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Treatment of three-quark problems in Faddeev theory*

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We propose a method that allows for the efficient solution of the three-body Faddeev equations in the presence of infinitely rising confinement interactions. Such a method is useful in calculations of nonrelativistic and especially semirelativistic constituent quark models. The convergence of the partial wave series is accelerated and possible spurious contributions in the Faddeev components are avoided.

We start from the total Hamiltonian of a nonrelativistic or a semirelativistic three-quark system, which can be written as

$$H = H^0 + v_\alpha + v_\beta + v_\gamma, \tag{1}$$

where H^0 is the three-body kinetic-energy operator and $v_\delta = v_\delta^c + v_\delta^{hf}$ represents the mutual quark-quark interactions containing both the confinement (v_δ^c) and hyperfine (v_δ^{hf}) potentials in the subsystems $\delta = \alpha, \beta, \gamma$

In the nonrelativistic case we may express the kinetic-energy operator by four equivalent forms

$$H^0 = \frac{p_\alpha^2}{2\mu_\alpha} + \frac{q_\alpha^2}{2M_\alpha} = \frac{p_\beta^2}{2\mu_\beta} + \frac{q_\beta^2}{2M_\beta} = \frac{p_\gamma^2}{2\mu_\gamma} + \frac{q_\gamma^2}{2M_\gamma} = \sum_{i=1}^{3} \frac{k_i^2}{2m_i}, \tag{2}$$

i.e. either through individual particle momenta $\mathbf{k}_i$ in the center-of-mass system or in terms of relative momenta $\mathbf{p}_\delta$ and $\mathbf{q}_\delta$ conjugate to the usual Jacobi coordinates $\mathbf{x}_\delta$ and $\mathbf{y}_\delta$, respectively ($\delta = \alpha, \beta, \gamma$). In Eq. (2), m_i denotes the individual particle mass, μ_δ the reduced mass in the two-body subsystem δ, and M_δ the reduced mass of this subsystem with the third particle δ. In the semirelativistic case the kinetic-energy operator takes the form

$$H^0 = \sum_{i=1}^{3} \sqrt{k_i^2 + m_i^2}, \tag{3}$$

where again $\mathbf{k}_i$ are the individual particle three-momenta in the frame with total three-momentum $\mathbf{P} = \sum_{i=1}^{3} \mathbf{k}_i = 0$. We note that a Hamiltonian as in Eq. (1) together with the relativistic kinetic-energy operator (3) represents an allowed mass

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operator in the point-form formalism of Poincaré-invariant quantum mechanics, irrespective of the dynamical origin of the interactions.

Strictly speaking the standard Faddeev scheme applies only for potentials falling of fast enough at large distances. This makes it necessary to modify the Faddeev formalism. Otherwise one risks unpleasant properties in the Faddeev components. In particular, for infinitely rising potentials spurious contributions are picked up and also the partial-wave series becomes slowly convergent.

One can circumvent these difficulties by performing the Faddeev decomposition in such a way that all the long-range potentials are included in a modified channel Green's operator. Specifically, in our case at least the long-range parts of the confinement interactions in all subsystems α, β, and γ should be included in the modified channel resolvent. One can attain this goal by adopting a different splitting of the total Hamiltonian into

$$H = H^c + \tilde{v}_\alpha + \tilde{v}_\beta + \tilde{v}_\gamma, \tag{4}$$

where

$$H^c = H^0 + \tilde{v}^c_\alpha + \tilde{v}^c_\beta + \tilde{v}^c_\gamma \tag{5}$$

contains, besides the kinetic energy, the long-range parts $\tilde{v}^c_\delta$ of the confining interactions v^c_δ in all subsystems. The potentials $\tilde{v}_\delta$ are the residual interactions containing the hyperfine potentials and the short-range parts of the confinement.

Based on Eqs. (4) and (5) we now decompose the total wave function into

$$|\Psi\rangle = |\tilde{\psi}_\alpha\rangle + |\tilde{\psi}_\beta\rangle + |\tilde{\psi}_\gamma\rangle, \tag{6}$$

where the modified Faddeev components are defined as

$$|\tilde{\psi}_\alpha\rangle = G^c(E)\tilde{v}_\alpha|\Psi\rangle \tag{7}$$

with

$$G^c(E) = (E - H^c)^{-1}. \tag{8}$$

They fulfill the integral equations

$$|\tilde{\psi}_\alpha\rangle = G^c_\alpha(E)\tilde{v}_\alpha(|\tilde{\psi}_\beta\rangle + |\tilde{\psi}_\gamma\rangle), \tag{9}$$

with α, β, γ again a cyclic permutation. The new channel resolvent is given by

$$G^c_\alpha(E) = (E - H^c - \tilde{v}_\alpha)^{-1}. \tag{10}$$

It exhibits just the desired property of including the long-range confining interactions in all subsystems α, β, γ. Only the short-range potential $\tilde{v}_\alpha$ remains in the modified Faddeev equations (9). Specifically, since now G^c_α contains also the long-range parts $\tilde{v}^c_\beta + \tilde{v}^c_\gamma$ of the confinement interactions in channels β and γ, the dependence of the component $|\tilde{\psi}_\alpha\rangle$ on the Jacobi coordinate $\mathbf{y}_\delta$ can never become a free motion. Rather the proper confinement-type asymptotic conditions are imposed on $|\tilde{\psi}_\alpha\rangle$. As a result, spurious contributions are avoided in the individual Faddeev components, and at the same time the partial-wave expansion converges much faster.

The splitting of the interactions in Eqs. (4) and (5) has to be done with care. In general, the interaction parts put into H^c must not produce any bound state. Otherwise the proper behavior of the Faddeev components $|\tilde{\psi}_\alpha\rangle$ would again be spoiled. Suppose the potentials contained in H^c would produce bound states. Then at the corresponding energies, the resolvent $G^c(E)$ would become singular. Consequently, according to Eq. (7), any large Faddeev component $|\widetilde{\psi}_\alpha\rangle$ could be generated even if the full solution $|\Psi\rangle$ remains infinitesimally small. Therefore, besides the true physical solutions of the Hamiltonian H, Eqs. (9) would also produce spurious solutions associated with the discrete eigenstates of the Hamiltonian H^c. These spurious solutions would occur for any $\tilde{v}_\alpha$, thus having no bearing for the physical spectrum of H. Of course, when adding up the three individual Faddeev components these spurious solutions would cancel out. However, they would cause numerical instabilities in the practical calculations. Therefore they should be avoided by not allowing H^c to produce any bound states.

In the case of confinement interactions the above requirement cannot strictly be met, since even the longest-range parts of the infinitely rising potential generate bound states. However, there is a practical way out: one needs to eliminate the bound states generated by H^c only in the region of physical interest. Outside that domain, i.e. reasonably far above the physical spectrum, they do not matter. In practice, upon splitting the interactions in the Hamiltonian (4) an auxiliary short-range potential is introduced with no effect on the physically interesting states. It only serves the purpose of cutting off the confinement interaction at short and intermediate distances thus avoiding low-lying bound states of H^c.

We solve Eqs. (9) along the Coulomb-Sturmian (CS) separable expansion approach. The further details of the method and the demonstration of its power in the example of the Goldstone-boson-exchange chiral quark model for baryons is given in Refs. [1] and [2].

References

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