



Poles as a link between QCD and scattering theory (old and contemporary knowledge)

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An overview of existing knowledge about definition of a resonance, and quantification of resonance signals have been given. A special attention has paid to explaining why the definition of a resonance is in principle ill defined mathematical problem [1], and how it is overcome in physics reality [2]. A notion of scattering and resolvent resonances has been introduced, their interconnection and differences have been discussed, and reasons were presented why a pole as a resonance signal is the most acceptable solution [3]. The importance of multi-channel analysis has been demonstrated for pole extraction giving the example of $N(1710) P_{11}$ resonance where single channel πN elastic data are insufficient to establish its existence. Only inclusion of inelastic channels (η production and/or $K\Lambda$ channels) is needed [4]. The dangers when using Breit-Wigner parameters for quantifying resonance properties have been discussed, and use of phase-shift as a link between QCD and scattering theory has been mentioned by using Lüscher's theorem [5]. The present state of the art of baryon spectroscopy has been presented by showing the highlights from the Camogli Workshop [6].

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Complete Experiments for Pion Photoproduction

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Abstract. The possibilities of a model-independent partial wave analysis for pion, eta or kaon photoproduction are discussed in the context of ‘complete experiments’. It is shown that the helicity amplitudes obtained from at least 8 polarization observables including beam, target and recoil polarization can not be used to analyze nucleon resonances. However, a truncated partial wave analysis, which requires only 5 observables will be possible with minimal model assumptions.

1 Introduction

Around the year 1970 people started to think about how to determine the four complex helicity amplitudes for pseudoscalar meson photoproduction from a complete set of experiments. In 1975 Barker, Donnachie and Storrow [1] published their classical paper on ‘Complete Experiments’. After reconsiderations and careful studies of discrete ambiguities [2–4], in the 90s it became clear that such a model-independent amplitude analysis would require at least 8 polarization observables which have to be carefully chosen. There are plenty of possible combinations, but all of them would require a polarized beam and target and in addition also recoil polarization measurements. Technically this was not possible until very recently, when transverse polarized targets came into operation at Mainz, Bonn and JLab and furthermore recoil polarization measurements by nucleon rescattering has been shown to be doable. This was the start of new efforts in different groups in order to achieve the complete experimental information and a model-independent partial wave analysis [5–8].

2 Complete experiments

A complete experiment is a set of measurements which is sufficient to predict all other possible experiments, provided that the measurements are free of uncertainties. Therefore it is first of all an academic problem, which can be solved by mathematical algorithms. In practise, however, it will not work in the same way and either a very high statistical precision would be required, which is very unlikely, or further measurements of other polarization observables are necessary. Both problems, first the mathematical problem but also the problem for a physical experiment can be studied with the help of state-of-the-art models like MAID or partial wave analyses (PWA) like SAID. With high precision calculations the complete sets of observables can be checked and with pseudo-data, generated from models and PWA, real experiments can be simulated under realistic conditions.

2.1 Coordinate Frames

Experiments with three types of polarization can be performed in meson photo-production: photon beam polarization, polarization of the target nucleon and polarization of the recoil nucleon. Target polarization will be described in the frame $\{x, y, z\}$, see Fig. 1, with the z -axis pointing into the direction of the photon momentum $\hat{k}$, the y -axis perpendicular to the reaction plane, $\hat{y} = \hat{k} \times \hat{q} / \sin \theta$, and the x -axis is given by $\hat{x} = \hat{y} \times \hat{z}$. For recoil polarization, traditionally the frame $\{x', y', z'\}$ is used, with the z' -axis defined by the momentum vector of the outgoing meson $\hat{q}$, the y' -axis is the same as for target polarization and the x' -axis given by $\hat{x}' = \hat{y}' \times \hat{z}'$.

The photon polarization can be linear or circular. For a linear photon polarization ($P_T = 1$) in the reaction plane ($\hat{x}, \hat{z}$), $\varphi = 0$. Perpendicular, in direction $\hat{y}$, the polarization angle is $\varphi = \pi/2$. Finally, for right-handed circular polarization, $P_\odot = +1$.

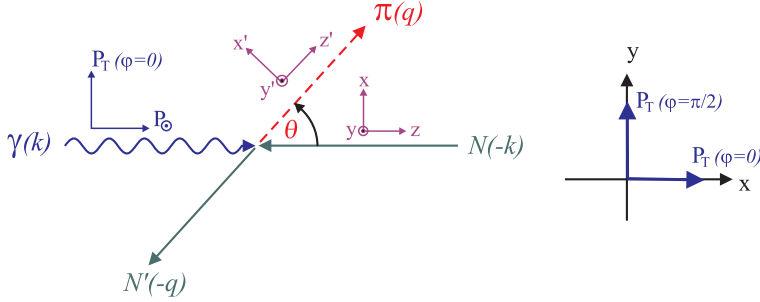


Fig. 1. Frames for polarization vectors in the CM.

The polarized differential cross section can be classified into three classes of double polarization experiments:
polarized photons and polarized target (types (S, BT))

$$\begin{aligned} \frac{d\sigma}{d\Omega} = \sigma_0 \{ & 1 - P_T \Sigma \cos 2\varphi + P_x (-P_T H \sin 2\varphi + P_\odot F) \\ & + P_y (T - P_T P \cos 2\varphi) + P_z (P_T G \sin 2\varphi - P_\odot E) \}, \end{aligned} \quad (1)$$

polarized photons and recoil polarization (types (S, BR))

$$\begin{aligned} \frac{d\sigma}{d\Omega} = \sigma_0 \{ & 1 - P_T \Sigma \cos 2\varphi + P_{x'} (-P_T O_{x'} \sin 2\varphi - P_\odot C_{x'}) \\ & + P_{y'} (P - P_T T \cos 2\varphi) + P_{z'} (-P_T O_{z'} \sin 2\varphi - P_\odot C_{z'}) \}, \end{aligned} \quad (2)$$

polarized target and recoil polarization (types (S, TR))

$$\frac{d\sigma}{d\Omega} = \sigma_0 \{ 1 + P_y T + P_{y'} P + P_{x'} (P_x T_{x'} - P_z L_{x'}) + P_{y'} P_y \Sigma + P_{z'} (P_x T_{z'} + P_z L_{z'}) \}. \quad (3)$$

In these equations σ_0 denotes the unpolarized differential cross section, Σ, T, P are single-spin asymmetries (S) , E, F, G, H the beam-target asymmetries (BT) ,

$O_{x'}, O_{z'}, C_{x'}, C_{z'}$ the beam-recoil asymmetries ($\mathcal{BR}$) and $T_{x'}, T_{z'}, L_{x'}, L_{z'}$ the target-recoil asymmetries ($\mathcal{TR}$). The polarization quantities are described in Fig. 1. The signs of the 16 polarization observables of Eq. (1,2,3) are in principle arbitrary, except for the cross section σ_0 , which is naturally positive. For the 15 asymmetries we use the sign convention of Barker et al. [1], which is also used by the MAID and SAID partial wave analysis groups. For other sign conventions, see Ref. [9].

2.2 Amplitude analysis

Pseudoscalar meson photoproduction has 8 spin degrees of freedom, and due to parity conservation it can be described by 4 complex amplitudes of 2 kinematical variables. Possible sets of amplitudes are: Invariant amplitudes A_i , CGLN amplitudes F_i , helicity amplitudes H_i or transversity amplitudes b_i . All of them are linearly related to each other and further combinations are possible. Most often in the literature the helicity basis was chosen and the 16 possible polarization observables can be expressed in bilinear products

$$O_i(W, \theta) = \frac{q}{k} \sum_{k, \ell=1}^4 \alpha_{k, \ell} H_k(W, \theta) H_\ell^*(W, \theta), \quad (4)$$

where O_1 is the unpolarized differential cross section σ_0 and all other observables are products of asymmetries with σ_0 , for details see Table 1.

From a complete set of 8 measurements $\{O_i(W, \theta)\}$ one can determine the moduli of the 4 amplitudes and 3 relative phases. But there is always an unknown overall phase, e.g. $\phi_1(W, \theta)$, which can not be determined by additional measurements. This is, however, not a principal problem as with the principally undetermined phase of a quantum mechanical wave function. Already in 1963 Goldberger et al. [10] discussed a method using the idea of a Hanbury-Brown and Twiss experiment, and very recently in 2012, Ivanov [11] discussed another method using vortex beams to measure the phase of a scattering amplitude. Both methods, however, are highly impractical for a meson photoproduction experiment.

Therefore, the complete information is contained in a set of 4 reduced amplitudes,

$$\tilde{H}_i(W, \theta) = H_i(W, \theta) e^{-i \phi_1(W, \theta)} \quad (5)$$

of which $\tilde{H}_1$ is a real function, the others are complex, resulting in a total of 7 real values for any given W and θ .

Figure 2 shows two of such amplitude analyses with a complete set of 8 observables and an overcomplete set of 10 observables. The data used for this analysis has been generated as pseudo-data from Monte-Carlo events according to the Maid2007 solution, see Sect. 3. The figure shows the real parts of two out of four reduced helicity amplitudes, $\text{Re} \tilde{H}_1$ and $\text{Re} \tilde{H}_4$. While the solution with the complete set of 8 observables results in a rather bad description of the true amplitudes, the solution of the overcomplete set gives a satisfactory result.

Table 1. Spin observables for pseudoscalar meson photoproduction involving beam, target and recoil polarization in 4 groups, $\mathcal{S}, \mathcal{BT}, \mathcal{BR}, \mathcal{TR}$. A phase space factor q/k has been omitted in all expressions and the asymmetries are given by $A = \hat{A}/\sigma_0$. In column 2 the observables are expressed in terms of the Walker helicity amplitudes [12] and in column 3 in $\sin \theta$ and $x = \cos \theta$ with the leading terms for an S, P wave truncation.

Spin Obs	Helicity Representation	Partial Wave Expansion
σ_0	$\frac{1}{2}(H_1 ^2 + H_2 ^2 + H_3 ^2 + H_4 ^2)$	$A_0^\sigma + A_1^\sigma x + A_2^\sigma x^2 + \dots$
$\hat{\Sigma}$	$\text{Re}(H_1 H_4^* - H_2 H_3^*)$	$\sin^2 \theta (A_0^\Sigma + \dots)$
$\hat{T}$	$\text{Im}(H_1 H_2^* + H_3 H_4^*)$	$\sin \theta (A_0^T + A_1^T x + \dots)$
$\hat{P}$	$-\text{Im}(H_1 H_3^* + H_2 H_4^*)$	$\sin \theta (A_0^P + A_1^P x + \dots)$
$\hat{G}$	$-\text{Im}(H_1 H_4^* + H_2 H_3^*)$	$\sin^2 \theta (A_0^G + \dots)$
$\hat{H}$	$-\text{Im}(H_1 H_3^* - H_2 H_4^*)$	$\sin \theta (A_0^H + A_1^H x + \dots)$
$\hat{E}$	$\frac{1}{2}(- H_1 ^2 + H_2 ^2 - H_3 ^2 + H_4 ^2)$	$A_0^E + A_1^E x + A_2^E x^2 + \dots$
$\hat{F}$	$\text{Re}(H_1 H_2^* + H_3 H_4^*)$	$\sin \theta (A_0^F + A_1^F x + \dots)$
$\hat{O}_{x'}$	$-\text{Im}(H_1 H_2^* - H_3 H_4^*)$	$\sin \theta (A_0^{O_{x'}} + A_1^{O_{x'}} x + A_2^{O_{x'}} x^2 + \dots)$
$\hat{O}_{z'}$	$\text{Im}(H_1 H_4^* - H_2 H_3^*)$	$\sin^2 \theta (A_0^{O_{z'}} + A_1^{O_{z'}} x + \dots)$
$\hat{C}_{x'}$	$-\text{Re}(H_1 H_3^* + H_2 H_4^*)$	$\sin \theta (A_0^{C_{x'}} + A_1^{C_{x'}} x + A_2^{C_{x'}} x^2 + \dots)$
$\hat{C}_{z'}$	$\frac{1}{2}(- H_1 ^2 - H_2 ^2 + H_3 ^2 + H_4 ^2)$	$A_0^{C_{z'}} + A_1^{C_{z'}} x + A_2^{C_{z'}} x^2 + A_3^{C_{z'}} x^3 + \dots$
$\hat{T}_{x'}$	$\text{Re}(H_1 H_4^* + H_2 H_3^*)$	$\sin^2 \theta (A_0^{T_{x'}} + A_1^{T_{x'}} x + \dots)$
$\hat{T}_{z'}$	$\text{Re}(H_1 H_2^* - H_3 H_4^*)$	$\sin \theta (A_0^{T_{z'}} + A_1^{T_{z'}} x + A_2^{T_{z'}} x^2 + \dots)$
$\hat{L}_{x'}$	$-\text{Re}(H_1 H_3^* - H_2 H_4^*)$	$\sin \theta (A_0^{L_{x'}} + A_1^{L_{x'}} x + A_2^{L_{x'}} x^2 + \dots)$
$\hat{L}_{z'}$	$\frac{1}{2}(H_1 ^2 - H_2 ^2 - H_3 ^2 + H_4 ^2)$	$A_0^{L_{z'}} + A_1^{L_{z'}} x + A_2^{L_{z'}} x^2 + A_3^{L_{z'}} x^3 + \dots$

2.3 Truncated partial wave analysis

Even with the help of unitarity in form of Watson's theorem, the angle-dependent phase $\phi_1(W, \theta)$ cannot be provided. This has very strong consequences, namely a partial wave decomposition would lead to wrong partial waves, which would be useless for nucleon resonance analysis. It becomes obvious in the following schematic formula

$$f_\ell(W) = \frac{2}{2\ell+1} \int \tilde{H}(W, \theta) e^{i\phi(W, \theta)} P_\ell(\cos \theta) d\cos \theta, \quad (6)$$

where the desired partial wave $f_\ell(W)$ cannot be obtained from the reduced helicity amplitudes $\tilde{H}(W, \theta)$ alone, as long as the angle dependent phase $\phi(W, \theta)$ is unknown.

Our main goal in the data analysis of photoproduction is the search for nucleon resonances and their properties. To better reach this goal, one can directly perform a partial wave analysis from the observables without going through the underlying helicity amplitudes. Such an analysis would be a truncated partial wave analysis (TPWA) with a minimal model dependence (i) from the truncation of the series at a maximal angular momentum $\ell_{\max}$ and (ii) from an overall unknown phase as in the case of the amplitude analysis in the previous paragraph. However, in the TPWA the overall phase would be only a function of energy and with additional theoretical help it can be constrained without strong model as-

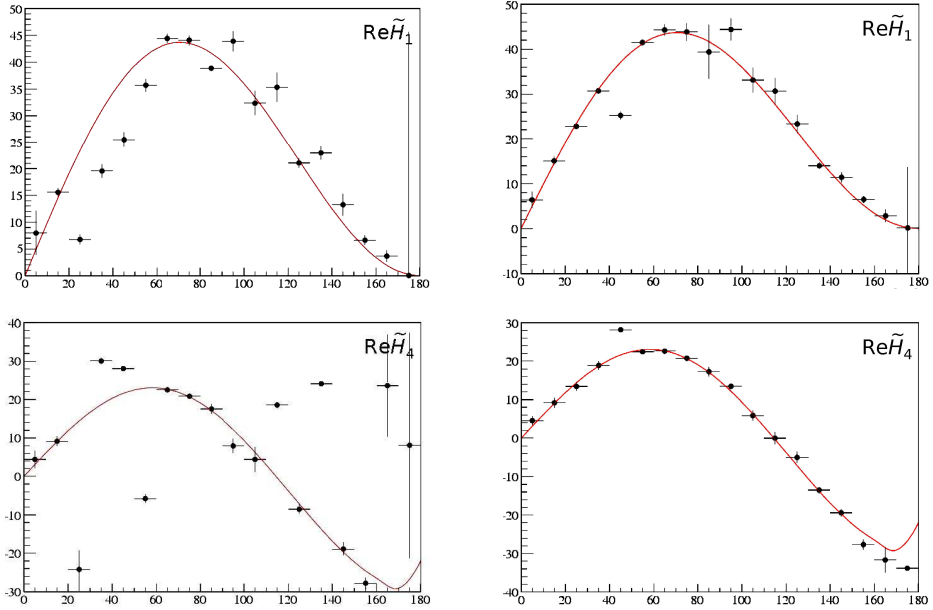


Fig. 2. Comparison of the reduced helicity amplitudes $\text{Re}\tilde{H}_1$ and $\text{Re}\tilde{H}_4$ between a pseudo-data analysis with a complete dataset of 8 observables: $\sigma_0, \Sigma, T, P, E, G, O_{x'}, C_{x'}$ (left 2 panels) and with an overcomplete dataset of 10 observables with additional F, H (right 2 panels) for $\gamma p \rightarrow \pi^0 p$ at $E = 320$ MeV as a function of the c.m. angle θ . The solid red curves show the MAID2007 solutions. Amplitudes are in units of $10^{-3}/m_{\pi^+}$.

sumptions. Such a concept was already discussed and applied for γ, π in the 80s by Grushin [13] for a PWA in the region of the $\Delta(1232)$ resonance.

Formally, the truncated partial wave analysis can be performed in the following way. All observables can be expanded either in a Legendre series or in a $\cos \theta$ series

$$O_i(W, \theta) = \frac{q}{k} \sin^{\alpha_i} \theta \sum_{k=0}^{2\ell_{\max} + \beta_i} A_k^i(W) \cos^k \theta, \quad (7)$$

$$A_k^i(W) = \sum_{\ell, \ell'=0}^{\ell_{\max}} \sum_{k, k'=1}^4 \alpha_{\ell, \ell'}^{k, k'} \mathcal{M}_{\ell, k}(W) \mathcal{M}_{\ell', k'}^*(W), \quad (8)$$

where k, k' denote the 4 possible electric and magnetic multipoles for each πN angular momentum $\ell \geq 2$, namely $\mathcal{M}_{\ell, k} = \{E_{\ell+}, E_{\ell-}, M_{\ell+}, M_{\ell-}\}$. For an S, P truncation ($\ell_{\max} = 1$) there are 4 complex multipoles $E_{0+}, E_{1+}, M_{1+}, M_{1-}$ leading to 7 free real parameters and an arbitrary phase, which can be put to zero for the beginning. In Table 1 we list the expansion coefficients for all observables that appear in an S, P wave expansion. Already from the 8 observables of the first two groups ($\mathcal{S}, \mathcal{BT}$) one can measure a set of 16 coefficients, from which we only need 8 well selected ones for a unique mathematical solution. This can be achieved by a measurement of the angular distributions of only 5 observables, e.g. $\sigma_0, \Sigma, T, P, F$ or $\sigma_0, \Sigma, T, F, G$. In the first example one gets even 10 coefficients, from which e.g.

A_1^P and A_0^F can be omitted. In the second case, there are 9 coefficients, of which A_0^F can be omitted. In practise one can select those coefficients, which have the smallest statistical errors, and therefore, the biggest impact for the analysis by keeping in mind that all discrete ambiguities are resolved.

As has been shown by Omelaenko [14] the same is true for any PWA with truncation at $\ell_{\max}$. For the determination of the $8\ell_{\max} - 1$ free parameters one has the possibility to measure $(8\ell_{\max}, 8\ell_{\max}, 8\ell_{\max} + 4, 8\ell_{\max} + 4)$ coefficients for types (S, BT, BR, TR) , respectively.

3 Partial wave analysis with pseudo-data

In a first numerical attempt towards a model-independent partial wave analysis, a procedure similar to the second method, the TPWA, described above, has been applied [6], and pseudo-data, generated for γ, π^0 and γ, π^+ have been analyzed.

Events were generated over an energy range from $E_{\text{lab}} = 200 - 1200$ MeV and a full angular range of $\theta = 0 - 180^\circ$ for beam energy bins of $\Delta E_\gamma = 10$ MeV and angular bins of $\Delta\theta = 10^\circ$, based on the MAID2007 model predictions [15]. For each observable, typically $5 \cdot 10^6$ events have been generated over the full energy range. For each energy bin a single-energy (SE) analysis has been performed using the SAID PWA tools [16].

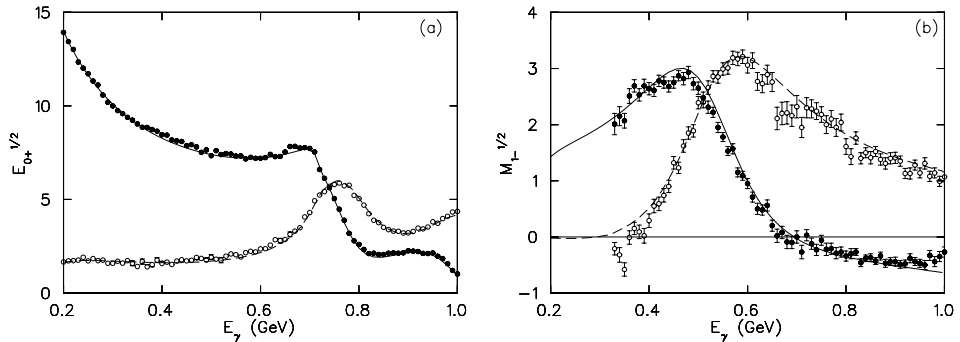


Fig. 3. Real and imaginary parts of (a) the S_{11} partial wave amplitude $E_{0+}^{1/2}$ and (b) the P_{11} partial wave amplitude $M_{1-}^{1/2}$. The solid (dashed) line shows the real (imaginary) part of the MAID2007 solution, used for the pseudo-data generation. Solid (open) circles display real (imaginary) single-energy fits (SE6p) to the following 6 observables without any recoil polarization measurement: $d\sigma/d\Omega$, two single-spin observables Σ , T and three beam-target double polarization observables E , F , G . Multipoles are in millifermi units.

A series of fits, SE4p, SE6p and SE8p have been performed [6] using 4, 6 and 8 observables, respectively. Here the example using 6 observables (σ_0 , Σ , T , E , F , G) is demonstrated, where no recoil polarization has been used. As explained before, such an experiment would be incomplete in the sense of an ‘amplitude analysis’, but complete for a truncated partial wave analysis. In Fig. 3 two multipoles $E_{0+}^{1/2}$ and $M_{1-}^{1/2}$ for the S_{11} and P_{11} channels are shown and the SE6p fits are compared to the MAID2007 solution. The fitted SE solutions are very close to the MAID

solution with very small uncertainties for the S_{11} partial wave. For the P_{11} partial wave we obtain a larger statistical spread of the SE solutions. This is typical for the $M_{1-}^{1/2}$ multipole, which is generally much more difficult to obtain with good accuracy [15], because of the weaker sensitivity of the observables to this magnetic multipole. But also this multipole can be considerably improved in an analysis with 8 observables [6].

4 Summary and conclusions

It is shown that for an analysis of N^* resonances, the amplitude analysis of a complete experiment is not very useful, because of an unknown energy and angle dependent phase that can not be determined by experiment and can not be provided by theory without a strong model dependence. However, the same measurements or even less will be very useful for a truncated partial wave analysis with minimal model dependence due to truncations and extrapolations of Watson's theorem in the inelastic energy region. A further big advantage of such a PWA is a different counting of the necessary polarization observables, resulting in very different sets of observables. While it is certainly helpful to have polarization observables from 3 or 4 different types, for a mathematical solution of the bilinear equations one can find minimal sets of only 5 observables from only 2 types, where either a polarized target or recoil polarization measurements can be completely avoided.

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