



Excited hyperons of the $N = 2$ band in the $1/N_c$ expansion

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Abstract. The spectrum of excited baryons in the $N = 2$ band is reanalyzed in the $1/N_c$ expansion method, with emphasis on hyperons. Predictions are made for the classification of these excited baryons into $SU(3)$ singlets, octets and decuplets.

1 Introduction

The $1/N_c$ expansion method [1, 2] where N_c is the number of colors, is a powerful and systematic tool for baryon spectroscopy. For N_f flavors, the ground state baryons display an exact contracted $SU(2N_f)$ spin-flavor symmetry in the large N_c limit of QCD [3, 4]. The Skyrme model, the strong coupling theory and the static quark model share a common underlying symmetry with QCD baryons in the large N_c limit [5].

The method has been successfully applied to ground state baryons ($N = 0$ band), in the symmetric representation **56** of $SU(6)$ [4, 6–9]. At $N_c \rightarrow \infty$ the ground state baryons are degenerate. At large, but finite N_c , the mass splitting starts at order $1/N_c$ as first observed in Ref. [5].

The extension of the $1/N_c$ expansion method to excited states requires the symmetry group $SU(2N_f) \times O(3)$ [10], in order to introduce orbital excitations. It happens that the experimentally observed resonances can approximately be classified as $SU(2N_f) \times O(3)$ multiplets, grouped into excitation bands, $N = 1, 2, 3, \dots$, each band containing a number of $SU(6) \times O(3)$ multiplets.

The situation is technically more complicated for mixed symmetric states than for symmetric states. Two approaches have been proposed so far. The first one is based on the Hartree approximation and describes the N_c quark system as a ground state symmetric core of $N_c - 1$ quarks and an excited quark [11].

The second procedure, where the Pauli principle is implemented to all N_c identical quarks has been proposed in Refs. [12, 13]. There is no physical reason to separate the excited quark from the rest of the system. The method can straightforwardly be applied to all excitation bands. It requires the knowledge of the matrix elements of all the $SU(2N_f)$ generators acting on mixed symmetric states described by the partition $(N_c - 1, 1)$. In both cases the mass splitting starts at order N_c^0 . The latest achievements for the ground state and the current status of large N_c QCD excited baryons ($N = 1, 2, 3, 4$) can be found in Ref. [14]. The N

= 1 band is the most studied. The $N = 2$ band received considerable attention too. Here we reanalyze the results of Ref. [15] for $N = 2$. The reason is that in a few octets an anomalous situation appeared where the hyperons Λ or Σ (presently degenerate) appeared slightly lighter than the nucleon in the same octet.

Here we use the data of the 2014 Particle Data Group [16] which includes changes due to a more complex analysis of all major photo-production of mesons in a coupled-channel partial wave analysis.

2 The Mass Operator

The general form of the mass operator, where the $SU(3)$ symmetry is broken, has first been proposed in Ref. [9] as

$$M = \sum_i c_i O_i + \sum_i d_i B_i. \quad (1)$$

The operators O_i are defined as the scalar products

$$O_i = \frac{1}{N_c^{n-1}} O_\ell^{(k)} \cdot O_{SF}^{(k)}, \quad (2)$$

where $O_\ell^{(k)}$ is a k -rank tensor in $SO(3)$ and $O_{SF}^{(k)}$ a k -rank tensor in $SU(2)$ -spin, but invariant in $SU(N_f)$. Thus O_i is rotational invariant. For the ground state one has $k = 0$. The excited states also require $k = 1$ and $k = 2$ terms. The $k = 1$ tensor has three components, which are the generators L^i of $SO(3)$. The components of the $k = 2$ tensor operator of $SO(3)$ read

$$L^{(2)ij} = \frac{1}{2} \{L^i, L^j\} - \frac{1}{3} \delta_{i,-j} \mathbf{L} \cdot \mathbf{L}. \quad (3)$$

The operators $O_{SF}^{(k)}$ are expressed in terms of the $SU(N_f)$ generators S^i , T^a and G^{ia} .

The operators B_i break the $SU(3)$ flavor symmetry and are defined to have zero expectation values for nonstrange baryons. The coefficients c_i encode the quark dynamics and d_i measure the $SU(3)$ breaking. They are obtained from a numerical fit. The most dominant operators considered in the mass formula together with the fitted coefficients are presented in Table 1.

For the [56]-plets the spin-orbit operator O_2 is defined in terms of angular momentum L^i components acting on the whole system as in Ref. [17] and is order $\mathcal{O}(1/N_c)$

$$O_2 = \frac{1}{N_c} \mathbf{L} \cdot \mathbf{S}, \quad (4)$$

while for the [70]-plets it is defined as a single-particle operator $\ell \cdot s$ of order $\mathcal{O}(N_c^0)$.

$$O_2 = \ell \cdot s = \sum_{i=1}^{N_c} \ell(i) \cdot s(i). \quad (5)$$

3 Matrix elements

The matrix elements of the $[56, 2^+]$ multiplet were derived in Ref. [17]. Details of the derivation of the matrix elements of O_i for $[70, \ell^+]$, as a function of N_c , can be found in Ref. [18]. Note that in the case of mixed symmetric states the matrix elements of O_6 are $\mathcal{O}(N_c^0)$, in contrast to the symmetric case where they are $\mathcal{O}(N_c^{-1})$, and non-vanishing only for octets, while for the symmetric case they are non-vanishing for decuplets. Thus, at large N_c the splitting starts at order $\mathcal{O}(N_c^0)$ for mixed symmetric states due both to O_2 and O_6 .

The $SU(3)$ flavor breaking operators B_i have the same definition for both the symmetric and mixed symmetric multiplets. The matrix elements of B_2 and B_3 for $[70, \ell^+]$ were first calculated in Ref. [15]. For practical purposes we have summarized these results by two simple analytic formulas valid at $N_c = 3$. The diagonal matrix elements of B_2 take the following form

$$B_2 = -n_s \frac{\langle L \cdot S \rangle}{6\sqrt{3}}, \quad (6)$$

where n_s is the number of strange quarks and $\langle L \cdot S \rangle$ is the expectation value of the spin-orbit operator acting on the whole system. Similarly the diagonal matrix elements of B_3 take the simple analytic form

$$B_3 = -n_s \frac{S(S+1)}{6\sqrt{3}}, \quad (7)$$

where S is the total spin. The contribution of B_3 is always negative, otherwise vanishing for nonstrange baryons. These formulas can be applied to $^2 8_J$, $^4 8_J$, $^2 10_J$ and $^2 1_{1/2}$ baryons of the $[70, \ell^+]$ multiplet. Presently the $SU(3)$ breaking operators B_2 and B_3 are included in the analysis of the $[70, \ell^+]$ multiplet, first considered in Ref. [15].

4 Fit and discussion

We have performed a consistent analysis of the experimentally known resonances supposed to belong either to the symmetric $[56, 2^+]$ multiplet or to the mixed symmetric multiplet $[70, \ell^+]$ with $\ell = 0$ or 2 , by using the same operator basis. Results of the fitted coefficients c_i and d_i are exhibited in Table 1 together with the values of χ_{dof}^2 for each multiplet.

The spin and flavor operators O_3 and O_4 are the dominant two-body operators and bring important $1/N_c$ corrections to the masses. The sum of c_3 and c_4 of $[70, \ell^+]$ is comparable to the value of c_3 in $[56, 2^+]$ where the equal contribution of O_3 and O_4 is included in c_3 . The contribution of the operator O_6 containing an $SO(3)$ tensor is important especially for $[70, \ell^+]$ multiplet. Together with the spin-orbit it may lead to the mixing of doublets and quartets to be considered in further studies when the accuracy of data will increase. The incorporation of B_2 and B_3 in the mass formula of the $[70, \ell^+]$ multiplet brings more insight into the $SU(6)$ multiplet classification of excited baryons in the $N = 2$ band.

Table 1. List of the dominant operators and their coefficients (MeV) from the mass formula (1) obtained in numerical fit for $[56, 2^+]$ in column 2 and for $[70, \ell^+]$ in column 3. The spin-orbit operator O_2 is defined by Eq. (4) for $[56, 2^+]$ and by Eq.(5) for $[70, \ell^+]$.

Operator	$[56, 2^+]$	$[70, \ell^+]$
$O_1 = N_c \mathbf{1}$	542 ± 2	631 ± 10
O_2 spin-orbit	7 ± 10	62 ± 26
$O_3 = \frac{1}{N_c} S^i S^i$	233 ± 11	91 ± 31
$O_4 = \frac{1}{N_c} \left[T^a T^a - \frac{1}{12} N_c (N_c + 6) \right]$		112 ± 22
$O_6 = \frac{1}{N_c} L^{(2)ij} G^{ia} G^{ja}$	6 ± 19	137 ± 55
$B_1 = n_s$	205 ± 14	35 ± 33
$B_2 = \frac{1}{N_c} (L^i G^{i8} - \frac{1}{2\sqrt{3}} L^i S^i)$	97 ± 40	-38 ± 121
$B_3 = \frac{1}{N_c} (S^i G^{i8} - \frac{1}{2\sqrt{3}} S^i S^i)$	197 ± 69	46 ± 159
χ^2_{dof}	1.63	1.67

4.1 The multiplet $[56, 2^+]$

The partial contribution and the calculated total mass obtained from the fit were presented in Table VI of Ref. [15] which we do not repeat here. The experimental masses were taken from the 2014 version of the Review of Particle Properties (PDG) [16], except for $\Delta(1905)5/2^+$ where we used the mass of Ref. [17] which gives a smaller χ^2_{dof} , but does not much change the fitted values of c_i and d_i . As expected, the most important sub-leading contribution comes from the spin operator O_3 . The contributions of the angular momentum-dependent operators O_2 and O_6 are comparable, but small. Among the SU(3) breaking terms, B_1 is dominant. An important remark is that in the $[56, 2^+]$ multiplet B_2 and B_3 lift the degeneracy of Λ and Σ baryons in the octets, which is not the case for the $[70, \ell^+]$ multiplet.

4.2 The multiplet $[70, \ell^+]$

As compared to Ref. [18] where only 11 resonances have been included in the numerical fit, here we consider 16 resonances, having a status of three, two or one star. This means that we have tentatively added the resonances $\Xi(2120)?^{*}$, $\Sigma(2070)5/2^{+*}$, $\Sigma(1940)?^{*}$, $\Xi(1950)?^{***}$ and $\Sigma(2080)3/2^{+**}$. The masses and the error bars considered in the fit correspond to averages over data from the particle listings, except for a few which favor specific experimental values cited in the headings of Table 2.

We have ignored the $N(1710)1/2^{+***}$ and the $\Sigma(1770)1/2^{+*}$ resonances, the theoretical argument being that their masses are too low, leading to unnatural sizes for the coefficients c_i or d_i [19]. On the experimental side one can justify the removal of the controversial $N(1710)1/2^{+***}$ resonance due to the latest GWU analysis of Arndt et al. [20] where it has not been seen. We have also ignored the

$\Delta(1750)1/2^{+*}$ resonance, because neither Arndt et al. [20] nor Anisovich et al. [21] find evidence for it.

Table 2. Partial contribution and the total mass (MeV) predicted by the $1/N_c$ expansion. The last two columns give the empirically known masses and status from the 2014 Review of Particles Properties [16] unless specified by (A) from [21], (L) from [22], (Z) from [23], (G1) from [24], (B) from [25], (AB) from [26], (G2) from [27].

	Part. contrib. (MeV)								Total(MeV)	Experiment(MeV)	Name, status
	c_1O_1	c_2O_2	c_3O_3	c_4O_4	c_6O_6	d_1B_1	d_2B_2	d_3B_3			
$^4N[70, 2^+]_{\frac{7}{2}^+}$	1892	62	113	28	-22	0	0	0	2073 ± 38	$2060 \pm 65(A)$	$N(1990)7/2^{+**}$
$^4\Lambda[70, 2^+]_{\frac{7}{2}^+}$						35	11	-17	2102 ± 19	$2100 \pm 30(L)$	$\Lambda(2020)7/2^{+**}$
$^4\Xi[70, 2^+]_{\frac{7}{2}^+}$						70	22	-34	2131 ± 8	2130 ± 8	$\Xi(2120)?^{?*$
$^4N[70, 2^+]_{\frac{5}{2}^+}$	1892	-10	113	28	57	0	0	0	2080 ± 32	2000 ± 50	$N(2000)5/2^{+**}$
$^4\Lambda[70, 2^+]_{\frac{5}{2}^+}$						35	-2	-17	2096 ± 10	2100 ± 10	$\Lambda(2110)5/2^{+***}$
$^4N[70, 2^+]_{\frac{3}{2}^+}$	1892	-62	113	28	0	0	0	0	1972 ± 29		
$^4\Lambda[70, 2^+]_{\frac{3}{2}^+}$					0	35	-11	-17	1979 ± 39		
$^4N[70, 2^+]_{\frac{1}{2}^+}$	1892	-93	113	28	-80	0	0	0	1861 ± 33	$1870 \pm 35(A)$	$N(1880)1/2^{+**}$
$^4\Lambda[70, 2^+]_{\frac{1}{2}^+}$						35	-16	-16	1869 ± 79		
$^2N[70, 2^+]_{\frac{5}{2}^+}$	1892	21	23	28	0	0	0	0	1964 ± 29	$1860 \pm {}^{120}_{60}(A)$	$N(1860)5/2^{+**}$
$^2\Sigma[70, 2^+]_{\frac{5}{2}^+}$					0	35	4	-3	2000 ± 18	$2051 \pm 25(G1)$	$\Sigma(2070)5/2^{+**}$
$^2N[70, 2^+]_{\frac{3}{2}^+}$	1892	-31	23	28	0	0	0	0	1912 ± 21	$1905 \pm 30(A)$	$N(1900)3/2^{+***}$
$^2\Sigma[70, 2^+]_{\frac{3}{2}^+}$					0	35	-6	-3	1938 ± 10	1941 ± 18	$\Sigma(1940)?^{?*$
$^2\Xi[70, 2^+]_{\frac{3}{2}^+}$					0	70	-11	-7	1964 ± 7	$1967 \pm 7(B)$	$\Xi(1950)?^{?***}$
$^4N[70, 0^+]_{\frac{3}{2}^+}$	1892	0	113	28	0	0	0	0	2033 ± 18	$2040 \pm 28(AB)$	$N(2040)3/2^{+**}$
$^4\Sigma[70, 0^+]_{\frac{3}{2}^+}$						35	0	-16	2052 ± 21	2100 ± 69	$\Sigma(2080)3/2^{+**}$
$^2\Delta[70, 2^+]_{\frac{5}{2}^+}$	1892	-21	23	140	0	0	0	0	2034 ± 31	1962 ± 139	$\Delta(2000)5/2^{+**}$
$^2\Sigma^*[70, 0^+]_{\frac{1}{2}^+}$	1892	0	23	140	0	35	0	-3	2087 ± 30	1902 ± 96	$\Sigma(1880)1/2^{+**}$
$^2\Lambda'[70, 0^+]_{\frac{1}{2}^+}$	1890	0	23	-84	0	35	0	-3	1863 ± 19	$1853 \pm 20(G2)$	$\Lambda(1810)1/2^{+***}$

The partial contributions and the calculated total masses obtained from the fit are presented in Table 2. Regarding the contribution of various operators we note that the good fit for $N(1880)1/2^{+**}$ was due to contribution of the spin-orbit operator O_2 of -93 MeV and of the operator O_6 which contributed with -80 MeV. The good fit also suggests that $\Sigma(1940)?^{?*}$ and $\Xi(1950)?^{***}$ assigned by us to the $^2[70, 2^+]3/2^+$ multiplet is reasonable, thus these resonances may have $J^P = 3/2^+$, to be experimentally confirmed in the future.

The $1/N_c$ expansion is based on the $SU(6)$ symmetry which naturally allows a classification of excited baryons into octets, decuplets and singlets. In Table 2 the experimentally known resonances are presented. In addition some predictions are made for unknown resonances. Many of the partners in a given $SU(3)$ multiplet are not known. Note that Λ and Σ are degenerate in our approach. Although the operators B_2 and B_3 have different analytic forms at arbitrary N_c [15] they become identical at $N_c = 3$ for Λ and Σ in octets, thus they cannot lift the degeneracy between these hyperons, contrary to the $[56, 2^+]$ multiplet.

The present findings can be compared to the suggestions for assignments in the $[70, \ell^+]$ multiplet made in Ref. [28] as educated guesses. The assignment of $\Sigma(1880)1/2^{+**}$ as a $[70, 0^+]1/2^+$ decuplet resonance is confirmed as well as the assignment of $\Lambda(1810)1/2^{+**}$ as a flavor singlet. We agree with Ref. [28] regarding $\Lambda(2110)5/2^{+***}$ as a partner of $N(2000)5/2^{+**}$ in a spin quartet, contrary to our previous work [15] where $\Lambda(2110)5/2^{+***}$ was a member of a spin doublet, together with $N(1860)5/2^{+**}$ and $\Sigma(2070)5/2^{+*}$. This helps to restore the correct hierarchy of masses in all octets. However we disagree with Ref. [28] that $N(1900)3/2^{+***}$ is a member of a spin quartet. We propose it as a partner of $\Sigma(1940)?^{?*}$ and $\Xi(1950)?^{***}$ in a spin doublet.

The problem of assignment is not trivial. Within the $1/N_c$ expansion method Ref. [17] suggests that $\Sigma(2080)3/2^{+**}$ and $\Sigma(2070)5/2^{+*}$ could be members of two distinct decuplets in the $[56, 2^+]$ multiplet.

Here the important result is that the hierarchy of masses as a function of the strangeness is correct for all multiplets.

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