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Some classical motions of three quarks tethered to the Torricelli string*

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In this talk we report some soon to be published results [1] of our studies of figure-eight orbits of three-bodies in three potentials: 1) the Newtonian gravity, i.e., the pairwise sum of $-1/r$ two-body potentials; 2) the pairwise sum of linearly rising r two-body potentials (a.k.a. the Δ string potential); 3) the Y-junction string potential [2] that contains both a genuine three-body part, as well as two-body contributions (this is the first time that the figure-eight has been found in these string potentials, to our knowledge). These three potentials share two common features, *viz.* they are attractive and symmetric under permutations of any two, or three particles. We were led to do this study after recognizing the existence of a dynamical symmetry underlying the remarkable regularity in the Y- and Δ string energy spectra [3].

We have found that a set of variables that consists of the "hyper-radius" $R = \sqrt{\rho^2 + \lambda^2}$, the "rescaled area of the triangle" $\frac{\sqrt{3}}{2R^2}|\boldsymbol{\rho} \times \boldsymbol{\lambda}|$) and the ("braiding") hyper-angle $\phi = \arctan\left(\frac{2\boldsymbol{\rho}\cdot\boldsymbol{\lambda}}{\lambda^2-\rho^2}\right)$ makes this permutation symmetry manifest; we use them to plot the motion of the numerically calculated figure-eight orbit. According to Ref. [4], H. Hopf was the first one to introduce these variables, Ref. [5]. As there are three independent three-body variables, and there are two independent permutation-symmetric three-body variables, R and the area the third variable cannot be permutation-symmetric. Moreover, it must be a continuous variable and not be restricted only to a discrete set of points, as is natural for permutations. We identify here the third independent variable as $\phi = \arctan\left(\frac{2\boldsymbol{\rho}\cdot\boldsymbol{\lambda}}{\lambda^2-\rho^2}\right)$ and show that it grows/descends (almost) linearly with the time t spent on the figure-eight trajectory and reaches $\pm 2\pi$ after one period T. Thus, on the figure-eight orbit ϕ is, for most practical purposes, interchangeable with the time variable t. The hyper-angle ϕ is the continuous braiding variable that interpolates smoothly be-

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tween permutations and thus plays a fundamental role in the braiding symmetry of the figure-eight orbits [6,8].

Then we constructed the hyper-angular momentum $G_3 = \frac{1}{2}\left(\mathbf{p}_\rho \cdot \boldsymbol{\lambda} - \mathbf{p}_\lambda \cdot \boldsymbol{\rho}\right)$ conjugate to ϕ, the two forming an (approximate) pair of action-angle variables for this periodic motion. Here we calculate numerically and plot the temporal variation of ϕ, as well as the hyper-angular momentum $G_3(t)$, the hyper-radius R and r. We show that the hyper-radius $R(t)$ oscillates about its average value $\overline{R}$ with the same angular frequency 3ϕ and phase, as the new ("reduced area") variable $r(t)$. Thus, we show that $\phi(t)$ is, for most practical purposes, interchangeable with the time variable t, in agreement with the tacit assumption(s) made in Refs. [7], [4], though the degree of linearity of this relationship depends on the precise functional form of the three-body potential.

As stated above, ϕ is not exactly proportional to time t, but contains some non-linearities that depend on the specifics of the three-body potential; consequently the hyper-angular momentum G_3 is not an exact constant of this motion, but oscillates about the average value $\overline{G}_3$, with the same basic frequency 3ϕ. Thus, the time-averaged hyper-angular momentum $\overline{G}_3$ is the action variable conjugate to the linearized hyper-angle ϕ'.

We used these variables to characterize two new planar periodic, but non-choreographic three-body motions with vanishing total angular momentum. One of these orbits corresponds to a modification of the figure-eight orbit with $\phi(t)$ that also grows more or less linearly in time, but has a more complicated periodicity pattern defined by the zeros of the area of the triangle formed by the three particles (also known as "eclipses", "conjunctions" or "syzygies"). Another new orbit has $\phi(t)$ that grows in time up to a point, then stops and "swings back". We show that this motion, and the other two, can be understood in view of the analogy between the three-body hyper-angular ("shape space") Hamiltonian on one hand and a variable-length pendulum in an azimuthally periodic in-homogeneous gravitational field, on the other.

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