

Game Theory Application for Performance Optimisation in Wireless Networks

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Abstract. Game theory has been recently introduced in wireless network design as a powerful modeling and analysing tool for competitive and completely distributed environments. It is a well-suited to describe mutual conflicting situations between multiple devices which attempt to communicate through a shared medium. In order to demonstrate suitability of a game-theoretic approach for optimisation of wireless networks, we first present the main idea, concepts and components of game theory. We then provide mapping of principles between the areas of game theory and wireless networks, and present some applications of game theory in wireless networks. We develop and implement a model for transmit power control optimisation in a wireless relay network consisting of wireless sensor network coordinator nodes using the category of potential games. In the game, we determine the Pareto efficient Nash equilibrium, which represents the optimal stable operating point of the network.

Keywords: game theory, wireless networks, relay network, potential games, optimisation

1 INTRODUCTION

As wireless communication networks are becoming more and more extensive and complex, their performance optimisation and management with conventional deterministic methods are no longer possible. Therefore, besides heuristic methods, other approaches are being increasingly used for these purposes, *game theory* [1] as a special field of applied mathematics being also one of them.

Game theory is a powerful tool to model situations where the success of an entity also depends on the decisions of other entities of the system. It can be used in the case when interests of one entity or group of entities intersect or influence over those of other entity or entities. Every intersection of interests is called a *conflict situation* [2] and represents a conflict between multiple entities present in the system. The behavior of participants in such conflict situation can be described by a *mathematical model*, which takes into consideration that actions or decisions made by participants obey certain *rules*. It is also assumed that participants act *rationally* [3], i.e., they always choose those actions which are promising the largest possible benefit, taking into consideration options of opponents and some external circumstances.

Game theory being a discipline for modeling situations in which decision makers interact between each other, it is also well suited for the use in wireless communication systems. In these systems, the interaction between individual devices occurs due to the

sharing of the radio channel. Assuming each device or node is capable of adapting independently utilizing game theory makes possible to steer the network to a stable operating point.

The intention of this paper is to present game theory as one of the possible methods for optimisation of radio resources in wireless networks. To this end we first present the main idea, concepts and components of game theory. We describe application of game theory in wireless networks and link various concepts and components of game theory with corresponding wireless networks concepts and components. As an example of using game theory, a *potential game*, which is particularly suitable for use in wireless networks, is applied to model optimisation of a transmit power control in coordinator nodes of a multi-hop hierarchical wireless sensor network (WSN).

2 BASIC CONCEPTS OF GAME THEORY

Game theory studies games in which result depends on the skills of players besides on pure chance. These games are called *strategic games* [4]. A *game* is actually a collection of rules and arrangements to be followed by players. A game consists of three basic components: a set of players, a set of strategies/actions for each player and a set of preferences over possible outcomes.

The participants of the game are *players*, which choose their *actions* in each *stage* of the game. The choices of players always influence the resulting *outcome* of the game for each player, thus they also

represent interaction between all players. Each player's choice of an action is called a *move*. *Strategy* of a player is a complete contingent plan that describes which action has to be chosen at each stage of the game, according to: (i) the rules (constraints) of the game; (ii) all possible contingencies that may arise; (iii) the previous actions of other players; (iv) any information that may become available during the game; (v) the player's preferences on the outcomes. The preference of a player is represented as his *payoff*. The payoff actually quantifies outcome of the game for each player.

2.1 Game representation

A game can be represented in various forms, for example normal, extensive, coalitional, etc. The form of game representation depends on the particular game and the purpose of analysis [5]. As an example, the normal form of a game is presented in the following for the potential game.

A game is usually presented in the normal form when each player only has a single move in the game and no information about the moves taken by other players. All players act simultaneously. The strategy of a player is represented by choosing an action a and is called a *pure strategy* s .

A game in the *normal form* is defined by $\Gamma = \langle N, A, U \rangle$, where $N = \{1, 2, \dots, n\}$ is the *set of players*, $A = \times_{i \in N} A_i$ is the *action space* (i.e. the set of all action profiles for each player), and $U = \{u_i\}: A \rightarrow \mathbb{R}^N$ is the *set of payoff or utility functions* for all players. $A_i = \{a_i\}$ is the finite *set of actions* of player i , and u_i is the *payoff or utility function* of player i .

An *action profile* $\mathbf{a} = (a_1, a_2, \dots, a_n)$ is an action vector, where each element of the vector represents one action for each player in the game. When each player chooses an action, the resulting action profile defines the outcome of the game. Often action profile is denoted as $\mathbf{a} = (a_i, a_{-i})$, where a_i is an action chosen by player i and a_{-i} are actions chosen by all the other players. Utility or payoff function is a function of action profiles and represents the player's preferences to certain action profiles. Utility is often a criterion for the payoff a player gets by a certain outcome of the game. The set of payoffs which is the result of action profile \mathbf{a} for the set of players N represents a *payoff or utility profile* $\mathbf{u} = (u_1(\mathbf{a}), u_2(\mathbf{a}), \dots, u_n(\mathbf{a}))$. The objective of a game is to maximize players' utilities or payoffs.

Table 1 shows a game with two players $N = \{1, 2\}$, denoted by I_1 (column player) and I_2 (row player) who have two actions each ($a_x(1), a_x(2)$). Utilities of players I_1 and I_2 are denoted by u_{1x} and u_{2x} , respectively. For example, if the action profile $\mathbf{a} = (a_1, a_2) = (a_1(1), a_2(2))$ is chosen, the corresponding utility profile is $\mathbf{u} = (u_1(\mathbf{a}), u_2(\mathbf{a})) = (u_{12}, u_{22}) = (0, 3)$.

Table 1: Two-player strategic game representation.

$I_2 \backslash I_1$	$a_2(1)$	$a_2(2)$
$a_1(1)$	$u_{11} = 2, u_{21} = 2$	$u_{12} = 0, u_{22} = 3$
$a_1(2)$	$u_{13} = 3, u_{23} = 0$	$u_{14} = 1, u_{24} = 1$

2.2 Types of games

According to their respective characteristics we distinguish different types of games. Some of the most representative types of games are briefly described in the following subsections.

2.2.1 Cooperative and non-cooperative games

In cooperative games players may enforce cooperation, while in non-cooperative they can not.

2.2.2 Games with complete and incomplete information

In a complete information game, players know other players, their action sets and all possible payoff or utility functions. All other games are incomplete information games.

2.2.3 Static or strategic games

Static games correspond to the description of subsection 2.1. If a player always chooses the same strategy, which determines the player's action in any game situation, the strategy is called a *pure strategy*. If a player chooses different strategies according to predefined probabilities, the player is said to play a *mixed strategy*. A mixed strategy of the player i is denoted by $\sigma_i \in \sigma(A_i)$, where $\sigma_i(a_i)$ is the probability of choosing action a_i when playing σ_i , $\sum_{a_i \in A_i} \sigma_i(a_i) = 1$ and $\sigma_i(a_i) \geq 0$. Profile σ is the *profile of mixed strategies* for all players. If $\sigma_i(a_i) = 0$ for all a_i except for one, the strategy is said to be pure.

2.2.4 Dynamic or extensive or sequential games

In dynamic games players act sequentially, taking decisions during the game based on some knowledge about other players' actions. At least one player has more than a single move, and the order of moves is very important. When a player is at a certain stage of a game, he chooses an action with respect to his and other players' choices in the previous stages of the game, i.e. according to his *position* in the game. The strategy of a player in such a game is a collection of rules describing which actions a player should take.

2.2.5 Repeated games

A class of dynamic games, in which the same decision is made by players repeatedly at regular intervals in the same environment, is called repeated games.

2.2.6 Potential games

In potential games [6] the utility function is a potential function and reflects the change in utility caused by each of the players when unilaterally changing their respective strategies. Thus, with one function, the utilities and behaviours of all players can be presented. A potential game is usually presented in a normal form.

Potential games with selfish strategies are of particular interest due to their specific property that they always achieve the Nash Equilibrium [7] (described in Section 2.3), which is found at some of the potential function's local optima. A frequently investigated class of potential games are ordinal potential games [6].

A game in the normal form $\Gamma = \langle N, A, U \rangle$ is an *ordinal potential game* if there exists a function $P: A \rightarrow \mathbb{R}$ (where \mathbb{R} represents real numbers), such that $u_i(a_i, a_{-i}) - u_i(a_i^*, a_{-i}) > 0 \leftrightarrow P(a_i, a_{-i}) - P(a_i^*, a_{-i}) > 0$ for all $i \in N$, all $a_{-i} \in A_{-i}$, and all $a_i, a_i^* \in A_i$. It means that the potential function increases (decreases) if the player obtains a greater (smaller) payoff by unilaterally changing his strategy. The algorithm above does not require that the game is finite which is another good property of these games.

In addition to the games described above there are also other types of games such as: coalitional, stochastic combinatorial, Bayesian, differential, evolutionary, bargaining, etc.

2.3 Nash equilibrium and Pareto optimum

In game theory there are several concepts used to determine which strategies or actions a player should choose to successfully complete the game. The concept of the *best response* of a player defines the player's action or strategy a_i^* to be the best response to the strategies a_{-i} chosen by the other players if it maximizes his payoff or utility: $u_i(a_i^*, a_{-i}) \geq u_i(a_i, a_{-i}) \forall a_i \neq a_i^*$. The strategy profile \mathbf{a}^* is a *Nash equilibrium* (NE) if none of the players can increase his payoff by changing his strategy unilaterally, thus there is no incentive to deviate from his strategy assuming that the other players do not deviate, i.e., for each player i : $u_i(a_i^*, a_{-i}^*) \geq u_i(a_i', a_{-i}^*) \forall a_i'$.

The game might have several NEs and there is also no guarantee that a NE will correspond to a desirable or efficient outcome. Several methods exist for identifying the efficiency of action profiles for a game, Pareto optimality being one of them. A strategy profile is said to be *Pareto optimal* (PO) if there is no other outcome or action profile \mathbf{a}' that makes every player at least as well off and at least one player strictly better off, such that $u_i(\mathbf{a}') \geq u_i(\mathbf{a}^*) \forall a_i'$. It is said that \mathbf{a}^* is *Pareto optimal Nash equilibrium* (PONE).

In Table 1, NE is presented by the action profile $(a_1(1), a_2(2))$, where both players get a payoff equal to one.

3 GAME THEORY IN WIRELESS NETWORKS

In wireless communications, the radio frequency spectrum is a limited natural resource shared by all devices within the same network. The efficient utilization of the frequency spectrum highly depends on other resources and operating parameters in these networks which are required for successful communication. These include transmit power, bandwidth, transmission time, nodes' energy, etc. Recently, game theory is being increasingly used as a tool to optimize the usage of these resources and parameters related to the resources in distributed systems such as wireless networks.

3.1 Mapping between game theory and wireless networks concepts

In order to apply game theory to optimisation problems in wireless networks, it is necessary to relate various game theory concepts to those of wireless networks. Thus, players represent network nodes, networks, service providers or customers in wireless networks domain. An action or a strategy is a choice or decision of the player related to the functionality of the application field under consideration and concerns allocation of the available resources (power, spectrum, bandwidth) in the game, e.g. setting the power level of nodes, accessing the wireless medium or not, etc. The utility function is a function modelled for a specific application and represents the metrics of quality of network performance or quality of service (QoS) for choosing certain strategies or actions. It aims at enhancing various network requirements such as SINR, throughput, bandwidth usage, etc. Payoff estimates the benefit of the utility functions based on QoS merits.

The main challenge of applying game theory to wireless networks is modelling of the appropriate game with the consideration of many restrictions called *parameterization of the game*. Not every optimisation problem can be already a game. There need to be multiple nodes involved in making decisions to create a game. Game players, actions and objectives need to be chosen and verified with extreme caution. Besides these factors, the key issue in parametrization of a game is a choice of utility or payoff functions. These functions can be used to analyze how the changes in players' strategies affect the performance of an individual player or the system as a whole. And as the objective of players is to maximize the performance of the system, utility functions need to have such ability that the system converges to the optimum state of operation according to the selected objectives. For example, the importance of sending a packet for a given user depends mostly on the situation if this packet has a crucial information, or if it makes no difference if it is immediately sent or not. This and similar problems are what the *mechanism design* [8] is coping with.

Nodes in wireless networks are typically characterised by a limited amount of energy. In this

respect, the energy consumption represents the *cost* for a player when applying game theory in wireless networks. So, nodes are choosing among possible strategies according to the cost they have to pay. The cost can also represent some QoS metric such as delay or the consumed bandwidth, etc. Generally, in modelling a game, we have to adjust the usage of scarce resources to players' costs, which is called *pricing* and constitutes a vital factor in the utility function. For example, in the case of energy consumption it has to be considered that devices with almost depleted batteries should have different evaluation of costs than devices with full batteries. Various pricing schemes are also used to stimulate cooperation and prevent selfish behaviour of nodes which may lead to a suboptimal equilibrium from the network perspective. Such pricing schemes are then called *incentive mechanisms* [9]. There are also other types of incentive mechanisms and they are all broadly divided into two categories. The first category is composed of *credit-exchange* mechanisms, which incorporate (i) the method of rewarding nodes by credits for cooperation with other nodes in offering their services, and (ii) the method of charging nodes for the same credits when requesting services from other nodes. The second category comprises *reputation-based* mechanisms incorporating (i) the method of giving a positive reputation to nodes which behave cooperative, and (ii) the method of giving a bad reputation to nodes which behave selfishly, and eventually, over time isolating nodes with bad reputation from the network.

The assumption of players' *rationality* in game theory may not be adequate in every situation as it does not necessarily lead to a socially optimal stable-state. The techniques of incentive mechanisms take also this into consideration and help to achieve the latter. In addition, in a game parameterization it is not always possible to assume that the game is infinite and with complete information. Realistic scenarios may require more complex models where the *dynamic* nature of wireless networks brings to the system imperfections or noise in actions observed by nodes. For example, at completeness assumption, a given node might be mistaken, when marking another node to behave selfishly, as such conclusion of the given node can be the consequence of the unexpected changes in the radio channel in wireless communication. In some occasions it can be also important that a given node is aware of its context, for instance that it is just about to be turned off or moved away, essentially being prevented from further interaction with another player. In such case, sending a packet immediately or at some later time is certainly of a great importance and assumption of infinity is not appropriate. These cases need to introduce the *finite-horizon* aspect and *incompleteness of information* to the game.

Game theory approaches can be used to optimise *node-level* as well as *network-wide* performance [10]. In this respect, *cooperative* or *non-cooperative* schemes

can be used for maximizing both individual node's payoff as well as the community welfare. In realistic wireless networking, most of such strategic situations require nodes to agree on sharing common resources in a distributed manner, so some researchers believe that non-cooperative schemes are more promising than cooperative. However, in many cases cooperation between nodes can achieve better overall results and can be also irreplaceable with non-cooperation aspects. In these cases, some communication between nodes is needed to exchange crucial information for cooperation (additional signalization or agreements are then needed). *Bargaining* and *coalition formation* are then considered as a choice. Furthermore, as there is a deep connection between cooperative and non-cooperative games, we can mix them in a single game but with extreme caution to make a reasonable game.

Often, performance optimising objectives involve more than only a single layer in the process of optimisation and various cross-layer techniques are thus needed. With the combination of various game-theoretic frameworks at different layers in wireless networks and an appropriate formulation of the action space game theory can be also used to achieve *cross-layer optimisation* [11].

When considering the fact that in future networks nodes will be autonomous agents and will make some decisions (such as changing the modulation scheme) on their own irrespective of distributed protocols running on the nodes, game theory can be also used for the optimisation purposes of such nodes' behaviour.

3.2 Game theory applications in wireless networks

Future generation wireless networks are expected to be distributed, self-organizing, dynamic networks. For such characteristics, these networks will require various mechanism and optimisation techniques, and game theory is expected to be one of them. In this respect, this section surveys some representative game-theory applications and game-theoretic approaches made so far in wireless networks.

3.2.1 Wireless ad hoc/sensor networks

At MAC layer, game theory is used for the implementation of the ALOHA and CSMA/CA medium-access protocols.

In multi-hop routing, cooperation between nodes is needed to forward packets in wireless networks. Game theory can be used as a tool to decide on the participation of nodes in packet forwarding, and to detect and isolate selfish nodes that do not want to cooperate in forwarding other nodes' packets.

In the case of ad hoc and sensor networks, routing models have to be different from those developed for conventional wireless networks. Since the role of each node, i.e. a device with limited power resources, in this case is of an extreme importance, energy consumption is considered as the most critical parameter, taken into account when designing routing schemes. With the help

of game theory, a game is formulated to perform routing of the network traffic based on the current amount of energy and the benefits experienced by nodes for routing.

3.2.2 Radio access/cellular networks

Transmit power control in wireless networks is required to limit interference between different users. Game theory can be used to form a game for minimizing/allocating/charging transmit power of nodes in OFDM and CDMA systems of radio access/cellular networks.

In MIMO systems, game theory is used to control co-channel interference of several links based on the ratio of data transmission rate and required transmit power.

Game theory can be also used for various connection admission control problems such as achieving QoS requirements, choosing a service provider, congestion avoidance/prevention control, efficient use of limited bandwidth, etc.

Cell selection is a process of determining the cell, which provides service to the mobile station. Optimising this process can be also performed based on game theory.

3.2.3 Transport/backbone networks

In these networks, the system capacity is of a particular importance. Therefore, in this case game theory can be used for throughput maximization. Moreover, it can be also used to select the most reliable path for traffic routing through wireless bachaul network.

3.2.4 Cognitive (radio) networks

The concept of cognitive communications is based on the assumption that devices are aware of the environment they are in and adapt their operational parameters according to that. It is introduced in various segments of future generations networks, from individual devices to access and also transport networks, but the actual implementation by different research groups is often based right on the use of game theory.

In a cognitive radio network, game theory is used to dynamically allocate the frequency sprectrum to secondary users, for transmit power control or even for changing the transmission parameters and characteristics such as modulation, waveform type, etc.

4 APPLICATION OF A POTENTIAL GAME FOR DISTRIBUTED TRANSMIT POWER CONTROL

Potential games are often used in engineering

applications, which also include optimisation of the radio resources usage in wireless networks [12]. In the following we present a model of an ordinal potential game with selfish strategies.

4.1 Transmit power control in a wireless relay network

In this section, the use of an ordinal potential game model is presented to model transmit power control in a wireless relay (forwarding) network depicted in Figure 1. Each of the relay nodes is at the same time also a coordinator node for a part (a cluster) of a wireless sensor network, together forming a multi-hop hierarchical wireless sensor network.

Following the example in [7], we model the potential game $\Gamma = \langle N, A, U \rangle$ in Matlab. Relay transceivers (Rx/Tx) $N = \{1, 2, \dots, 9\}$ in the presented chain are forwarding network traffic to the receiver at the end of the chain (denoted by 10), which is not a player and represents only the transition point to the external network. Transmit powers of transmitters, represented by A , are changed during the game for optimisation purposes. The corresponding potential functions of the players are denoted by U . The distances between consecutive nodes in the chain are set to 1 km. Because the relay nodes N play also the role of the coordinating nodes of WSN clusters, the traffic towards the transition node increases. This implies that the relay nodes closer to the transition node require a higher signal-to-noise ratio (SINR). The bandwidth of the links between the nodes is set to $B = 1 \text{ MHz}$. The traffic load of each WSN is assumed to be 1 Mbit/s . Figure 2 shows the setting of the individual nodes' transmit power (denoted with numbers from 1 to 9) through several iterations, at the incoming load of each WSN of 1 Mbit/s , which requires capacities of individual links to be $C = 1/2/3/4/5/6/7/8/9 \text{ Mbit/s}$. The potential functions of the players reflect the minimum transmit power required (i) to overcome the path loss, thermal noise and interference made by other transmitters, and (ii) to meet the required SINR. We assume that we have "ideally" oriented receiver antennas with which we achieve that transmitters can only cause interference to receivers further along the chain towards the transition node. Transmitters adjust their transmit power unilaterally, without knowing the transmit powers of other transmitters. This means they do not consider strategies of other transmitters, which corresponds to the definition of selfish strategies. The powers of transmitters are initially set to zero and increase with the

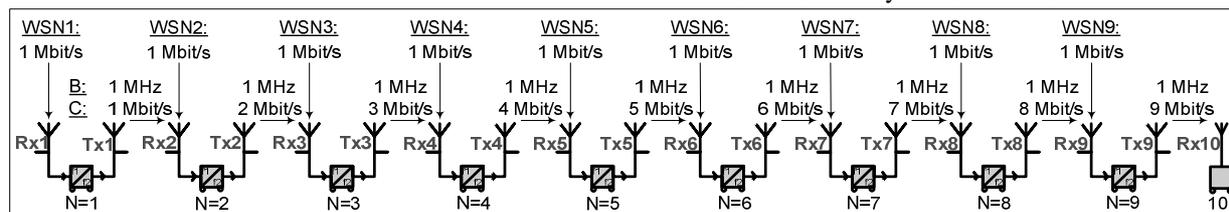


Figure 1: Transmit power control in a wireless relay network using game theory.

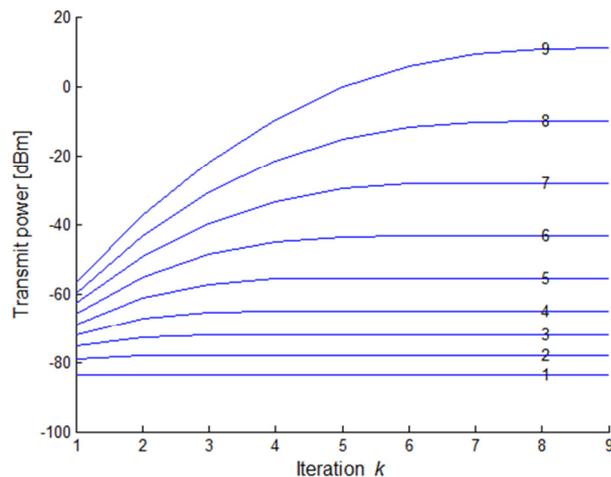


Figure 2: Powers of transmitters through several iterations.

influence of interference of other transmitters in the chain. In the case that transmit powers do not already meet the desired network conditions, the player's utility, which is reflected by the potential function of the player, is negative. When the utility of each node reaches a value of zero, the network is set to a steady operating point and transmit powers of all transmitters are optimally set. Curves in Figure 2 show that the network meets the stable operating point after nine iterations of setting the transmit power on each transmitter, which then presents PONE [7]. Nine iterations are required to overcome the increasing of the interference made by all transmitters following a particular transmitter in the chain.

5 CONCLUSION

By introducing of the heterogeneous networks and the concept of cognitive communications in networks, game theory is becoming more and more important in the field of wireless networks as it represents a powerful tool for modeling, analyzing and optimisation of the network performance. In this paper we present the main concepts of game theory, which we then extend to the use of game theory in wireless networks. Through a discussion we emphasise the challenge of choosing the correct type of a game for a particular situation and identifying the most appropriate incentive mechanisms and players' utility or payoff functions when using game theory in wireless networks. As an example we show the use of a special category of games, i.e. potential games, to model distributed nodes' transmit power control in a multi-hop hierarchical wireless sensor network, with the aim to optimise the transmit power of each node.

REFERENCES

- [1] D.Fudenberg, J. Tirole: Game Theory. MIT Press, 1991.
- [2] R. B. Myerson: Game Theory – Analysis of conflict, Harvard University Press, 1991.
- [3] M. J. Osborne, A. Rubinstein: A course in game theory. The MIT Press, 1994.
- [4] S. Smith: The mathematics of games of pure chance and games of pure strategy. Department of mathematics, Saint Joseph's University, Philadelphia, USA, http://www.sju.edu/~smith/Current_Courses/games.pdf.
- [5] R. Gibbons: Game Theory For Applied Economists. Princeton University Press, 1992.
- [6] D. Monderer, L. S. Shapley: Potential games. Games and Economic Behavior, 14 (1), pp. 124–143, 1996.
- [7] R. W. Thomas: Cognitive networks, Ph.D. Dissertation, Virginia Polytechnic and State University, Blacksburg, VA, USA, 2007.
- [8] D. Miller, S. Tilak, T. Fountain: "Token" equilibria in sensor networks with multiple sponsors. Proc. Workshop on Stochasticity in Distributed systems, 2005.
- [9] M. Felegyhazi, J. Hubaux, L. Buttyan: Nash Equilibria of Packet Forwarding Strategies in Wireless Ad Hoc Networks. IEEE Transaction on Mobile Computing 5 (5) (2006) 463-476.
- [10] R. Valli, P. Dananjayan: A Non-Cooperative Game Theoretical Approach For Power Control In Virtual MIMO Wireless Sensor Network. International Journal of UbiComp (IJU) 1 (3) (2010) 44-55.
- [11] X. Lin, N. B. Shroff, R. Srikant: A Tutorial on Cross-Layer Optimization in Wireless Networks. IEEE Journal on Selected Areas in Communications 24 (8) (2006) 1452-1463.
- [12] A. B. MacKenzie, L. A. DaSilva: Game Theory for Wireless Communications. Morgan and Claypool Publishers, 2006.

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