PROBABILISTIC STUDY OF THE PHENOMENON OF CRUSHED GRAINS USING THE MODEL OF WEIBULL

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Abstract

The crushing of grains in a granular medium is a very important phenomenon; it is a source of both physical and mechanical changes to these materials. A statistical study of the mechanical properties of a material was used to characterize the distribution and fracture mechanics in order to quantify the evolution of these distributions with sample sizes (grains and whole grains).

This work presents the results of an experimental study made on the crushing of individual grains of limestone and samples consisting of multiple grains subjected to a uniaxial loading in order to highlight the influence of the loading and the grain size on the rate of crushing. A statistical study using the Weibull method allowed us to model the problem and quantify the rate of breakage for the two cases.

The results obtained show that the rate of deflection depends on the grain size and the intensity of the applied load. Statistical modelling using the Weibull method gave us acceptable results in both cases.

keywords

grain, sample, crushing, statistical study

INTRODUCTION

The effect of grain crushing is a specific problem for the behaviour of granular materials subjected to high stresses. The breaking or crushing of a grain is of great importance in understanding the deformation properties of soils or rocks. It has been observed that the effect of the crushing of particles is advanced as a major cause of the compressibility and the deterioration of the mechanical properties of granular materials subjected to high stresses [1].

Studies, usually based on comparisons of grain size distribution curves before and after loading, performed on grain samples showed that the phenomenon of crushing the grains depends on the physical and mechanical characteristics of the grains. The authors of [2,3,4] studied the influence of grain size, shape and size distribution. Their findings, in agreement with all the results available on the mechanical behaviour of granular media under high stresses, suggest that the effect of grain crushing and the compressibility of the medium increases with increasing grain size, the uniformity of the sample, the angularity of the particles, the pressure containment, and the deviator stress for a given confining pressure.

Based on these experiments, parameters such as the total deal-breaker factor [5], the granular stability (aggregate stability) [6], the probability of a crash and the breaker particle factor [7] were identified. These parameters were found to be of great importance for an understanding of the behaviour of granular materials crashing. It has been shown that the stresses of the resistance to the crushing of particles follow the Weibull distribution [8,9]. For soil particles of size "d", loaded diametrically between two flat plates, McDowell and Amon [8] showed that the survival probability $P_s(d)$ is given according to the reference diameter d_0, the stress σ_0 that gives a probability of survival of 37% for the particle diameter d_0 tested, and "m", which is the Weibull modulus.

This work focuses on determining the probability of the survival of grains made of limestone subjected to unidirectional loading using the Weibull statistical method. It consists of submitting a singular grain to a unidirectional loading and then samples of multiple grains until break up. Laboratory tests provide the pairs (σ; $P_s(d)$) that allow us to determine the Weibull modulus for each case separately. The results obtained show that the probability of survival, either of the crash of a singular grain or in the case of samples formed by multiple grains, can be represented by the Weibull statistical distribution.

1 IDENTIFICATION AND CHARACTERIZATION OF MATERIALS

The materials used are aggregates obtained from crushing rock coming from Adrar Oufarnou career. The deposit of rock in question is located at a distance of 8 km north of the city of Bejaia in Algeria. The deposit is formed by lands of lower leas, which are represented by grey limestone, a light-grey, pinkish compact and massive to small and fat crystals dolomitized by locals for the production of aggregates and stones used in construction. All the seated limestone is traversed by folions. The massif is formed at its base by marls, marly limestone and the dogger by massive limestone and dolomite from the lower leas to the top.

The analysis performed by the specialized laboratory of the National Office of Geological and Mining Research (ORGM) on rock samples in order to identify the mineralogical and chemical characteristics of the deposit produced the results in Table1.

Table 1. Chemical constituents of the material.

Component	Chemical symbols	Percentage
silicon oxide	SiO_2	~1.75%
iron oxide	FeO_3,	~0.2%
titanium oxide	TiO_2	<0.5%
Aluminum oxide	Al_2O_2	~0.05%
manganese oxide	MnO	~0.05%
magnesium oxide	MgO	0.62%
sodium oxide	Na_2O	<0.5%
potassium oxide	K_2O	<0.5%
Sulfuric oxide	SO_3	<0.5%
lime	(CaO)	54%
fire losses PAF (organic materials and other)	-	43%

The tests results carried out with the Los Angeles (LA) and Micro-Deval (MDE) on the material gave mean values of LA (%) = 31 and of MDE (%) = 19. Tests of the fragmentability according to the EN standard (NF P94-066) and the degradability DG (NF P94-067) made into a 10/20mm fraction of this material were Fr = 3 and DG = 1. The average absolute density is approximately 2.7g/cm^3.

2 DESCRIPTION OF THE EXPERIMENTAL DEVICE

The experimental device shown in Figure (1) is designed specifically for a study of the characteristics of the crushing of granular materials that come from crushing limestone.

Figure 1. Presentation of the samples used.

Actually, this material is used to adapt a brain hydraulics used to apply loads on the device, which is used in turn to hold the samples. Overall, we used three types of fixtures for the various tests.

The first assembly consists of two hard steel plates. The thickness of each is 6 mm; they are used to apply stress to the grains. These will be placed between two plates and subjected to a normal load for the crushing grain by grain. And for the second installation it is a hard steel cylinder of 5.5 mm in diameter, with a height that respects the ratio H/d between 3 and 5, and steel piston lasts of 5.3 mm in diameter, used to transmit loads to the sample. This arrangement is used for crushing tests of the samples of grain classes 3.5mm, 5mm and 6.3-8mm. The third device is a second identical assembly with different dimensions used to crush grain samples of classes 8-10 mm and 14mm. The diameter of the cylinder is 10.5mm and the piston is 10.3 mm, able to maintain the ratio H/d at between 3 and 5.

The grains used in the tests will be coated with a colour pigment on the surface for the easy identification of possible breaks after the application of a load on different samples. This coloration is such that it has no effect on the surface of the grains.

The load is applied with hydraulic system equipment that allows automatic control of the load applied to the sample of the material in question and facilitates the reading values of the applied force.

3 CRUSH BY GRAIN

In this case grains of limestone were used more or less rounded to the different size classes after they were coated with a colour pigment on the surface. Note that the choice of a standard sample to be tested in the laboratory is necessary if the size effects are considered solely responsible for the variations in the mechanical properties in order to avoid effects related to the shape.

The tensile strength of the soil particles and the rock is usually measured by the diametral compression between plates until fracture [10].

The crushing test consists of subjecting the individual grains to a normal force. The grain is placed between two steel plates and using the hydraulic cylinder a load is exerted. For a particle of diameter d, diametrically compressed by a force F, a tensile strength σ feature is induced that can be defined as follows:

$$\sigma = \frac{F}{d^2} \qquad (1)$$

The dimension d is usually the distance between two plates, but for particles that are roughly spherical the nominal size can be taken as the average of three measurements along the orthogonal directions [9].

The tensile strength corresponds to the maximum value of σ at which the particle undergoes catastrophic failure [9].

For each size class we conducted 40 tests, such that each test effort is exerted on the grain in question until rupture. The tensile strength is taken as the point at which the grain undergoes a catastrophic failure. The results for the five classes tested are represented in Figure (3).

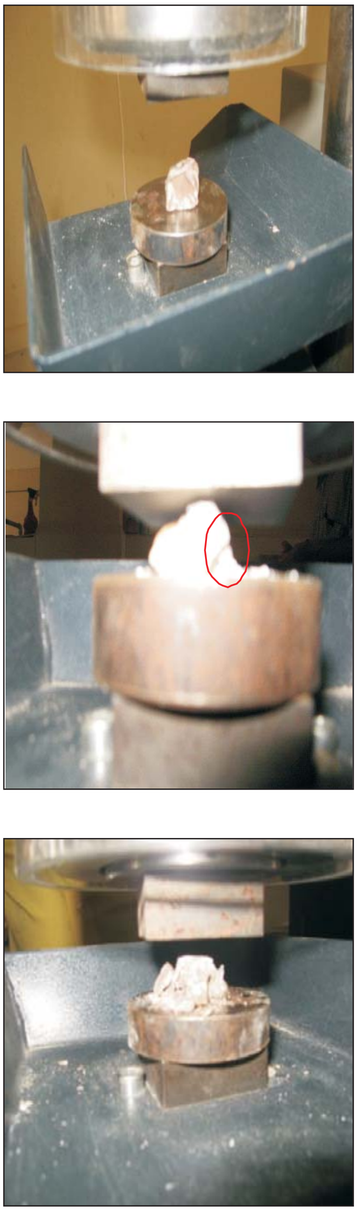

Figure 2. Tests crushing grain by grain.

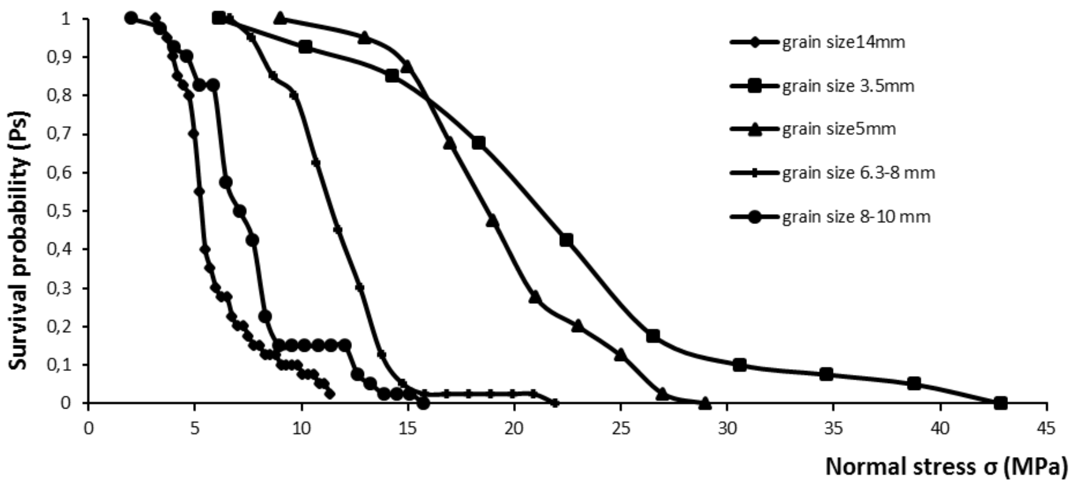

Figure 3. Representation of the percentage survival of grains of limestone according to the load exerted on the crushed limestone grains (singular grain).

4 RESULTS AND DISCUSSION

After submitting a grain to a normal force, it was noted that for all grains, the first cracks appear on the angularities in contact with the steel plates. The fracture occurs in the form of a shell for some grains and finally the total failure and catastrophic fracture occurs.

Figure (3) shows the representation of the percentage survival of particles as a function of the applied stress. The relative frequency is calculated using the median value. The results obtained show a dispersion of the values of the compressive strength for the same size class and that for different classes studied. These results are similar to those obtained for cereal grains and sand silica gel reported by [10], and the ballast carried out by [11]. The variation of the resistance to the crushing of the grains can be explained by the fact that the state of the grains (shape) is not the same before their submission to the loading, so that the state of the cracking varies from one grain to another, which is reflected by the change in the resistance, and thus spreading curves $P_s = f(\sigma)$. These generally have the same form for different classes.

The calculation of the probability of survival for the grains was carried out using the following formula:

$$P_S = 1 - P_C \qquad (2)$$

- P_c : cumulative percentage of broken grains;
- P_s : percentage of grains that survive;

The minimum resistance of the crash or of the appearance of the first break differs from one class to the other; its value increases with the decrease in the grain diameters, *d*, such that the minimum resistances obtained are shown in Table 2:

Table 2. Minimum stress of rupture by size range.

Diameter of particle size *d* (mm)	3.5	5	6.3-8	8-10	14
minimum stress (MPa)	6.12	9	5.6	3.4	1.4

The compressive strength presents a scale effect: the larger the sample size, the lower is the resistance [10]. Granular materials from original rocks are generally porous with a low tenacity (full of cracks and pores). When two geometrically identical blocks are cut from a single block of fragile rock, they have a dispersion of strength due to the dispersion of the size of the existing defects. When the grains of soil are healthy, small and very rounded, they can withstand very high stresses [12].This has been explained by several authors with the fact that the probability of the existence of defects in a big grain is more than for a small one. When the soil grains are hard, strong and very rounded, they can withstand high stresses without being crushed. The angular particles of freshly extracted materials undergo fragmentation under the normal pressures due to the rupture of sharp angularities.

5 STATISTICAL MODELLING

In order to understand the physical behaviour of soils, it is important to define the degree of crushing of the grains of a soil element and then to quantify it, because the crushing of grain soils during loading is a source of modification of the physical and mechanical characteristics [1]. In order to quantify the grain-crushing phenomenon we made a representation of the results of crash tests for singular grains with the graphs P_s = f(σ) is shown.

To achieve a model that supports the quantization rate of crushing grains, we used the Weibull statistical method. This method takes into account the dispersion of the responses measured during the tests. The perspective used in the model is that the rupture is initiated at a defect of the material that spreads instantaneously to produce the break. This is taken as being fragile. The heterogeneity of the material is modelled by defects that are characterized by their sensitivity to rupture under a mechanical stress.

In the spirit of the Weibull statistical method for any constraint σ of the pair (σ; $P_s(d)$) we calculate $x_i = \ln(\sigma_i)$ and $P_s = 1 - P_c$ and $y_i = \ln(-\ln(P_s(\sigma_i))$. The graphical representation of the cloud point Fig (3) shows linearity, so that the points obtained are sufficiently aligned and can be presented by the regression lines of y in x.

The regression equation obtained was:

$$Y = m \cdot x - m \cdot \ln(\sigma_0) \qquad (3)$$

The regression lines obtained give us the values of the Weibull parameters m for each class studied, such that the latter is the slope of the line and ($-m \cdot \ln(\sigma_0)$) is the intercept. The Weibull modulus m is a measure of the dispersion; the lower the modulus, the higher is the dispersion.

Weibull proposed a simplified form for calculating the probability of failure:

$$P_C(\sigma, V) = 1 - e^{\left(-\left(\frac{V}{V_0}\right) f(\sigma)\right)} \qquad (4)$$

where V is the sample volume, V_0 is the reference volume and σ is the tensile strength. McDowell and Humphreys [10] studied the crushing of singular-grain serials (corn flask, corn Cripps and sand silica gel) and concluded that the failure of the latter can be quantified using the Weibull statistical method. They proposed the following formula:

$$P_S(d) = e^{\left[-\left(\frac{d}{d_0}\right)^3 \left(\frac{\sigma}{\sigma_0}\right)^m\right]} \qquad (5)$$

where σ_0 corresponds to the stress for a survival of 37% for a particle diameter d_0 of the reference. The value thereof is approximately equal to the average stress, and m is the Weibull modulus, which decreases with the growth of the variability stresses.

McDowell and Bolton [13] have suggested that the tensile strength should be proportional to the average stress of crushing the grain. McDowell and Humphrys [10] showed that the value of the crush strength is measured according to the size, so that the average resistance is given by:

$$\sigma_{av} = d^{-b} \qquad (6)$$

where b is a parameter determined experimentally to be equal to $3/m$.

McDowell and Humphrys [10] proposed that for the singular grain crash for a number of materials equation (5) can be written as follows:

$$P_S(d) = e^{\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right]} \qquad (7)$$

Wee Lim et al. [11] showed that σ_0 is proportional to the grain diameter and approximately equal to the average resistance with $\sigma_{av} = d^{-3/m}$

In the case of identical grains of limestone, it was considered that $d/d_0 = 1$, and then equation (5) can be reduced to the following formula:

$$P_S(d) = e^{\left[-\left(\frac{\sigma}{\sigma_{50}}\right)^m\right]} \qquad (8)$$

The results for all the cases studied show that we can take $\sigma_0 \approx \sigma_{50}$.

So in this case, the survival rate of the grain can be approximated by the empirical formula shown in (8).

The linearization curves P_s = f(σ) using the Weibull method Fig. (4) for the limestone grains show a linear tendency, and the Weibull modulus that corresponds to the slope of the regression line varies according to the sample size, such that the values obtained for each class are represented in Table 3:

Table 3. Weibull modulus obtained for each size class studied.

Diameter of particle size d (mm)	3.5	5	6.3-8	8-10	14
Weibull modulus obtained "m"	2.969	5.66	4.28	3.17	2.37

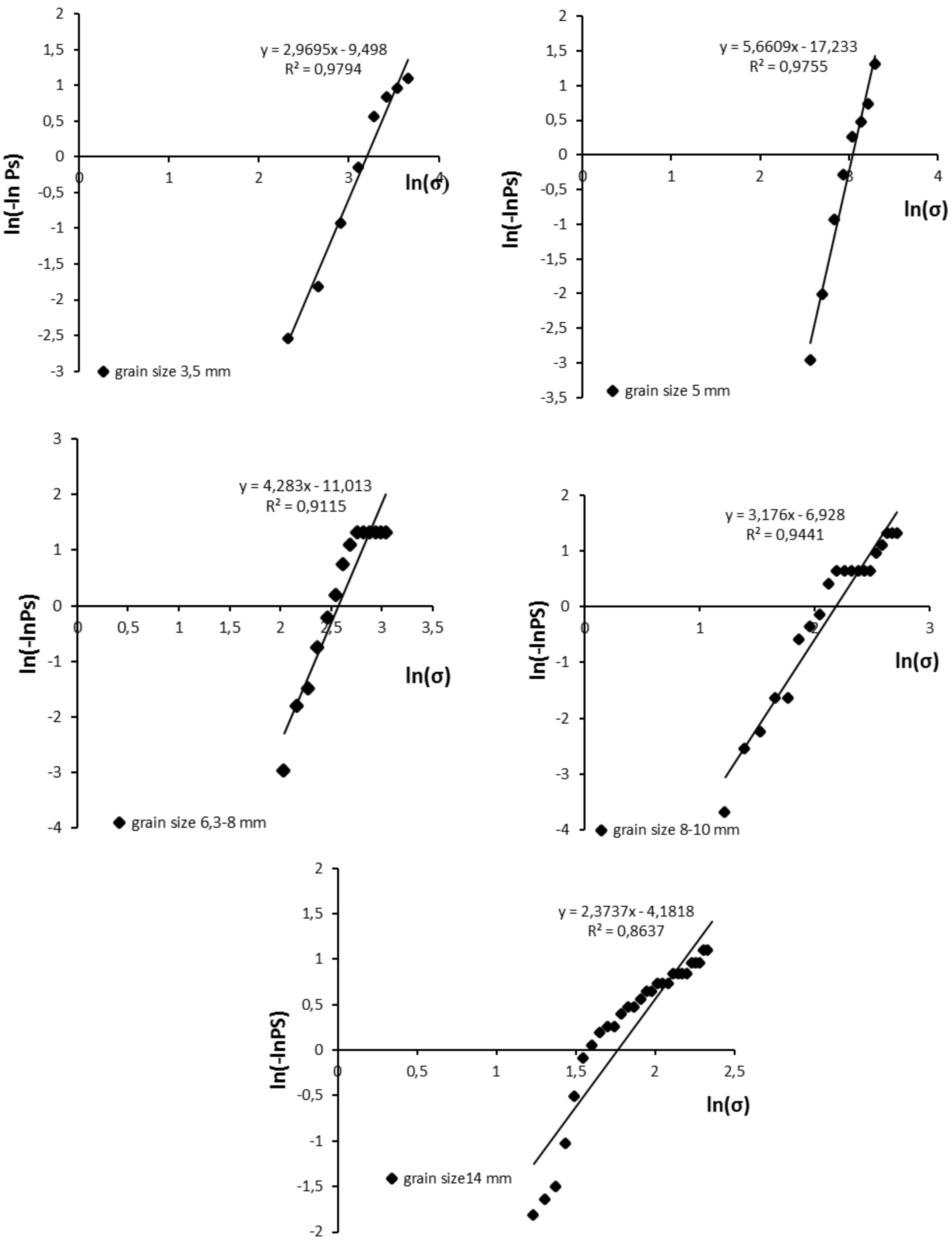

Figure 4. Graphical representation of the survival Weibull probability (crushing grain by grain).

Note that the Weibull modulus increases with a reduction of the grain size for the four classes studied. This means that higher when the grain size is important, the greater dispersion of values of resistance is important. In fact the Weibull modulus decreases with an increasing of the distribution of the values of the stress σ [8, 11]. The results confirm that the Weibull modulus cannot be a parameter belonging to the materials, but depends on the grain size and the initial state of the material. It is possible that the shape parameter is overestimated because it is biased, but in this study it is not taken in consideration that this parameter is overestimated and should be corrected.

5.1 CRUSHING OF A SAMPLE CONSISTING OF SEVERAL GRAINS

In this case we pass from a study of a singular grain to a study of one of medium to several grains in order to calculate the probability of the rupture of the grains in a granular medium and their quantification.

Several studies have been conducted in the direction of quantifying the rate of crushing of the grains. Usually, these studies are based on a comparison of the grain size distribution curves of samples of materials before and after their submission to the load. Several authors have defined the rate of deflection from developing or spreading the grading curve [14, 15, 16]. Then, all the parameters are determined from the percentage corresponding to a given selected grain size. We distinguish the methods of a single factor, on the one hand [2, 17], and on the other hand the methods with several other factors, for example, the Marsal method that allows determining the percentage of grains weight affected during the test. In our case we conducted another way, as we took for each size class a known number of grains, according to the diameters, the number of grains taken for each sample is as follows: (25 grains of the class for 14mm, 30 grains for class 8-10, and 6.3-8, 100 for class 5 mm and 200 grains for Class 3.5mm). The number of grains was chosen to meet the ratio H/d between 3 and 5. And then we calculate the number of grain that has apparent breaks.

Granular systems are disordered and heterogeneous systems formed by many interacting elements; they have two complementary phases, a poly-disperse distribution of sizes, varied forms of particles, interactions that are highly nonlinear and an intrinsically disordered geometry [18]. For these reasons, we must use a standard sample in the laboratory for the effects of size to be considered solely responsible for the variations in the mechanical properties.

5.2 OBSERVATIONS ON SAMPLES

The observations made on all samples submitted to unidirectional loading showed that the contacts between the grains are the cause of most breaks noticed in most grains. When two bodies in contact are pressed against each other, they deform locally to create a contact area of a characteristic dimension smaller than the small radius of curvature of two bodies at the contact point, the area on which the contact force acts. The induced stresses are thus concentrated in a small region around the contact point that leads to breakage, in most cases it is the sharp points that break, and their intensity decreases rapidly when it is removed (after the break). In the aggregate, there are two types of changes possible under mechanical stress: they break giving elements of any size or they wear by friction giving essentially fine elements [12].

Figure 5. Crushing tests of samples composed of multiple grains.

The phenomenon of grain crushing occurs at much higher values in the case of crushing grain by grain. This can be explained in terms of the rearrangement of the grains of the samples. This leads us to say that the grains begin to crash when they have a tight configuration and activation of the contact forces between the grains. So in this case, the force applied on the set (sample) has not yet reached the inherent resistance of the grains. The crushing of the grains appears when the constraint imposed on the grains is beyond their own strength [12]. We can also add that when a grain in a granular medium is surrounded by other grains and can generate forces of containment on the latter, it will prevent or at least delay the break.

6 EXPERIMENTAL RESULTS

The test results of unidirectional loading on the samples consisting of several grains of the material Fig. (6) show a distribution similar to the grain-by-grain crushing (grain singular), but with much higher stress values. During the loading tests we can distinguish three phases, the first one happens without apparent breaks, this is the phase of the rearrangement of the grain. In this case the load applied is dissipated to create closer contacts of the grains against each other, and this is reflected in most cases by a slight change in volume. Above a certain load we obtain a significant rate of grain crushing. This is the second phase. The last one is the phase or structure in which the sample has a new configuration (new size). New contacts and a new charge distribution reduce the rate of deflection. Hagerty et al [19] observed the same phases.

The choice of grains of the same shape (rounded) and the same size constitutes samples that are more or less of uniform size for different classes of limestone aggregates used in the tests. It was found that the crushing of the grains does not change after obtaining a certain rate. This is probably due to the new configuration size of the samples, so that smaller particles generated by the crushing of the larger ones will rearrange to fill the inter-granular voids. This will lead to the creation of other points of contact, and a new redistribution of stresses within the sample. A particle of a given size suffers less in the case of breakage, because it will fragment into smaller particles that will create a new scale of the sample and therefore new contacts. In the case of graduated soil there are many particles so many contacts and inter-particle charge in the contact is less than in a soil with uniform graduation [14, 20].

The observation of the samples after their submission to the various loads has shown that most cracks are located at the angularities. This can be explained by the fact that particles with angularities break more easily, as long as the loads are concentrated along the latter, causing fracturing of the particles. The stress can also focus on the corner points of contact, causing breakage points [17].

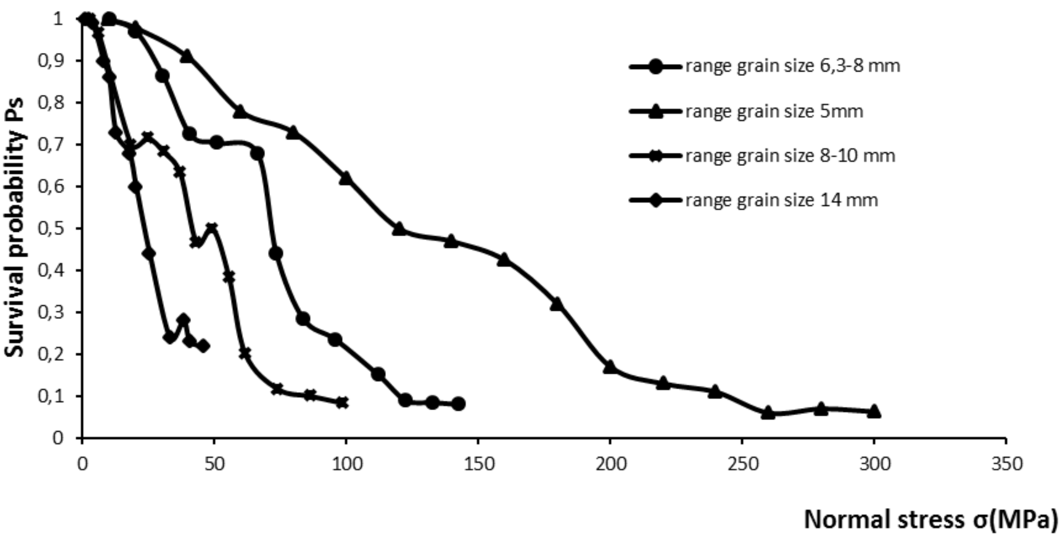

Figure 6. Representation of the survival percentage of limestone grains accordingto the load exerted on the crushed grains (samples consisting of several grains).

7 STATISTICAL MODELLING

Statistical modelling aims to quantify the rate of crushing of grains in samples subjected to a load. For this we plotted the curves P_s = f(σ) corresponding to each size class. The results of the statistical study gave the values σ and P_s from Eq. (2).

In the same way, for any constraint σ we calculate $x_i = \ln(\sigma_i)$ and $P_s = 1\text{-}P_c$ and $y_i = \ln(\text{-}\ln(P_s(\sigma_i))$ in the spirit of the Weibull statistical method. The graphical representation of the cloud point Fig (7) shows linearity, so that the points obtained are sufficiently aligned and can be represented by the regression lines of y=f(x).

Fig. 7 shows the linearization of the curves in Fig. 6 using the Weibull method, obtained for samples composed of multiple grains. This linearization is obtained by calculating ln(σ) and ln(-ln(P_s)), respectively, which correspond to the natural logarithm of the stress and the survival probability of the particles to obtain the Weibull modulus m.

The results obtained confirm that the Weibull method are applicable for modelling the phenomenon of crushing the grains in a granular medium.

Because of the uniformity of the samples used for various tests, we can consider reports V/V_0 = 1, where V

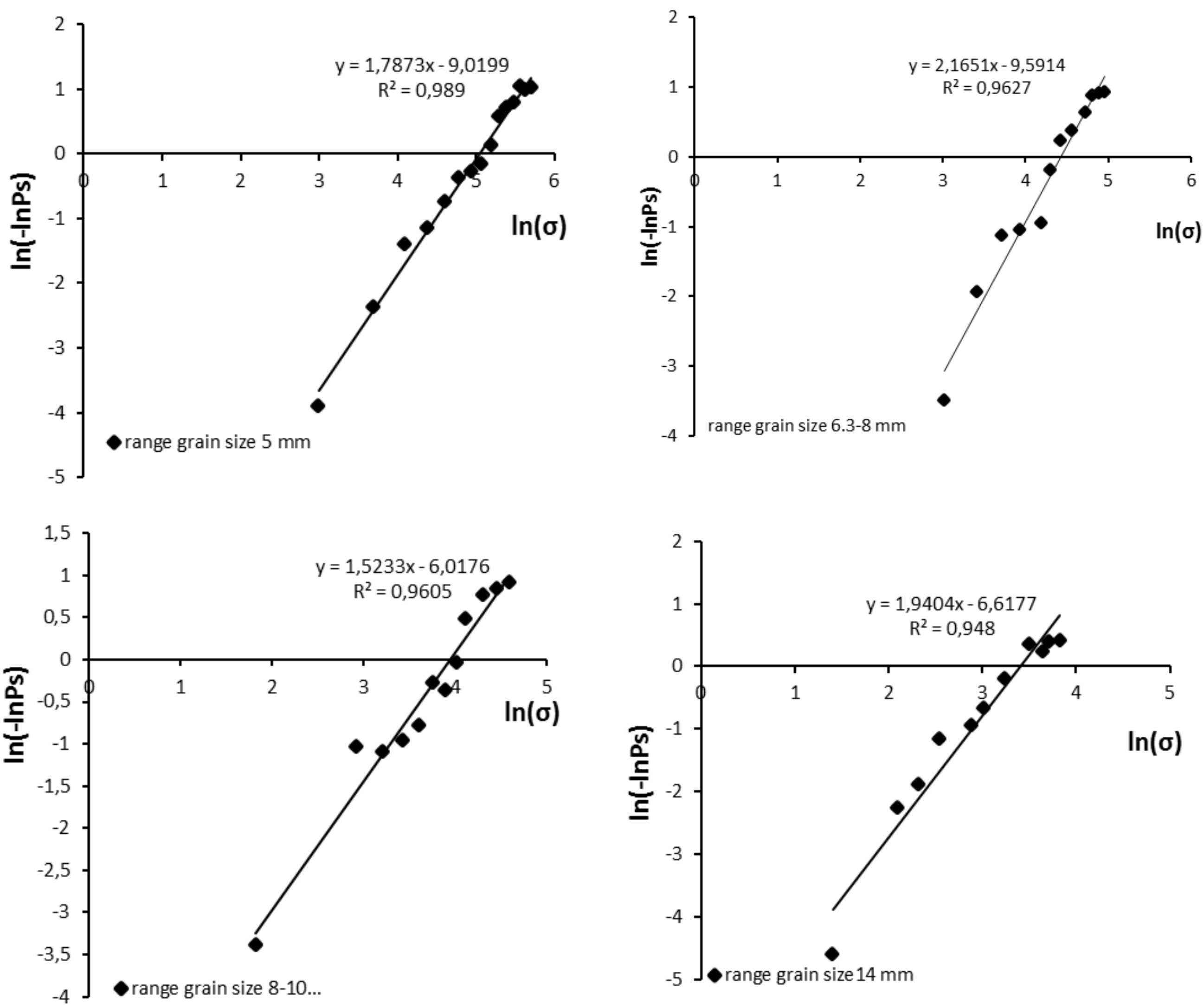

Figure 7. Representation of the survival Weibull probability (sample consisting of several grains).

and V_0 are respectively the volumes of the sample tested and the volume of the reference sample. The Weibull modulus for each class is obtained from the slopes of the regression lines, and σ_0 in our case is taken as the stress at which about 50% of the particles survive. In this case the probability of the survival of the grains can be calculated using equation (7).

To verify the validity of the model we can perform the Kolmogorov-Smirnov test on all the classes studied. In this test, the calculations about the laws of probability are on distribution functions: measuring the difference between the theoretical distribution function and the distribution function observed. In this study, instead of conduct tests, it was limited to the presentation of the comparison of the experimental values with those obtained by the theoretical formula. The results showed that the differences are not significant.

The comparison of results obtained in the laboratory with those calculated by the formula (7) for each class (fig. 8) shows that they have the same appearance as those obtained from the experiment with a slight lag. This means that the formula obtained with the following reasoning may represent the quantity of grain crushed under a known load. We can therefore say that the Weibull method can be generalized to estimate the probability of crushing the grains in a medium composed of multiple grains.

CONCLUSION

The diametral compression tests conducted on five classes of different size, *d*, of limestone grain for the crash of singular grains showed the sensitivity of grain crushing. It was noted that the rate of deflection

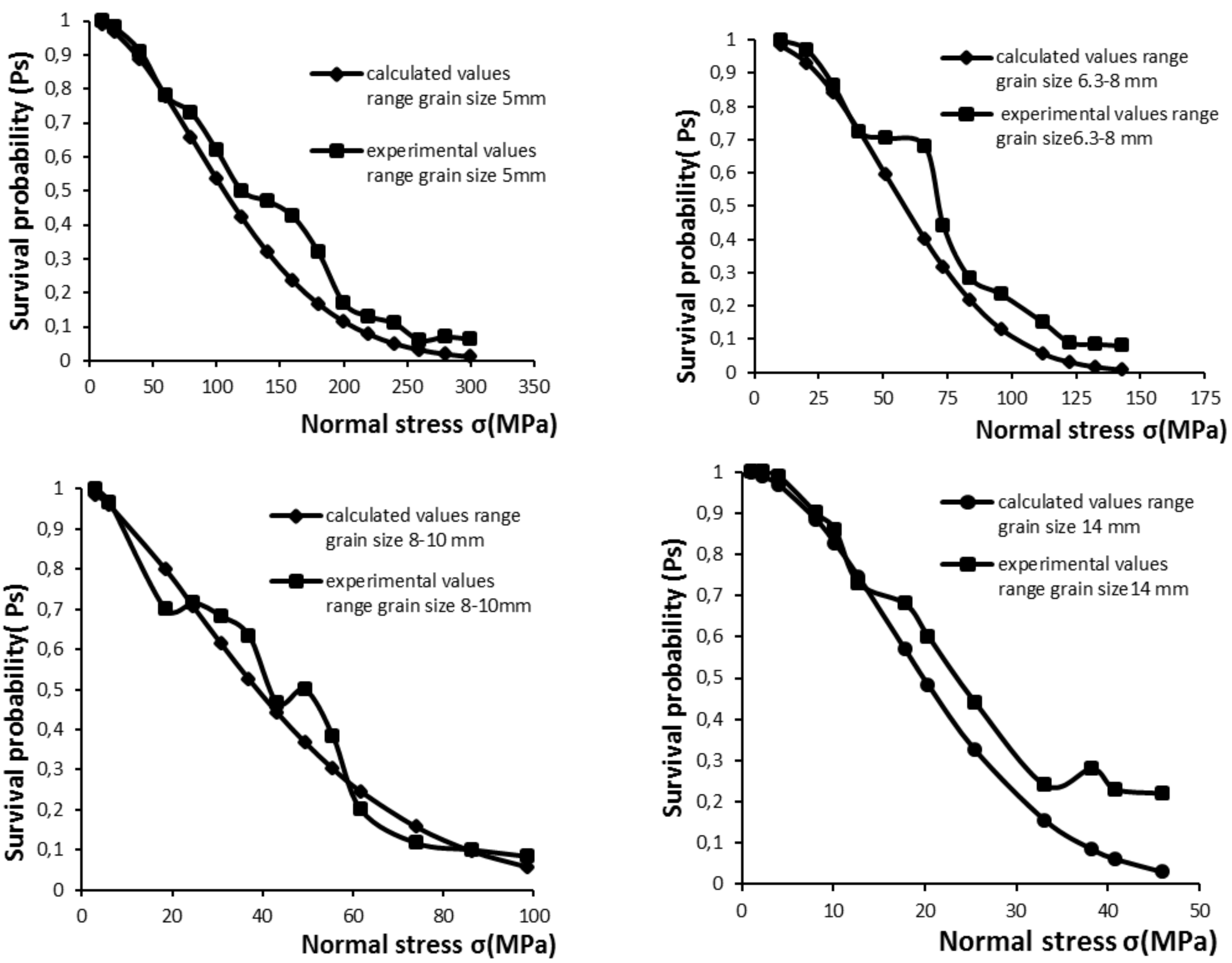

Figure 8. Comparison of experimental results with those calculated for the crash of samples consisting of several grains of limestone.

increases with the increase in the grain size and the applied load. In general, the first grain crash took place at the angularities. These superficial cracks are caused by contacts between the grains and the press platens for the crash grain by grain. After a certain threshold load, brutal destruction of the grain may occur. The representation of the percentage survival of particles as a function of applied stress showed dispersion values of the compressive strength for the same class size for different classes studied. The minimum resistance of the crash or of the appearance of the first fracture differs from one class to another increase with an decreasing diameter, *d*, of the grain. For each class under test, the stress distribution of the crash follows the Weibull distribution.

The uniaxial loading tests performed on samples consisting of several grains of the same limestone have shown that the rate of deflection also depends in this case on the diameter of the grain and the applied load, in the same way as in the case of crushing of singular grains. The stresses of the first breaks are higher compared to the minimum stress fracture of one element of the same diameter. This can be explained by the grain rearrangement of the samples. This leads us to say that the grains begin to crash when they have a tight configuration and by the activation of contact forces between the grains. The representation of the probability of survival according to the applied stress on the sample also shows a dispersion of resistance values for samples of the same class. The Weibull statistical method gave satisfactory results on the statistical modelling of the crushing phenomenon. So we conclude that the probability of survival of the grains in a granular medium follows the Weibull distribution.

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