Fusion of Low Carbon Steel Scrap in the Middle Carbon Steel Melt

Taljenje niskougljičnog čeličnog otpatka u talini srednje ugljičnog čelika

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A quasi three-dimensional mathematical model of fusion of cylindrical steel scrap in converter melt was developed. The model was solved using the implicit alternating direction method. The obtained algorithm was programmed in ASCII FORTRAN for the computer SPERRY 1100/72. In the model temperature dependent thermophysical properties of material were incorporated. That gives to the model a nonlinearity. On the basis of the model it was concluded that the addition of 1% of low carbon steel scrap decreases the temperature of a middle carbon steel melt for cca 20°C. This is in good agreement with experimental data from literature. The mathematical model was tested for one-dimensional exact solution using the Bessel functions. A good agreement was found.

Key words: fusion, steel scrap, mathematical model

Razvijen je i istražen kvazitrodimenzijski matematički model taljenja valjkastog čeličnog otpatka u talini kod konverterskog procesa. Matematički model riješen je implicitnom metodom promjenljivog smjera. Dobiveni algoritam programiran je u programskom jeziku ASCII FORTRAN za računalo SPERRY 1100/72. U matematički model inkorporirana su temperaturno ovisna toplofizička svojstva materijala, što modelu daje nelinearnost. Na temelju matematičkog modela zaključeno je da 1% niskougljičnog čeličnog otpatka snizi temperaturu srednje ugljične čelične taline za cca 20°C, što se dobro slaže s eksperimentalnim podacima iz literature. Matematički model testiran je na jednodimenzionalnom egzaktnom rješenju pomoću Besselovih funkcija. Konstatirano je njihovo međusobno dobro slaganje.

Ključne riječi: taljenje, čelični otpadak, matematički model

1 Introduction

From the increase of the share of metalic scrap in the charge, an increased economy of steel manufacturing in the converter processes is expected. The share of steel scrap should approach that in the openhearth process. Steel scrap, usually as low carbon steel refuse, is a very economical means of melt cooling. Iron ore is only better means of cooling. Data in ref.1 show that 1% of steel scrap decreases the bath temperature for 12 to 15°C, while 1% iron ore decreases the temperature for 30 to 40°C. However, production expirience shows that steel scrap is better than iron ore. For example, by using steel the quantity of metal ejected from the converter is diminished, the resistance of refractory lining is increased and a better utilization of excess heat in the bath obtained. The regime of scrap fusion depends in significant degree on scrap size, and affects the bath temperature, the slag forming processes, as well as the oxidation of carbon and the metal desulfurization. For instance, small sized scrap melts faster and cools quickly the bath. This decreases the rate of slag forming, carbon oxidation, desulfurization, and lower the quantity of blown oxygen. Very large pieces of scrap don't melt completly during the processing in the oxygen converter. A mathematical model of low carbon steel scrap melting in the carbon steel melt was developed and tested with the aim to determine the optimal scrap size. Since the bath temperature is above 1550°C the so-called diffusion melting is not considered. In **figure 1** the investigated system is illustrated. It consists of a volume element of melt in which cylindrical steel scrap is immersed.

2 Mathematical model

For the start of the melting of a cylindrical scrap piece in a volume element of melt in **figure 1** the Fourier's partial differential equation of heat conduction has the form[2]:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) \tag{1}$$

Since for the horizontal axis of the system r = 0 the equation (1) is modified according to L'Hospital's in the form:

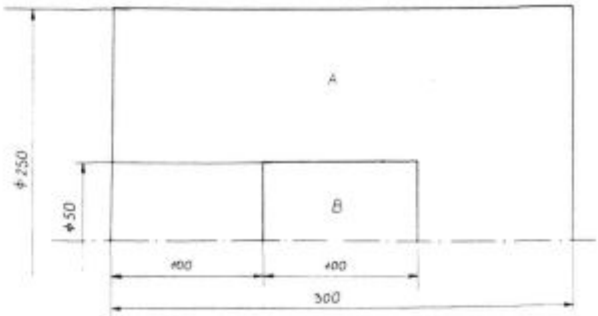

Figure 1: Volume element of melt with cylindrical steel scrap

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$$\frac{\partial T}{\partial t} = a\,(2\frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2}) \qquad (2)$$

The basic assumption for the validity of the differential equations (1) and (2) is that scrap piece is immersed is physicaly realistic since the difference in density of steel scrap (7860 kg/m^3) and melt (7507 kg/m^3) is very small. Considering the system in **figure 1**, it can be concluded that mathematical model is quasi three-dimensional. In the time $t = 0$ the temperature of the melt is T_L, and that of the scrap piece is T_s. The initial temperature at the steel scrap/melt boundary interface is obtained by solving the Fourier's differential equation for heat flow through the contact area of two semifinite medias[3]:

$$T_i = T_s + \frac{T_L - T_s}{1 + \frac{k_s}{k_m}\sqrt{\frac{a_m}{a_s}}} \qquad (3)$$

On the contact steel scrap/melt area a continuous heat flow occurs with boundary condition of the fourth kind:

$$k_m \frac{\partial T_m}{\partial n} = k_s \frac{\partial T_s}{\partial n} \qquad (4)$$

In developing the model it was asummed that thermal properties of low carbon steel scrap (0,2% C) and middle carbon steel melt (0,6% C) are temperature dependent[4].

3 Implicit alternating direction method

The differential heat flow equations (1) and (2) with the corresponding initial and boundary conditions were numerically solved using the implicit alternating direction method[5] and dividing the time interval into two steps.

In the first half of the time interval the equation is solved implicitly for the z and explicitly for the r direction. The procedure is reversed in the second half of time interval.

Consequently, for the differential equation (1) and first half of time interval $\Delta t/2$ we obtain:

$$\partial_r^2 T_{i,j}^n + \frac{T_{i,j+1}^n - T_{i,j-1}^n}{2r_j\Delta r} + \partial_z^2 T_{i,j}^* = \frac{1}{a_{i,j,n}}\frac{T_{i,j}^* - T_{i,j}^n}{\Delta t/2} \qquad (5)$$

Whereas for the second $\Delta t/2$ we obtain

$$\partial_r^2 T_{i,j}^{n+1} + \frac{T_{i,j+1}^{n+1} - T_{i,j-1}^{n+1}}{2r_j\Delta r} + \partial_z^2 T_{i,j}^* = \frac{1}{a_{i,j,n}}\frac{T_{i,j}^{n+1} - T_{i,j}^*}{\Delta t/2} \qquad (6)$$

The numerical solution of the differential equation (2) of heat flow for first $\Delta t/2$ is:

$$4\,\frac{T_{i,2}^n - T_{i,1}^n}{(\Delta r)^2} + \partial_z^2 T_{i,1}^* = \frac{1}{a_{i,j,n}}\frac{T_{i,1}^* - T_{i,1}^n}{\Delta t/2} \qquad (7)$$

and for second $\Delta t/2$:

$$4\,\frac{T_{i,2}^{n+1} - T_{i,1}^{n+1}}{(\Delta r)^2} + \partial_z^2 T_{i,1}^* = \frac{1}{a_{i,j,n}}\frac{T_{i,1}^{n+1} - T_{i,1}^*}{\Delta t/2} \qquad (8)$$

The solution of equations (1) and (2) for the net point (i, j) in the melt and scrap piece as well as for net points on their boundary surface are given in Appendix 2.

Generally it holds:

$$\begin{aligned}
&b_1v_1 + c_1v_1 &&= d_1\\
&a_2v_1 + b_2v_2 + c_2v_3 &&= d_2\\
&\quad a_3v_3 + b_3v_3 + c_3v_4 &&= d_3\\
&\quad \text{--------------}\\
&\qquad a_iv_{i-1} + b_iv_i + c_iv_{i+1} &&= d_i \quad (9)\\
&\quad \text{------------------}\\
&\qquad a_{N-1}v_{N-2} + b_{N-1}v_{N-1} + c_{N-1}v_N &&= d_{N-1}\\
&\qquad a_Nv_{N-1} + b_Nv_N = &&\ d_N
\end{aligned}$$

where v is the unknown temperature, and N is a real number. On the base of the presented algorithm of fusion a computer program was written in ASCII FORTRAN and solved on SPERRY 1100/72 computer.

4 Discusion

The simulation of fusion of a low carbon steel scrap in the carbon steel melt is carried out by space steps $\Delta z = \Delta r = 1$ cm and the time step $\Delta t = 30$ s till $t_{max} = 630$ s. The initial melt temperatures of 1700°C for the melt, 25°C for the scrap piece and 883°C for the boundary surface were assumed. On the basis of succesive temperatures prints out for particular net points the fusion time of 540 s was obtained for a low carbon steel cylinder of size ϕ 50 x 100 mm. The weight of the scrap piece was 4,3% of total weight of the melt. Thus, it can be concluded that 1% of steel scrap decrease the temperature of steel melt for 20 to 22°C, a value in good agreement with published experimental data[1]. Three-dimensional mathematical model of fusion of scrap piece was tested through the one-dimensional exact solution of the fusion of the low carbon steel cylinder and a good agreement was established. The derivation of the exact solution using Bessel functions is given in Appendix 3.

5 Conclusions

A quasi three-dimensional mathematical model of fusion of low carbon steel scrap piece in carbon steel melt in oxygen converter was developed. The model was checked in the base of experience data. The simulation of the fusion is carried out on cylindrical piece of diameter of 50 mm and length of 100 mm. The fusion time of 540 s was calculated. Also it has been established that the addition of 1% of steel scrap decrease the melt temperature for cca 20°C, a value in good agreement with experimental data, and also with the exact one-dimensional solution of the equations, which are the model base.

Appendix 1

Abbreviations used:

a - temperature conductivity

a_i, b_i, c_i, d_i - coefficients adjoining to unknowns in tridiagonal system of algebric equations
c_p - specific heat at constant pressure
k - thermal conductivity
n - vertical direction
r - space coordinate
t - time
T - temperature
v_i - unknown in system of simultaneous algebric equations
z - space coordinate

Appendix 2

Constant whivh appear in tridiagonal coefficients

$p_1 = \frac{a\Delta t}{2(\Delta r)^2}$

$p_2 = \frac{a\Delta t}{4r_j\Delta r}$

$p_3 = p_1 - p_2$

$p_4 = p_1 + p_2$

$p_5 = \frac{\Delta t(k_A + k_B}{2c(\Delta r)^2}$

$p_6 = \frac{\Delta t(k_A + k_B}{4cr_j\Delta r}$

$c = \frac{k_A}{a_A} + \frac{k_B}{a_B}$

$q_1 = \frac{a\Delta t}{2(\Delta z)^2}$

$q_2 = \frac{k_A\Delta t}{c(\Delta t)^2}$

$q_3 = \frac{k_B\Delta t}{c(\Delta t)^2}$

$q_4 = \frac{\Delta t\ (k_A + k_B)}{2c(\Delta z)^2}$

$q_5 = \frac{k_A\Delta t}{c(\Delta r)^2}$

$q_6 = \frac{k_B\Delta t}{c(\Delta r)^2}$

Tridiagonal coefficients

1. Point (i,j) in the melt or scrap piece

– first $\Delta t/2$:

$a_i = c_i = -q_1$
$b_i = 1 + 2q_1$
$d_i = p_3T^n_{i,j-1} + (1-2p_1)T^n_{i,j} + p_4T^n_{i,j+1}$ (10)

– second $\Delta t/2$:

$a_j = -p_3$
$b_j = 1 + 2p_1$
$c_j = -p_4$
$d_j = q_1T^*_{i-1,j} + (1-2q_1)T^*_{i,j} + q_1T^*_{i+1,j}$ (11)

2. Point (i,j) on the boundary surface parallel to r axis separating the materials A (left) and B (right)

– first $\Delta t/2$:

$a_i = -q_2$
$b_i = 1 + q_2 + q_3$
$c_i = -q_3$
$d_i = (p_5-p_6)T^n_{i,j-1} + (1-2p_5)T^n_{i,j} + (p_5+p_6)T^n_{i,j+1}$ (12)

– second $\Delta t/2$:

$a_j = -(p_5-p_6)$
$b_j = 1+2p_5$
$c_j = -(p_5+p_6)$
$d_j = q_2T^*_{i-1,j} + (1-q_2-q_3)T^*_{i,j} + q_3T^*_{i+1,j}$ (13)

3. Point (i,j) on the boundary surface parallel z axis separating the materials A (down) and B (up)

– first $\Delta t/2$:

$a_i = c_i = -q_4$
$b_i = 1+2q_4$
$d_i = (q_5-q_6)T^n_{i,j-1} + (1-2p_5)T^n_{i,j} + (q_6+p_6)T^n_{i,j+1}$ (14)

– second $\Delta t/2$:

$a_j = p_6 - q_5$
$b_j = 1+2p_5$
$c_j = -(q_6-p_6)$
$d_j = q_4T^*_{i-1,j} + (1-2q_4)T^*_{i,j} + q_4T^*_{i+1,j}$ (15)

4. Point (i,1) out of the boundary surface

– first $\Delta t/2$:

$a_i = c_i = -q_1$
$b_i = 1+2q_1$
$d_i = (1-4p_1)T^*_{i,1} + 4p_1T^n_{i,2}$ (16)

– second $\Delta t/2$:

$b_j = 1+4p_1$
$c_j = 4p_1$
$d_j = q_1T^*_{i-1,1} + (1-2q_1)T^*_{i,1} + q_1T^*_{i+1,1}$ (17)

5. Point (i,1) on the boundary surface which separates the materials A (left) and B (right)

– first $\Delta t/2$:

$a_i = -q_2$
$b_i = 2q_4 + 1$
$c_i = -q_3$
$d_i = (1-4p_5)T^n_{i,1} + 4p_5T^n_{i,2}$ (18)

– second $\Delta t/2$:

$b_j = 4p_5 + 1$
$c_j = -4p_5$
$d_j = q_2T^*_{i-1,1} + (1-2q_4)T^*_{i,1} + q_3T^*_{i+1,1}$ (19)

Appendix 3

One-dimensional mathematical model of fusion of steel cylinder consists of the solution of a differential equation of heat conduction with adequate initial and boundary conditions

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \qquad 0<r<1,\ t>0 \quad (20)$$

$T(r,0) = T_0$
$T(1,t) = T_L$
$|T(r,t)| < M$
where M is a positive real number.

It is convenient to consider insted of the equation (20) the equation

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \tag{21}$$

and then to repleace t by at.

Applying the Laplace transform[6], we find

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - s\theta = -T_0 \tag{22}$$

$\theta(1,s) = T_L/s$, $\theta(r,s)$ is connected.

The general solution of this equation is given in terms of Bessel functions as

$$\theta(r,s) = c_1 J_0 (ir\sqrt{s}) + c_2 Y_0 (ir\sqrt{s}) + \frac{T_0}{s} \tag{23}$$

Since $Y_0(ir\sqrt{s})$ is unbounded as $r \to 0$, we must choose $c_2 = 0$. Than

$$\theta(r,s) = c_1 J_0 (ir\sqrt{s}) + \frac{T_0}{s} \tag{24}$$

From the boundary conditions we find

$$\theta(1,s) = c_1 J_0 (i\sqrt{s}) + \frac{T_0}{s} = \frac{T_L}{s} \tag{25}$$

$$c_1 = \frac{T_L - T_0}{sJ_0 (i\sqrt{s})} \tag{26}$$

Thus $$\theta(r,s) = \frac{T_0}{s} + (T_L - T_0)\frac{J_0 (ir\sqrt{s})}{sJ_0 (i\sqrt{s})} \tag{27}$$

After complex inversion this equation acquires the form

$$T(r,t) = T_0 + (T_L - T_0)\frac{1}{2\pi 1}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{e^{st} J_0 (ir\sqrt{s})}{sJ_0 (i\sqrt{s})\, ds} \tag{28}$$

and the final solution is obtained as

$$T(r,t) = T_L - 2(T_L - T_0)\sum_{n=1}^{\infty} \frac{e{-}a\lambda_n^2 t\, J_0(\lambda_n^r)}{\lambda_n J_1 (\lambda_n)} \tag{29}$$

where $\lambda_1, \lambda_2, ..., \lambda_n, ...$ are positive zeros of equation $J_0(\lambda_n) = 0$, which are given in **Table 1**.

Table 1: Zeros of equation $J_0(\lambda_n) = 0$[7].

n	λ_n
1	2,40482 55577
2	5,52007 81103
3	8,65372 79129
4	11,79153 44391
5	14,93091 77086
6	18,07106 39679
7	21,21163 66299
8	24,35247 15308
9	27,49347 91320
10	30,63460 64684

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