Nonlinear Co-Integration Between Unemployment and Economic Growth in South Africa

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In this paper, a momentum threshold autoregressive (MTAR) model is used to evaluate nonlinear equilibrium reversion between unemployment and economic growth for South African data between the periods 2000–2013. To attain this objective we estimate the first-difference and the gap model variations of Okun's specification. For the latter model variation, we employ three de-trending methods to obtain the relevant 'gap' data; namely, the Hodrick-Prescott (HP) filter, the Baxter-King (BK) filter and the Butterworth (BW) digital filter. A common finding from our empirical analysis is that Okun's law holds concretely for South African data regardless of the model specification or the de-trending technique that is used. Moreover, our analysis proves that unemployment granger causes economic growth in the long-run, a result which may account for the jobless-growth phenomenon experienced by South Africa over the last decade or so.

Key Words: unemployment, economic growth, Okun's law, South Africa, MTAR model, nonlinear unit root tests, nonlinear co-integration, nonlinear Granger tests, Hodrick-Prescott filter, Baxter-King filter, Butterworth high-pass filter

JEL Classification: C22, C51, E23, E24

Introduction

High economic growth in conjunction with low unemployment under a low inflation environment can be deemed as the ultimate objective of macroeconomic policy in South Africa. Over the last decade or so, two prominent macroeconomic policy frameworks have embodied these objectives, those being, monetary policy's 'inflation-targeting' regime and fiscal policy's Accelerated and Shared Growth Initiative of South Africa (ASGISA). Implemented in February 2002 and still in use to date, the inflation-target policy rule specifies that the South African Reserve Bank (SARB) should contain inflation at levels of between 3 and 6 percent, whereas the ASGISA initiative seeks to halve unemployment and attain a

6% economic growth rate by the year 2014. The assumed compatibility of the aforementioned policy objectives is inevitable demonstrated as monetary policy in South Africa is designated towards manipulating nominal variables like interest rates and inflation as a means of influencing real variables such as output growth and employment. Ultimately, the success of disinflation policy is reflected in its effect on unemployment and output growth. However, up-to-date South Africa has not only to managed to achieve arguably the highest economic growth rates in Africa since 1994, but the economy simultaneously boasts one of the highest youth unemployment rates in the world. So even though the South African Reserve Bank (SARB) can be credited for containing inflation within its set target which has been accompanied with steadily improved economic growth, such acquired growth has been characterized by what is popularly referred to as a 'jobless growth' syndrome (Hodge 2009). A mystery is warranted since the 'jobless growth' phenomenon contradicts the epic rise of unemployment caused by the sharp decline of real output experienced worldwide during the great depression. Therefore, a classical challenge for academics and policymakers alike is to provide an adequate account of unemployment-growth correlations in the South African economy.

The question regarding the linkage between economic growth and unemployment gained prominence after Okun (1962) depicted the extent to which the unemployment rate is negatively correlated with output growth. By analyzing data over the period of 1947 to 1960, Okun (1962) documented that unemployment in the United States tends to fall by a one percentage point for every 3 percentage point rise in output growth. Thereafter, the United States was dubbed as having an estimated 'Okun coefficient' of 3 and a plethora of subsequent authors sought to estimate Okun's coefficient by either adopting a single-country approach (see Caraiani 2010; Ahmed, Khali, and Saeed 2011), panel-data approach (see Dixon and Shepard 2002, 1997; Lal et al. 2010) or a multi-regional approach (see Freeman 2000; Adanu 2005; Villaverde and Maza 2009). The appeal of Okun's relationship is attributed to its simplicity and its extensive empirical support qualifies it to belong at the core of modern macroeconomics (Jardin and Gaetan 2011). As noted by Silvapulle, Moosa, and Silvapulle (2004), estimating the Okun coefficient has important implications for the business cycle since it relates the level of activity in the labour market to the level of activity in the product market. Whilst Okun's law implies that more labour is typically required for increased productivity levels, Okun's coefficient serves as an indication of the cost of unemployment in terms of output growth (Noor, Nor, and Ghani 2007). And in consolidation with the Phillips curve; Okun's relationship assists macroeconomic policy in determining the optimal or desirable growth rate as a prescription for reducing unemployment (Moosa 1997). Overall, Okun's law is recommended as 'a rule of thumb' which provides policymakers with an understanding of how different markets adjust, and thus allowing for correct policies to be selected when facing shocks (Pereira, Bento in Silva 2009).

In reality, Okun's law is more of a statistical relationship rather than a structural feature of the macroeconomy (Knotek 2007). The development of a pure theoretical foundation for Okun's relationship has been largely neglected in the academic literature, such that empirically, no functional form has been dominantly preferred to any other on the basis of theory (Weber and West 1996). As a consequence, the empirical examination of Okun's law is typically subject to revisions with the comovement between output growth and unemployment frequently being analyzed under different settings. So while there is no contention on the importance of Okun's law, debates have evolved on the econometric techniques used to establish this relationship; how the cyclical components are extracted; and whether a dynamic or static specification is adopted (Turturean 2007). Recently, the possibility of asymmetric behaviour between economic growth and the unemployment rate has added a new dimension in the development of the academic literature. Take for instance Jardin and Gaetan (2011) who consider asymmetries in Okun's relationship as being important because asymmetric behaviour can adequately account for the varying effectiveness of structural and stabilization policies.

Other commentators, such as Geldenhuys and Marnikov (2007), consider the impact of asymmetric behaviour on policy forecasting practices. In particular, these authors argue that if Okun's relationship is indeed found to be asymmetric, forecasts based on linear estimates of Okun's coefficient can lead to biased error terms. And yet another cluster of authors can also be identified, who advocate on the necessity of incorporating asymmetries in Okun's relationship as a means of reinforcing asymmetric behaviour in the Phillips curve. The rationale behind this line of thought is that if Okun's coefficient changes between regimes, then the sacrifice ratios should also change between regimes. In other words, different degrees of gradualism in the disinflation process may imply different impacts on unemployment for the same reduction in inflation (Beccarini and Gros 2008).

Our study contributes to the literature by addressing the economic significance of asymmetric behaviour in Okun's relationship for South African data. To this end, our study makes use of the momentum threshold (MTAR) autoregressive framework of Enders and Granger (1998). The logic behind the choice of our adopted approach can be described as follows. Engle and Granger (1987) argue that evidence of unit roots between a pair of time series variables necessitates the use of co-integration analysis prior to the estimation of any regression formed by the variables. According to the authors, the presence of co-integration would then imply that the variables follow a common long-run trend and the OLS estimation of the time series will not yield spurious results. This is an important implication for our case study since previous empirical works have cautioned of unit root I(1) behaviour in output growth and unemployment variables for South African data (see Hodge 2006; Burger and Marnikov 2006; Gupta and Uliwingiye 2010). And yet it should also be noted that these conclusions are based on studies which assume a linear data generating process (DGP) among the series. Of recent, it has become widely accepted that standard unit root tests, suffer from low power when a linear approximation of an otherwise nonlinear time series is used to evaluate the integration properties of a time series (Enders and Granger 1998). A similar contention has risen for co-integration analysis, in which researchers like Enders and Dibooglu (2001) prove that the implicit assumption of symmetric adjustment is problematic if the adjustment towards long-run equilibrium is not linear. In particular, the authors argue that the presence of nonlinearities between a pair of time series signifies a high probability of nonlinear adjustment processes towards the long-run equilibrium for the data. With this in mind, our paper probes into the possibility of asymmetric behaviour between the unemployment rate and output growth using the MTAR model. We choose this model because it represents a simple yet flexible framework that can simultaneously facilitate for (1) nonlinear unit root tests, (2) nonlinear co-integration analysis; and (3) nonlinear causality analysis.

Therefore, against this backdrop, we present the remainder of the paper as follows. The following section of the paper presents the empirical framework of the study whereas section four presents the empirical results of the study. The paper is concluded in section five by providing policy recommendations and suggesting avenues for future research.

Empirical Framework

Our paper uses two classes of Okun's law specifications; namely, the first differences model and the gap model. To ensure that we obtain a balanced, robust view on the estimation results, we specify the Okun's specifications on both the direct and the reverse regressions of unemployment on output growth. For instance, in specifying the 'first differences' version of Okun's law, the link between the unemployment rate (*ur*) and economic growth (*gdp*) is represented as:

$$\begin{pmatrix} \Delta gdp_t \\ \Delta ur_t \end{pmatrix} = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix} \begin{pmatrix} \Delta ur_t \\ \Delta gdp_t \end{pmatrix} + \begin{pmatrix} \xi_{t1} \\ \xi_{t2} \end{pmatrix}, \tag{1}$$

where Δ is the first difference operator such that $\Delta gdp_t = gdp_t - gdp_{t-1}$ and $\Delta ur_t = ur_t - ur_{t-1}$. On the other hand, the 'gap model' measures these variables in terms of their deviations from long-run trends and is specified as:

$$\begin{pmatrix} gdp_t^c \\ ur_t^c \end{pmatrix} = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix} \begin{pmatrix} ur_t^c \\ gdp_t^c \end{pmatrix} + \begin{pmatrix} \xi_{t1} \\ \xi_{t2} \end{pmatrix}, \tag{2}$$

where $ur_t^c \equiv ur_t - ur_t^*$ and $gdp_t^c \equiv gdp_t - gdp_t^*$ are representative of the cyclical components of the unemployment rate and real output, respectively; with gdp_t^* denoting a measure of potential output gap and ur_t^* the unemployment gap variable. Having specified our baseline theoretical models, we can proceed to introduce co-integration analysis amongst the variables. We, therefore, take heed of Enders and Granger (1998) and model asymmetric adjustment between the unemployment and real output growth variables by allowing the residual deviations (i. e. ξ_{ti}) from the long-run equilibrium of regressions (1) and (2) to behave as a TAR process. Formally, these residuals are modelled as follows:

$$\delta\xi_{ti} = I_t\rho_1\xi_{t-1} + (1 - I_t)\rho_2\xi_{t-1} + \sum_{i=1}^{p} \beta_i \Delta\xi_{t-1} + \varepsilon_t. \tag{3}$$

In our paper, we identify four types of co-integration relations which govern the asymmetric dynamics within Okun's law, namely; TAR with a zero threshold; consistent TAR with a nonzero threshold; MTAR with a zero threshold; and consistent MTAR with a nonzero threshold. In the TAR model with a zero threshold, the indicator function, I_t, is set according to:

$$I_t = \begin{cases} 1, & if\backslash\xi_{t-1} \geq 0 \\ 0, & if\ \xi_{t-1} < 0 \end{cases}. \tag{4}$$

Under the TAR model with a nonzero threshold, we set I_t, as:

$$I_t = \begin{cases} 1, & if\backslash\xi_{t-1} \geq \tau \\ 0, & if\ \xi_{t-1} < \tau \end{cases}, \tag{5}$$

where τ is the value of the threshold variable. Enders and Granger (1998) suggest the use of a grid search procedure, as demonstrated in Hansen (1997), to derive a consistent estimate of the threshold i.e. the threshold estimate yielding the lowest RSS is considered the true threshold estimate. The TAR models are designed to capture potential asymmetric deep movements in the residuals if, for example, positive deviations are more prolonged than negative deviations (Enders and Dibooglu 2001). Enders and Granger (1998) and Caner and Hansen (2001) suggest that by permitting the Heaviside indicator function, It, to rely on the first differences of the residuals, $\Delta\xi_{t-1}$, a MTAR version of equation (11) can be developed. The implication of the MTAR model is that correction mechanism dynamic since by using $\Delta\xi_{t-1}$, it is possible to access if the momentum of the series is larger in a given direction relative to the direction in the alternative direction. In other words, the MTAR model can effectively capture large and smooth changes in a series whereas the TAR model shows the 'depth' of the swings in equilibrium relationship. In modelling MTAR threshold co-integration with a zero threshold, the indicator function M_t, is set as:

$$M_t = \begin{cases} 1, & if\backslash\xi_{t-1} \geq 0 \\ 0, & if\ \xi_{t-1} < 0 \end{cases}. \tag{6}$$

While in the MTAR model with a nonzero threshold, M_t, is set as:

$$I_t = \begin{cases} 1, & if\backslash\xi_{t-1} \geq \tau \\ 0, & if\ \xi_{t-1} < \tau \end{cases}. \tag{7}$$

For both TAR and MTAR specifications, Enders and Silkos (1998) demonstrate that a sufficient condition for stationary of ξ_{t-1} is that $\rho_1, \rho_2 < 0$. If ξ_{t-1} is found to be stationary, the least squares estimates of ρ_1 and ρ_2 have an asymptotic multivariate normal distribution for any given value of a consistently estimated threshold. Moreover, the null hypothesis of no co-integration (i. e. H_{01}: $\rho_1 = \rho_2 = 0$) can be formally tested using a standard F-statistic for both TAR and MTAR models. If the null hypothesis of no co-integration is rejected, it is possible to test for the null hypothesis of symmetric adjustment (i. e. H_{02}: $\rho_1 = \rho_2$) against the alternative of asymmetric adjustment (i. e. H_{12}: $\rho_1 \neq \rho_2$) using a similar F-test. The empirical F-distribution for the null hypothesis; $\rho_1 = \rho_2 = 0$ is tabulated in Enders and Dibooglu (2001) whereas Enders and Siklos (2001) report critical values for testing the null hypothesis of $\rho_1 \neq \rho_2$. If both null hypotheses of no co-integration and no asymmetric co-integration can be simultaneously rejected, the granger representation theorem is satisfied and thus an associated error correction model can be estimated for the pair of time series variables. Thus in validating the presence of threshold co-integration, the error correction model can be modified to take into account asymmetries as in Blake and Fomby (1997). In our study we augment each of our threshold co-integration regressions with thresholds error correction specifications. In particular, the TAR-TEC model can be expressed as:

$$\begin{pmatrix} \Delta gdp_t \\ \Delta ur_t \end{pmatrix} = \lambda_{11} I_t \xi_{t-1} + \lambda_{12}(1 - I_t)\xi_{t-1} + \sum_{i=1}^{p} \alpha_{1i} \Delta gdp_{t-1} + \sum_{i=1}^{p} \beta_{1i} \Delta ur_{t-1}. \tag{8}$$

Whereas the MTAR-TEC model is specified as:

$$\begin{pmatrix} \Delta gdp_t \\ \Delta ur_t \end{pmatrix} = \lambda_{21} M_t \xi_{t-1} + \lambda_{22}(1 - M_t)\xi_{t-1} + \sum_{i=1}^{p} \alpha_{2i} \Delta gdp_{t-1} + \sum_{i=1}^{p} \beta_{2i} \Delta ur_{t-1}, \tag{9}$$

where the indicator functions for the TAR and MTAR model specifications are represented by I_t and M_t respectively. Through the above described systems of error correction models, two types of joint hypotheses can be tested. Firstly, the presence of asymmetries between the variables could initially be examined by examining the signs on the coefficients of the error correction terms. This involves testing the null hypothesis of H_{03}: $\lambda_{i1}\xi_{t-1} = \lambda_{i2}\xi_{t-1}$ against the alternative H_{13}: $\lambda_{i1}\xi_{t-1} \neq \lambda_{i2}\xi_{t-1}$. The second type of hypothesis tested is that of granger causality effects which relatively examines whether all Δgdp_{t-k} and Δur_{t-k} are statistically different from zero. Granger tests are used to examine whether the lagged values of one variable do not improve on the explanation or 'granger-cause' another variable. In particular, the null hypothesis that urt does not lead to gdpt can be denoted as: $H_{04}: \alpha_i = 0, i = 1, \ldots, k$; whereas the null hypothesis that gdp_t does not lead to ur_t is: $H_{05}: \beta i = 0, i = 1, \ldots, k$. All aforementioned hypotheses are based on a standard F-test. Furthermore, three types of joint hypotheses can be formed from the TEC model. Firstly, granger causality tests can be implemented by testing whether all Δgdp_{t-k} and Δur_{t-k} are statistically different from zero based on a standard F-test and if the λ coefficients of the error correction are also significant.

Empirical Analysis

EMPIRICAL DATA

The data used in the empirical analysis consists of the annual percentage change in the real gross domestic product which is gathered from the South African Reserve Bank (SARB) online database whereas the unemployment rate for all persons aged above 15 years of age is collected from various issues of the quarterly labour force surveys (QLFS) as complied by Statistics South Africa (STATSSA). Our empirical analysis uses quarterly adjusted data obtained for the periods extending from 2000 to 2014. The choice of our sample period and periodicity reflects the limitations in the availability of the time-series data on unemployment and economic growth for South Africa. Although it would be desirable to employ a longer span of data, the available data provides the advantage of avoiding the issue of potential structural breaks related to South Africa's political and structural reforms such as those experienced in 1994. Moreover, we take note that while our data is relatively short, it is, however, up-to-date and further eliminates the problem of data unreliability associated with the South African unemployment series before 2000. Further given that gross domestic product is available on a quarterly basis and the unemployment rate is limited to half-yearly data, we use cubic spline interpolation to convert the half-yearly unemployment data into quarterly data over the same time period. We favour the use of cubic spline interpolation over other time series data conversion techniques due to its computational accuracy and stability of computation. Moreover, cubic spline interpolations satisfy the further condition at the end point.

As a part of our data construction, we introduce the de-trending methods used to extract the 'potential output' and 'unemployment gap' variables necessary to estimate the gap version of Okun's specification. The construction of these 'gap variables' is necessary since there exists no observable data on the trend components of the unemployment and output growth variables. Also taking into consideration that a majority of these de-trending techniques are not without scepticism, it is standard practice to apply a variety/different de-trending techniques to ensure robustness in the regressions analysis. Therefore in following along this course of reasoning, our study considers three alternative de-trending techniques, namely the Hodrick-Prescott (HP) filter; the Baxter-King (BK) filter and the Butterworth (BW) digital filter as respectively introduced by Hodrick and Prescott (1997), Baxter and King (1999); and Pollock (2000). The purpose of using these three de-trending techniques is to enable a robust analysis concerning the sensitivity of the estimated Okun's coefficient to the different choices of our gap variable estimates.

UNIT ROOT TESTS

In testing for unit roots, we begin on the simple premise of subjecting a univariate time series, y_t, to the following generalized autoregression:

$$Y_t = \varphi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2). \tag{10}$$

Heuristically, one can test the null hypothesis of a unit root as $H_0: \varphi = 1$ against the alternative hypothesis of an otherwise stationary series. However, as previously discussed, there exists a problem of low power associated with traditional unit root tests when the underlying data generating process of time series is proven to be asymmetric. Therefore, in order to accommodate asymmetric behaviour in the unit root testing procedure, we re-formulate regression (10) in terms of first differences. This enables us to follow in pursuit of Enders and Granger (1998) and specify the unit root testing regressions for the TAR model with a zero threshold and a consistent threshold estimate, respectively, as:

$$\Delta y_t = \varepsilon_t(\varepsilon_{t-1} < 0) + \varepsilon_t(\varepsilon_{t-1} \geq 0) + \upsilon_t, \tag{11}$$

$$\Delta y_t = \varepsilon_t(\varepsilon_{t-1} < \tau) + \varepsilon_t(\varepsilon_{t-1} \geq \tau) + \upsilon_t, \tag{12}$$

Whereas the MTAR version of the unit root test regression with a zero threshold and a consistent threshold estimate threshold are, respectively, specified as:

$$\Delta y_t = \varepsilon_t(\Delta\varepsilon_{t-1} < 0) + \varepsilon_t(\Delta\varepsilon_{t-1} \geq 0) + \upsilon_t, \tag{13}$$

TABLE 1 Nonlinear Unit Root Tests

Variable	Model	Lag	Asymmetry test (i. e. $\rho_1 = \rho_2$)	Unit root test (i. e. $\rho_1 = \rho_2 = 0$)	Decision
gdp	TAR	2	0.94	12.63***	Linear $I(0)$
			(3.32)*	(16.46)***	Nonlinear $I(0)$
	C-TAR	2	3.94*	15.59***	Nonlinear $I(0)$
			(7.87)*	(21.28)***	Nonlinear $I(0)$
	MTAR	2	0.95	12.13***	Linear $I(0)$
			(9.46)**	(22.89)***	Nonlinear $I(0)$
	C-MTAR	2	4.90*	16.03***	Nonlinear $I(0)$
			(6.67)*	(19.96)***	Nonlinear $I(0)$
ur	TAR	0	2.45	2.86*	Linear $I(0)$
			(4.96)*	(7.22)**	Nonlinear $I(0)$
	C-TAR	0	2.37	2.81*	Linear $I(0)$
			(5.21)*	(7.40)**	Nonlinear $I(0)$
	MTAR	0	2.59	2.94*	Linear $I(0)$
			(3.44)*	(6.17)**	Nonlinear $I(0)$
	C-MTAR	0	2.70	3.00*	Linear $I(0)$
			(3.37)*	(6.12)**	Nonlinear $I(0)$

NOTES Significance level codes: ***, ** and * denote the 1%, 5% and 10% significance levels respectively. Tests statistics for the first differences of the variables, i. e. Δgdp_t and Δur_t are given in parenthesis.

$$\Delta y_t = \varepsilon_t(\Delta\varepsilon_{t-1} < \tau) + \varepsilon_t(\Delta\varepsilon_{t-1} \geq \tau) + \upsilon_t. \tag{14}$$

Thereafter, two hypotheses can be formed from regressions (11)–(14). The first hypothesis tests for asymmetries within the time series. To this end, we test the null hypothesis of no asymmetric effects as H_{00}: $\rho_1 = \rho_2$ against the alternative hypothesis of an asymmetric data generating process (i. e. H_{01}: $\rho_1 \neq \rho_2$). Subsequent to testing for asymmetric effects, we then proceed to test for unit root behaviour within the time series. Pragmatically, the null hypothesis of a unit root is tested as H_{10}: $\rho_1 = \rho_2 = 0$ against the alternative hypothesis of an otherwise stationary asymmetric process (i. e. H_{10}: $\rho_1 \neq \rho_2 \neq 0$). The aforementioned tests of asymmetry and unit root behaviour are performed on time series variables of economic growth and the unemployment rate. The lag length of the threshold models which facilitate these tests are determined by the AIC information criterion.

As is evident from table 1, the empirical test results obtained for the time series in their levels are quite mixed. For instance, in scanning through the model tests conducted on the unemployment variable, we find that we cannot reject the null hypothesis of a symmetric process and yet we are able to reject the null hypothesis of a unit root process for same time series. Thus for the unemployment variable in its levels, we conclude a linear, stationary data generating process for the series. However, for the output growth variable in its levels, we conversely find that the C-TAR and C-MTAR versions of the employed tests simultaneously reject both null hypotheses of symmetry and unit root behaviour. This particular result implies a nonlinear, nonstationary data generating process for the output growth variable in its levels. And yet, in turning to the empirical results obtained for the time series in their first differences, our analysis reveals a common finding of a nonlinear yet stationary process for all variables under all model specifications. All in all, we can conclude that all utilized time series appear to be both nonlinear yet stationary processes in their first differences. Therefore, the results obtained from our preliminary unit root analysis paves the way for the threshold co-integration analysis which we conduct next.

CO-INTEGRATION ANALYSIS

Having investigated the integration properties of the unemployment and economic growth variables, we proceed to investigate threshold co-integration and error correction effects amongst the times series. However, prior to estimating any threshold models, we must first test a number of hypotheses to select which models best capture asymmetric behaviour in Okun's specification. To this end, we employ three threshold tests which have been previously discussed previously discussed. To recall, (1) we test for co-integration effects; (2) we test for threshold co-integration effects and (3) we test for threshold error correction effects. The results of these tests are reported in table 2. In referring to these results, we find that at least one type of threshold model manages to reject all three hypotheses at least a 10 percent significance level for all variations of Okun's law. This is quite an encouraging result since it implies that the data displays at least one form of nonlinearity for each version of Okun's specification.

Another interesting result is that the MTAR specification is most suitable for modelling nonlinear behaviour between unemployment and economic growth for South African data. The only exception holds for the CF filter estimates which favour a TAR model specification. Furthermore, all estimated versions of Okun's law unveil significant asymmetric co-integration behaviour only when output growth is placed as the dependent variable in the regression.

In summing up the test results reported in table 2, we can draw two broad conclusions thus far. Firstly, our analysis infers significant asymmetric behaviour between unemployment and economic growth for South African data. In this respect, our results adhere with those obtained in Geldenhuys and Marnikov (2007). However, in slightly differing from Geldenhuys and Marnikov (2007), we find smooth nonlinear adjustment behaviour in the data as opposed to an abrupt one. This result is expected since the otherwise abrupt nonlinearity is most suited for data containing structural break periods. Seeing that our data does not cover such periods, it therefore becomes reasonable that we detect smooth nonlinear behaviour among the data. Our second conclusion is that we establish economic growth as being the driving variable in the asymmetric relationship detected between the time series. This is worth observing since it serves as a guideline on how to estimate each of the selected threshold regressions. In our instance, we specify the MTAR models under the assumption that economic growth is regressed on the unemployment rate. This is of course with the exception of the CF filter regression in which we model TAR nonlinearity and yet retain economic growth as the dependent variable in the regression. Our estimation results of the first difference model specifications are reported below in table 3 whereas the results obtained for the gap model versions are reported in table 4.

Starting with the results reported in table 3 for first differences model, we take note of a long-run coefficient estimate of –0.09. Technically speaking, the magnitude of this coefficient estimate as obtained under both first difference models implies that a 1 percent decrease in the unemployment rate is associated with a –0.09 percent increase in productivity output. This result is seemingly plausible as it does not violate traditional theory of a negative unemployment-growth co-relationship as initially postulated by Okun (1962). Furthermore, the magnitude of this relationship is consistent with some of the Okun coefficient estimates obtained in previous studies. Among these previous studies are the works of Adanu (2005) who obtain a similar estimate of –0.09 for Alberta province in Canada; Villaverde and Maza (2009) who find a –0.08 estimate for a regional group of Spanish data and also Geldenhuys and Marnikov (2007) who obtain an estimate of –0.11 for South African data.

In moving on to examining the regime switching behaviour among the co-integration error terms, we firstly note that all threshold estimates

TABLE 2 Threshold Cointergation and Error Correction Tests

Model	(1)	(2)	TAR-TEC $H_0^{(1)}$	TAR-TEC $H_0^{(2)}$	TAR-TEC $H_0^{(3)}$	MTAR-TEC $H_0^{(1)}$	MTAR-TEC $H_0^{(2)}$	MTAR-TEC $H_0^{(3)}$
First differences	Δgdp_t	Δur_t	25.36 (0.00)***	4.10 (0.05)*	0.47 (0.50)	32.71 (0.00)***	9.16 (0.01)**	2.47 (0.13)*
	Δur_t	Δgdp_t	41.82 (0.00)***	0.68 (0.42)	0.01 (0.91)	50.82 (0.00)***	1.66 (0.21)	0.01 (0.95)
HP filter	gdp_t^c	ur_t^c	6.84 (0.01)**	1.07 (0.31)	0.66 (0.43)	6.15 (0.01)**	0.16 (0.69)	0.25 (0.62)
	ur_t^c	gdp_t^c	4.36 (0.02)*	0.22 (0.64)	2.78 (0.11)*	4.46 (0.02)*	1.19 (0.28)	0.49 (0.49)
BK filter	gdp_t^c	ur_t^c	28.51 (0.00)***	3.56 (0.07)*	2.94 (0.11)*	33.43 (0.00)***	6.70 (0.02)*	1.59 (0.23)
	ur_t^c	gdp_t^c	27.28 (0.00)***	0.01 (0.91)	0.23 (0.64)	32.79 (0.00)***	0.09 (0.76)	1.10 (0.32)
BW filter	gdp_t^c	ur_t^c	26.51 (0.00)***	4.34 (0.05)*	0.65 (0.43)	34.03 (0.00)***	9.29 (0.01)**	3.51 (0.08)*
	ur_t^c	gdp_t^c	54.27 (0.00)***	1.06 (0.31)	0.01 (0.94)	55.93 (0.00)***	0.96 (0.34)	0.66 (0.43)
			C-TAR-TEC $H_0^{(1)}$	C-TAR-TEC $H_0^{(2)}$	C-TAR-TEC $H_0^{(3)}$	C-MTAR-TEC $H_0^{(1)}$	C-MTAR-TEC $H_0^{(2)}$	C-MTAR-TEC $H_0^{(3)}$
First differences	Δgdp_t	Δur_t	29.08 (0.00)**	6.84 (0.02)*	0.79 (0.39)	32.75 (0.00)***	9.19 (0.01)**	2.78 (0.11)*
	Δur_t	Δgdp_t	42.23 (0.00)***	0.86 (0.36)	0.96 (0.34)	67.86 (0.00)***	8.18 (0.01)**	1.85 (0.19)
HP filter	gdp_t^c	ur_t^c	6.84 (0.01)**	1.06 (0.31)	0.01 (0.98)	10.04 (0.00)***	5.27 (0.03)*	3.75 (0.07)*
	ur_t^c	gdp_t^c	5.20 (0.01)*	1.47 (0.24)	3.64 (0.07)*	6.85 (0.01)*	4.81 (0.04)*	5.26 (0.03)**
BK filter	gdp_t^c	ur_t^c	28.74 (0.00)***	3.71 (0.07)*	1.08 (0.32)	33.91 (0.00)***	7.01 (0.01)*	1.82 (0.20)
	ur_t^c	gdp_t^c	27.71 (0.00)***	0.27 (0.61)	0.23 (0.64)	32.79 (0.00)***	0.09 (0.76)	1.10 (0.32)
BW filter	gdp_t^c	ur_t^c	32.08 (0.00)***	8.35 (0.01)**	1.27 (0.28)	33.28 (0.00)***	8.77 (0.01)**	2.22 (0.15)
	ur_t^c	gdp_t^c	56.83 (0.00)***	1.99 (0.17)	0.24 (0.63)	60.65 (0.00)***	2.58 (0.12)	0.44 (0.52)

NOTES Column headings are as follows: (1) dependent variable, (2) independent variable. Significance level codes: ***, ** and * denote the 1%, 5% and 10% significance levels respectively.

TABLE 3 Threshold Co-Integration and Error Correction Estimates for First Difference Model Specification

	MTAR-TEC		C-MTAR-TEC	
	Y	X	Y	X
	Δgdp	Δur	Δgdp	Δur
β_i	−0.09 (0.00)***		−0.09 (0.00)***	
$\rho_1\xi_{t-1}$	−0.72 (0.01)**		−0.72 (0.01)**	
$\rho_1\xi_{t-1}$	−1.76 (0.00)***		−1.76 (0.00)***	
τ	0		0.11	
$\Delta\Delta gdp^{+}_{t-k}$	−0.39 (0.47)	−1.18 (0.31)	−0.38 (0.47)	−1.26 (0.27)
$\Delta\Delta gdp^{-}_{t-k}$	−0.30 (0.36)	−0.50 (0.47)	−0.29 (0.36)	−0.47 (0.50)
$\Delta\Delta ur^{+}_{t-k}$	−0.04 (0.64)	−0.80 (0.00)***	−0.04 (0.66)	−0.80 (0.00)***
$\Delta\Delta ur^{-}_{t-k}$	−0.09 (0.28)	−0.99 (0.00)***	−0.09 (0.29)	−0.99 (0.00)***
$\lambda^{+}\xi_{t-1}$	0.21 (0.83)	2.39 (0.27)	0.19 (0.85)	2.53 (0.23)
$\lambda^{-}\xi_{t-1}$	−1.82 (0.00)***	−1.05 (0.14)*	−1.81 (0.00)***	−1.06 (0.13)*
R^2	0.80	0.86	0.80	0.85
DW	1.61	2.42	1.61	2.39
p-value	0.37	0.31	0.35	0.31
LB	0.31	0.55	0.27	0.59
JB	3.59	3.82	3.65	3.98

NOTES Significance level codes: ***, ** and * denote the 1%, 5% and 10% significance levels respectively. DW and LB respective denote the Durbin Watson and Ljung-Box test statistics for autocorrelation whereas JB denotes the Jarque-Bera normality test of the residuals.

are encouragingly close to zero in value. Moreover, the threshold error term estimates satisfy the convergence condition of error term stationarity i. e. $\rho_1, \rho_2 < 0$ and $(1 - \rho_1)(1 - \rho_2) < 1$. In further diagnosing these co-integration threshold error terms, we observe that negative deviations are eliminated quicker than positive ones. We can make such inference since the estimate of 1 is of a lower absolute value in comparison to its 2 counterpart. Notably, Harris and Silverstone (2001) make similar inferences in their study for both US and UK data. In addition, our estimates of the threshold error correction terms also bear a slight resemblance to those obtained in Harris and Silverstone (2001), in the sense of producing correct negative estimates in the lower regimes of the estimated models. However in differing from these authors, we are able to obtain significant values for the estimates of the threshold error correction terms and thus we can draw meaningful interpretations of the error correction coefficients. In this respect, we not only discover that the long-run error correction terms for both MTAR-TEC and C-MTAR-TEC models are almost identical in magnitude, but we more importantly note that the speed of adjustment in both models is quicker when there is a shock to economic growth as opposed to a shock to the unemployment rate. Meanwhile, we are only able to identify significant short-run effects for the lagged coefficients of the economic growth variable when shock has been induced on the unemployment rates, whilst we are find no short-run effects for shocks to economic growth variable.

In diverting our attention to the empirical results of the estimated gap versions of Okun's law as reported in table 5, we generally observe that the regression estimates, more or less, bear close resemblance to those attained for the first difference models. For instance, the long-run regression coefficient obtained from the gap version models produce similarly negative estimates, albeit the magnitude of these estimates vary between 0.09 and 0.98 for the different de-trending methods employed. In further considering the absolute coefficient values of the threshold error terms formed by the long-run regressions, we note that the gap model estimates also bear similarities to those obtained for the first difference models. Specifically, we observe that the absolute values of p_1 are significantly higher when the unemployment rate is the driving variable, whilst the values of p_2 are higher when the unemployment rate is the dependent variable in the co-integration system. As previously explained, this result infers that negative shocks are eliminated quicker when economic growth is the driving variable, whereas positive shocks are eliminated quicker when the unemployment rate is the dependent variable.

However, after scrutinizing through the threshold error correction model estimates, we find the estimates from the gap models to be less encouraging. This especially becomes apparent when mainly considering the long-run error correction terms, from which we observe that only two

TABLE 4 Threshold Co-Integration and Threshold Error Correction Estimates for First De-Trended Model Specification

	HP filter				BK filter		BW filter	
	C-MTAR-TEC		C-MTAR-TEC		TAR-TEC		MTAR-TEC	
	Y	X	Y	X	Y	X	Y	X
	Δgdp	Δur	Δgdp	Δur	Δgdp	Δur	Δgdp	Δur
β_i	−0.2 (0.02)**		−0.15 (0.01)		−0.09 (0.03)*		−0.10 (0.01)**	
$\rho_1\xi_{t-1}$	−0.13 (0.66)		−0.88 (0.01)**		−0.97 (0.01)**		−0.73 (0.01)**	
$\rho_1\xi_{t-1}$	−0.98 (0.00)***		−0.16 (0.48)		−1.68 (0.00)***		−1.77 (0.00)***	
τ	−0.286		−1.747		0		0.254	
$\Delta\Delta gdp^+_{t-k}$	0.32 (0.27)	0.37 (0.55)	0.44 (0.45)	0.48 (0.11)	−0.30 (0.40)	−0.56 (0.34)	−0.32 (0.55)	−1.55 (0.19)
$\Delta\Delta gdp^-_{t-k}$	0.32 (0.61)	−0.26 (0.84)	−1.09 (0.10)*	−1.29 (0.00)***	−1.22 (0.05)*	0.32 (0.74)	−0.30 (0.35)	−0.30 (0.66)
$\Delta\Delta ur^+_{t-k}$	0.08 (0.63)	−1.07 (0.00)***	−0.48 (0.23)	0.13 (0.54)	0.06 (0.73)	−1.14 (0.00)***	−0.02 (0.82)	−0.78 (0.00)***
$\Delta\Delta ur^-_{t-k}$	−0.09 (0.42)	−0.39 (0.14)	−0.48 (0.07)*	−0.19 (0.16)	0.12 (0.50)	−0.36 (0.22)	−0.10 (0.23)	−0.99 (0.00)***
$\lambda^+\xi_{t-1}$	0.09 (0.83)	−1.63 (0.07)*	−0.64 (0.03)	−0.05 (0.73)	−0.09 (0.91)	1.82 (0.19)	0.08 (0.94)	3.03 (.017)
$\lambda^-\xi_{t-1}$	−0.88 (0.07)*	−0.53 (0.61)	0.12 (0.59)	0.18 (0.12)*	−0.44 (0.57)	−1.54 (0.24)	−1.83 (0.00)***	−1.16 (0.12)
R^2	0.54	0.57	0.61	0.50	0.46	0.80	0.80	0.85
DW	2.10	1.56	1.43	1.68	2.42	1.85	1.60	2.63
p-value	0.89	0.23	0.09	0.28	0.39	0.62	0.31	0.13
LB	0.54	0.62	0.25	0.18	0.50	0.58	0.23	0.44
JB	3.56	4.10	3.89	4.86	3.79	4.26	3.98	4.58

NOTES Significance level codes: ***, ** and * denote the 1%, 5% and 10% significance levels respectively. DW and LB respective denote the Durbin Watson and Ljung-Box test statistics for autocorrelation whereas JB denotes the Jarque-Bera normality test of the residuals.

models manage to produce negative and significant estimates i. e. the HP and BW filter specifications with economic growth placed as the driving variables in both models. Therefore, we are restricted to interpreting the error correction coefficient estimates solely for these two model specifications. In drawing inference from these estimates, we conclude equilibrium reverting behaviour over the business cycle for the HP filter model when a shock has been induced on either the economic growth or the unemployment variables. Similarly, for the BW filter estimates, long-run equilibrium reversions occur only in the event of a shock to economic growth. It is also interesting to find that for both cases of the first difference models, we obtain significant short-run coefficient estimates of the lagged unemployment variable when a shock has been induced on the unemployment rate. Thus we collectively observe a distinct pattern over the business cycle, in which the unemployment rate is a driving factor of equilibrium conversions over the short-run whilst economic growth is responsible for equilibrium adjustment over the long-run.

Having established various forms of threshold co-integration within Okun's law for the data implies that there must exist some form of causality between the variables in the granger sense. However, the direction of causality cannot be assumed a prior and thus should be investigated through a formal analysis. We are permitted to examine causality effects amongst the variables via a standard *F*-test. The construction of these tests has been adequately discussed in the previous section of the paper. Table 7 reports the results of the causal analysis. The most striking feature of our obtained results is that, in all cases save one, we are able to reject the null hypothesis of unemployment not causing output growth at conventional levels of significance. Conversely, we fail to reject the null hypothesis of economic growth not leading the unemployment rate. We have noted an exceptional case for the HP filter model with economic growth as the driving variable, in which we detect no causal effects within the data. In summing up these results, we can safely assume that our results depict unidirectional causality running from the unemployment rate to economic growth for the data as a whole. This result is plausible seeing that we have already established that economic growth is regressed as being the dependent on the unemployment rate but not vice versa.

Concluding Remarks

The goal of this paper was to examine nonlinear co-integration and causality effects in Okun's law for South African data dating between the periods of 2000 and 2013. This objective was facilitated through the use of MTAR modelling techniques. We favour this approach on the premise of allowing for unit root testing, co-integration analysis and causality analysis under a single, comprehensive framework. Moreover, our study dif-

TABLE 5 Granger Causality Tests

Model		Y	X	H_{03}: $Y \rightarrow X$	H_{03}: $X \rightarrow Y$	Decision
First differences	MTAR-TEC	*gdp*	*ur*	1.11 (0.35)	34.71 (0.00)***	$ur \rightarrow gdp$ $gdp \neq ur$
	C-MTAR-TEC	*gdp*	*ur*	1.19 (0.33)	35.24 (0.00)***	$ur \rightarrow gdp$ $gdp \neq ur$
HP filter	C-MTAR-TEC	*gdp*	*ur*	0.65 (0.54)	0.36 (0.70)	$ur \rightarrow gdp$ $gdp \neq ur$
CF filter	C-MTAR-TEC	*ur*	*gdp*	3.97 (0.04)**	1.50 (0.25)	$ur \rightarrow gdp$ $gdp \neq ur$
BW filter	C-TAR-TEC	*gdp*	*ur*	0.49 (0.62)	12.61 (0.00)***	$ur \rightarrow gdp$ $gdp \neq ur$
	C-MTAR-TEC	*gdp*	*ur*	1.27 (0.31)	32.37 (0.00)***	$ur \rightarrow gdp$ $gdp \neq ur$

NOTES Significance level codes: ***, ** and * denote the 1%, 5% and 10% significance levels respectively. Definitions of notations: →, ↔ and ≠ represent unidirectional causality, bi-directional causality and no causality, respectively.

fers from previous South African case studies as we are able to introduce nonlinearity in a strict co-integration sense. Having applied the MTAR framework to South African unemployment and economic growth data has produced a number of interesting policy considerations. First of all, in quantifying the long run correlation coefficient, we find negative Okun coefficients ranging from −0.09 to −0.20 for all estimated threshold models. Clearly, these observations have far reaching ramifications as they give rise to the intriguing possibility of a long run trade-off between unemployment and economic growth. However, the aforementioned observations are of limited policy value in absence of knowing the causal relations amongst the variables.

In examining the empirical results obtained from the causal analysis, we discover that during abrupt shocks to the economy there are no causal effects between the variables. This essentially means that in the event of sharp or anticipated shocks to the economy there is very little that policy intervention can do for long-run equilibrium restoration between unemployment and economic growth. However, during smooth shocks, unemployment granger causes economic growth thus allowing for direct labour policies to have an impact on output productivity. We substantiate these smooth shocks as carefully implemented and monitored policies directives which are aimed at narrowing the existing gap between the demand and supply within South African labour markets. Inclusive of such shocks are policy programmes aimed at improving the higher education system through intensifying further education and training (FET) programmes and the recently proposed 'target wage subsidy' programme which is intended to facilitate for the school-to-work transition within the youth population. We also note that under no circumstance does economic growth granger cause unemployment thus insinuating that policies aimed directly at improving economic growth such as foreign exchange policies would exert little or no influence on eradicating unemployment over the long run. This is particularly worth noting since it has been previously assumed that the stability of the exchange rate would lead to a direct improvement of employment growth in import-competitive and export-oriented sectors, especially the manufacturing sectors. Our study implies that, whilst these macro-policies may create a sustainable environment for improved economic growth, they are of little use with regards to directly eradicating unemployment. Therefore, the overall finding of uni-directional causality from unemployment to economic growth provides an adequate explanation for the 'job-less' growth pandemic experienced in South Africa over the last two decades or so.

In recent South African recession periods, unemployment has continued to rise despite economic growth seemingly returning to its previous long-run trend. Deriving from our study, there exist two rational explanations to this pandemic. Firstly, negative shocks to economic growth are eradicated quicker than negative shocks to unemployment. This implies that in the event of smooth shocks to output productivity, it should be expected that economic growth should return back to its long-run steady state at a quicker rate than its unemployment counterpart. Secondly, our general finding of causality running from unemployment to economic growth highlights the ineffectiveness of macroeconomic policies aimed at reducing unemployment through improved productivity growth. Specifically, our empirical estimates suggest that smooth unemployment shocks, in the form of structural labour policies, would help stabilize the structural and cyclical components of unemployment over both the short and the long run. Therefore, for the specific case of South Africa, the job-less growth pandemic can be attributed to structural factors underlying the overwhelming unemployment rates facing the economy. Specifically, structural unemployment in South Africa is a result of a mismatch between jobs offered by employers and potential workers. Thus factors which can minimize the extent of structural unemployment within the economy need to be addressed in policies. These factors include the rigidity of labour markets, real wage rigidity, high minimum wages compared to relatively low productivity and other factors which would lead to job creation and job security. Overall, we conclude that labour policies aimed at stabilizing and eradicating structural unemployment within the economy may be a panacea towards simultaneously reducing overall unemployment and boosting economic growth over the long run.

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