



## 7 General Majorana Neutrino Mass Matrix from a Low Energy SU(3) Family Symmetry with Sterile Neutrinos

A. Hernández-Galeana\*

Departamento de Física, Escuela Superior de Física y Matemáticas, I.P.N.,  
U. P. "Adolfo López Mateos". C. P. 07738, México, D.F., México

**Abstract.** Within the framework of a local SU(3) family symmetry model, we report a general analysis of the mechanism for neutrino mass generation and mixing, including light sterile neutrinos. In this scenario, ordinary heavy fermions, top and bottom quarks and tau lepton, become massive at tree level from Dirac See-saw mechanisms implemented by the introduction of a new set of SU(2)<sub>L</sub> weak singlet vector-like fermions, U, D, E, N, with N a sterile neutrino. Right-handed and the N<sub>L,R</sub> sterile neutrinos allow the implementation of a 8 × 8 general Majorana neutrino mass matrix with four or five massless neutrinos at tree level. Hence, light fermions, including light neutrinos get masses from radiative corrections mediated by the massive SU(3) gauge bosons. We report the corresponding Majorana neutrino mass matrix up to one loop. Previous numerical analysis of the free parameters show out solutions for quarks and charged lepton masses within a parameter space region where the vector-like fermion masses M<sub>U</sub>, M<sub>D</sub>, M<sub>E</sub>, and the SU(3) family gauge boson masses lie in the low energy region of  $\mathcal{O}(1 - 20)$  TeV, with light neutrinos within the correct order of square neutrino mass differences:  $m_2^2 - m_1^2 \approx 7 \times 10^{-5} \text{ eV}^2$ ,  $m_3^2 - m_1^2 \approx 2 \times 10^{-3} \text{ eV}^2$ , and at least one sterile neutrino of the order  $\approx 0.5 \text{ eV}$ . A more precise fit of the parameters is still needed to account also for the quark and lepton mixing.

**Povzetek.** Avtor pojasnjuje pojav družin pri leptonih tako, da uporabi za opis družin model z lokalno simetrijo SU(3). Trem družinam kvarkov in leptonov doda še družinski triplet desnoročnih nevtrinov, ki nosi samo družinski naboj, levoročni in desnoročni U in prav tak D kvark, ki nosijo poleg barve le hiper naboj, levoročni in desnoročni nevtrino, ki ne nosita nobenega naboja, ter levoročni in desnoročni elektron s hipernabojem (-2). Vsi ti novi delci so masivni. Novi fermioni poskrbijo na drevesnem nivoju samo za maso tretje družine kvarkov in leptonov. Lahkim fermionom, tudi lahkim nevtrinom, priskrbijo maso popravki v naslednjih redih pri interakciji z masivnimi bozoni, ki nosijo družinsko kvantno število. Avtor izračuna masno matriko 8x8 za Majoranane nevtrine do prvega reda. Proste parametre modela določi z izmerjenimi masami in mešalnimi matrikami. Po dosedanjih izračunih so primerne vrednosti za mase fermionov M<sub>U</sub>, M<sub>D</sub>, M<sub>E</sub> in za maso družinskega tripleta umeritvenega bozona v intervalu  $\mathcal{O}(1 - 20)$  TeV, za izmerjene masne razlike lahkih nevtrinov  $m_{22} - m_{12} \approx 7 \times 10^{-5} \text{ eV}^2$ ,  $m_{32} - m_{12} \approx 2 \times 10^{-3} \text{ eV}^2$  lahko avtor poskrbi s še vsaj enim sterilnim nevtrinom, ki ima maso  $\approx 0.5 \text{ eV}$ . Avtor pričakuje, da bo z bolj natančnimi izračuni lahko s pomočjo tega modela pojasnil mešalne matrike kvarkov in leptonov.

\* E-mail: albino@esfm.ipn.mx

## 7.1 Introduction

Although the standard picture with three light flavor neutrinos has been successful to describe the neutrino oscillation data. On the other hand, there have been recent hints from the LSND and MiniBooNe short-baseline neutrino oscillation experiments[1,2] on the possible existence of at least one light sterile neutrino in the eV scale, which mix with the active neutrinos. On the other hand, an explanation of the strong hierarchy of quark and charged lepton masses is still a big challenge in particle physics. This hierarchy have suggested to many model building theorists that light fermion masses could be generated from radiative corrections, while those of the top and bottom quarks and the tau lepton are generated at tree level. This may be understood as the breaking of a symmetry among families, a horizontal symmetry.

In this report we update the general features of a "Beyond the Standard Model"(BSM) proposal which introduces a SU(3) [3] gauged family symmetry<sup>1</sup> commuting with the Standard Model group. Previous reports[4] within this scenario showed that this model has the features and particle content to account for a realistic spectrum of charged fermion masses and quark mixing. This BSM model introduce a hierarchical mass generation mechanism in which the light fermions obtain masses through one loop radiative corrections, mediated by the massive bosons associated to the SU(3) family symmetry that is spontaneously broken, while the masses for the top and bottom quarks as well as for the tau lepton, are generated at tree level from "Dirac See-saw"[5] mechanisms implemented by the introduction of a new generation of SU(2)<sub>L</sub> weak singlets vector-like fermions.

*The SU(3) family symmetry model allows one to address the problem of quark and lepton masses and mixing, including active and light sterile neutrinos.*

## 7.2 SU(3) flavor symmetry model

### 7.2.1 Fermion content

Before "Electroweak Symmetry Breaking"(EWSB) all ordinary, "Standard Model"(SM) fermions remain massless, and the global symmetry in this limit of all quarks and leptons massless, including R-handed neutrinos, is:

$$SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{e_R} \quad (7.1)$$

$$\supset SU(3)_{q_L+u_R+d_R+l_L+e_R+\nu_R} \equiv SU(3) \quad (7.2)$$

We define the gauge group symmetry  $G \equiv SU(3) \otimes G_{SM}$ , where Eq.(7.2) defines the SU(3) gauged family symmetry, and  $G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  is the "Standard Model" gauge group, with  $g_H$ ,  $g_s$ ,  $g$  and  $g'$  the corresponding

<sup>1</sup> See [3,4] and references therein for some SU(3) family symmetry models.

coupling constants. The content of fermions assumes the ordinary quarks and leptons assigned under G as:

$$\psi_q^o = (3, 3, 2, \frac{1}{3})_L \quad , \quad \psi_u^o = (3, 3, 1, \frac{4}{3})_R \quad , \quad \psi_d^o = (3, 3, 1, -\frac{2}{3})_R$$

$$\psi_l^o = (3, 1, 2, -1)_L \quad , \quad \psi_e^o = (3, 1, 1, -2)_R \quad ,$$

where the last entry corresponds to the hypercharge  $Y$ , and the electric charge is defined by  $Q = T_{3L} + \frac{1}{2}Y$ . The model also includes two types of extra fermions:

- Right handed neutrinos  $\Psi_\nu^o = (3, 1, 1, 0)_R$  required to cancel anomalies[6], and
- the  $SU(2)_L$  singlet vector-like fermions:

$$U_{L,R}^o = (1, 3, 1, \frac{4}{3}) \quad , \quad D_{L,R}^o = (1, 3, 1, -\frac{2}{3}) \quad (7.3)$$

$$N_{L,R}^o = (1, 1, 1, 0) \quad , \quad E_{L,R}^o = (1, 1, 1, -2) \quad , \quad (7.4)$$

which conserve the previous anomaly cancellation. The transformation of these vector-like fermions allows the mass invariant mass terms

$$M_U \bar{U}_L^o U_R^o + M_D \bar{D}_L^o D_R^o + M_E \bar{E}_L^o E_R^o + \text{h.c.} \quad , \quad (7.5)$$

and

$$m_D \bar{N}_L^o N_R^o + m_L \bar{N}_L^o (N_L^o)^c + m_R \bar{N}_R^o (N_R^o)^c + \text{h.c.} \quad (7.6)$$

These  $SU(2)_L$  weak singlets vector-like fermions have been introduced to give masses at tree level only to the third family of known fermions through Dirac See-saw mechanisms.  $M_U, M_D, M_E$  play a crucial role to implement a hierarchical spectrum for quarks and charged lepton masses and mixing, meanwhile  $m_D, m_L, m_R$  play a similar role for neutrino masses and lepton mixing, all together with the radiative corrections.

### 7.3 SU(3) family symmetry breaking

The corresponding SU(3) gauge bosons are defined through their couplings to fermions as

$$\begin{aligned} i\mathcal{L}_{\text{int}} = & \frac{g_H}{2} (f_1^o \gamma_\mu f_1^o - f_2^o \gamma_\mu f_2^o) Z_1^\mu + \frac{g_H}{2\sqrt{3}} (f_1^o \gamma_\mu f_1^o + f_2^o \gamma_\mu f_2^o - 2f_3^o \gamma_\mu f_3^o) Z_2^\mu \\ & + \frac{g_H}{\sqrt{2}} (f_1^o \gamma_\mu f_2^o Y_1^+ + f_1^o \gamma_\mu f_3^o Y_2^+ + f_2^o \gamma_\mu f_3^o Y_3^+ + \text{h.c.}) \quad (7.7) \end{aligned}$$

$f_1^o = u^o, d^o, e^o, \nu_e^o$ ,  $f_2^o = c^o, s^o, \mu^o, \nu_\mu^o$  and  $f_3^o = t^o, b^o, \tau^o, \nu_\tau^o$ . To implement a hierarchical spectrum for charged fermion masses, and simultaneously to achieve

the SSB of SU(3), we introduce the flavon scalar fields:  $\eta_i = (3, 1, 1, 0)$ ,  $i = 1, 2, 3$ , transforming as the fundamental representation under SU(3) and being standard model singlets, with the "Vacuum Expectation Values" (VEV's):

$$\langle \eta_1 \rangle^T = (\Lambda_1, 0, 0) \quad , \quad \langle \eta_2 \rangle^T = (0, \Lambda_2, 0) \quad , \quad \langle \eta_3 \rangle^T = (0, 0, \Lambda_3) . \quad (7.8)$$

Actually, let us point out here that only two scalar flavons in the fundamental representation are needed to completely break down the SU(3) symmetry. The most convenient way to accomplish the spontaneous breaking of the SU(3) family symmetry is under current study. Thus, the contribution to the horizontal gauge boson masses from Eq.(7.8) read

- $\eta_1 : \frac{g_{H_1}^2 \Lambda_1^2}{2} (Y_1^+ Y_1^- + Y_2^+ Y_2^-) + \frac{g_{H_1}^2 \Lambda_1^2}{4} (Z_1^2 + \frac{Z_2^2}{3} + 2Z_1 \frac{Z_2}{\sqrt{3}})$
- $\eta_2 : \frac{g_{H_2}^2 \Lambda_2^2}{2} (Y_1^+ Y_1^- + Y_3^+ Y_3^-) + \frac{g_{H_2}^2 \Lambda_2^2}{4} (Z_1^2 + \frac{Z_2^2}{3} - 2Z_1 \frac{Z_2}{\sqrt{3}})$
- $\eta_3 : \frac{g_{H_3}^2 \Lambda_3^2}{2} (Y_2^+ Y_2^- + Y_3^+ Y_3^-) + g_{H_3}^2 \Lambda_3^2 \frac{Z_2^2}{3}$

Therefore, neglecting tiny contributions from electroweak symmetry breaking, we obtain the gauge boson mass terms

$$\begin{aligned} & (M_1^2 + M_2^2) Y_1^+ Y_1^- + (M_1^2 + M_3^2) Y_2^+ Y_2^- + (M_2^2 + M_3^2) Y_3^+ Y_3^- \\ & + \frac{1}{2} (M_1^2 + M_2^2) Z_1^2 + \frac{1}{2} \frac{M_1^2 + M_2^2 + 4M_3^2}{3} Z_2^2 + \frac{1}{2} (M_1^2 - M_2^2) \frac{2}{\sqrt{3}} Z_1 Z_2 \quad (7.9) \end{aligned}$$

$$M_1^2 = \frac{g_{H_1}^2 \Lambda_1^2}{2} \quad , \quad M_2^2 = \frac{g_{H_2}^2 \Lambda_2^2}{2} \quad , \quad M_3^2 = \frac{g_{H_3}^2 \Lambda_3^2}{2} \quad (7.10)$$

	$Z_1$	$Z_2$
$Z_1$	$M_1^2 + M_2^2$	$\frac{M_1^2 - M_2^2}{\sqrt{3}}$
$Z_2$	$\frac{M_1^2 - M_2^2}{\sqrt{3}}$	$\frac{M_1^2 + M_2^2 + 4M_3^2}{3}$

**Table 7.1.**  $Z_1 - Z_2$  mixing mass matrix

From the diagonalization of the  $Z_1 - Z_2$  squared mass matrix, we obtain the eigenvalues

$$\begin{aligned} M_-^2 &= \frac{2}{3} \left( M_1^2 + M_2^2 + M_3^2 - \sqrt{(M_2^2 - M_1^2)^2 + (M_3^2 - M_1^2)(M_3^2 - M_2^2)} \right) \\ M_+^2 &= \frac{2}{3} \left( M_1^2 + M_2^2 + M_3^2 + \sqrt{(M_2^2 - M_1^2)^2 + (M_3^2 - M_1^2)(M_3^2 - M_2^2)} \right) \end{aligned}$$

$$M_{Y_1}^2 Y_1^+ Y_1^- + M_{Y_2}^2 Y_2^+ Y_2^- + M_{Y_3}^2 Y_3^+ Y_3^- + M_-^2 \frac{Z_-^2}{2} + M_+^2 \frac{Z_+^2}{2} \quad (7.11)$$

where

$$M_{Y_1}^2 = M_1^2 + M_2^2 \quad , \quad M_{Y_2}^2 = M_1^2 + M_3^2 \quad , \quad M_{Y_3}^2 = M_2^2 + M_3^2 \quad (7.12)$$

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} \quad (7.13)$$

$$\cos \phi \sin \phi = \frac{\sqrt{3}}{4} \frac{M_1^2 - M_2^2}{\sqrt{(M_2^2 - M_1^2)^2 + (M_3^2 - M_1^2)(M_3^2 - M_2^2)}} \quad ,$$

with the hierarchy  $M_1, M_2 \gg M_W^2$ . Due to the  $Z_1 - Z_2$  mixing we diagonalize the propagators involving  $Z_1$  and  $Z_2$  gauge bosons according to Eq.(7.13):

$$Z_1 = \cos \phi Z_- - \sin \phi Z_+ \quad , \quad Z_2 = \sin \phi Z_- + \cos \phi Z_+$$

$$\langle Z_1 Z_1 \rangle = \cos^2 \phi \langle Z_- Z_- \rangle + \sin^2 \phi \langle Z_+ Z_+ \rangle$$

$$\langle Z_2 Z_2 \rangle = \sin^2 \phi \langle Z_- Z_- \rangle + \cos^2 \phi \langle Z_+ Z_+ \rangle$$

$$\langle Z_1 Z_2 \rangle = \cos \phi \sin \phi (\langle Z_- Z_- \rangle - \langle Z_+ Z_+ \rangle)$$

## 7.4 Electroweak symmetry breaking

Recently ATLAS[7] and CMS[8] at the Large Hadron Collider announced the discovery of a Higgs-like particle, whose properties, couplings to fermions and gauge bosons will determine whether it is the SM Higgs or a member of an extended Higgs sector associated to a BSM theory. The electroweak symmetry breaking in the SU(3) family symmetry model involves the introduction of two triplets of  $SU(2)_L$  Higgs doublets.

To achieve the spontaneous breaking of the electroweak symmetry to  $U(1)_Q$ , we introduce the scalars:  $\Phi^u = (3, 1, 2, -1)$  and  $\Phi^d = (3, 1, 2, +1)$ , with the VEVs:

$$\langle \Phi^u \rangle = \begin{pmatrix} \langle \Phi_1^u \rangle \\ \langle \Phi_2^u \rangle \\ \langle \Phi_3^u \rangle \end{pmatrix} \quad , \quad \langle \Phi^d \rangle = \begin{pmatrix} \langle \Phi_1^d \rangle \\ \langle \Phi_2^d \rangle \\ \langle \Phi_3^d \rangle \end{pmatrix} \quad , \quad (7.14)$$

$$\langle \Phi_i^u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{ui} \\ 0 \end{pmatrix} \quad , \quad \langle \Phi_i^d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{di} \end{pmatrix} \quad , \quad (7.15)$$

<sup>2</sup> Notice that in the limit  $M_1^2 = M_2^2$ ;  $\sin \phi = 0$ ,  $\cos \phi = 1$

contribute to the  $W$  and  $Z$  boson masses:

$$\frac{g^2}{4} (v_u^2 + v_d^2) W^+ W^- + \frac{(g^2 + g'^2)}{8} (v_u^2 + v_d^2) Z^2$$

$v_u^2 = v_{u1}^2 + v_{u2}^2 + v_{u3}^2$ ,  $v_d^2 = v_{d1}^2 + v_{d2}^2 + v_{d3}^2$ . Hence, if we define  $M_W = \frac{1}{2} g v$ , we may write  $v = \sqrt{v_u^2 + v_d^2} \approx 246$  GeV.

## 7.5 Tree level neutrino masses

Now we describe briefly the procedure to get the masses for ordinary fermions. The analysis for quarks and charged leptons has already discussed in [4]. Here, we introduce the procedure for neutrinos.

### 7.5.1 Tree level Dirac neutrino masses

With the fields of particles introduced in the model, we may write the Dirac type gauge invariant Yukawa couplings

$$h_D \bar{\Psi}_l^o \Phi^u N_R^o + h_1 \bar{\Psi}_\nu^o \eta_1 N_L^o + h_2 \bar{\Psi}_\nu^o \eta_2 N_L^o + h_3 \bar{\Psi}_\nu^o \eta_3 N_L^o + M_D \bar{N}_L^o N_R^o + h.c. \quad (7.16)$$

$h_D, h_1, h_2$  and  $h_3$  are Yukawa couplings, and  $M_D$  a Dirac type, invariant neutrino mass for the sterile neutrinos  $N_{L,R}^o$ . After electroweak symmetry breaking, we obtain in the interaction basis  $\Psi_{\nu L,R}^o = (v_e^o, v_\mu^o, v_\tau^o, N^o)_{L,R}$ , the mass terms

$$h_D [v_1 \bar{v}_{eL}^o + v_2 \bar{v}_{\mu L}^o + v_3 \bar{v}_{\tau L}^o] N_R^o + [h_1 \Lambda_1 \bar{v}_{eR}^o + h_2 \Lambda_2 \bar{v}_{\mu R}^o + h_3 \Lambda_3 \bar{v}_{\tau R}^o] N_L^o + M_D \bar{N}_L^o N_R^o + h.c. \quad (7.17)$$

### 7.5.2 Tree level Majorana masses:

Since  $N_{L,R}^o$ , Eq.(7.4), are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L \bar{\Psi}_l^o \Phi^u (N_L^o)^c + m_L \bar{N}_L^o (N_L^o)^c + h.c. \quad (7.18)$$

and

$$h_{1R} \bar{\Psi}_\nu^o \eta_1 (N_R^o)^c + h_{2R} \bar{\Psi}_\nu^o \eta_2 (N_R^o)^c + h_{3R} \bar{\Psi}_\nu^o \eta_3 (N_R^o)^c + m_R \bar{N}_R^o (N_R^o)^c + h.c., \quad (7.19)$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L \left[ v_1 \bar{\nu}_{eL}^o + v_2 \bar{\nu}_{\mu L}^o + v_3 \bar{\nu}_{\tau L}^o \right] (N_L^o)^c + m_L \bar{N}_L^o (N_L^o)^c + \text{h.c.}, \quad (7.20)$$

$$+ \left[ h_{1R} \Lambda_1 \bar{\nu}_{eR}^o + h_{2R} \Lambda_2 \bar{\nu}_{\mu R}^o + h_{3R} \Lambda_3 \bar{\nu}_{\tau R}^o \right] (N_R^o)^c + m_R \bar{N}_R^o (N_R^o)^c + \text{h.c.}, \quad (7.21)$$

	$(\nu_{eL}^o)^c$	$(\nu_{\mu L}^o)^c$	$(\nu_{\tau L}^o)^c$	$(N_L^o)^c$	$\nu_{eR}^o$	$\nu_{\mu R}^o$	$\nu_{\tau R}^o$	$N_R^o$
$\overline{\nu_{eL}^o}$	0	0	0	$h_L v_1$	0	0	0	$h_D v_1$
$\overline{\nu_{\mu L}^o}$	0	0	0	$h_L v_2$	0	0	0	$h_D v_2$
$\overline{\nu_{\tau L}^o}$	0	0	0	$h_L v_3$	0	0	0	$h_D v_3$
$\overline{N_L^o}$	$h_L v_1$	$h_L v_2$	$h_L v_3$	$m_L$	$h_1 \Lambda_1$	$h_2 \Lambda_2$	$h_3 \Lambda_3$	$m_D$
$\overline{(\nu_{eR}^o)^c}$	0	0	0	$h_1 \Lambda_1$	0	0	0	$h_{1R} \Lambda_1$
$\overline{(\nu_{\mu R}^o)^c}$	0	0	0	$h_2 \Lambda_2$	0	0	0	$h_{2R} \Lambda_2$
$\overline{(\nu_{\tau R}^o)^c}$	0	0	0	$h_3 \Lambda_3$	0	0	0	$h_{3R} \Lambda_3$
$\overline{(N_R^o)^c}$	$h_D v_1$	$h_D v_2$	$h_D v_3$	$m_D$	$h_{1R} \Lambda_1$	$h_{2R} \Lambda_2$	$h_{3R} \Lambda_3$	$m_R$

**Table 7.2.** Tree Level Majorana masses

Thus, in the basis

$$\Psi_\nu^{oT} = \left( \nu_{eL}^o, \nu_{\mu L}^o, \nu_{\tau L}^o, N_L^o, (\nu_{eR}^o)^c, (\nu_{\mu R}^o)^c, (\nu_{\tau R}^o)^c, (N_R^o)^c \right), \quad (7.22)$$

the Generic  $8 \times 8$  tree level Majorana mass matrix for neutrinos  $\mathcal{M}_\nu^o$ , from Table 7.2,  $\bar{\Psi}_\nu^o \mathcal{M}_\nu^o (\Psi_\nu^o)^c$ , read

$$\mathcal{M}_\nu^o = \begin{pmatrix} \mathcal{M}_L^o & \mathcal{M}_D^o \\ \mathcal{M}_D^{oT} & \mathcal{M}_R^o \end{pmatrix} \quad (7.23)$$

where

$$\mathcal{M}_L^o = \begin{pmatrix} 0 & 0 & 0 & \alpha_1 \\ 0 & 0 & 0 & \alpha_2 \\ 0 & 0 & 0 & \alpha_3 \\ \alpha_1 & \alpha_2 & \alpha_3 & m_L \end{pmatrix}, \quad \mathcal{M}_R^o = \begin{pmatrix} 0 & 0 & 0 & \beta_1 \\ 0 & 0 & 0 & \beta_2 \\ 0 & 0 & 0 & \beta_3 \\ \beta_1 & \beta_2 & \beta_3 & m_R \end{pmatrix} \quad (7.24)$$

and

$$\mathcal{M}_D^o = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ b_1 & b_2 & b_3 & m_D \end{pmatrix}, \quad (7.25)$$

$$\alpha_i = h_L v_i, \quad a_i = h_D v_i, \quad b_i = h_i \Lambda_i, \quad \beta_i = h_{iR} \Lambda_i \quad (7.26)$$

Diagonalization of  $\mathcal{M}_V^{(o)}$ , Eq.(7.23), yields four zero eigenvalues, associated to the neutrino fields:

$$\frac{a_2}{ap} v_{eL}^o - \frac{a_1}{ap} v_{\mu L}^o, \quad \frac{a_1 a_3}{ap a} v_{eL}^o + \frac{a_2 a_3}{ap a} v_{\mu L}^o - \frac{a_p}{a} v_{\tau L}^o, \quad (7.27)$$

$$\frac{b_2}{bp} v_{eR}^o - \frac{b_1}{bp} v_{\mu R}^o, \quad \frac{b_1 b_3}{bp b} v_{eR}^o + \frac{b_2 b_3}{bp b} v_{\mu R}^o - \frac{b_p}{b} v_{\tau R}^o, \quad (7.28)$$

$$ap = \sqrt{a_1^2 + a_2^2}, \quad bp = \sqrt{b_1^2 + b_2^2}, \quad a = \sqrt{a_1^2 + a_2^2 + a_3^2}, \quad b = \sqrt{b_1^2 + b_2^2 + b_3^2}.$$

Assuming for simplicity  $\frac{h_{1R}}{h_1} = \frac{h_{2R}}{h_2} = \frac{h_{3R}}{h_3} \equiv c_R$ , that is

$$\frac{\alpha_i}{a_i} = \frac{h_L}{h_D} = c_L, \quad \frac{\beta_i}{b_i} = \frac{h_{iR}}{h_i} = c_R,$$

the Characteristic Polynomial for the nonzero eigenvalues of  $\mathcal{M}_V^o$  reduce to the one of the matrix  $m_4^3$ , Eq.(7.29), where

$$m_4 = \begin{pmatrix} 0 & \alpha & 0 & a \\ \alpha & m_L & b & m_D \\ 0 & b & 0 & \beta \\ a & m_D & \beta & m_R \end{pmatrix}, \quad U_4 = \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix} \quad (7.29)$$

$$\alpha = \sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}, \quad \beta = \sqrt{\beta_1^2 + \beta_2^2 + \beta_3^2}.$$

$$U_4^T m_4 U_4 = \text{Diag}(m_5^o, m_6^o, m_7^o, m_8^o) \equiv d_4, \quad m_4 = U_4 d_4 U_4^T \quad (7.30)$$

Eq.(7.30) impose the constrains

$$u_{11}^2 m_5^o + u_{12}^2 m_6^o + u_{13}^2 m_7^o + u_{14}^2 m_8^o = 0 \quad (7.31)$$

$$u_{31}^2 m_5^o + u_{32}^2 m_6^o + u_{33}^2 m_7^o + u_{34}^2 m_8^o = 0 \quad (7.32)$$

$$u_{11} u_{31} m_5^o + u_{12} u_{32} m_6^o + u_{13} u_{33} m_7^o + u_{14} u_{34} m_8^o = 0, \quad (7.33)$$

<sup>3</sup> The relation  $a b = \alpha \beta$  would yield five massless neutrinos at tree level.



corresponding to the  $(m_4)_{11} = (m_4)_{33} = (m_4)_{13} = 0$  zero entries, respectively.

Therefore,  $\mathcal{M}_\nu^o$  is diagonalized by the orthogonal matrix

$$\mathbf{U}_\nu^o = \begin{pmatrix} \frac{a_2}{ap} & \frac{a_1 a_3}{a ap} & 0 & 0 & \frac{a_1}{a} u_{11} & \frac{a_1}{a} u_{12} & \frac{a_1}{a} u_{13} & \frac{a_1}{a} u_{14} \\ -\frac{a_1}{ap} & \frac{a_2 a_3}{a ap} & 0 & 0 & \frac{a_2}{a} u_{11} & \frac{a_2}{a} u_{12} & \frac{a_2}{a} u_{13} & \frac{a_2}{a} u_{14} \\ 0 & -\frac{ap}{a} & 0 & 0 & \frac{a_3}{a} u_{11} & \frac{a_3}{a} u_{12} & \frac{a_3}{a} u_{13} & \frac{a_3}{a} u_{14} \\ 0 & 0 & 0 & 0 & u_{21} & u_{22} & u_{23} & u_{24} \\ 0 & 0 & \frac{b_2}{bp} & \frac{b_1 b_3}{b bp} & \frac{b_1}{b} u_{31} & \frac{b_1}{b} u_{32} & \frac{b_1}{b} u_{33} & \frac{b_1}{b} u_{34} \\ 0 & 0 & -\frac{b_1}{bp} & \frac{b_2 b_3}{b bp} & \frac{b_2}{b} u_{31} & \frac{b_2}{b} u_{32} & \frac{b_2}{b} u_{33} & \frac{b_2}{b} u_{34} \\ 0 & 0 & 0 & -\frac{bp}{b} & \frac{b_3}{b} u_{31} & \frac{b_3}{b} u_{32} & \frac{b_3}{b} u_{33} & \frac{b_3}{b} u_{34} \\ 0 & 0 & 0 & 0 & u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix} \quad (7.34)$$

$$(\mathbf{U}_\nu^o)^T \mathcal{M}_\nu^o \mathbf{U}_\nu^o = \text{Diag}(0, 0, 0, 0, m_5^o, m_6^o, m_7^o, m_8^o) \quad (7.35)$$

## 7.6 One loop neutrino masses

After tree level contributions the fermion global symmetry, Eq.(7.1), is broken down to

$$\text{SU}(2)_{q_L} \otimes \text{SU}(2)_{u_R} \otimes \text{SU}(2)_{d_R} \otimes \text{SU}(2)_{l_L} \otimes \text{SU}(2)_{\nu_R} \otimes \text{SU}(2)_{e_R} . \quad (7.36)$$

Therefore, in this scenario light neutrinos may get extremely small masses from radiative corrections mediated by the SU(3) heavy gauged bosons.

### 7.6.1 One loop Dirac Neutrino masses

After the breakdown of the electroweak symmetry, neutrinos may get tiny Dirac mass terms from the generic one loop diagram in Fig. 7.1, The internal fermion line in this diagram represent the tree level see-saw mechanisms, Eqs.(7.16-7.21). The vertices read from the SU(3) family symmetry interaction Lagrangian

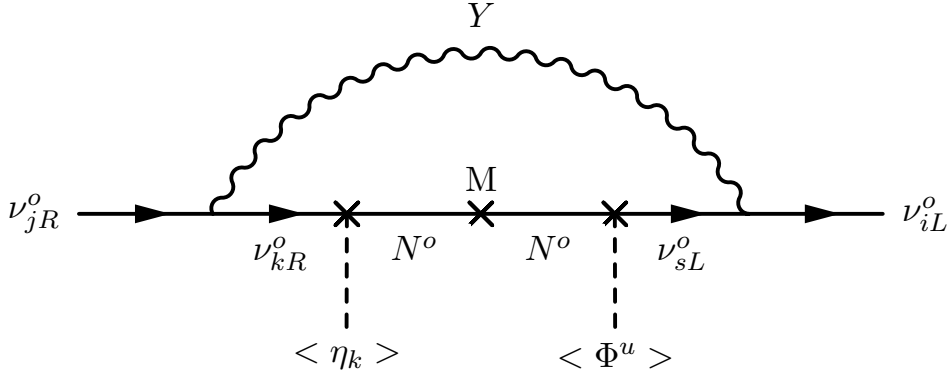
$$i\mathcal{L}_{\text{int}} = \frac{g_H}{2} (\bar{\nu}_e^o \gamma_\mu \nu_e^o - \bar{\nu}_\mu^o \gamma_\mu \nu_\mu^o) Z_1^\mu + \frac{g_H}{2\sqrt{3}} (\bar{\nu}_e^o \gamma_\mu \nu_e^o + \bar{\nu}_\mu^o \gamma_\mu \nu_\mu^o - 2\bar{\nu}_\tau^o \gamma_\mu \nu_\tau^o) Z_2^\mu \\
 + \frac{g_H}{\sqrt{2}} (\bar{\nu}_e^o \gamma_\mu \nu_\mu^o Y_1^+ + \bar{\nu}_e^o \gamma_\mu \nu_\tau^o Y_2^+ + \bar{\nu}_\mu^o \gamma_\mu \nu_\tau^o Y_3^+ + \text{h.c.}) \quad (7.37)$$

The contribution from these diagrams may be written as

$$c_Y \frac{\alpha_H}{\pi} m_\nu(M_Y)_{ij} \quad , \quad \alpha_H = \frac{g_{\Gamma H}^2}{4\pi} \quad , \quad (7.38)$$

$$m_\nu(M_Y)_{ij} \equiv \sum_{k=5,6,7,8} m_k^o u_{ik}^o u_{jk}^o f(M_Y, m_k^o) \quad , \quad (7.39)$$

$$f(M_Y, m_k^o) = \frac{M_Y^2}{M_Y^2 - m_k^{o2}} \ln \frac{M_Y^2}{m_k^{o2}} \quad ,$$



**Fig. 7.1.** Generic one loop diagram contribution to the Dirac mass term  $m_{ij} \bar{\nu}_{iL}^o \nu_{jR}^o$ .  $M = M_D, m_L, m_R$

	$\nu_{eR}^o$	$\nu_{\mu R}^o$	$\nu_{\tau R}^o$	$N_R^o$
$\bar{\nu}_{eL}^o$	$D_{\nu 11}$	$D_{\nu 12}$	$D_{\nu 13}$	0
$\bar{\nu}_{\mu L}^o$	$D_{\nu 21}$	$D_{\nu 22}$	$D_{\nu 23}$	0
$\bar{\nu}_{\tau L}^o$	$D_{\nu 31}$	$D_{\nu 32}$	$D_{\nu 33}$	0
$\bar{N}_L^o$	0	0	0	0

**Table 7.3.** One loop Dirac mass terms  $\frac{\alpha_H}{\pi} D_{\nu ij} \bar{\nu}_{iL}^o \nu_{jR}^o$

$$m_\nu(M_Y)_{i,4+j} = \frac{a_i b_j}{a b} \mathcal{F}_\nu(M_Y) \quad (7.40)$$

$$\mathcal{F}_\nu(M_Y) = u_{11} u_{31} m_5^o f(M_Y, m_5^o) + u_{12} u_{32} m_6^o f(M_Y, m_6^o)$$

$$+ u_{13} u_{33} m_7^o f(M_Y, m_7^o) + u_{14} u_{34} m_8^o f(M_Y, m_8^o) \quad (7.41)$$

$$D_{\nu 11} = \frac{a_1 b_1}{ab} \left[ \frac{1}{4} \mathcal{F}_\nu(M_{Z_1}) + \frac{1}{12} \mathcal{F}_\nu(M_{Z_2}) + \mathcal{F}_{\nu,m} \right] + \frac{1}{2} \left[ \frac{a_2 b_2}{ab} \mathcal{F}_\nu(M_{Y_1}) + \frac{a_3 b_3}{ab} \mathcal{F}_\nu(M_{Y_2}) \right],$$

$$D_{\nu 12} = \frac{a_1 b_2}{ab} \left[ -\frac{1}{4} \mathcal{F}_\nu(M_{Z_1}) + \frac{1}{12} \mathcal{F}_\nu(M_{Z_2}) \right],$$

$$D_{\nu 13} = \frac{a_1 b_3}{ab} \left[ -\frac{1}{6} \mathcal{F}_\nu(M_{Z_2}) - \mathcal{F}_{\nu,m} \right],$$

$$D_{\nu 21} = \frac{a_2 b_1}{ab} \left[ -\frac{1}{4} \mathcal{F}_\nu(M_{Z_1}) + \frac{1}{12} \mathcal{F}_\nu(M_{Z_2}) \right],$$

$$D_{\nu 22} = \frac{a_2 b_2}{ab} \left[ \frac{1}{4} \mathcal{F}_\nu(M_{Z_1}) + \frac{1}{12} \mathcal{F}_\nu(M_{Z_2}) - \mathcal{F}_{\nu,m} \right] + \frac{1}{2} \left[ \frac{a_1 b_1}{ab} \mathcal{F}_\nu(M_{Y_1}) + \frac{a_3 b_3}{ab} \mathcal{F}_\nu(M_{Y_3}) \right],$$

$$D_{\nu 23} = \frac{a_2 b_3}{ab} \left[ -\frac{1}{6} \mathcal{F}_\nu(M_{Z_2}) + \mathcal{F}_{\nu,m} \right],$$

$$D_{\nu 31} = \frac{a_3 b_1}{ab} \left[ -\frac{1}{6} \mathcal{F}_\nu(M_{Z_2}) - \mathcal{F}_{\nu,m} \right],$$

$$D_{\nu 32} = \frac{a_3 b_2}{ab} \left[ -\frac{1}{6} \mathcal{F}_\nu(M_{Z_2}) + \mathcal{F}_{\nu,m} \right],$$

$$D_{\nu 33} = \frac{1}{3} \frac{a_3 b_3}{ab} \mathcal{F}_\nu(M_{Z_2}) + \frac{1}{2} \left[ \frac{a_1 b_1}{ab} \mathcal{F}_\nu(M_{Y_2}) + \frac{a_2 b_2}{ab} \mathcal{F}_\nu(M_{Y_3}) \right],$$

$$\mathcal{F}_\nu(M_{Z_1}) = \cos^2 \phi \mathcal{F}_\nu(M_-) + \sin^2 \phi \mathcal{F}_\nu(M_+)$$

$$\mathcal{F}_\nu(M_{Z_2}) = \sin^2 \phi \mathcal{F}_\nu(M_-) + \cos^2 \phi \mathcal{F}_\nu(M_+)$$

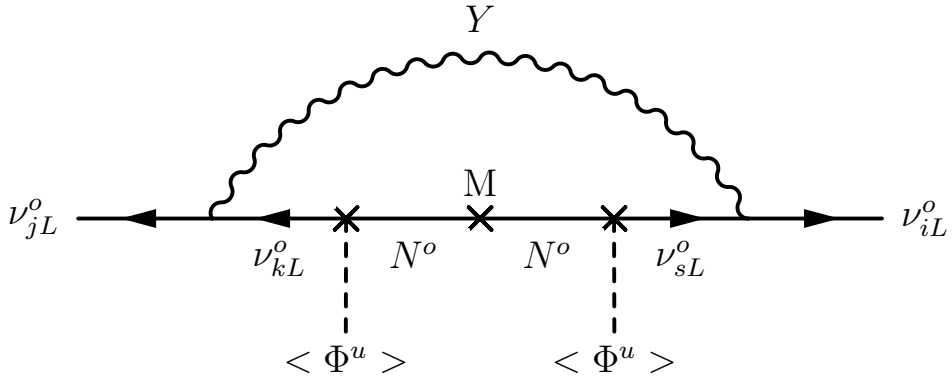
$$\mathcal{F}_{\nu,m} = \frac{1}{2\sqrt{3}} \cos \phi \sin \phi [\mathcal{F}_\nu(M_-) - \mathcal{F}_\nu(M_+)], \quad (7.42)$$

### 7.6.2 One loop L-handed Majorana masses

Neutrinos also obtain one loop corrections to L-handed and R-handed Majorana masses from the diagrams of Fig. 7.2 and Fig. 7.3, respectively. A similar procedure as for Dirac Neutrino masses leads to the one loop Majorana mass terms

$$m_\nu(M_Y)_{i,j} = \frac{a_i a_j}{a^2} \mathcal{G}_\nu(M_Y) \tag{7.43}$$

$$\begin{aligned} \mathcal{G}_\nu(M_Y) = m_5^o u_{11}^2 f(M_Y, m_5^o) + m_6^o u_{12}^2 f(M_Y, m_6^o) + m_7^o u_{13}^2 f(M_Y, m_7^o) \\ + m_8^o u_{14}^2 f(M_Y, m_8^o) \end{aligned} \tag{7.44}$$



**Fig. 7.2.** Generic one loop diagram contribution to the L-handed Majorana mass term  $m_{ij} \bar{\nu}_{iL}^o (\nu_{jL}^o)^T$ .  $M = M_D, m_L, m_R$

	$\nu_{eL}^o$	$\nu_{\mu L}^o$	$\nu_{\tau L}^o$	$N_L^o$
$\nu_{eL}^o$	$L_{\nu 11}$	$L_{\nu 12}$	$L_{\nu 13}$	0
$\nu_{\mu L}^o$	$L_{\nu 12}$	$L_{\nu 22}$	$L_{\nu 23}$	0
$\nu_{\tau L}^o$	$L_{\nu 13}$	$L_{\nu 23}$	$L_{\nu 33}$	0
$N_L^o$	0	0	0	0

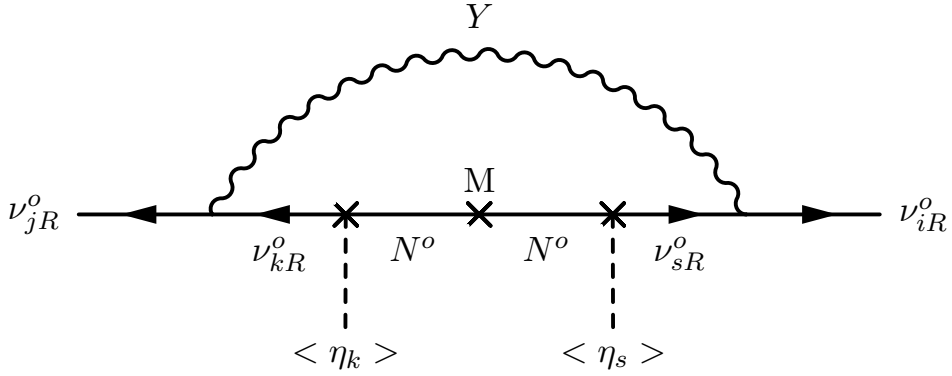
**Table 7.4.** One loop L-handed Majorana mass terms  $\frac{\alpha_H}{\pi} L_{\nu ij} \bar{\nu}_{iL}^o (\nu_{jL}^o)^T$

$$\begin{aligned}
L_{\nu 11} &= \frac{a_1^2}{a^2} \left[ \frac{1}{4} \mathcal{G}_\nu(M_{Z_1}) + \frac{1}{12} \mathcal{G}_\nu(M_{Z_2}) + \mathcal{G}_{\nu,m} \right], \\
L_{\nu 22} &= \frac{a_2^2}{a^2} \left[ \frac{1}{4} \mathcal{G}_\nu(M_{Z_1}) + \frac{1}{12} \mathcal{G}_\nu(M_{Z_2}) - \mathcal{G}_{\nu,m} \right], \\
L_{\nu 33} &= \frac{1}{3} \frac{a_3^2}{a^2} \mathcal{G}_\nu(M_{Z_2}), \\
L_{\nu 12} &= \frac{a_1 a_2}{a^2} \left[ -\frac{1}{4} \mathcal{G}_\nu(M_{Z_1}) + \frac{1}{12} \mathcal{G}_\nu(M_{Z_2}) + \frac{1}{2} \mathcal{G}_\nu(M_1) \right], \\
L_{\nu 13} &= \frac{a_1 a_3}{a^2} \left[ -\frac{1}{6} \mathcal{G}_\nu(M_{Z_2}) + \frac{1}{2} \mathcal{G}_\nu(M_2) - \mathcal{G}_{\nu,m} \right], \\
L_{\nu 23} &= \frac{a_2 a_3}{a^2} \left[ -\frac{1}{6} \mathcal{G}_\nu(M_{Z_2}) + \frac{1}{2} \mathcal{G}_\nu(M_3) + \mathcal{G}_{\nu,m} \right] \\
\mathcal{G}_\nu(M_{Z_1}) &= \cos^2 \phi \mathcal{G}_\nu(M_-) + \sin^2 \phi \mathcal{G}_\nu(M_+) \\
\mathcal{G}_\nu(M_{Z_2}) &= \sin^2 \phi \mathcal{G}_\nu(M_-) + \cos^2 \phi \mathcal{G}_\nu(M_+) \\
\mathcal{G}_{\nu,m} &= \frac{1}{2\sqrt{3}} \cos \phi \sin \phi [\mathcal{G}_\nu(M_-) - \mathcal{G}_\nu(M_+)], \tag{7.45}
\end{aligned}$$

### 7.6.3 One loop R-handed Majorana masses

$$m_\nu(M_Y)_{4+i,4+j} = \frac{b_i b_j}{b^2} \mathcal{H}_\nu(M_Y) \tag{7.46}$$

$$\begin{aligned}
\mathcal{H}_\nu(M_Y) &= m_5^0 u_{31}^2 f(M_Y, m_5^0) + m_6^0 u_{32}^2 f(M_Y, m_6^0) + m_7^0 u_{33}^2 f(M_Y, m_7^0) \\
&\quad + m_8^0 u_{34}^2 f(M_Y, m_8^0) \tag{7.47}
\end{aligned}$$



**Fig. 7.3.** Generic one loop diagram contribution to the R-handed Majorana mass term  $m_{ij} \tilde{\nu}_{iR}^o (\nu_{jR}^o)^T$ .  $M = M_D, m_L, m_R$

	$\nu_{eR}^o$	$\nu_{\mu R}^o$	$\nu_{\tau R}^o$	$N_R^o$
$\nu_{eR}^o$	$R_{\nu 11}$	$R_{\nu 12}$	$R_{\nu 13}$	0
$\nu_{\mu R}^o$	$R_{\nu 12}$	$R_{\nu 22}$	$R_{\nu 23}$	0
$\nu_{\tau R}^o$	$R_{\nu 13}$	$R_{\nu 23}$	$R_{\nu 33}$	0
$N_R^o$	0	0	0	0

**Table 7.5.** One loop R-handed Majorana mass terms  $\frac{\alpha_H}{\pi} R_{\nu ij} \tilde{\nu}_{iR}^o (\nu_{jR}^o)^T$

$$R_{\nu 11} = \frac{b_1^2}{b^2} \left[ \frac{1}{4} \mathcal{H}_\nu(M_{Z_1}) + \frac{1}{12} \mathcal{H}_\nu(M_{Z_2}) + \mathcal{H}_{\nu, m} \right],$$

$$R_{\nu 22} = \frac{b_2^2}{b^2} \left[ \frac{1}{4} \mathcal{H}_\nu(M_{Z_1}) + \frac{1}{12} \mathcal{H}_\nu(M_{Z_2}) - \mathcal{H}_{\nu, m} \right],$$

$$R_{\nu 33} = \frac{1}{3} \frac{b_3^2}{b^2} \mathcal{H}_\nu(M_{Z_2}),$$

$$R_{\nu 12} = \frac{b_1 b_2}{b^2} \left[ -\frac{1}{4} \mathcal{H}_\nu(M_{Z_1}) + \frac{1}{12} \mathcal{H}_\nu(M_{Z_2}) + \frac{1}{2} \mathcal{H}_\nu(M_1) \right],$$

$$R_{\nu 13} = \frac{b_1 b_3}{b^2} \left[ -\frac{1}{6} \mathcal{H}_\nu(M_{Z_2}) + \frac{1}{2} \mathcal{H}_\nu(M_2) - \mathcal{H}_{\nu, m} \right],$$

$$R_{\nu 23} = \frac{b_2 b_3}{b^2} \left[ -\frac{1}{6} \mathcal{H}_\nu(M_{Z_2}) + \frac{1}{2} \mathcal{H}_\nu(M_3) + \mathcal{H}_{\nu, m} \right]$$

$$\mathcal{H}_\nu(M_{Z_1}) = \cos^2 \phi \mathcal{H}_\nu(M_-) + \sin^2 \phi \mathcal{H}_\nu(M_+)$$

$$\mathcal{H}_\nu(M_{Z_2}) = \sin^2 \phi \mathcal{H}_\nu(M_-) + \cos^2 \phi \mathcal{H}_\nu(M_+)$$

$$\mathcal{H}_{\nu,m} = \frac{1}{2\sqrt{3}} \cos \phi \sin \phi [\mathcal{H}_\nu(M_-) - \mathcal{H}_\nu(M_+)] , \quad (7.48)$$

where  $\mathcal{F}_{\nu,m}$ ,  $\mathcal{G}_{\nu,m}$  and  $\mathcal{H}_{\nu,m}$ , Eqs.(7.42,7.45,7.48), come from  $Z_1 - Z_2$  mixing diagram contributions.

Thus, in the  $\Psi_\nu^0$  basis, Eq.(7.22), we may write the one loop contribution for neutrinos as  $\Psi_\nu^0 \mathcal{M}_{1\nu}^0 (\Psi_\nu^0)^c$ ,

$$\mathcal{M}_{1\nu}^0 = \begin{pmatrix} L_{\nu 11} & L_{\nu 12} & L_{\nu 13} & 0 & D_{\nu 11} & D_{\nu 12} & D_{\nu 13} & 0 \\ L_{\nu 12} & L_{\nu 22} & L_{\nu 23} & 0 & D_{\nu 21} & D_{\nu 22} & D_{\nu 23} & 0 \\ L_{\nu 13} & L_{\nu 23} & L_{\nu 33} & 0 & D_{\nu 31} & D_{\nu 32} & D_{\nu 33} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ D_{\nu 11} & D_{\nu 21} & D_{\nu 31} & 0 & R_{\nu 11} & R_{\nu 12} & R_{\nu 13} & 0 \\ D_{\nu 12} & D_{\nu 22} & D_{\nu 32} & 0 & R_{\nu 12} & R_{\nu 22} & R_{\nu 23} & 0 \\ D_{\nu 13} & D_{\nu 23} & D_{\nu 33} & 0 & R_{\nu 13} & R_{\nu 23} & R_{\nu 33} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \frac{\alpha_H}{\pi} \quad (7.49)$$

#### 7.6.4 Neutrino mass matrix up to one loop

Finally, we obtain the general symmetric Majorana mass matrix for neutrinos up to one loop

$$\mathcal{M}_\nu = (\mathbf{U}_\nu^0)^T \mathcal{M}_{1\nu}^0 \mathbf{U}_\nu^0 + \text{Diag}(0, 0, 0, 0, m_5^0, m_6^0, m_7^0, m_8^0) , \quad (7.50)$$

where explicitly

$$(\mathcal{U}_\nu^o)^T \mathcal{M}_1^o \mathcal{U}_\nu^o = \begin{pmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15} & N_{16} & N_{17} & N_{18} \\ N_{12} & N_{22} & N_{23} & N_{24} & N_{25} & N_{26} & N_{27} & N_{28} \\ N_{13} & N_{23} & N_{33} & N_{34} & N_{35} & N_{36} & N_{37} & N_{38} \\ N_{14} & N_{24} & N_{34} & N_{44} & N_{45} & N_{46} & N_{47} & N_{48} \\ N_{15} & N_{25} & N_{35} & N_{45} & N_{55} & N_{56} & N_{57} & N_{58} \\ N_{16} & N_{26} & N_{36} & N_{46} & N_{56} & N_{66} & N_{67} & N_{68} \\ N_{17} & N_{27} & N_{37} & N_{47} & N_{57} & N_{67} & N_{77} & N_{78} \\ N_{18} & N_{28} & N_{38} & N_{48} & N_{58} & N_{68} & N_{78} & N_{88} \end{pmatrix} \frac{\alpha_H}{\pi} \quad (7.51)$$

**Majorana L-handed:**

$$N_{11} = \frac{a_1^2 a_2^2}{a_p^2 a^2} (\mathcal{G}_{Z_1} - \mathcal{G}_1) \quad (7.52)$$

$$N_{12} = -\frac{a_1 a_2 a_3}{2a^3} \left[ \frac{a_2^2 - a_1^2}{a_p^2} (\mathcal{G}_{Z_1} - \mathcal{G}_1) + \mathcal{G}_2 - \mathcal{G}_3 - 6\mathcal{G}_m \right] \quad (7.53)$$

$$\begin{aligned}
 N_{22} = \frac{a_3^2}{a^2} \left[ \frac{1}{4} \frac{(a_2^2 - a_1^2)^2}{a_p^2 a^2} (\mathcal{G}_{Z_1} - \mathcal{G}_1) + \frac{a_2^2}{a^2} (\mathcal{G}_2 - \mathcal{G}_3) \right. \\
 \left. + \frac{a_p^2}{4a^2} (\mathcal{G}_1 + 3\mathcal{G}_{Z_2} - 4\mathcal{G}_2) - 3 \frac{a_2^2 - a_1^2}{a^2} \mathcal{G}_m \right] \quad (7.54)
 \end{aligned}$$

**Dirac:**

$$\begin{aligned}
 N_{13} = \frac{1}{2ap \, bp \, a \, b} \{ (a_1^2 b_1^2 + a_2^2 b_2^2) \mathcal{F}_1 + a_3 b_3 (a_2 b_2 \mathcal{F}_2 + a_1 b_1 \mathcal{F}_3) \\
 + 2a_1 b_1 a_2 b_2 \mathcal{F}_{Z_1} \} \quad (7.55)
 \end{aligned}$$

$$\begin{aligned}
 N_{14} = \frac{1}{2ap \, bp \, a \, b} \frac{b_3}{b} \{ b_1 b_2 (a_2^2 - a_1^2) \mathcal{F}_1 + a_3 b_3 (a_2 b_1 \mathcal{F}_2 - a_1 b_2 \mathcal{F}_3) \\
 + a_1 a_2 (b_1^2 - b_2^2) \mathcal{F}_{Z_1} + 6a_1 a_2 \, bp^2 \mathcal{F}_m \} \quad (7.56)
 \end{aligned}$$



$$\begin{aligned}
 N_{23} = \frac{1}{2ap bp ab} \frac{a_3}{a} \{ & a_1 a_2 (b_2^2 - b_1^2) \mathcal{F}_1 + a_3 b_3 (a_1 b_2 \mathcal{F}_2 - a_2 b_1 \mathcal{F}_3) \\
 & + b_1 b_2 (a_1^2 - a_2^2) \mathcal{F}_{Z_1} + 6b_1 b_2 ap^2 \mathcal{F}_m \} \quad (7.57)
 \end{aligned}$$

$$\begin{aligned}
 N_{24} = \frac{1}{ap bp a^2 b^2} \\
 \left\{ a_3 b_3 [a_1 b_1 a_2 b_2 \mathcal{F}_1 + \frac{1}{4} (a_1^2 - a_2^2) (b_1^2 - b_2^2) \mathcal{F}_{Z_1} + \frac{3}{4} ap^2 bp^2 \mathcal{F}_{Z_2}] \right. \\
 \left. + \frac{1}{2} (a_3^2 b_3^2 + ap^2 bp^2) (a_1 b_1 \mathcal{F}_2 + a_2 b_2 \mathcal{F}_3) + 3a_3 b_3 (a_1^2 b_1^2 - a_2^2 b_2^2) \mathcal{F}_m \right\}
 \end{aligned}$$

**Majorana R-handed:**

$$N_{33} = \frac{b_1^2 b_2^2}{b_p^2 b^2} (\mathcal{H}_{Z_1} - \mathcal{H}_1) \quad (7.58)$$

$$N_{34} = -\frac{b_1 b_2 b_3}{2b^3} \left[ \frac{b_2^2 - b_1^2}{b_p^2} (\mathcal{H}_{Z_1} - \mathcal{H}_1) + \mathcal{H}_2 - \mathcal{H}_3 - 6\mathcal{H}_m \right] \quad (7.59)$$

$$\begin{aligned}
 N_{44} = \frac{b_3^2}{b^2} \left[ \frac{1}{4} \frac{(b_2^2 - b_1^2)^2}{b_p^2 b^2} (\mathcal{H}_{Z_1} - \mathcal{H}_1) + \frac{b_2^2}{b^2} (\mathcal{H}_2 - \mathcal{H}_3) \right. \\
 \left. + \frac{b_p^2}{4b^2} (\mathcal{H}_1 + 3\mathcal{H}_{Z_2} - 4\mathcal{H}_2) - 3 \frac{b_2^2 - b_1^2}{b^2} \mathcal{H}_m \right] \quad (7.60)
 \end{aligned}$$

**Majorana L-handed and Dirac:**

$$N_{15} = \mathcal{G}_{15} u_{11} + m_{13} u_{31} \quad ; \quad N_{16} = \mathcal{G}_{15} u_{12} + m_{13} u_{32} \quad (7.61)$$

$$N_{17} = \mathcal{G}_{15} u_{13} + m_{13} u_{33} \quad ; \quad N_{18} = \mathcal{G}_{15} u_{14} + m_{13} u_{34} \quad (7.62)$$

$$\mathcal{G}_{15} = -\frac{a_1 a_2}{2 a_p a} \left[ \frac{a_2^2 - a_1^2}{a^2} (\mathcal{G}_{Z_1} - \mathcal{G}_1) + \frac{a_3^2}{a^2} (\mathcal{G}_3 - \mathcal{G}_2) + 2 \frac{(2 a_3^2 - a_p^2)}{a^2} \mathcal{G}_m \right]$$

$$m_{13} = \frac{1}{2 a_p a b^2} \left\{ b_1 b_2 (a_2^2 - a_1^2) \mathcal{F}_1 + a_3 b_3 (a_2 b_1 \mathcal{F}_2 - a_1 b_2 \mathcal{F}_3) \right. \\ \left. + a_1 a_2 (b_1^2 - b_2^2) \mathcal{F}_{Z_1} + 2 a_1 a_2 (b p^2 - 2 b_3^2) \mathcal{F}_m \right\}$$

$$N_{25} = \mathcal{G}_{25} u_{11} + m_{23} u_{31} \quad ; \quad N_{26} = \mathcal{G}_{25} u_{12} + m_{23} u_{32} \quad (7.63)$$

$$N_{27} = \mathcal{G}_{25} u_{13} + m_{23} u_{33} \quad ; \quad N_{28} = \mathcal{G}_{25} u_{14} + m_{23} u_{34} \quad (7.64)$$

$$\mathcal{G}_{25} = \frac{a_3}{4 a_p a^4} \\ \left\{ (a_2^2 - a_1^2)^2 (\mathcal{G}_{Z_1} - \mathcal{G}_1) + 2 a_2^2 (a_3^2 - a_p^2) (\mathcal{G}_3 - \mathcal{G}_2) - a_p^4 (\mathcal{G}_{Z_2} - \mathcal{G}_1) \right. \\ \left. - 2 a_p^2 (a_3^2 - a_p^2) (\mathcal{G}_{Z_2} - \mathcal{G}_2) + 4 (a_2^2 - a_1^2) (a_3^2 - 2 a_p^2) \mathcal{G}_m \right\}$$

$$m_{23} = \frac{1}{a_p a^2 b^2} \\ \left\{ a_3 [a_1 b_1 a_2 b_2 \mathcal{F}_1 + \frac{1}{4} (a_1^2 - a_2^2) (b_1^2 - b_2^2) \mathcal{F}_{Z_1} + \frac{1}{4} a p^2 (b p^2 - 2 b_3^2) \mathcal{F}_{Z_2}] \right. \\ \left. + \frac{1}{2} b_3 (a_3^2 - a p^2) (a_1 b_1 \mathcal{F}_2 + a_2 b_2 \mathcal{F}_3) + a_3 [a_1^2 (3 b_1^2 - b^2) + a_2^2 (b^2 - 3 b_2^2)] \mathcal{F}_m \right\}$$

**Dirac and Majorana R-handed:**

$$N_{35} = m_{31} u_{11} + \mathcal{H}_{35} u_{31} \quad , \quad N_{36} = m_{31} u_{12} + \mathcal{H}_{35} u_{32}$$

$$N_{37} = m_{31} u_{13} + \mathcal{H}_{35} u_{33} \quad , \quad N_{38} = m_{31} u_{14} + \mathcal{H}_{35} u_{34}$$

$$m_{31} = \frac{1}{2b_p a^2 b} \left\{ a_1 a_2 (b_2^2 - b_1^2) \mathcal{F}_1 + a_3 b_3 (a_1 b_2 \mathcal{F}_2 - a_2 b_1 \mathcal{F}_3) \right. \\ \left. + b_1 b_2 (a_1^2 - a_2^2) \mathcal{F}_{Z_1} + 2b_1 b_2 (ap^2 - 2a_3^2) \mathcal{F}_m \right\}$$

$$\mathcal{H}_{35} = -\frac{b_1 b_2}{2b_p b} \left[ \frac{b_2^2 - b_1^2}{b^2} (\mathcal{H}_{Z_1} - \mathcal{H}_1) + \frac{b_3^2}{b^2} (\mathcal{H}_3 - \mathcal{H}_2) + 2 \frac{(2b_3^2 - b_p^2)}{b^2} \mathcal{H}_m \right]$$

$$N_{45} = m_{32} u_{11} + \mathcal{H}_{45} u_{31} \quad , \quad N_{46} = m_{32} u_{12} + \mathcal{H}_{45} u_{32} \quad (7.65)$$

$$N_{47} = m_{32} u_{13} + \mathcal{H}_{45} u_{33} \quad , \quad N_{48} = m_{32} u_{14} + \mathcal{H}_{45} u_{34} \quad (7.66)$$

$$m_{32} = \frac{1}{b_p a^2 b^2} \left\{ b_3 [a_1 b_1 a_2 b_2 \mathcal{F}_1 + \frac{1}{4} (a_1^2 - a_2^2) (b_1^2 - b_2^2) \mathcal{F}_{Z_1} + \frac{1}{4} b_p^2 (ap^2 - 2a_3^2) \mathcal{F}_{Z_2}] \right. \\ \left. + \frac{1}{2} a_3 (b_3^2 - b_p^2) (a_1 b_1 \mathcal{F}_2 + a_2 b_2 \mathcal{F}_3) + b_3 [b_1^2 (3a_1^2 - a^2) + b_2^2 (a^2 - 3a_2^2)] \mathcal{F}_m \right\}$$

$$\mathcal{H}_{45} = \frac{b_3}{4b_p b^4} \left\{ (b_2^2 - b_1^2)^2 (\mathcal{H}_{Z_1} - \mathcal{H}_1) + 2b_2^2 (b_3^2 - b_p^2) (\mathcal{H}_3 - \mathcal{H}_2) - b_p^4 (\mathcal{H}_{Z_2} - \mathcal{H}_1) \right. \\ \left. - 2b_p^2 (b_3^2 - b_p^2) (\mathcal{H}_{Z_2} - \mathcal{H}_2) + 4(b_2^2 - b_1^2) (b_3^2 - 2b_p^2) \mathcal{H}_m \right\}$$

**Majorana L-handed, Dirac and Majorana R-handed:**

$$N_{55} = \mathcal{G}_{55} u_{11}^2 + 2 m_{33} u_{11} u_{31} + \mathcal{H}_{55} u_{31}^2 \quad (7.67)$$

$$N_{56} = \mathcal{G}_{55} u_{11} u_{12} + m_{33} (u_{11} u_{32} + u_{12} u_{31}) + \mathcal{H}_{55} u_{31} u_{32} \quad (7.68)$$

$$N_{57} = \mathcal{G}_{55} u_{11} u_{13} + m_{33} (u_{11} u_{33} + u_{13} u_{31}) + \mathcal{H}_{55} u_{31} u_{33} \quad (7.69)$$

$$N_{58} = \mathcal{G}_{55} u_{11} u_{14} + m_{33} (u_{11} u_{34} + u_{14} u_{31}) + \mathcal{H}_{55} u_{31} u_{34} \quad (7.70)$$

$$N_{66} = \mathcal{G}_{55} u_{12}^2 + 2 m_{33} u_{12} u_{32} + \mathcal{H}_{55} u_{32}^2 \quad (7.71)$$

$$N_{67} = \mathcal{G}_{55} u_{12} u_{13} + m_{33} (u_{13} u_{32} + u_{12} u_{33}) + \mathcal{H}_{55} u_{32} u_{33} \quad (7.72)$$

$$N_{68} = \mathcal{G}_{55} u_{12} u_{14} + m_{33} (u_{14} u_{32} + u_{12} u_{34}) + \mathcal{H}_{55} u_{32} u_{34} \quad (7.73)$$

$$N_{77} = \mathcal{G}_{55} u_{13}^2 + 2 m_{33} u_{13} u_{33} + \mathcal{H}_{55} u_{33}^2 \quad (7.74)$$

$$N_{78} = \mathcal{G}_{55} u_{13} u_{14} + m_{33} (u_{14} u_{33} + u_{13} u_{34}) + \mathcal{H}_{55} u_{33} u_{34} \quad (7.75)$$

$$N_{88} = \mathcal{G}_{55} u_{14}^2 + 2 m_{33} u_{14} u_{34} + \mathcal{H}_{55} u_{34}^2 \quad (7.76)$$

$$\mathcal{G}_{55} = \frac{a_1^2 a_2^2}{a^4} \mathcal{G}_1 + \frac{a_1^2 a_3^2}{a^4} \mathcal{G}_2 + \frac{a_2^2 a_3^2}{a^4} \mathcal{G}_3 + \frac{(a_2^2 - a_1^2)^2}{4 a^4} \mathcal{G}_{Z_1} + \frac{(2a_3^2 - a_p^2)^2}{12 a^4} \mathcal{G}_{Z_2} \\ + \frac{(a_2^2 - a_1^2)(2a_3^2 - a_p^2)}{a^4} \mathcal{G}_m$$

$$m_{33} = \frac{1}{a^2 b^2} \\ \left\{ a_1 b_1 a_2 b_2 \mathcal{F}_1 + \frac{1}{4} (a_1^2 - a_2^2) (b_1^2 - b_2^2) \mathcal{F}_{Z_1} + \frac{1}{12} (a p^2 - 2a_3^2) (b p^2 - 2b_3^2) \mathcal{F}_{Z_2} \right. \\ \left. + a_3 b_3 (a_1 b_1 \mathcal{F}_2 + a_2 b_2 \mathcal{F}_3) + [a_1^2 b_1^2 - a_2^2 b_2^2 + a_3^2 (b_2^2 - b_1^2) + b_3^2 (a_2^2 - a_1^2)] \mathcal{F}_m \right\}$$

$$\mathcal{H}_{55} = \frac{b_1^2 b_2^2}{b^4} \mathcal{H}_1 + \frac{b_1^2 b_3^2}{b^4} \mathcal{H}_2 + \frac{b_2^2 b_3^2}{b^4} \mathcal{H}_3 + \frac{(b_2^2 - b_1^2)^2}{4 b^4} \mathcal{H}_{Z_1} + \frac{(2b_3^2 - b_p^2)^2}{12 b^4} \mathcal{H}_{Z_2} \\ + \frac{(b_2^2 - b_1^2)(2b_3^2 - b_p^2)}{b^4} \mathcal{H}_m$$

### 7.6.5 Quark $(V_{CKM})_{4 \times 4}$ and $(V_{PMNS})_{4 \times 8}$ mixing matrices

Within this SU(3) family symmetry model, the transformation from massless to physical mass fermion eigenfields for quarks and charged leptons is

$$\psi_L^o = V_L^o V_L^{(1)} \Psi_L \quad \text{and} \quad \psi_R^o = V_R^o V_R^{(1)} \Psi_R,$$

and for neutrinos  $\Psi_\nu^o = U_\nu^o U_\nu \Psi_\nu$ . Recall now that vector like quarks, Eq.(7.3), are SU(2)<sub>L</sub> weak singlets, and hence, they do not couple to W boson in the interaction basis. In this way, the interaction of L-handed up and down quarks;  $f_{uL}^o{}^T = (u^o, c^o, t^o)_L$  and  $f_{dL}^o{}^T = (d^o, s^o, b^o)_L$ , to the W charged gauge boson is

$$\frac{g}{\sqrt{2}} \bar{f}_{uL}^o \gamma_\mu f_{dL}^o W^{+\mu} = \frac{g}{\sqrt{2}} \bar{\Psi}_{uL} [(V_{uL}^o V_{uL}^{(1)})_{3 \times 4}]^T (V_{dL}^o V_{dL}^{(1)})_{3 \times 4} \gamma_\mu \Psi_{dL} W^{+\mu}, \quad (7.77)$$

g is the SU(2)<sub>L</sub> gauge coupling. Hence, the non-unitary  $V_{CKM}$  of dimension  $4 \times 4$  is identified as

$$(V_{CKM})_{4 \times 4} = [(V_{uL}^o V_{uL}^{(1)})_{3 \times 4}]^T (V_{dL}^o V_{dL}^{(1)})_{3 \times 4} \quad (7.78)$$

Similar analysis of the couplings of active L-handed neutrinos and L-handed charged leptons to W boson, leads to the lepton mixing matrix

$$(U_{PMNS})_{4 \times 8} = [(V_{eL}^o V_{eL}^{(1)})_{3 \times 4}]^T (U_\nu^o U_\nu)_{3 \times 8} \quad (7.79)$$

## 7.7 Conclusions

We reported an updated and general analysis for the generation of neutrino masses and mixing within the SU(3) family symmetry model. The right handed neutrinos  $(\nu_e \nu_\mu \nu_\tau)_R$ , and the vector like completely sterile neutrinos  $N_{L,R}$ , the flavon scalar fields and their VEV's introduced to break the symmetries:  $\Phi^u, \Phi^d, \eta_1, \eta_2$  and  $\eta_3$ , all together, yields a  $8 \times 8$  general Majorana neutrino mass matrix with four or five massless neutrinos at tree level. Therefore, light neutrinos get tiny masses from radiative corrections mediated by the heavy SU(3) gauge bosons. Neutrino masses and mixing are extremely sensitive to the parameter space region, and a global fit for all quark masses and mixing together with neutrino masses and lepton mixing is in progress.

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