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Scattering of nucleon on a superheavy neutron ⋆

Norma Mankoč Borštnik[a] and Mitja Rosina[a,b]

[a]Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, P.O. Box 2964, 1001 Ljubljana, Slovenia
[b]J. Stefan Institute, 1000 Ljubljana, Slovenia

Abstract. The scattering cross section of a superheavy baryon on a nucleon is estimated. The possibility that such a superheavy baryon (from a higher quark family) might be a viable candidate for the dark matter, is discussed.

1 Introduction

The purpose of this talk is twofold.

(i) Scattering of a light cluster on a superheavy cluster is a challenging few-body problem. The energy scales and consequently the sizes of both clusters differ by 5-6 orders of magnitude. Due to colour neutrality of unperturbed clusters, the strong interaction acts only at a very short distance via the virtual colour-octet colour-octet Van der Waals excitation. The novel feature is the van der Waals interaction at contact separation. Moreover, due to the small size of the superheavy cluster the effective quark-quark interaction is expected to be coulomb-like and this feature might be tested even in bottomium collisions.

(ii) We want to show that clusters of strongly interacting particles are viable candidates for dark matter provided their masses are large enough. Then both the number density of dark matter particles is small and their cross section is small due to their small size.

We require that the number of collisions of dark matter particles against the detector is either consistent with the DAMA experiment [1] (if confirmed) or lower (if DAMA is not confirmed). It turns out that superheavy quarks must have a mass of about 100 TeV or more in order to have a low enough collision rate by weak interaction. Surprisingly, at this mass the strong cross section is much smaller than the weak cross section and can be neglected.

As an example we take the superheavy quarks from the unified *Spin-Charge-Family* theory [2–6] which has been developed by one of the authors (SNMB) in the recent two decades. For a short review, we invite the reader to read the Bled 2010 Proceedings [7]. In this theory eight families of quarks and leptons are predicted, with the fifth family decoupled from the lower ones and therefore rather stable. The most promising candidates for dark matter are the superheavy neutrons (the $n_5 = u_5 d_5 d_5$ clusters) of the fifth family.

⋆ Talk delivered by M. Rosina

There is a danger in this proposal. Either the charged baryon $u_5u_5u_5$ or the charged baryon $d_5d_5d_5$ could be the lightest, depending on whether u_5 or d_5 is lighter. Charged clusters cannot, of course, constitute dark matter. Forming the atoms with the first family electrons they would have far too large scattering amplitude to be consistent with the properties of dark matter. However, if one takes into account also the electro-weak interaction between quarks, then the neutral baryon $n_5 = u_5d_5d_5$ can be the lightest, provided the u-d mass difference is not too large. We have put limits on the u-d quark mass differences in ref. [7] and we briefly repeat the result (choosing $\alpha_{EM} = 1/128,\ \alpha_W = 1/32,\ \alpha_Z = 1/24$).

For superheavy quarks, the colour interaction is assumed to be coulombic and we solve the Hamiltonian for the three-quark system

$$H = 3m_5 + \sum_i \frac{\mathbf{p}_i^2}{2m_5} - \frac{(\sum_i \mathbf{p}_i)^2}{6m_5} - \sum_{i<j} \frac{2}{3}\frac{\alpha_s}{r_{ij}}.$$

For the choice of the average quark mass $m_5 = 100$ TeV and $\alpha_s = 1/13$ the binding energy is $E_0 = -\eta\alpha_s^2 m_5 = -0.39$ TeV and the average quark momentum $p = \sqrt{2m_5\, E_{kin}/3} = 5.1$ TeV . (The coefficient η has been obtained variationally).

The electroweak interaction prefers the neutral $u_5d_5d_5$ and it cannot decay into $d_5d_5d_5$ or $u_5u_5d_5$ provided

$$-0.026\,\text{TeV} \ < m_{u5} - m_{d5} < \ 0.39\,\text{TeV}.$$

This limits are not very narrow, but they are narrow compared to the mass scale of $m_5 = 100$ TeV.

2 The weak $(u_5d_5d_5)$ – $(u_1d_1d_1)$ cross section

It is easy to calculate the scattering amplitude since the superheavy neutron is a point particle compared to the range of the weak interaction and its quark structure is not important. Only Z-exchange matters since there is not enough energy to excite $u_5d_5d_5$ into $d_5d_5d_5$ or $u_5u_5d_5$ via W-exchange. We consider only the scattering on neutron (the “charge” of proton almost happens to cancel!). Also, we consider only the Fermi (vector) matrix element, since it adds coherently in heavy nuclei, while the Gamov-Teller (axial) has many cancellations in spin coupling.

$$\mathcal{M} = [\frac{1}{2}t_0(1) - \sin^2\vartheta_W e(1)]\ \frac{g_Z^2}{m_Z^2}\ [\frac{1}{2}t_0(5) - \sin^2\vartheta_W e(5)] = \frac{G_F}{2\sqrt{2}}$$

$$\sigma_n = 2\pi|\mathcal{M}|^2\,\frac{4\pi p_1^2}{(2\pi)^3 v^2} = \frac{m_{n1}^2}{\pi}\,|\mathcal{M}|^2 = \frac{G_F^2 m_{n1}^2}{8\pi} = 1.9\times 10^{-13}\text{fm}^2\,.$$

We should note that the cross section does not depend on the mass m_{n5} provided it is much larger than m_{n1} of the first family. For a heavy target

$$\sigma_A = \sigma_n\,(A-Z)^2A^2$$

The rate at a detector of ${}^{23}_{11}\mathrm{Na}\ {}^{127}_{53}\mathrm{I}$ per kilogram of detector is

$$R_{1kg} = \sigma_A \, N_A \, \frac{\rho_{n5} \times v}{m_{n5}}$$

$$R_{1kg} = \sigma_n \, [(A_{Na} - Z_{Na})^2 A_{Na}^2 + (A_I - Z_I)^2 A_I^2] \, \frac{N_{avogadro}}{A_{Na} + A_I} \, \frac{\rho_{n5} \times v}{m_{n5}} = 1.3/\mathrm{day}$$

We used the data $\rho_{n5} = 0.3\,\mathrm{GeV\,cm}^{-3}$, $m_{n5} = 300\,\mathrm{TeV}$, $v = 230\,\mathrm{km/s}$.

This can be compared to the rate claimed by the DAMA collaboration:

$$\Delta R_{1kg}(\mathrm{DAMA}) = 0.02/\mathrm{day}, \quad R_{1kg}(\mathrm{DAMA}) \sim (0.1 \leftrightarrow 1)/\mathrm{day}.$$

This comparison was used to decide about the choice of m_5 in our example. If DAMA results are not confirmed, m_5 should be even larger.

3 The strong MESON – meson cross section

This Section is a **lesson** for a future realistic calculation of the $(u_5 d_5 d_5) - (u_1 d_1 d_1)$ scattering . We want to show that for superheavy quarks the strong cross section is much smaller than the weak cross section and can be neglected. For this purpose we need only an estimate and not a detailed calculation. Meson-meson scattering offers a good estimate since the baryon in a quark-diquark approximation resembles a meson. However, this lesson is very relevant for botomium scattering and for future heavy baryons in the 10-100 GeV region.

Here we present the trial functions of the light and heavy meson, together with relevant quantities such as the chromomagnetic dipole moment D of the heavy meson sitting in the dipole field G of the light meson. Note that m and M are quark masses and $\alpha = \frac{4}{3}\alpha_s$.

$$\mathbf{r} = \mathbf{r_q} - \mathbf{r_{\bar{q}}}, \quad b = 1/(\tfrac{1}{2}m)\alpha \qquad \mathbf{R} = \mathbf{R_Q} - \mathbf{R_{\bar{Q}}}, \quad B = 1/(\tfrac{1}{2}M)\alpha \ll b$$

$$\psi_0 = (2/\sqrt{4\pi b^3}) \, \exp(-r/b) \qquad \Psi_0 = (2/\sqrt{4\pi B^3}) \, \exp(-R/B)$$

$$\psi_z = \frac{2^{-3/2}}{\sqrt{4\pi f^3}} \, (r/f) \cos\vartheta \, \exp(-r/f) \qquad \Psi_z = \frac{2^{-3/2}}{\sqrt{4\pi B^3}} \, (R/B) \cos\Theta \, \exp(-R/B)$$

$$\epsilon_0 = -(1/2)(\tfrac{1}{2}m)\alpha^2 \qquad E_0 = -(1/2)(\tfrac{1}{2}M)\alpha^2$$

$$\epsilon_{z,kin} = +(1/8)(\tfrac{1}{2}m)\alpha^2 \, (b/f)^2 \qquad E_z = -(1/8)(\tfrac{1}{2}M)\alpha^2$$

$$G_z = \langle \psi_z | z/(r/2)^3 | \psi_0 \rangle = \gamma/\sqrt{fb^3} \qquad D = \langle \Psi_z | Z | \Psi_0 \rangle = \beta B$$

$$\gamma = 16\sqrt{2}/3 = 7.542 \qquad \beta = 2^{15/2}/3^5 = 0.745$$

The meson wavefunctions get "decorated" with colour factors

$$\phi_0 = \psi_0 \, \frac{(r[gb] + g[br] + b[rg])}{\sqrt{3}}, \quad \phi_{z3} = \psi_z \, \frac{(r[gb] - g[br])}{\sqrt{2}}$$

$$\Phi_0 = \Psi_0 \, \frac{(r[gb] + g[br] + b[rg])}{\sqrt{3}}, \quad \Phi_{z3} = \Psi_z \, \frac{(r[gb] - g[br])}{\sqrt{2}}$$

We write explicitly only the spatial excitation in the z-direction and colour excitation in the "third colour" $\omega = 3$. Others behave similarly.

We shall need the colour matrix element

$$\left\langle \frac{r[gb] - g[br]}{\sqrt{2}} \right| \frac{\lambda_q^3}{2} - \frac{\lambda_{\bar{q}}^3}{2} \left| \frac{r[gb] + g[br] + b[rg]}{\sqrt{3}} \right\rangle = \sqrt{\frac{2}{3}}$$

For color neutral hadrons, the dominant term in the expansion yields the effective dipole–dipole, colour-octet – colour-octet potential

$$\hat{V}_{\text{dipole}} = \alpha_s \left(\mathbf{R_Q} \frac{\overrightarrow{\lambda_Q}}{2} + \mathbf{R_{\bar{Q}}} \frac{\overrightarrow{\lambda_{\bar{Q}}}}{2} \right) \left(\frac{\mathbf{r_q}}{r_q^3} \frac{\overrightarrow{\lambda_q}}{2} + \frac{\mathbf{r_{\bar{q}}}}{r_{\bar{q}}^3} \frac{\overrightarrow{\lambda_{\bar{q}}}}{2} \right),$$

The perturbation term between the unperturbed ground state and the virtual excitation is then

$$V'_{z,3} = \alpha_s \langle \Psi_z \psi_z | \left\{ \frac{Z}{2} \right\} \sqrt{\frac{2}{3}} \left\{ \frac{z/2}{(r/2)^3} \right\} \sqrt{\frac{2}{3}} | \Psi_0 \psi_0 \rangle = \frac{\alpha_s \, D_z \, G_z}{6}$$

$$V'_{x,\omega} = V'_{y,\omega} = V'_{z,\omega} \equiv V' \quad \text{equal for all } \omega \, .$$

The second order perturbation theory then gives the effective potential between the two clusters

$$V_{eff} = -24 \, \frac{{V'}^2}{(E_z - E_0) + \epsilon_{z,kin}}$$

We have neglected $\epsilon_{z,pot}$ and ϵ_0. The factor 24 comes from 3 spacial and 8 colour degrees of freedom.

$$V_{eff} = -\frac{2}{3} \, \frac{(\alpha_s D_z G_z)^2}{(3/8)(\frac{1}{2}M)(4\alpha_s/3)^2 + (1/8)(\frac{1}{2}m)(4\alpha_s/3)^2(b/f)^2}$$

$$V_{eff} = -\frac{2(\beta\gamma \, B)^2}{fb^3(M + (1/3)m(b/f)^2)}$$

Note that α_s has canceled. Minimization with respect to f gives $f/b = \sqrt{m/3M} << 1$. Finally, we get

$$V_{eff} = -\frac{\sqrt{3}\beta^2\gamma^2}{b^3} \left(\frac{m}{M}\right)^{3/2} \frac{B}{m}$$

Here we took the distance between the two clusters $U = 0$. We assume

$$V_{eff}(U) = V_{eff}(U = 0) \exp(-2U/b).$$

In Born approximation (with the mass of the lighter cluster $m_q + m_{\bar{q}} = 2m$) we get

$$a = \frac{(2m)}{2\pi} \int V_{eff}(U) d^3U = \sqrt{3}\beta^2\gamma^2 \left(\frac{m}{M}\right)^{3/2} B.$$

Let us give a numerical example with the choice

$$m = 300\,\text{MeV}, \quad M = \tfrac{1}{2}m_Q = 100\,\text{TeV}, \quad m/M = 3 \cdot 10^{-6}, \quad \alpha_s = 1/13$$

$$a = \sqrt{3}\beta^2\gamma^2 (\frac{m}{M})^{3/2}\, B = 1.1 \cdot 10^{-11}\ \text{fm}$$

$$\sigma = 4\pi a^2 = 1.5 \cdot 10^{-21}\ \text{fm}^2$$

4 Conclusion

Regarding the weak interaction, the scattering rate of superheavy clusters is inversely proportional to their mass because (i) their weak cross section is independent of the heavy mass if it is large enough and (ii) because their number density is inversely proportional to their mass for the known dark matter density. This argument requires the superheavy quark mass to be about 100 TeV (if DAMA experiment is confirmed) or more.

For such a heavy mass, **the strong cross section is MUCH SMALLER than the weak cross section.** The reason is (i) the small size of the heavy hadron, $B = 3.8 \cdot 10^{-5}$ fm and moreover, (ii) the suppression factor $(m/M)^3$ which is a consequence of colour neutrality of both clusters so that they interact only by induced color dipoles (“van der Waals interaction”).

The lesson from the heavy hadron – light hadron scattering will be useful also for not-so-exotic processes such as botomium and bbb scattering.

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