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6 Gravitational Effects for Dirac Particles *

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Abstract. We present an update on recent advances in the theory of Dirac particles in curved space-times. The basic formulation behind the covariant coupling of the Dirac bispinor to space-time geometry is briefly reviewed including the appropriate covariant action principle. A number of central-field problems have recently been analyzed; all of these depend on a concrete, explicit evaluation of the spin connection matrices for particular space-time geometries; the relevant results are discussed. The generalization of the formalism to tachyonic spin-1/2 particles is rather straightforward and allows the identification of the leading interaction terms for high-energy tachyons, which approach the light cone. The combination of quantum electrodynamics on curved space-time backgrounds may seem like a far-fetched field of research, but recent claims in the field have shaken the foundations of fundamental principles of general relativity. We show that a careful consideration of the vacuum polarization integral, with a gravitational effective mass, restores the validity of the weak equivalence principle in deep gravitational potentials.

Povzetek. Poročava o nedavnem napredku v teoriji Diracovih delcev v ukrivljenem prostoru-času. Na kratko predstaviva kovariantno sklopitev Diracovega bispinorja z geometrijo prostor-časa s in ustrezno kovariantno akcijo. Na kratko predstaviva nove dosžke pri iskanju rešitev za Diracov delec v več različnih centralnih potencialih, ki so se pojavili v zadnjem času. Vsi uporabijo matrike spinskih povezav. Ta pristop posplošiva na tahionske delce s spinom 1/2, kar nama omogoči prepoznati vodilne člene interakcije za skoraj brezmasne tahione na svetlobnemu stožcu. Povezava kvantne elektrodinamike v ukrivljenem prostoru-času se zdi zanimiva ob trditvah o morebitni neveljavnosti splošne teorije relativnosti. Vendar s skrbno obravnavo polarizacije vakuuma (z gravitacijsko efektivno maso) pokaževa veljavnost načela šibke ekvivalence v globokih gravitacijskih potencialih in s tem zmotnost teh trditev.

6.1 Introduction

The coupling of a spin-1/2 particle to the gauge fields via the covariant derivative, within the Dirac equation, has been central to the formulation of the Standard Model of Elementary Interactions, and to the understanding of the properties of antiparticles. It is much less common wisdom how to couple a Dirac particle to a curved space-time geometry. Some naive guesses fail. In flat space, the Clifford algebra of Dirac matrices was formulated [1,2] to fulfill the fundamental anticommutator relation

$$\{\gamma^\mu, \gamma^\nu\} = \eta^{\mu\nu} \tag{6.1}$$

* Talk delivered by U.D. Jentschura

where $\eta^{\mu\nu}$ is the flat space-time metric $\eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. In curved space, this relation has to be generalized to

$$\{\overline{\gamma}^{\mu}(x), \overline{\gamma}^{\nu}(x)\} = \overline{g}^{\mu\nu}(x)\,, \tag{6.2}$$

where $\overline{g}^{\mu\nu}(x)$ is the curved space-time metric, and the Dirac matrices become coordinate-dependent. (We choose to denote the curved-space metric by $\overline{g}$ in order to avoid confusion with the flat-space counterpart, which is usually denoted by g in elementary physics.)

However, one would be mistaken to simply replace $\gamma^{\mu} \to \overline{\gamma}^{\mu}$ in the Dirac equation in order to couple the Dirac particle to space-time curvature, or, to simple insert the gravitational potential $V = -G m_1 m_2 / r$ into the Dirac equation by hand. Both approaches fail because they are not covariant with respect to Lorentz transformations of curved space-time. In particular, the simple insertion of the gravitational potential into the Dirac equation would lead to a different equation of motion for the Dirac particle under a change of the space-time coordinates, which is unacceptable. The requirement of covariance under local Lorentz transformations leads to the definition of the spin connection matrices, and to the covariant derivative for a spinor in curved space time.

Here, we briefly review some recent works on related topics, which are based on the concrete evaluation of the spin connection matrices for particular space-time geometries, and discuss an application to quantum electrodynamics in curved space-time, namely, the gravitational correction to vacuum polarization. We use units with $\hbar = c = \epsilon_0 = 1$ and employ the standard representation for the Dirac matrices [3,4] (the "standard" speed of light is sometimes denoted as c_0, for reasons apparent from the context of the discussion on conceivable tiny deviations induced by quantum phenomena).

6.2 Dirac Particles and Curved Space–Time

In order to fix ideas [5], let us recall that the vierbein e^{A}_{μ} (the "square root of the metric") describes the connection of the curved-space and flat-space metrics,

$$\overline{g}_{\mu\nu}(x) = e^{A}_{\mu}\, e^{B}_{\nu}\, \eta_{AB}\,, \qquad \eta_{AB} = \mathrm{diag}(1,-1,-1,-1)\,. \tag{6.3}$$

The completeness of the vierbein implies that both the "local" (nonholonomic) as well as the "global" (holonomic) indices can be raised and lowered using the metric(s) η and $\overline{g}$. In particular, one has

$$e^{\mu}_{A}\, e_{\mu B} = \eta_{AB}\,, \qquad e^{A}_{\mu}\, e_{\nu A} = \overline{g}_{\mu\nu}(x) \tag{6.4}$$

The connection of the flat-space ($\widetilde{\gamma}$) and curved-space ($\overline{\gamma}$) Dirac matrices is given as follows,

$$\overline{\gamma}_{\mu}(x) = e^{A}_{\mu}\, \widetilde{\gamma}_{A}\,, \qquad \{\widetilde{\gamma}^{A}, \widetilde{\gamma}^{B}\} = \eta^{AB}\,, \qquad \{\overline{\gamma}_{\mu}(x), \overline{\gamma}_{\nu}(x)\} = \overline{g}_{\mu\nu}(x)\,. \tag{6.5}$$

Local Lorentz transformations lead to a reparameterization of the "internal" space,

$$e'^{\mu A}(x) = \Lambda^{A}{}_{B}(x)\, e^{\nu B}(x)\,, \qquad e'^{\mu}_{A}(x) = \Lambda_{A}{}^{B}(x)\, e^{\nu}_{B}(x)\,, \tag{6.6}$$

The Ricci rotation coefficient ω_ν^{AB} is obtained from the covariant derivative of an anholonomic basis vector,

$$\vec{e}_A = e^\mu_A \, \vec{e}_\mu \,, \qquad \vec{e}^{\,B} = e^{\mu B} \, \vec{e}_\mu \,,$$

$$\partial_\nu \vec{e}^{\,B} = \left(\nabla_\nu \, e^{\mu B}\right) \vec{e}_\mu = e^A_\mu \, \left(\nabla_\nu \, e^{\mu B}\right) \vec{e}_A \equiv \omega_\nu^{AB} \, \vec{e}_A \,. \tag{6.7}$$

It is given, in terms of the vierbein and Christoffel symbols, as follows,

$$\omega_\nu^{AB} = e^A_\mu \, \nabla_\nu \, e^{\mu B} = e^A_\mu \, \partial_\nu \, e^{\mu B} + e^A_\mu \, \Gamma^\mu_{\nu\lambda} \, e^{\lambda B} \,. \tag{6.8}$$

A local spinor Lorentz transformation with generators $\Omega^{AB}(x)$ transforms the bispinor ψ according to

$$\psi'(x') = S(\Lambda(x)) \, \psi(x) = \left(1 - \frac{i}{4} \, \Omega^{AB}(x) \, \widetilde{\sigma}_{AB}\right) \, \psi(x) \tag{6.9}$$

The flat-space spin matrices $\widetilde{\sigma}_{AB}$ are given as

$$\widetilde{\sigma}_{AB} = \frac{i}{2} \, [\widetilde{\gamma}_A, \widetilde{\gamma}_B] \tag{6.10}$$

The covariant derivative ∇_μ of a spinor contains the spin connection matrix $\Gamma_\mu(x)$,

$$\Gamma_\mu(x) = \frac{i}{4} \, \omega_\mu{}^{AB}(x) \, \widetilde{\sigma}_{AB} \,,$$

$$\nabla_\mu \psi(x) = \left(\partial_\mu - \frac{i}{4} \, \omega_\mu{}^{AB}(x) \, \widetilde{\sigma}_{AB}\right) \psi(x) = (\partial_\mu - \Gamma_\mu) \, \psi(x) \tag{6.11}$$

A change of the vierbein according to Eq. (6.6) leads to a different form of the Ricci rotation coefficients, and of the spin connection matrices,

$$\nabla'_\mu \psi(x) = \left(\partial_\mu - \frac{i}{4} \, \omega'_\mu{}^{AB}(x) \, \widetilde{\sigma}_{AB}\right) \psi(x) = \left(\partial_\mu - \Gamma'_\mu\right) \psi(x) \tag{6.12}$$

but the covariance with respect to the local Lorentz transformation is ensured by the relationship [5],

$$\nabla'_\mu \, [S(\Lambda(x))\psi(x)] = S(\Lambda(x)) \, \nabla_\mu \, \psi(x) \,, \tag{6.13}$$

in accordance with the underlying idea of the covariant derivative. From the action

$$S = \int d^4x \, \sqrt{-\det \overline{g}(x)} \; \overline{\psi}(x) \, \left(\frac{i}{2} \gamma^\mu(x) \overleftrightarrow{\nabla}_\mu - m\right) \psi(x) \,,$$

one derives the gravitationally coupled Dirac equation

$$(i\gamma^\mu \, \nabla_\mu - m)\psi(x) = 0 \,. \tag{6.14}$$

It is straightforward to generalize this formalism to tachyons, which in flat spacetime are described by the tachyonic Dirac equation,

$$(i\widetilde{\gamma}^\mu \, \nabla_\mu - \widetilde{\gamma}^5 \, m)\psi(x) = 0 \,. \tag{6.15}$$

In curved space-time [6,7], the generalization of the γ^5 matrix reads as

$$\overline{\gamma}^5(x) = \frac{i}{4!} \frac{\epsilon_{\mu\nu\rho\delta}}{\sqrt{-\det \overline{g}(x)}} \widetilde{\gamma}^\mu(x)\, \widetilde{\gamma}^\nu(x)\, \widetilde{\gamma}^\rho(x)\, \widetilde{\gamma}^\delta(x)\,, \tag{6.16}$$

the action becomes

$$S = \int d^4x\, \sqrt{-\det \overline{g}(x)}\; \overline{\psi}(x)\, \overline{\gamma}^5(x)\, \left(\frac{i}{2} \gamma^\rho(x) \overleftrightarrow{\nabla}_\rho - \overline{\gamma}^5(x)\, m \right) \psi(x)\,,$$

from which one derives the gravitationally coupled tachyonic Dirac equation as

$$\left[i\overline{\gamma}^\mu\, \nabla_\mu - \overline{\gamma}^5(x)\, m \right]\, \psi(x) = 0\,. \tag{6.17}$$

Based on this formalism, a number of very concrete and definite problems have recently been investigated [8–10,7], mainly for time-independent, central-field curved-spacetime configurations. The Dirac bispinor ψ describes both particle ("electron") as well as antiparticle ("positron") states. A symmetry of particle and anti-particle solutions has been uncovered in Ref. [8] for the Schwarzschild space-time geometry; it implies that, on the level of Newtonian and Einsteinian geometrodynamics, antiparticles are attracted in central gravitational fields in the same way as particles are (including all relativistic corrections of motion, and within a quantum dynamical formalism). A conceivable deviation of the gravitational interactions for particles and antiparticles therefore would be indicative of a fifth fundamental force [8]. We also found the nonrelativistic limit of the Dirac-Schwarzschild Hamiltonian and identified the gravitational spin-orbit coupling, and gravitational zitterbewegung term [9]. The leading relativistic corrections terms are obtained after a Foldy–Wouthuysen transformation [11], and read [9]

$$\begin{aligned} H_{FW} &= \beta \left(m + \frac{\vec{p}^{\,2}}{2m} - \frac{\vec{p}^{\,4}}{8m^3} \right) - \beta\, \frac{m\, r_s}{2\, r} \\ &+ \beta \left(-\frac{3r_s}{8m} \left\{ \vec{p}^{\,2}, \frac{1}{r} \right\} + \frac{3\pi r_s}{4m} \delta^{(3)}(\vec{r}) + \frac{3r_s}{8m}\, \frac{\vec{\Sigma} \cdot \vec{L}}{r^3} \right)\,. \end{aligned} \tag{6.18}$$

Here, $r_s = 2GM$ is the Schwarzschild radius. The spectrum of a purely gravitationally coupled bound state in a central field was studied, including the relativistic corrections [12], and the analogue of the electromagnetic fine-structure constant for gravity was identified [12]. Furthermore, it has recently been clarified, based on tachyonic gravitationally coupled Dirac equation (6.17), that the leading term in gravitational central fields actually is *attractive* for tachyons, in full agreement with the fact that tachyons become "luxons" in the high-energy domain (they approach the light cone), and they thus are attracted to gravitational centers much like photons. However, several correction terms are repulsive for tachyons, in contrast to their attractive counterparts for tardyons [7]. Namely, according to Ref. [7], One finds for tardyons (in the high-energy limit, with $\mathcal{E} = -\vec{\Sigma} \cdot \vec{p}$)

$$\begin{aligned} \mathcal{H}_{ds} = \beta \Bigg(& \mathcal{E} + \frac{m^2}{2\mathcal{E}} - \frac{1}{2} \left\{ \mathcal{E}, \frac{r_s}{r} \right\} + \frac{9}{32} \left\{ \mathcal{E}, \frac{r_s^2}{r^2} \right\} \\ & - \frac{7\, m^2}{64} \left\{ \frac{1}{\mathcal{E}}, \frac{r_s^2}{r^2} \right\} + \frac{3\, m^2}{16} \frac{r_s}{r} \frac{1}{\mathcal{E}} \frac{r_s}{r} \Bigg)\,, \end{aligned} \tag{6.19}$$

while for tachyons

$$\mathcal{H}_{tg} = \beta \left(\mathcal{E} - \frac{m^2}{2\mathcal{E}} - \frac{1}{2} \left\{ \mathcal{E}, \frac{r_s}{r} \right\} + \frac{9}{32} \left\{ \mathcal{E}, \frac{r_s^2}{r^2} \right\} + \frac{7\,m^2}{64} \left\{ \frac{1}{\mathcal{E}}, \frac{r_s^2}{r^2} \right\} - \frac{3\,m^2}{16} \frac{r_s}{r} \frac{1}{\mathcal{E}} \frac{r_s}{r} \right) . \tag{6.20}$$

Here, "ds" and "tg" refer to the identifications of the Hamiltonians as "Dirac Schwarzschild" and "tachyonic gravitational", respectively. The final two terms in these Hamiltonians have opposite signs, indicating a difference in the gravitational interaction for tachyons and tardyons.

We should clarify that, in order to couple a Dirac particle to space-time curvature, it is not necessary to quantize space-time. The spin connection matrices mediate the coupling to the "classical" (not quantum) space-time geometry, and they ensure the covariance of the covariant derivative under local Lorentz transformations (in a nonholonomic basis).

6.3 Speed of Light in Deep Gravitational Potentials

First, it's necessary to remember that the speed of light is not as "constant" as one would a priori assume, when expressed in global coordinates. According to Eq. (5) of Ref. [13], the space-time metric for static, weak gravitational fields reads as

$$ds^2 = (1 + 2\,\Phi_G(\vec{r}))\; dt^2 - (1 - 2\,\Phi_G(\vec{r}))\; d\vec{r}^{\,2} , \tag{6.21}$$

where Φ_G is the gravitational potential. Light travels on a null geodesic, with $ds^2 = 0$, and so

$$\left(\frac{d\vec{r}}{dt} \right)^2 = \frac{1 + 2\,\Phi_G(\vec{r})}{1 - 2\,\Phi_G(\vec{r})} \approx 1 + 4\,\Phi_G(\vec{r}) . \tag{6.22}$$

The local speed of light, expressed in terms of the global coordinates, thus is

$$\left| \frac{d\vec{r}}{dt} \right| = 1 + 2\,\Phi_G(\vec{r}) , \qquad \Delta c = 2\,\Phi_G(\vec{r}) = (1 + \gamma)\,\Phi_G(\vec{r}) < 0 . \tag{6.23}$$

In a central field, we have $\Phi_G(\vec{r}) = -GM/r$. Deviations from $\gamma = 1$ parameterize departures from standard geometrodynamics [14–16]. For further discussion, we also refer to Chap. 4.4 on page 196 ff. of Ref. [17], Eq. (4.43) of Ref. [18] and Sec. 4.5.2 of Ref. [18], as well as Ref. [19]. The effect parameterized by Eq. (6.23) is known as the Shapiro time delay [20–24].

Some attention [25] was recently directed to a recent paper [26] where it was claimed that quantum electrodynamics, when considering the gravitational correction to the fermion propagators, yields an additional correction to the speed of light, parameterized as

$$\delta c_\gamma = \frac{9}{64}\,\alpha\,\frac{\Phi_G(\vec{r})}{c_0^2} < 0 , \tag{6.24}$$

slowing down photons as compared to other high-energy particles, which approach the "unperturbed light cone". The reason for the special role of photons is claimed to be due to the fact that vacuum polarization, on shell, receives a tiny correction due to the gravitational interactions of the fermions in the loop, which in turn displaces the photon ever so slightly from the flat space-time mass shell.

First doubts arise because the value

$$\gamma - 1 = \chi\,\alpha = \frac{9\,\alpha}{64} = 1.03 \times 10^{-3}\,. \tag{6.25}$$

is in disagreement with the bounds set by radar reflection from the the Cassini observations [27] in superior conjunction, which reads as follows,

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}\,. \tag{6.26}$$

Further conceptual difficulties result because, one might otherwise perform a thought experiment and enter a region of deep gravitational potential with three freely falling, propagating wave packets, one describing a photon, the others describing a very highly energetic neutrino and a very highly energetic electron. The latter two propagate at a velocity (infinitesimally close to) the flat-space speed of light c_0. If a correction of the form δc_γ exists, then photons will have been decelerated to a velocity $c_0 - |\delta c_\gamma|$ within the gravitational potential, whereas *both* fermions retain a velocity (infinitesimally close to) c_0. If we regard the photons as particles, then we could argue that a "force" must have acted onto the photon, causing deceleration, even though the particles were in free fall, leading to violating of the weak equivalence principle. However, the claim (6.24) is of quantum origin and therefore beyond the realm of applicability of standard classical general relativity; it is thus hard to refute based on first principles.

It thus remains to calculate the leading correction to vacuum polarization in gravitational fields using the gravitationally coupled Dirac equation. According to Eq. (12) of Ref. [8], the leading term is

$$H = \vec{\alpha} \cdot \vec{p} + \beta\, m\, w(\vec{r})\,, \qquad w(\vec{r}) \approx 1 + \Phi_G\,, \tag{6.27}$$

leading to an effective mass of the fermion,

$$m_{\rm eff} = m\, w(r) \approx m\,(1 + \Phi_G)\,, \tag{6.28}$$

which needs to be inserted into the covariant representation of the one-loop vacuum insertion into the photon propagator,

$$\frac{g_{\mu\nu}}{k^2} \to \frac{g_{\mu\nu}}{k^2\,[1 + \overline{\omega}^R(k^2)]}\,, \qquad k^2 = \omega^2 - \vec{k}^2\,, \tag{6.29}$$

$$\overline{\omega}^R(k^2) = \frac{\alpha k^2}{3\pi} \int\limits_{4\,m_{\rm eff}^2}^{\infty} \frac{dk'^2}{k'^2}\, \frac{1 + 2m_{\rm eff}^2/k'^2}{k'^2 - k^2} \sqrt{1 - \frac{4m_{\rm eff}^2}{k'^2}}\,. \tag{6.30}$$

The asymptotic forms

$$\overline{\omega}^{R}(k^2) = \frac{\alpha}{15\pi}\frac{k^2}{m_{\rm eff}^2} + \mathcal{O}(k^4)\,, \qquad k^2 \to 0\,, \tag{6.31a}$$

$$\overline{\omega}^{R}(k^2) = -\frac{\alpha}{3\pi}\ln\left(-\frac{k^2}{m_{\rm eff}^2}\right) + \frac{5\alpha}{3\pi} + \mathcal{O}\left(\frac{\ln(-k^2)}{k^2}\right)\,, \qquad k^2 \to \infty\,, \tag{6.31b}$$

imply that the correction on the mass shell, $\overline{\omega}^{R}(0) = 0$, invalidating the claim made in Ref. [26].

6.4 Conclusions

In Sec. 6.2, we have studied the gravitational coupling of Dirac particles to curved space-time backgrounds, and found that the covariant coupling to space-time implies the use of spin connection matrices; naive prescriptions based on the insertion of the gravitational potential into the Dirac equation can only be valid in an approximate sense. The central idea behind the covariant coupling is the covariance of the covariant derivative in spinor space, given in Eq. (6.13), from which by an explicit evaluation of the spin connection matrices, the results given in Eqs. (6.18), (6.19) and (6.20) can be derived.

The gravitational correction to vacuum polarization is discussed in Sec. 6.3, and a recent claim [26] regarding an additional modification of the speed of light in deep gravitational potentials [parameterized by the γ parameter, see Eq. (6.25)] is refuted. The vacuum polarization tensor in gravitational backgrounds is obtained, within the leading approximation, by a substitution of the gravitationally shifted effective electron mass into the fermion propagator of the one-loop vacuum polarization integral [see Eq. (6.29)]. In summary, we have shown that one can apply the gravitational coupling of Dirac particles in order to solve a number of problems of practical interest, including central-field problems and variations thereof, and potential gravitational corrections to quantum-field theoretical phenomena, like vacuum polarization.

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References

1. P. A. M. Dirac, Proc. Roy. Soc. London, Ser. A **117**, 610 (1928).
2. P. A. M. Dirac, Proc. Roy. Soc. London, Ser. A **118**, 351 (1928).
3. J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

4. J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
5. M. Bojowald, *Canonical Gravity and Applications* (Cambridge University Press, Cambridge, 2011).
6. D. R. Brill and J. A. Wheeler, Rev. Mod. Phys. **29**, 465 (1957).
7. J. H. Noble and U. D. Jentschura, Phys. Rev. A **92**, 012101 (2015).
8. U. D. Jentschura, Phys. Rev. A **87**, 032101 (2013), [Erratum Phys. Rev. A **87**, 069903(E) (2013)].
9. U. D. Jentschura and J. H. Noble, Phys. Rev. A **88**, 022121 (2013).
10. U. D. Jentschura and J. H. Noble, J. Phys. A **47**, 045402 (2014).
11. L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).
12. U. D. Jentschura, Phys. Rev. A **90**, 022112 (2014).
13. J. F. Donoghue and B. R. Holstein, Am. J. Phys. **54**, 827 (1986).
14. C. M. Will, Living Rev. Relativity **4**, 4 (2001).
15. C. M. Will, Living Rev. Relativity **9**, 3 (2006).
16. C. M. Will, Living Rev. Relativity **17**, 4 (2014).
17. H. C. Ohanian and R. Ruffini, *Gravitation and Spacetime* (W. W. Norton, New York and London, 1994).
18. T. Padmanabhan, *Gravitation: Foundations and Frontiers* (Cambridge University Press, Cambridge, 2010).
19. N. Ashby, Phys. Today **55**, 41 (2002).
20. I. I. Shapiro, Phys. Rev. Lett. **13**, 789 (1964).
21. I. I. Shapiro, G. H. Pettengill, M. E. Ash, M. L. Stone, W. B. Smith, R. P. Ingalls, and R. A. Brockelman, Phys. Rev. Lett. **20**, 12651269 (1968).
22. I. I. Shapiro, Rev. Mod. Phys. **71**, S41 (1999).
23. M. J. Longo, Phys. Rev. Lett. **60**, 173 (1988).
24. L. M. Krauss and S. Tremaine, Phys. Rev. Lett. **60**, 176 (1988).
25. A. Migdall, prinvate communication (2015).
26. J. D. Franson, New J. Phys. **16**, 065008 (2014).
27. B. Bertotti, L. Iess, and P. Tortura, Nature (London) **425**, 374376 (2003).