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# NJL Model and the Nuclear Tightrope

Steve A. Moszkowski*

UCLA, Los Angeles, USA

## 1 Introduction

Tightrope is balancing act. There are actually two aspects to this:
**I. Large two nucleon scattering lengths** and
**II. Small Nuclear Binding Energies relative to Rest Energy.**
Both of these were known since the 1930's. However, the NJL Model can help to get more basic understanding.

## 2 Large Two Nucleon Scattering Lengths

Large scattering length = small binding (or antibinding).
For T=0, S=1 (d), we get binding = 2.22 MeV, a = 5.4 fm , while for T=1, S=0 (pp), we get antibinding = 0.1 MeV, a = -23 fm. Clearly, it requires only a slight change in the potential to get zero binding.
Splitting (to both sides of tightrope!) is due to spin-dependence. Without it we would not be here! But its role in quark-nuclear physics is unclear. Neglect spin-dependence for now.

## 3 Scalar Meson Exchange with NJL Model

For a review of the NJL model, see Klevansky [1] and Vogl and Weise [2]. We will not discuss the model here, but only mention two important consequences for the Sigma (Scalar Meson) Exchange Interaction:
1. The mass of the sigma is:

$$m_\sigma = 2\, m_q = 2M/N_c \tag{1}$$

so that the $q-\bar{q}$ forms a state with zero binding relative to the constituent quarks. (This is if we neglect any explicit chiral symmetry breaking, which means that the current quark mass, and thus also the pion mass, is neglected.)
2. The strength of the equivalent Yukawa interaction is:

$$\frac{g^2}{4\pi} \approx \pi\, N_c\,, \tag{2}$$

* E-mail: stevemos@ucla.edu

(provided the NJL Cutoff is at $m_\sigma$)
This is not far from the strength required to get $a = \infty$ and $N_b = \frac{N_c - 1}{2}$ deeply bound states.
Some other points: OPEP (with empirical pion mass) gives only 30 percent of binding.
We need a repulsion to get rid of deeply bound states. Goldstone-Boson exchange can lead to such a repulsion, see Bartz and Stancu [3], though it is not the only possible explanation.

# 4 Small Nuclear Binding Energies Relative to Rest Energy

## 4.1 Known Results

BE/A of nuclei ranges up to 8.5 MeV.
BE/A of nuclear matter ≈ 16 MeV.
Rest Energy/A = 938 MeV.
**Binding energies are only about 1 percent of rest energies!**

## 4.2 NJL Model For Nuclear Matter

We are actually describing quark matter. There is no confinement or quark clustering in the NJL model.
Consider first a toy model in two dimensions.

$$T = \frac{g_c \rho}{2} \qquad \text{for small } \rho \tag{3}$$

$$W = -\frac{(g - g_c)\rho}{2} \tag{4}$$

This expression for W applies for $\rho$ up to the value where $m_q = 0$. $g_c$ is the critical value of g necessary to just give two body binding in 2D.

$$W = c_1 \rho^{1/2} - c_2 + c_3 \rho^{-1} \qquad \text{for larger } \rho \tag{5}$$

**We get saturation, but with zero quark mass!**
For a more realistic model in three dimensions, the calculations are more complicated, but one still gets saturation with zero quark mass, similar to 2D.

## 4.3 Generalized NJL Model (With J. da Providencia)

Assume $q - \bar{q}$ coupling gets stronger with density:
$g_s = \frac{g}{[1 - b(g - g_c)^2 \rho^2]}$ This still preserves chiral symmetry (with dependence on $\rho^2$). Effective scalar coupling $g_s = (b+1)(g - g_c)$ but we need vector meson with coupl. $g_v = b(g - g_c)$ to get same low $\rho$ result. We (somewhat arbitrarily) identify b with $b = N_b = \frac{N_c - 1}{2}$ We can solve the Generalized NJL model numerically. Note that the correction opposes chiral symmetry restoration.

We can make a low density expansion. For energy per particle and neglecting all kinetic energies:

$$\frac{W}{m} \approx -\frac{g\rho}{2} + bg^2\rho^2 + \ldots \tag{6}$$
$$\frac{W}{M} \approx -\frac{g\rho}{2N_c} + \frac{(N_c-1)g^2\rho^2}{2\,N_c} + \ldots \tag{7}$$

Here $m$ denotes the constituent quark mass and $M = N_c\, m$ the nucleon mass. For the effective mass, which is the ratio of either mass in the medium to that in free space, we have:

$$m^* = 1 - g\rho + \ldots \tag{8}$$
$$g\rho_0 = \frac{1}{2(N_c-1)} = (1-m^*) \tag{9}$$

Apart from kinetic energies, the saturation energy per nucleon is:

$$\frac{W_0}{M} = -\frac{1}{8N_c(N_c-1)} \tag{10}$$

For $N_c = 3, W_0 = -20MeV$ (CLOSE to empirical value!)

### 4.4 Connection with Relativistic Mean Field Theory at Low Density

In the relativistic mean field approach, the nuclear matter energy per particle, (neglecting kinetic energy) is given by:

$$\frac{W(m^*,\hat{\rho})}{M} = m^* - 1 + \frac{B_\nu\,\hat{\rho}}{2} + \frac{(1-m^*)^2}{2B_s\hat{\rho}\,m^{*\,\alpha_s}} \tag{11}$$

Here $m^*$ denotes the effective mass in units of the free nucleon mass. The Walecka and Zimanyi-Moszkowski derivative coupling models [4] correspond to $\alpha_s = 0, 2$ respectively. If $B \ll 1$, then $B_\nu \approx B_s \approx B$. We then obtain, for small densities:

$$m^* = 1 - B\hat{\rho} + \ldots \tag{12}$$
$$\frac{W}{M} = \frac{\alpha_s B^2}{2}(-2\hat{\rho} + \hat{\rho}^2) + \ldots \tag{13}$$

Comparing the effective mass, with that from the generalized NJL model, we see that:

$$B = \frac{1}{2(N_c-1)} \tag{14}$$
$$W_0 = -M\frac{\alpha_s B^2}{2} = -M\frac{\alpha_s}{8(N_c-1)^2} \tag{15}$$

For $\alpha_s = 1$, we reproduce the results of the generalized NJL model, at least for large $N_c$. This is intermediate between the original Walecka model and the derivative coupling model and is close to the hybrid model used by Glendenning, Weber and S.M. [5]. Of course, the mean field models, unlike the generalized NJL model, lead to finite energies at all densities, but the GNJL is slightly less phenomenological.

## 5 Open Problems

NJL is like a quark shell model, see Petry et. al. [6] and Talmi [7]. How to include effect of quark clustering, without losing NJL simplifications?
Relation of Effective Vector repulsion to short range correlations?
Can Goldstone Boson Exchange do the job, or do we need non-localities, as in Moscow potential?
Where does the density dependence of $g_s$ come from?

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