto predstavljajo overjen optimizacijski postopek za oblikovanje gonilnikov.

### 1 OBLIKOVANJE AKSIALNIH GONILNIKOV

Oblikovanja lopatic gonilnika (slika 1) se lahko lotimo na različne načine. Poznamo neposredne in nasprotno metode.

### 0 INTRODUCTION

Nowadays, powerful computers, which are able to calculate and visualise the flow of water through axial and radial runners, have completely changed the development process of water turbines. The main advantage of numerical tests is that the amount of expensive and time consuming experimental work can be reduced. The optimum shape of the runner blades is essential for high energy transformation in the runner and for avoiding losses.

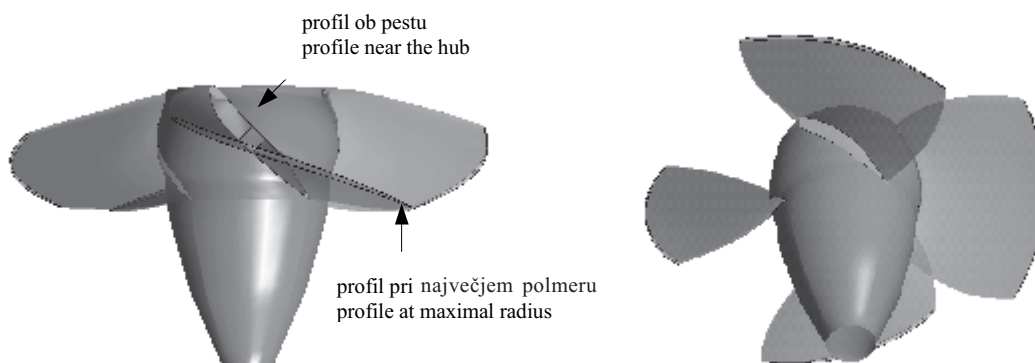
This paper describes the following theories:

- the design of axial runners,
- flow analyses in axial runners
  - the experimental analysis (velocity field measurement)
  - the numerical method
  - the analysis of energetic and cavitation characteristics,
- the optimization method
  - the genetic algorithm,

which when merged together define a very effective procedure for developing water turbines.

### 1 THE DESIGN OF AXIAL RUNNERS

New runner blade shapes (Figure 1) can be developed using various direct or inverse design methods.



Sl. 1. Štirilopatični aksialni gonilnik  
Fig. 1. Four-bladed axial runner

Oblikovanje aksialnega gonilnika začnemo z določitvijo osnovnih parametrov turbine:

- nominalni in največji padec,
- imenska moč turbine,
- predvideni izkoristek,
- predvideni kavitacijski koeficient.

Osnova izračuna lopatice aksialnega gonilnika je analiza ravninske lopatične vrste ([1] in [2]). Račun temelji na uporabi teoretičnih hidrodinamičnih karakteristik ravninskih lopatičnih vrst, ki jih obteka idealna kapljevina. Realno smer toka dobimo z upoštevanjem posebnega koeficienta, ki upošteva vpliv viskoznosti.

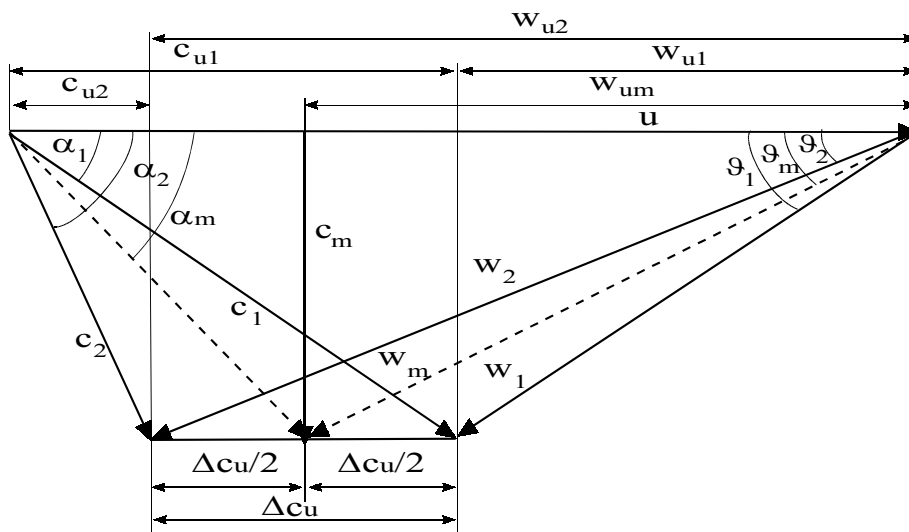
Zamisel, da lahko poenostavimo analizo toka skozi rotor, temelji na glavnih predpostavkah o toku skozi aksialni gonilnik. V pretočnem delu turbine, pred gonilnikom in za njim, je tok osnosimetričen. To zelo dobro velja v bližini optimalnega obratovalnega režima. V medlopatičnem območju se tok ne razlikuje dosti od osnosimetričnega, zato lahko vzamemo, da so tokovne ploskve valjne [3]. Če upoštevamo tudi hipotezo o valjnih tokovnih ploskvah, lahko preidemo iz 3-D na 2-D obravnavo toka v aksialnih gonilnikih (sl. 2).

The development of a new runner starts with a definition of the following parameters:

- nominal and maximum head,
- nominal power,
- predicted efficiency,
- predicted cavitation coefficient.

The design procedure is based on the use of theoretical hydrodynamic cascade characteristics for a potential flow and an empirical flow correction due to viscosity effects ([1] and [2]). The resulting cascade and profile camber result in the optimum for a given set of inlet parameters.

The simplification of the flow analysis is based on the fundamental features of the flow in axial turbines. Inside the channel, in front of the runner and behind it, the flow is axisymmetric, especially at the optimum operating point. Inside the runner the stream surfaces are cylindrical, which is a consequence of the axisymmetric flow [3]. Taking the hypothesis of cylindrical stream surfaces into account, the 3-D flow can be analysed as 2-D flow through axial runners (Figure 2).



Sl. 2. Hitrostni trikotniki na vstopu (1) v gorilnik in izstopu (2) iz gonilnika  
Fig. 2. Velocity triangles at the inlet (1) and outlet (2) of the runner

Pomembna koeficienta, ki določata vstopne in izstopne razmere, sta: koeficient meridianske hitrosti:

$$K_{cm} = \frac{c_m}{c_{msr}}$$

kjer sta  $c_m$  meridianska hitrost in  $c_{msr}$  povprečna meridianska hitrost in koeficient obodne komponente absolutne hitrosti na izstopu iz gonilnika, ki je definiran kot:

$$K_{2u} = \frac{c_{2u}}{\Delta c_u}$$

kjer je  $c_{2u}$  obodna komponenta izstopne absolutne hitrosti,  $\Delta c_u$  pa je sprememba obodne komponente absolutne hitrosti. Vrednosti koeficienta  $K_{2u}$  so okoli nič ali negativne pri pestu, do 0,4 na zunanem valjnem rezu. Računalniški program, ki izračuna optimalno obliko aksialnega gonilnika pri danih pogojih, poskrbi tudi za pripravo datotek, ki jih lahko uporabimo neposredno za izdelavo računske mreže s programom CFX-TASCgrid. Pripraviti je treba le nekaj podatkov o želenem številu elementov za računsko območje in predpisati zgoščitve mreže na mestih, kjer pričakujemo velike gradiente hitrosti ali tlaka.

Ko so pripravljene še robni pogoji, je mogoče začeti z analizo toka v aksialnem gonilniku.

## 2 ANALIZA TOKA V AKSIALNIH GONILNIKIH

Za numerični izračun toka v aksialnih vodnih turbinah smo uporabili programski paket CFX-TASCflow. Omenjeni programski paket rešuje povprečene Navier-Stokesove enačbe na poljubnem 3-D območju.

Pomemben del optimizacijskega postopka je avtomatična in parametrizirana izdelava računske mreže. Po vsaki posamezni obliki rotorskih lopatic in celotnega računskega območja je treba izdelati računsko mrežo.

V primeru aksialnih rotorjev je območje obravnave omejeno le na medlopatični prostor med dvema lopaticama gonilnika [4]. Glavno območje je torej omejeno s tlačno in sesalno stranjo dveh sosednjih lopatic rotorja, delom pesta med lopaticama ter mirujočo zunanjo steno. Posebno pozornost je treba posvetiti izdelavi mreže okoli vstopnega roba lopatic, kjer pričakujemo velike gradiente tlaka in hitrosti. Na tem mestu je nujno potrebna zadosti gosta mreža in tudi pravilno oblikovani elementi, kajti zelo hitro lahko pride do prekrivanja elementov, kar vodi do računskih problemov. Temu se izognemo s pravokotno mrežo, ki jo zagotovimo s posebno generacijo zlepkov, ki povezujejo vstopna robova dveh sosednjih lopatic.

Numerična analiza toka v aksialnih gonilnikih je bila narejena na štirih različnih kaplanovih gonilnikih. V vseh primerih so bili izračuni narejeni za

Important coefficients that are responsible for the inlet and outlet conditions are the meridional velocity coefficient:

where  $c_m$  is the meridional velocity and  $c_{msr}$  is the average meridional velocity; and the coefficient of the outlet vortex, defined as:

where  $c_{2u}$  is the circumferential component of absolute velocity at the outlet,  $\Delta c_u$  is the difference between the inlet and outlet circumferential component of absolute velocity. The value of the coefficient  $K_{2u}$  is close to zero, or it is a linear distribution from  $-0.4$  (near the hub) to  $0.4$  (on the peripheral cylindrical section). The computer program for designing the optimum shape of the axial runner also prepares the files necessary for the grid generation procedure with the CFX-TASCgrid code. Before the solver can start the calculation, some data about the number of elements and the density of the elements has to be determined.

The flow analysis in the axial runner can be started when the boundary condition is prepared.

## 2 FLOW ANALYSES IN THE AXIAL RUNNERS

The computer code CFX-TASCflow was used for the flow analyses in all parts of the turbine. The governing equations solved by CFX-TASCflow are the Reynolds stress-averaged Navier-Stokes equations.

An important part of the optimization procedure is the automatic and parametric grid generation. After each design of the runner blades and all the computational domains, including the channel in front of and behind the runner, the computational grid is prepared.

In the case of an axial runner, the computational domain is limited to be between two runner blades [4]. The boundaries of the computational domain are the pressure side of the blade, the suction side of the blade, a part of the hub and the peripheral stationary wall. The grid generation near the inlet part of the runner blade has to be done with great care because in this region high pressure gradients and velocities are expected. In this area a very fine grid is necessary, as are elements with proper angles, in order to avoid computational problems. The internal angles in the elements should be as orthogonal as possible; this can be achieved by using special splines between the leading edges of two neighbouring runner blades.

The numerical flow analysis was done for four Kaplan runners with different specific speeds. In all

model enake izmere, razlikovali so se le po osnovnih parametrih, padcu, pretoku in vrtljajih ter številu gonilnikovih lopatic. Vsak gonilnik je treba obravnavati za različna relativna odprtja vodilnika in z ustreznimi koti lopatic.

Rezultati analize toka v različnih aksialnih gonilnikih kažejo, da se koeficient meridianske hitrosti od pesta do zunanjega profila razlikuje tudi za več ko samo 10 do 20 %, kakor je bila osnovna teoretična predpostavka. Zato je treba dejanske razmere upoštevati pri pripravi glavnih parametrov za oblikovanje gonilnikovih lopatic.

Glavni kriterij uspešnosti gonilnika je hidravlični izkoristek, ki ga lahko izračunamo iz dobljenih rezultatov numerične analize [5].

Zato je pomembno preveriti točnost rezultatov s primerjavo numeričnih in eksperimentalnih rezultatov. Na sliki 3 vidimo primerjavo treh različnih gonilnikov, ki so bili numerično analizirani in tudi eksperimentalno preverjeni. Primerjava relativnega izkoristka pokaže, da absolutne vrednosti izkoristka niso povsem točne in se razlikujejo za toliko, kolikor vplivov preostalih delov turbin smo zanemarili. Natančnost meritev izkoristka je ocenjena na 0,3 %. Numerično dobljeni izkoristki se razlikujejo za 3 %. S sklopljenimi izračuni vseh delov turbine bi lahko natančnost izračuna izkoristka izboljšali za 2 %.

Iz primerjave vidimo, da lahko med sabo primerjamo različne gonilnike pri enakih obratovalnih pogojih, kar pomeni, da ima gonilnik z največjim izmerjenim izkoristkom tudi izračunan izkoristek največji v primerjavi z drugimi numerično analiziranimi.

Ker so poleg energetske in kavitacijske karakteristike pomembne tudi primerjave kinematike toka pred in za gonilnikom aksialnih turbin, imamo

cases the analyses were done for the same size model, and for different flow rates, heads, rotational speeds and number of blades. The individual runner has to be analysed for different relative guide-vane openings and an appropriate angle of the runner blades.

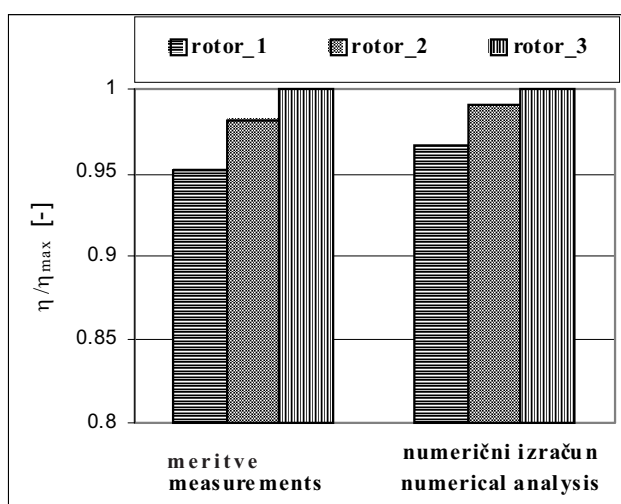
The results of the flow analysis show that the coefficient of meridional velocity can vary by more than 20 % from the hub to the peripheral profile than was believed from theoretical predictions, which is why the real conditions have to be considered when preparing basic parameters for the design of runner blades.

The efficiency with which the available hydraulic energy is converted into mechanical energy is the most important measure for determining the quality of the runner [5].

This is why comparing numerical results with experimental results tests the accuracy of the numerical method. In Figure 3 the numerically and experimentally obtained efficiencies for three different runners are presented. The comparison shows that using numerical results for the sorting of designed runners is successful. The efficiency measurement can be predicted up to 0.3 % accuracy; however, the accuracy of the numerically obtained efficiency is about 3 %. With the coupled calculation of the whole turbine the accuracy of the numerically obtained efficiency can be improved by approximately 2 %.

From this comparison we found that different runners can be compared for equal operation conditions and the best experimentally obtained runner also has the highest numerically obtained efficiency.

In addition to the energetic and cavitation characteristics, a comparison of the numerical results with the experimental results is a good test of the accuracy of the numerical method. On the test rig a lot of energetic and cavitation



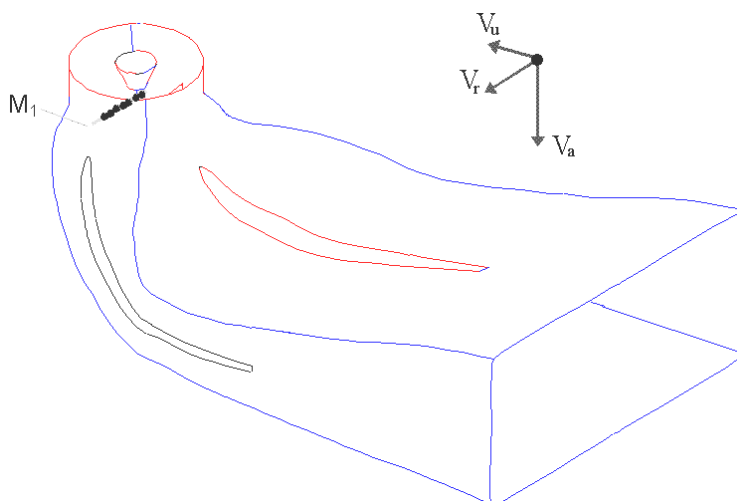
Sl. 3. Primerjava relativnega izkoristka ( $\eta/\eta_{max}$  - izkoristek gonilnika, deljen z izkoristkom najboljšega gonilnika) za tri gonilnike

Fig. 3. Comparison of the relative efficiency ( $\eta/\eta_{max}$  - efficiency of the runner divided by the efficiency of the best runner) for three different runners

tudi primerjavo porazdelitve vseh treh komponent hitrosti, dobljenih z meritvami in numerično analizo (sl. 5.).

## 2.1 Meritve

Meritve so bile opravljene na modelu štirilopatične kaplanove turbine v Turboinštitutu. Hitrosti so bile izmerjene s petluknjično valjno sondo. Merilno mesto M1 na izstopu iz gonilnika oz. na vstopu v sesalno cev je prikazano na sliki 4. S sondo je bila izmerjena hitrost v 12 merilnih točkah, merjeno od stene do osi. Merilno mesto je bilo 100 mm pod pestom. Izmerjena je bila porazdelitev vseh treh komponent hitrosti pri enem obratovalnem režimu, največjem pretoku. Natančnost meritev je ocenjena na 1 % v glavnem toku, v bližini sten pa je natančnost ocenjena na 3 %.



Sl. 4. Merilno mesto M1 pri meritvi hitrosti na izstopu iz gonilnika  
Fig. 4. The measurement section M1 at the outlet of the runner

## 2.2 Primerjava rezultatov meritev in numeričnega izračuna

Pri rezultatih meritev opazimo za pestom večje območje majhnih hitrosti, pri numeričnem rezultatu pa je to območje manjše. Pri numerični analizi toka je to posledica neupoštevanja vodoravnega rebra v sesalni cevi, ki vpliva na tokovne razmere za pestom.

Iz primerjave vseh treh komponent absolutne hitrosti na izstopu iz gonilnika vidimo, da se izračunane hitrosti dobro ujemajo z izmerjenimi, razen v bližini sten, kar še posebej velja za obodno komponento absolutne hitrosti. Vzrokov za neujemanje rezultatov je lahko več. Prvi izhaja iz dejstva, da se s sondo ne da izmeriti hitrosti do stene, drugi vzrok lahko iščemo v matematičnem modelu za turbulentni tok, kjer se računa porazdelitev hitrosti ob steni s stenskim funkcijami in tretji vzrok je lahko vpliv gostote računske mreže na rezultate izračuna.

measurements as well as some measurements of the flow kinematics at the outlet of the runner were made (Figure 5).

## 2.1 Measurements

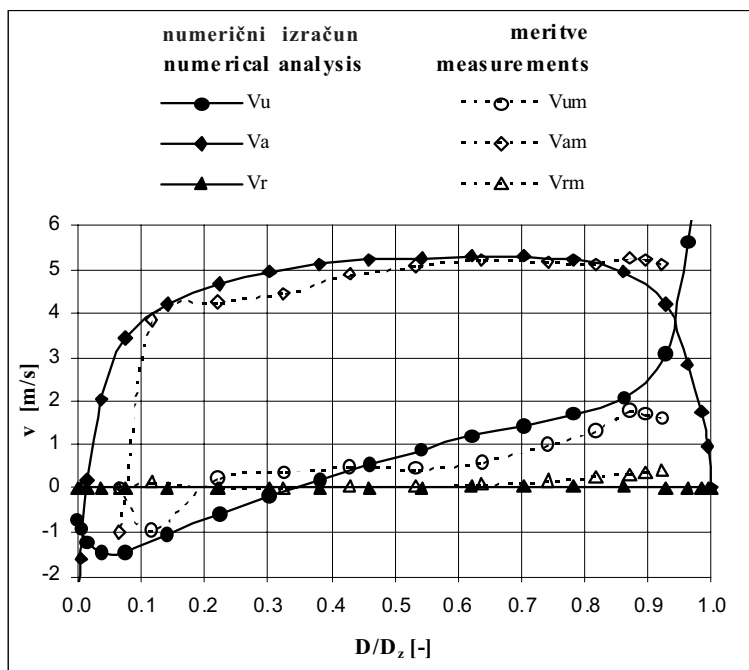
Measurements were carried out on a model of a four-bladed Kaplan turbine at the Turboinstitute. The velocities were measured with a five-hole cylindrical probe. The measurement section M1 at the inlet of the draft tube is shown in Figure 4. The traverse M1 with 12 measuring points from the wall to the centre of the inlet section was chosen at a distance of 100 mm under the runner hub. Measurements were performed at the one operating point at full load. As a result, distributions of the axial, radial and circumferential velocity components along the traverse were obtained. The measurement can be predicted with an accuracy of up to 1% in the main flow; however, the accuracy of the measurements near the wall is about 2%.

## 2.2 Comparison between experimental and numerical results

From the experimental results we can see, behind the hub, the area of small velocities, and in the case of numerical analysis this area is smaller. This is the consequence of the horizontal pier in the draft tube, which was not taken into account in the numerical analysis.

From the comparison of all the velocity components it is clear that there is good agreement between the numerical and the experimental results, except near the wall, where the velocity measurements are less accurate. The reason for some of the differences between the numerical and experimental results can be found in the mathematical model and in the density of the numerical grids.





Sl. 5. Primerjava izračunanih in izmerjenih komponent hitrosti na izstopu iz gonilnika:  $v_u$  (obodna),  $v_a$  (aksialna),  $v_r$  (radialna) – numerični rezultati;  $v_{um}$ ,  $v_{am}$ ,  $v_{rm}$  – eksperimentalni rezultati  
 Fig. 5. Comparison of numerically and experimentally obtained velocity components:  $v_u$  (circumferential),  $v_a$  (axial),  $v_r$  (radial) - numerical results;  $v_{um}$ ,  $v_{am}$ ,  $v_{rm}$  - experimental results

### 3 OPTIMIZACIJSKA METODA

Genetski algoritmi, ki se uporabljajo v tehniki, temeljijo na preprosti analogiji iz narave, kjer se živa bitja razvijajo na podlagi naravne izbire. V naravi je glavni kriterij razvoja preživetje, v tehniki pa lahko pogoj preživetja poljubno definiramo. Postavimo neki kriterij, ki določa uspešnost ali neuspešnost posameznikov v posameznih generacijah. Glavno vodilo pri razvoju omenjene optimizacijske metode je bila zahteva po čim bolj grobi metodi, zlasti pri iskanju celotnega ekstrema, ki ga iščemo.

V tehniki je pomembno, kako problem definiramo, da lahko umetni sistem deluje na temelju naravne izbire. Najprej je treba predmet, ki ga želimo optimalizirati, zapisati podobno, kakor je v kromosomih genska zasnova živih bitij. Genski zapis je lahko nek binaren niz, ki z določeno spremembo določa predmet optimizacije

Naša naloga je poiskati geometrijsko obliko aksialnega gonilnika, ki bo zadoščala energetskim in kavitacijskim kriterijem, zato moramo obliko gonilnika zapisati kot niz števil. V tem primeru lahko osnovne parametre, ki jih potrebujemo v programu za oblikovanje aksialnega gonilnika, uporabimo v genskem zapisu. Ta zapis program za oblikovanje spremeni v geometrijsko obliko gonilnika, ki je pripravljena za ocenjevanje. Ker se neposredno iz oblike ne da oceniti, ali dobljena geometrijska oblika izpolnjuje vse kriterije, je treba uporabiti program za analizo toka skozi gonilnik [6]. Rezultati izračuna omogočajo na podlagi porazdelitve

### 3 OPTIMIZATION METHOD

Genetic algorithms are search algorithms based on the mechanics of natural selection and natural genetics. For a natural process the survival of the fittest individuals is the main criteria, for engineering systems the survival criteria can be defined in a very arbitrary manner. The criteria have to determine the success of the individuals in each generation. The main goal in the development of the optimization procedure is the demand for a robust method, especially in searching for the global extreme.

In the technique, the definition of the problem is very important, because an artificial system has to operate on the basis of natural selection. The object, which has to be optimized, needs to be presented in a similar way as the genotype in chromosomes. The genotype can be a binary string, which defines the object in connection with the appropriate transformation.

The basic idea is to find the shape of the axial runner with demanding energetic and cavitation characteristics, which is why the binary string for the runner shape has to be determined. In this case the design parameters can be used in the genotype. This string can be converted into the runner geometry using the design program, and the runner is prepared for evaluation. Having only the shape of the runner is not enough to know all the characteristics, so a numerical flow analysis is necessary to obtain the criteria for evaluation [6]. From the distribution of the

hitrosti in tlakov izračunati izkoristek, kar je neposreden kriterij ocenjevanja kakovosti posamezne geometrijske oblike.

Uspešnost optimizacijske metode je odvisna od načina parametrizacije geometrijske oblike. Za določitev oblike aksialnega gonilnika uporabimo veliko število neodvisnih parametrov. Nekateri parametri so odvisni samo od obratovalnega režima, drugi pa so odvisni tudi od relativnega premera, pri katerem analiziramo posamezne profile.

Najprej izberemo število in razporeditev relativnih premerov, kjer sekamo lopatico gonilnika z valjem. Potem za vsak relativni premer predpišemo porazdelitev posameznih parametrov: koeficient meridianske hitrosti (dva parametra), koeficient izstopnega vrtinca (dva parametra), delitveno razmerje (dva parametra), relativna debelina profila (dva parametra) in lego največje ukrivljenosti (dva parametra).

Porazdelitev posameznih parametrov je lahko linearna funkcija relativnega premera ali pa funkcija višjega reda. V našem primeru smo uporabili samo linearne porazdelitve desetih parametrov. Vrednost posameznega parametra je med nič in devet. Primer genskega zapisa za posamezno geometrijsko obliko aksialnega gonilnika je predstavljen v naslednji preglednici.

|   |   |                                 |
|---|---|---------------------------------|
| 1 | } | Koeficient meridianske hitrosti |
| 4 |   |                                 |
| 8 | } | Koeficient izstopnega vrtinca   |
| 7 |   |                                 |
| 3 | } | Delitveno razmerje              |
| 4 |   |                                 |
| 4 | } | Relativna debelina              |
| 9 |   |                                 |
| 8 | } | Položaj največje ukrivljenosti  |
| 2 |   |                                 |

Sl. 6. Primer genskega zapisa

Predstavljeni parametri definirajo obliko aksialnega gonilnika in so osnova za pripravo računske mreže, ki jo avtomatično izdelamo za poljuben niz parametrov. Osnova genetične optimizacijske metode so operatorji ([7] in [8]), ki se uporabljajo za izbiro in reprodukcijo ter bistveno prispevajo k grobosti in uspešnosti algoritma. Najbolj klasičen operator za reprodukcijo, ki se uporablja pri genetičnih algoritmih, je dvotočkovno križanje (two point cross-over). Dve točki sta poljubno izbrani in genetični parametri, ki vplivajo na oblikovanje, se zamenjajo med starševskimi vektorji ([9] in [10]). Usmerjeno križanje je malo spremenjeno in upošteva smer izboljšave, ki jo lahko dobimo s primerjavo vrednosti dveh posameznikov. Izboljšani operator usmerjenega križanja se imenuje operator razvojnega usmerjenega križanja.

Večkriterijska metoda je predstavljena kot:

velocity and pressure, the efficiency of the runner can be obtained. This efficiency can be used as a quality criterion of the runner.

The success of the optimization procedure depends on the parametrization of the geometry. To define the shape of the axial runner a lot of independent parameters were used. Some of them depend only on the operating regime; all others depend on the relative diameter of the analysed profile.

First, the number and the distribution of the relative parameters is chosen. Next, the distribution of each parameter is defined for all diameters: coefficient of meridional velocity (two parameters), coefficient of outlet vortex (two parameters), chord-pitch ratio (two parameters), maximum thickness of the relative profile (two parameters) and the position of maximum curvature (two parameters).

The distribution of the parameters can be a linear function of the relative diameter or a high-order function. In our case only the linear distribution was taken into account with the ten parameters. The value of each parameter was between zero and nine. An example of one genotype is presented in the following table.

|   |   |                                    |
|---|---|------------------------------------|
| 1 | } | Coefficient of meridional velocity |
| 4 |   |                                    |
| 8 | } | Coefficient of outlet vortex       |
| 7 |   |                                    |
| 3 | } | Chord – pitch ratio                |
| 4 |   |                                    |
| 4 | } | Relative thickness                 |
| 9 |   |                                    |
| 8 | } | Position of maximal curvature      |
| 2 |   |                                    |

Fig. 6. Example of genotype

The presented parameters define the complete shape of the axial runner and also provide all the details for the generation of the automatic computational grid for an arbitrary array of parameters. The key points of the GA are the operators ([7] and [8]) used for selection and reproduction, which strongly influence the robustness and the efficiency of the algorithm (Goldberg 1988). A two-points crossover is the most classical operator for reproduction, and is one of the operators that offers the highest robustness to the search. The two points are randomly chosen and the genetic materials (the design variables) are exchanged between the parent variable vectors ([9] and [10]). The directional crossover is slightly different and assumes that a "direction of improvement" can be detected by comparing the fitness values of two reference individuals.

The multi-objective optimization problem can be expressed as follows:

$$\begin{aligned} \text{maks/max } F_i(\bar{x}) & \text{ za/for } i = 1, n \\ g_j \leq 0 & \text{ za/for } j = 1, m \end{aligned}$$

kjer je razvidno, da v splošnem rešitev ni enolična, razen če so funkcije linearno odvisne. Z vpeljavo prevladujočega načela, ki ga je vpeljal Pareto, lahko rešitve razdelimo na dve podskupini: *prevladujoče* in *neprevladujoče*. V drugo skupino spadajo rešitve, pri katerih se eden izmed kriterijev ne more povečevati, ne da bi se drugi zmanjševal.

Bolj formalno lahko to trditev napišemo tako, da rešitev  $\bar{x}$  prevladuje nad rešitvijo  $\bar{y}$ , če velja naslednja zveza:

$$\bar{x} > \bar{y} \Leftrightarrow (\forall i, F_i(\bar{x}) \geq F_i(\bar{y})) \cap (\exists j, F_j(\bar{x}) > F_j(\bar{y}))$$

Uporaba načela, ki ga je predlagal Pareto, se v glavnem razlikuje od običajnega postopka pri operatorjih za izbiro.

Optimizacija aksialnega gonilnika je bila uporabljena pri oblikovanju šestlopatičnega kaplanovega gonilnika. Oblikovanje gonilnika je bilo parametrizirano, tako da smo s spremembo desetih parametrov lahko spreminjali obliko gonilnikovih lopatic.

Pri posameznih primerih iz začetnih generacij opazimo iz porazdelitve tlaka, da vpadni koti niso najboljши in zato dobimo večji tlak na sesalni, namesto na tlačni strani. Zaradi integracije tlaka, s katero izračunamo navor na gredi gonilnika, je tudi izkoristek manjši in ne izpolnjuje kriterijev za zadovoljivo rešitev.

Rezultati meritev izkoristka, ki so bili dobljeni z meritvami na modelu kaplanove turbine, so prikazani na sliki 7. Vidimo lahko, da je izkoristek novega gonilnika večji, še posebej pri majhnih pretokih.

Na sliki 7 sta predstavljeni karakteristiki dveh gonilnikov. Izkoristek novega je za pol do več ko dva

and it is obvious that in general the solution is not unique if the functions are not linearly dependent. With the introduction of the Pareto dominance concept it is possible to divide any group of solutions into two subgroups: the *dominated* and the *non-dominated*. Solutions belonging to the second group are the "efficient" solutions, i.e. the ones for which it is not possible to increase any objective value without deteriorating the values of the remaining objectives.

In more formal terms and in the case of maximization problems it is possible to pretend that the solution  $\bar{x}$  dominates  $\bar{y}$  if the following relation is true:

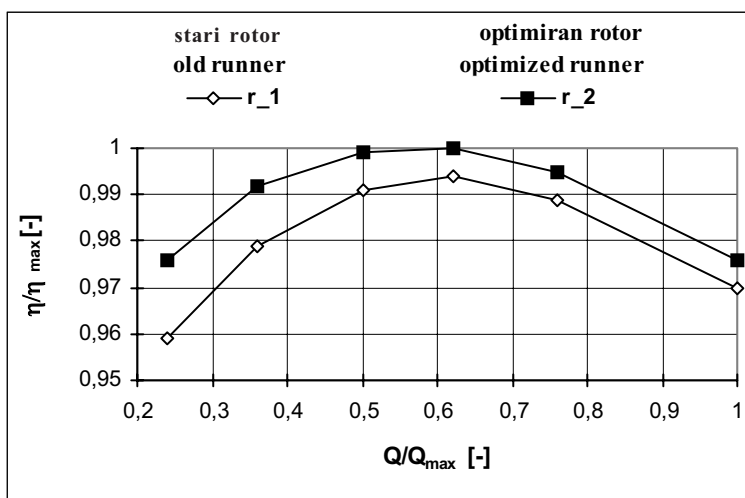
Pareto-GA algorithms mainly differ from classical GAS in terms of the selection process, even though other specific operators might be designed.

The optimization of the axial runner was used for the design of the six-bladed Kaplan runner. The shape of the runner was parameterised and ten parameters were used for the design of the runner blades.

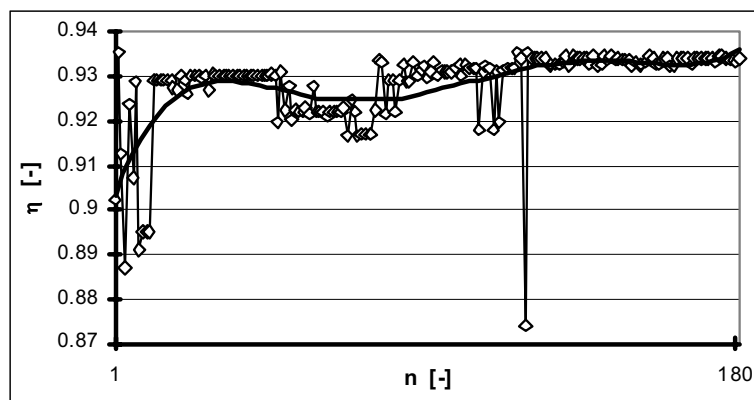
In some cases, from the first few generations the distribution of the pressure show higher pressure on the suction side than on the pressure side, which is the result of having the wrong inlet angles. Calculating the torque on the shaft using the integration of the pressure shows that the efficiency is not high enough for an optimized solution.

The results of efficiency measurements obtained on the model of the Kaplan turbine are shown in Figure 7. The efficiency of the optimized runner is better than the original, especially for the lower flow rate.

Figure 7 compares the performances of two runners for various operating conditions. The efficiency of the new runner is higher, from about



Sl. 7. Primerjava izkoristka pri starem ( $r_1$ ) in gonilniku po opravljeni optimizaciji ( $r_2$ )  
Fig. 7. Comparison of the old ( $r_1$ ) and new, optimized ( $r_2$ ) runner efficiency



Sl. 8. Porazdelitev izkoristka - 18 generacij po 10 posameznikov  
 Fig. 8. Efficiency - 18 generations and 10 individuals in each generation

odstotka višji od osnovnega, odvisno od obratovalnega režima.

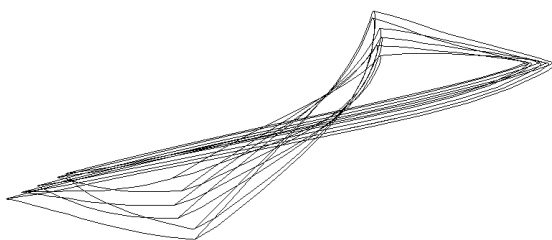
Rezultati analize enega izmed kriterijev pokažejo, da je konvergenca monotona, le od pete do devete generacije pride do manjšega padca izkoristka, kar se vidi na sliki 8.

Analiza spreminjanja geometrijske oblike skozi generacije (sl. 9, 10, 11) kaže na dejstvo, da se v začetnih generacijah najprej popravljajo parametri, ki vplivajo na zvitost lopatic, v zadnjih generacijah pa se spreminjajo v glavnem le parametri, ki spreminjajo dolžine profilov. To je posledica teorije, ki je uporabljena pri metodi oblikovanja rotorskih lopatic, saj temelji na znanih vstopnih in izstopnih hitrostnih trikotnikih, ki direktno vplivajo na kote kaskad posameznih profilov na rotorskih lopaticah.

half a percent to more than two percent, depending on the operating point.

The results of the efficiency distribution for all the individuals show a relatively monotone convergence, except for between the fifth and ninth generations, where some smaller efficiency can be observed (Figure 8).

From the analysis of the geometry for all the generations (Figure 9, Figure 10, Figure 11) we can conclude that in the first generation the parameters that influence the skewness are dominant. In the last few generations the parameters only influence the length of the runner blades. This is a consequence of the design theory based on the known inlet and outlet-velocity triangles, which have a direct influence on the cascade angles.



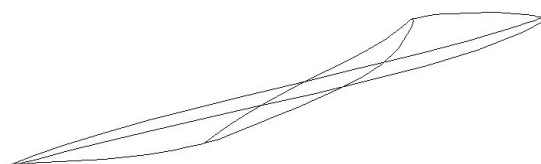
Sl. 9. Primerjava geometrijske oblike gonilnikov prve generacije

Fig. 9. The geometry of the runner blades from the first generation



Sl. 10. Primerjava geometrijske oblike gonilnikov enajste generacije

Fig. 10. The geometry of the runner blades from the eleventh generation



Sl. 11. Primerjava geometrijske oblike gonilnikov zadnje generacije  
 Fig. 11. The geometry of the runner blades from last generation

## 4 SKLEP

Prispevek predstavlja povezavo doslej ločenih metod, pripravo vsake od omenjenih metod za uporabo v skupnem programskem sklopu, dobro izbiro parametrov, ki pomenijo genski zapis posameznega gonilnika ter določitev primernih kriterijev za izbiro uspešnih geometrijskih oblik lopatic aksialnega gonilnika.

Vse skupaj združeno v celoto postane uspešno, učinkovito in hkrati preprosto orodje v rokah inženirjev, kot pomoč pri razvoju hidravličnih oblik.

Metoda je z majhnimi spremembami primerna tudi za aksialne črpalke, radialne gonilnike, ventilatorje in tudi za optimizacijo vseh mirujočih delov v hidravličnih strojih.

## 4 CONCLUSIONS

This paper describes the connection between the different methods in the coupled optimization computer code, the appropriate choosing of the parameters used in the genotype for each axial runner, and the definition of the successful criteria in order to obtain an optimized axial runner.

Together these become a useful and efficient tool for design engineers in the field of turbo machinery.

The presented method can also be useful for other types of turbines, pumps and fans; and also for the optimization of other parts in hydraulic machines.

5 OZNAKE  
5 NOTATION

|                                    |                 |                   |                                    |
|------------------------------------|-----------------|-------------------|------------------------------------|
| premer                             | D               | m                 | diameter                           |
| ukrivljenost profila               | f               | m                 | profile skewness                   |
| funkcija pripravljenosti           | F               | -                 | fitness function                   |
| koeficient izstopnega vrtinca      | $K_{2u}$        | -                 | coefficient of outlet vortex       |
| koeficient meridianske hitrosti    | $K_{cm}$        | -                 | coefficient of meridional velocity |
| dolžina profila                    | l               | m                 | profile length                     |
| pretok                             | Q               | m <sup>3</sup> /s | flow rate                          |
| delitev                            | t               | m                 | pitch                              |
| obodna hitrost                     | u               | m/s               | circumferential velocity           |
| valjne komponente hitrosti         | $v_a, v_r, v_u$ | m/s               | cylindrical velocity components    |
| vektor spremenljivk (genski zapis) | x               | -                 | vector of variables (genotype)     |
| relativna hitrost                  | w               | m/s               | relative velocity                  |
| vstopni kot (absolutna hitrost)    | $\alpha$        | °                 | inlet angle (absolute velocity)    |
| izkoristek                         | $\eta$          | -                 | efficiency                         |
| vstopni kot (relativna hitrost)    | $\vartheta$     | °                 | inlet angle (relative velocity)    |

6 LITERATURA  
6 REFERENCES

- [1] Rabbe, J., Hydro power, *VDI Verlag*.
- [2] Barlit, V., V. (1977) Hidravličeskie turbiny, *Višjsja škola*, Kijev.
- [3] Bohl, W. (1977) Strömungsmaschinen - Aufbau und Wirkungsweise, *Vogel Verlag*, Würzburg 1977.
- [4] Jošt, D., A. Lipej, K. Oberdank, M. Jamnik, B. Velenšek (1996) Numerical flow analysis of a Kaplan turbine. V: Cabrera, E. (ur.) Espert, V. (ur.), Martnez, F. (ur.). Hydraulic machinery and cavitation, *Proceedings of the 18<sup>th</sup> IAHR symposium hydraulic machinery and cavitation*, Valencia, Spain, Dordrecht: Kluwer, 1123-1132. [COBISS-ID 2327579].
- [5] Poloni, C., A. Lipej (1997) Progettazione di una turbina Kaplan con codici fluidodinamici viscosi tridimensionali e algoritmi di ottimizzazione, Congresso nazionale ATI, Cernobbio, Italy, *SGEditoriali*, Padova, 1033-1044.
- [6] TASCflow USRES MANUAL, Version 2.2, (1993), Waterloo, Canada.
- [7] Goldberg, D. E. (1988) Genetic algorithms in search, optimisation and machine learning, *Addison Wesley*, Reading Mass, USA.
- [8] Poloni, C., D. Ng, M. Fearon (1996) Parallelisation of genetic algorithm for aerodynamic design optimisation, adaptive computing in engineering design and control, *Plymouth University*, UK.
- [9] Mosetti, G., C. Poloni (1993) Aerodynamic shape optimization by means of genetic algorithm, *5<sup>th</sup> International Symposium on Computational Fluid Dynamics*, Sendai, Japan.
- [10] Lipej, A., C. Poloni (1998) Design of Kaplan runner using genetic algorithm optimisation, *XIX IAHR Symposium*, Section on hydraulic machinery and cavitation, Singapore.

Avtorjev naslov: Dr. Andrej Lipej  
Turboinštitut  
Rovšnikova 7  
1210-Ljubljana-Šentvid  
andrej.lipej@turboinstitut.si

Author's Address: Dr. Andrej Lipej  
Turboinstitute  
Rovšnikova 7  
1210-Ljubljana-Šentvid, Slovenia  
andrej.lipej@turboinstitut.si

Prejeto: 13.12.2002  
Received:

Sprejeto: 29.5.2003  
Accepted:

Odprt za diskusijo: 1 leto  
Open for discussion: 1 year

## Osebnosti vesti

### Personal Events

#### Doktorati, magisteriji, diplome

##### DOKTORATI

Na Fakulteti za strojništvo Univerze v Ljubljani je z uspehom zagovarjal svojo doktorsko disertacijo, in sicer:

*dne 4. marca 2003: mag. Martin Furlan*, z naslovom: "Karakterizacija magnetnega hrupa enosmernega elektromotorja".

Na Fakulteti za strojništvo Univerze v Mariboru je z uspehom zagovarjal svojo doktorsko disertacijo, in sicer:

*dne 21. marca 2003: spec. Marjan Leber*, z naslovom: "Model dinamičnega osvajanja izdelkov".

S tem sta navedena kandidata dosegla akademsko stopnjo doktorja tehničnih znanosti.

##### MAGISTERIJI

Na Fakulteti za strojništvo Univerze v Ljubljani so z uspehom zagovarjali svoja magistrska dela, in sicer:

*dne 5. marca 2003: Primož Čermelj*, z naslovom: "Raziskava prenosa vibracij preko enorednih krogličnih ležajev";

*dne 24. marca 2003: Jure Klemenčič*, z naslovom: "Raziskava laserskega tvorjenja kapljic iz žice majhnega premera";

*dne 25. marca 2003: Aleš Brezovar*, z naslovom: "Optimalno razporejanje operacij v maloserijski proizvodnji", in

*dne 26. marca 2003: Janez Marko Slabe*, z naslovom: "Lasersko legiranje in navarjanje tankih površinskih slojev".

Na Fakulteti za strojništvo Univerze v Mariboru so z uspehom zagovarjali svoja magistrska dela, in sicer:

*dne 14. marca 2003: Peter Tibaut*, z naslovom: "Uporaba različnih turbulentnih modelov pri določanju faktorjev ugodja z numeričnim modeliranjem naravne in prisilne konvekcije";

*dne 17. marca 2003: Simona Gobec*, z naslovom: "Primerjava postopkov mokre oksidacije in ozoniranja na primeru modelnega polutanta"; in

*dne 28. marca 2003: Marija Beras*, z naslovom: "Razvoj novih pH in O<sub>2</sub> optičnih senzorjev".

S tem so navedeni kandidati dosegli akademsko stopnjo magistra tehničnih znanosti.

##### DIPLOMIRALISO

Na Fakulteti za strojništvo Univerze v Mariboru so pridobili naziv univerzitetni diplomirani inženir strojništva:

*dne 27. marca 2003: Damijan DOMINKO, Dejan STRELEC.*

\*

Na Fakulteti za strojništvo Univerze v Ljubljani so pridobili naziv diplomirani inženir strojništva:

*dne 13. marca 2003: Jože GRUBAR, Milan IVANČIČ, Tomaž KOSMAČ, Božidar LAVRENČIČ, Štefan SRŠA, Marjan STARIČ, Iztok ŠALAMON;*

*dne 14. marca 2003: Marjan HRIBLJAN, Franci MLAKAR, Janez RUPAR, Štefan ŠIMUNIČ, Damir TOPOLOVŠEK.*

Na Fakulteti za strojništvo Univerze v Mariboru so pridobili naziv diplomirani inženir strojništva:

*dne 27. marca 2003: Anita MEŽNAR, Janez SKRINJAR, Anton ŠKAMLEC, Simon ULEN, Gregor VRANIČ.*

## Navodila avtorjem

### Instructions for Authors

Članki morajo vsebovati:

- naslov, povzetek, besedilo članka in podnaslove slik v slovenskem in angleškem jeziku,
- dvojezične preglednice in slike (diagrami, risbe ali fotografije),
- seznam literature in
- podatke o avtorjih.

Strojniški vestnik izhaja od leta 1992 v dveh jezikih, tj. v slovenščini in angleščini, zato je obvezen prevod v angleščino. Obe besedili morata biti strokovno in jezikovno med seboj usklajeni. Članki naj bodo kratki in naj obsegajo približno 8 tipkanih strani. Izjemoma so strokovni članki, na željo avtorja, lahko tudi samo v slovenščini, vsebovati pa morajo angleški povzetek.

#### Vsebina članka

Članek naj bo napisan v naslednji obliki:

- Naslov, ki primerno opisuje vsebino članka.
- Povzetek, ki naj bo skrajšana oblika članka in naj ne presega 250 besed. Povzetek mora vsebovati osnove, jedro in cilje raziskave, uporabljeno metodologijo dela, povzetek rezultatov in osnovne sklepe.
- Uvod, v katerem naj bo pregled novejšega stanja in zadostne informacije za razumevanje ter pregled rezultatov dela, predstavljenih v članku.
- Teorija.
- Eksperimentalni del, ki naj vsebuje podatke o postavitvi preskusa in metode, uporabljene pri pridobitvi rezultatov.
- Rezultati, ki naj bodo jasno prikazani, po potrebi v obliki slik in preglednic.
- Razprava, v kateri naj bodo prikazane povezave in posplošitve, uporabljene za pridobitev rezultatov. Prikazana naj bo tudi pomembnost rezultatov in primerjava s poprej objavljenimi deli. (Zaradi narave posameznih raziskav so lahko rezultati in razprava, za jasnost in preprostejšo bralčevo razumevanje, združeni v eno poglavje.)
- Sklepi, v katerih naj bo prikazan en ali več sklepov, ki izhajajo iz rezultatov in razprave.
- Literatura, ki mora biti v besedilu oštevilčena zaporedno in označena z oglatimi oklepaji [1] ter na koncu članka zbrana v seznamu literature. Vse opombe naj bodo označene z uporabo dvignjene številke<sup>1</sup>.

#### Oblika članka

Besedilo naj bo pisano na listih formata A4, z dvojnimi presledki med vrstami in s 3 cm širokim robom, da je dovolj prostora za popravke lektorjev. Najbolje je, da pripravite besedilo v urejevalniku Microsoft Word. Hkrati dostavite odtis članka na papirju, vključno z vsemi slikami in preglednicami ter identično kopijo v elektronski obliki.

Prosimo, da ne uporabljate urejevalnika LaTeX, saj program, s katerim pripravljamo Strojniški vestnik, ne uporablja njegovega formata. V urejevalniku LaTeX oblikujte grafe, preglednice in enačbe in jih stiskajte na kakovostnem laserskem tiskalniku, da jih bomo lahko presneli.

Enačbe naj bodo v besedilu postavljene v ločene vrstice in na desnem robu označene s tekočo številko v okroglih oklepajih

#### Enote in okrajšave

V besedilu, preglednicah in slikah uporabljajte le standardne označbe in okrajšave SI. Simbole fizikalnih veličin v besedilu pišite poševno (kurzivno), (npr.  $v$ ,  $T$ ,  $n$  itn.). Simbole enot, ki sestojijo iz črk, pa pokončno (npr.  $\text{ms}^{-1}$ , K, min, mm itn.).

Vse okrajšave naj bodo, ko se prvič pojavijo, napisane v celoti v slovenskem jeziku, npr. časovno spremenljiva geometrija (CSG).

Papers submitted for publication should comprise:

- Title, Abstract, Main Body of Text and Figure Captions in Slovene and English,
- Bilingual Tables and Figures (graphs, drawings or photographs),
- List of references and
- Information about the authors.

Since 1992, the Journal of Mechanical Engineering has been published bilingually, in Slovenian and English. The two texts must be compatible both in terms of technical content and language. Papers should be as short as possible and should on average comprise 8 typed pages. In exceptional cases, at the request of the authors, speciality papers may be written only in Slovene, but must include an English abstract.

#### The format of the paper

The paper should be written in the following format:

- A Title, which adequately describes the content of the paper.
- An Abstract, which should be viewed as a miniversion of the paper and should not exceed 250 words. The Abstract should state the principal objectives and the scope of the investigation, the methodology employed, summarize the results and state the principal conclusions.
- An Introduction, which should provide a review of recent literature and sufficient background information to allow the results of the paper to be understood and evaluated.
- A Theory
- An Experimental section, which should provide details of the experimental set-up and the methods used for obtaining the results.
- A Results section, which should clearly and concisely present the data using figures and tables where appropriate.
- A Discussion section, which should describe the relationships and generalisations shown by the results and discuss the significance of the results making comparisons with previously published work. (Because of the nature of some studies it may be appropriate to combine the Results and Discussion sections into a single section to improve the clarity and make it easier for the reader.)
- Conclusions, which should present one or more conclusions that have been drawn from the results and subsequent discussion.
- References, which must be numbered consecutively in the text using square brackets [1] and collected together in a reference list at the end of the paper. Any footnotes should be indicated by the use of a superscript<sup>1</sup>.

#### The layout of the text

Texts should be written in A4 format, with double spacing and margins of 3 cm to provide editors with space to write in their corrections. Microsoft Word for Windows is the preferred format for submission. One hard copy, including all figures, tables and illustrations and an identical electronic version of the manuscript must be submitted simultaneously.

Please do not use a LaTeX text editor, since this is not compatible with the publishing procedure of the Journal of Mechanical Engineering. Graphs, tables and equations in LaTeX may be supplied in good quality hard-copy format, so that they can be copied for inclusion in the Journal.

Equations should be on a separate line in the main body of the text and marked on the right-hand side of the page with numbers in round brackets.

#### Units and abbreviations

Only standard SI symbols and abbreviations should be used in the text, tables and figures. Symbols for physical quantities in the text should be written in Italics (e.g.  $v$ ,  $T$ ,  $n$ , etc.). Symbols for units that consist of letters should be in plain text (e.g.  $\text{ms}^{-1}$ , K, min, mm, etc.).

All abbreviations should be spelt out in full on first appearance, e.g., variable time geometry (VTG).



**Slike**

Slike morajo biti zaporedno oštevilčene in označene, v besedilu in podnaslovu, kot sl. 1, sl. 2 itn. Posnete naj bodo v kateremkoli od razširjenih formatov, npr. BMP, JPG, GIF. Za pripravo diagramov in risb priporočamo CDR format (CorelDraw), saj so slike v njem vektorske in jih lahko pri končni obdelavi preprosto povečujemo ali pomanjšujemo.

Pri označevanju osi v diagramih, kadar je le mogoče, uporabite označbe veličin (npr.  $t$ ,  $v$ ,  $m$  itn.), da ni potrebno dvojezično označevanje. V diagramih z več krivuljami, mora biti vsaka krivulja označena. Pomen oznake mora biti pojasnjen v podnaslovu slike.

Vse označbe na slikah morajo biti dvojezične.

Za vse slike po fotografskih posnetkih je treba priložiti izvorne fotografije ali kakovostno narejen posnetek. V izjemnih primerih so lahko slike tudi barvne.

**Preglednice**

Preglednice morajo biti zaporedno oštevilčene in označene, v besedilu in podnaslovu, kot preglednica 1, preglednica 2 itn. V preglednicah ne uporabljajte izpisanih imen veličin, ampak samo ustrezne simbole, da se izognemo dvojezični podvojitvi imen. K fizikalnim veličinam, npr.  $t$  (pisano poševno), pripišite enote (pisano pokončno) v novo vrsto brez oklepajev.

Vsi podnaslovi preglednic morajo biti dvojezični.

**Seznam literature**

Vsa literatura mora biti navedena v seznamu na koncu članka v prikazani obliki po vrsti za revije, zbornike in knjige:

- [1] Tamg, Y.S., Y.S. Wang (1994) A new adaptive controller for constant turning force. *Int J Adv Manuf Technol* 9(1994) London, pp. 211-216.
- [2] Čuš, F., J. Balič (1996) Rationale Gestaltung der organisatorischen Abläufe im Werkzeugwesen. *Proceedings of International Conference on Computer Integration Manufacturing*, Zakopane, 14.-17. maj 1996.
- [3] Oertli, P.C. (1977) Praktische Wirtschaftskybernetik. *Carl Hanser Verlag*, München.

**Podatki o avtorjih**

Članku priložite tudi podatke o avtorjih: imena, nazive, popolne poštne naslove, številke telefona in faksa ter naslove elektronske pošte.

**Sprejem člankov in avtorske pravice**

Uredništvo Strojniškega vestnika si pridržuje pravico do odločanja o sprejemu članka za objavo, strokovno oceno recenzentov in morebitnem predlogu za krajšanje ali izpopolnitev ter terminološke in jezikovne korekture.

Avtor mora predložiti pisno izjavo, da je besedilo njegovo izvorno delo in ni bilo v dani obliki še nikjer objavljeno. Z objavo preidejo avtorske pravice na Strojniški vestnik. Pri morebitnih kasnejših objavah mora biti SV naveden kot vir.

Rokopisi člankov ostanejo v arhivu SV.

Vsa nadaljnja pojasnila daje:

Uredništvo  
STROJNIŠKEGA VESTNIKA  
p.p. 197/IV  
1001 Ljubljana  
Telefon: (01) 4771-757  
Telefaks: (01) 2518-567  
E-mail: strojniksi.vestnik@fs.uni-lj.si

**Figures**

Figures must be cited in consecutive numerical order in the text and referred to in both the text and the caption as Fig. 1, Fig. 2, etc. Figures may be saved in any common format, e.g. BMP, GIF, JPG. However, the use of CDR format (CorelDraw) is recommended for graphs and line drawings, since vector images can be easily reduced or enlarged during final processing of the paper.

When labelling axes, physical quantities, e.g.  $t$ ,  $v$ ,  $m$ , etc. should be used whenever possible to minimise the need to label the axes in two languages. Multi-curve graphs should have individual curves marked with a symbol, the meaning of the symbol should be explained in the figure caption.

All figure captions must be bilingual.

Good quality black-and-white photographs or scanned images should be supplied for illustrations. In certain circumstances, colour figures may be considered.

**Tables**

Tables must be cited in consecutive numerical order in the text and referred to in both the text and the caption as Table 1, Table 2, etc. The use of names for quantities in tables should be avoided if possible: corresponding symbols are preferred to minimise the need to use both Slovenian and English names. In addition to the physical quantity, e.g.  $t$  (in Italics), units (normal text), should be added in new line without brackets.

All table captions must be bilingual.

**The list of references**

References should be collected at the end of the paper in the following styles for journals, proceedings and books, respectively:

- [1] Tamg, Y.S., Y.S. Wang (1994) A new adaptive controller for constant turning force. *Int J Adv Manuf Technol* 9(1994) London, pp. 211-216.
- [2] Čuš, F., J. Balič (1996) Rationale Gestaltung der organisatorischen Abläufe im Werkzeugwesen. *Proceedings of International Conference on Computer Integration Manufacturing*, Zakopane, 14.-17. maj 1996.
- [3] Oertli, P.C. (1977) Praktische Wirtschaftskybernetik. *Carl Hanser Verlag*, München.

**Author information**

The following information about the authors should be enclosed with the paper: names, complete postal addresses, telephone and fax numbers and E-mail addresses.

**Acceptance of papers and copyright**

The Editorial Committee of the Journal of Mechanical Engineering reserves the right to decide whether a paper is acceptable for publication, obtain professional reviews for submitted papers, and if necessary, require changes to the content, length or language.

Authors must also enclose a written statement that the paper is original unpublished work, and not under consideration for publication elsewhere. On publication, copyright for the paper shall pass to the Journal of Mechanical Engineering. The JME must be stated as a source in all later publications.

Papers will be kept in the archives of the JME.

You can obtain further information from:

Editorial Board of the  
JOURNAL OF MECHANICAL ENGINEERING  
P.O.Box 197/IV  
1001 Ljubljana, Slovenia  
Telephone: +386 (0)1 4771-757  
Fax: +386 (0)1 2518-567  
E-mail: strojniksi.vestnik@fs.uni-lj.si