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11 Deriving Veneziano Model in a Novel String Field Theory Solving String Theory by Liberating Right and Left Movers*

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Abstract. Bosonic string theory with the possibility for an arbitrary number of strings —i.e. a string field theory— is formulated by a Hilbert space (a Fock space), which is just that for massless noninteracting scalars. We earlier presented this novel type of string field theory, but now we show that it leads to scattering just given by the Veneziano model amplitude. Generalization to strings with fermion modes would presumably be rather easy. It is characteristic for our formulation /model that: 1) We have thrown away some null set of information compared to usual string field theory, 2)Formulated in terms of our "objects" (= the non-interacting scalars) there is no interaction and essentially no time development(Heisenberg picture), 3) so that the S-matrix is in our Hilbert space given as the unit matrix, $S = 1$, and 4) the Veneziano scattering amplitude appear as the overlap between the initial and the final state described in terms of the "objects". 5) The integration in the Euler beta function making up the Veneziano model appear from the summation over the number of "objects" from one of the incoming strings which goes into a certain one of the two outgoing strings.

Povzetek. Novo bozonsko teorijo strun sta avtorja formulirala na Hilbertovem (Fockovem) prostoru brezmasnih skalarjev, ki ne interagirajo. Teorija dopušča posplošitev na poljubno število strun, tedaj na strunsko teorijo polja. Avtorja v tem prispevku pokažeta, da je njuna sipalna amplituda enaka amplitudi v Venezianovem modelu. Kaže, da je posplošitev njune bozonske teorije strun na fermionsko enostavna. Bistveno za njuno teorijo strun je: 1) V primerjavi z običajno strunsko teorijo ima njuna teorija manjše število privzetkov. 2) Njuno struno sestavljajo enaki skalarni "objekti", med katerimi ni nobene interakcije in v bistvu tudi nobenega časovnega razvoja (Heisenbergova slika). 3) Sipalna matrika S je v njunem Hilbertovem prostoru kar enotska matrika, $S = 1$. 4) Venezianova sipalna amplituda sledi iz prekrivanja med začetnim in končnim stanjem njunih skalarnih "objektov". 5) Integriranje Eulerjeve beta funkcije Venezianovega modela sledi v njunem primeru iz seštevanja po skalarnih "objektih" ene od prihajajočih strun, ki gre v eno od dveh odhajajočih strun.

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11.1 Introduction...

We have already earlier put forward [1–3] ideas towards a novel string field theory (meaning a second quantized theory of strings from string theory [4–7,9]), which of course means the theory[10–18] in which you can describe several strings like one describes several particles at a time in in quantum field theory. It may be understood that we as other string field theories in or theory have a Hilbert space or Fock space the vectors of which describe states of the whole universe in the string theory. Our model is similar to the formulation of Thorn [19] and also use a discretization like ourselves, but we discretize after having separated right and left movers as we shall see below. This Hilbert space, which describes the states of the universe, turns in our model/formulation out to be really surprisingly simple in as far as it is simply the second quantized Fock space of a non-interacting massless scalar 25 +1 dimensional particle theory in the bosonic string case!

Let it be immediately be stated that although our formulation/model is supposed just a rewriting of string theory - and thus in its goal there is nothing new fundamentally - it is definitely new because we throw away compared to usual string theory and usual string field theory as Kakku and Kikkawa's and Witten's a *null set of information*. The information, which we throw away is the one about how the different pieces of strings hang together. That is to say we rather only keep the information about where in target space time you will find a string and where not. Due to this throwing away of information and other technically doubtful treatment of the string theory by us it is a priori no longer guaranteed that our string field theory appearing as just the non-interacting massless scalar theory in 25+1 dimensions is indeed just a rewriting of string theory. Rather one should see our progresses such as the derivation of the string spectrum [3] in reproducing usual properties of string theory from our model/formulation as tests that indeed our model is in spite of the null set of information thrown away indeed the full string theory.

The major achievement in the present work is also such a test, namely testing that our model/formulation leads to the Veneziano model scattering amplitude for scattering of strings formulated in our novel string field theory.

The particles that formally occurs in the construction of our Hilbert space or Fock space of our model or formulation of string theory we call "even objects" and each such "even object" has in our formulation a kind of momentum variable set J^μ (it is proportional to a contribution to the total momentum of the string to which it belongs). Really this J^μ has as some technical details got its longitudinal momentum (in target space time of 25 +1 dimensions) component $J^+ = J^0 + J^{25}$ fixed by what corresponds to a gauge choice in the string parameterization to be

$$J^+ = \frac{a\alpha'}{2}, \tag{11.1}$$

(We shall below that we end up being driven to also allow $J^+ = -\frac{a\alpha'}{2}$) and its infinite momentum frame energy proportional component $J^- = J^0 - J^{25}$ is written just by the mathematical expression ensuring the light-likeness

$$(J^\mu)^2 = \eta_{\mu\nu} J^\mu J^\nu = 0 \tag{11.2}$$

of the "even object" momentum-like J^μ variable. Thus the only genuine degrees of freedom components of this even object variables J^μ are the "transverse" components corresponding to the first 24 components, namely those having $\mu = i$ where $i = 1, 2, 3, ..., 24$, i.e. J^i. In addition the "even objects" have 24 conjugate momenta Π^i, conjugate to the J^i's, so that

$$[\Pi^i, J^j] = i\delta_{ij} \tag{11.3}$$

for Π^i and J^j belonging to the same even object of course.

Our Hilbert space for states of the universe corresponds now simply to a set of harmonic oscillators, one for every set of J^i-value combinations (of 24 real numbers), and the creation operator for an "even object" with its J^i's being J^i is denoted $a^\dagger(J^i)$. Since there is a calculational relation between the set J^i of the transverse components and the full 26-vector J^μ given by adding the equations (11.1, 11.2), we could equally well use as the symbol in the creation and annihilation operators J^μ as the symbol J^i, and so we have by just allowing both notations $a^\dagger(J^i) = a^\dagger(J^\mu)$, where it is understood that the J^μ is calculated from the only important transverse components J^i. Similarly the destruction operators are $a(J^\mu) = a(J^i)$ and we shall think of the Hilbert space describing the states of the Universe (in a string theory world) as having basis vectors of the type

$$a^\dagger(J^i(1))a^\dagger(J^i(2)) \cdots a^\dagger(J^i(L))|0> . \tag{11.4}$$

To tell the truth we though better reveal the little technical detail, that this simple situation with only one type of " even objects" that can exist in the states described by (J^i, Π^i) is only true for the case of a string theory model *with open strings*, while we for the case of a string theory with only closed strings must have *two kind of even objects* that can be put into the 24 or 26=25+1 dimensional (Minkowski) space, one right denoted by R and one left denoted by L. So in the only closed string case we could even naturally consider it that the two types of even objects "live" in two different Minkowski spaces - one R and one L-. The figure 11.1 illustrates these two slightly different cases. The connection between the strings present in a given state of the universe and the even objects corresponding to a set of strings is a priori not completely trivial and has to be described. It is not hundred percent true that the strings consist of even objects, but there is so much about it that there is an actually infinite number of even objects corresponding to each string present. This divergent number of many even objects in a string is given as a function of the small parameter a already mentioned in formula (11.1).

11.2 Correspondence from Strings to Objects

The crux of the matter in the formulation of our string field theory model or formulation is to put forward the rule for how a given string state is translated into a state described in terms of a state of what we call "even objects":

In the case of a theory with only closed strings we shall make use of the *solution in the conformal gauge for the 26-position fields* $X^\mu(\sigma, \tau)$ *in terms of right and left movers*. Remember that the time-development of a string in string theory is

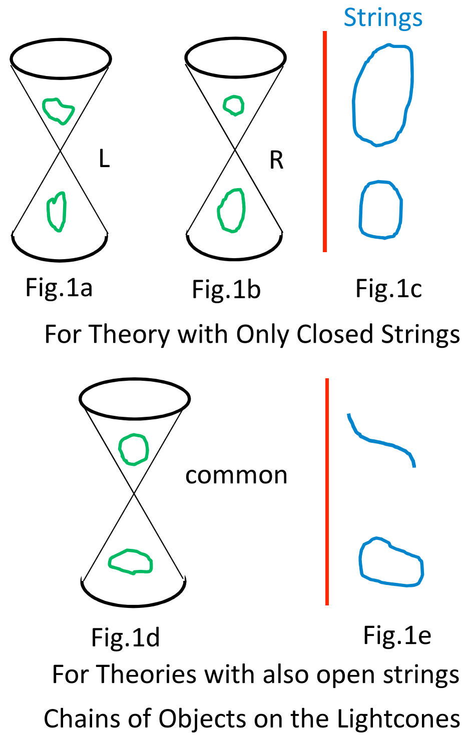


Fig. 11.1.

described by letting its timetrack - which of course becomes a two-dimensional surface in the 25+1 dimensional "target space" - be parameterized by the two real variables called σ and τ. At first one may think of these parameters as parameterizing the timetrack surface in an arbitrary way and therefore one even has to have an action - the Nambu(-Goto) action - chosen so as to be invariant under reparameterization, meaning that one goes over to a new set of coordinates parameterizing the timetrack $(\sigma', \tau') = (\sigma'(\sigma, \tau), \tau'(\sigma, \tau))$. This requirement of reparametrization invariance fixes up to an overall constant the action to be given by the area of the timetrack surface

$$\text{Single string action} = S_{Nambu} \propto \text{area} = \int \sqrt{\det \begin{pmatrix} (\dot{X}^\mu)^2 & \dot{X}^\mu \cdot X'^\mu \\ \dot{X}^\mu \cdot X'^\mu & (X'^\mu)^2 \end{pmatrix}} d\sigma d\tau, \tag{11.5}$$

where we have as usual denoted

$$\dot{X}^\mu(\sigma, \tau) \stackrel{def}{=} \frac{\partial X^\mu(\sigma, \tau)}{\partial \tau} \tag{11.6}$$

$$X'^\mu(\sigma, \tau) \stackrel{def}{=} \frac{\partial X^\mu(\sigma, \tau)}{\partial \sigma}. \tag{11.7}$$

For an open string the timetrack is like a band extending in the time direction, while for a closed string the track is topologically like a tube/cylinder also extending roughly in time direction.

Now one usually in steps fix the "gauge" meaning the parameterization, i.e. the choice of a new set of coordinates which we again may call (σ, τ)(leaving out the prime on (σ', τ')). The first step in the gauge choosing is what is called conformal gauge choice and corresponds to arranging the coordinate equal constant curves to be orthogonal seen from the external/target space of 25+1 dimensions. Often in literature one works with Euclideanized σ and τ as if the string timetrack were a two dimensional Euclidean space, but thinking physically on a true string the space felt by a being living attached onto the string would be a 1+1 dimensional space time with one time dimension and one spatial dimension. For the thinking of the present article and our foregoing works on our novel string field theory we shall take this latter - more physical - point of view that the internal space time is indeed a space-*time*. We think of τ as the time coordinate and of σ as the spatial coordinate along the string.

After having chosen the "conformal gauge" the equation of motion derived from the Nambu action at first simplifies and together with the constraints appearing due to the reparameterization symmetry of the original action we can summarize the equations in the conformal gauge:

$$\Box X^{\mu}(\sigma, \tau) = 0(\text{equation of motion}) \tag{11.8}$$

$$(\dot{X}^{\mu}(\sigma, \tau))^2 - (X'^{\mu}(\sigma, \tau))^2 = 0 \text{ (constraint)} \tag{11.9}$$

$$\dot{X}^{\mu}(\sigma, \tau) \cdot X'^{\mu}(\sigma, \tau) = 0(\text{constraint also}). \tag{11.10}$$

Here the D'Alambertian

$$\Box = \partial_{\tau}^2 - \partial_{\sigma}^2 = (\partial_{\tau} - \partial_{\sigma})(\partial_{\tau} + \partial_{\sigma}), \tag{11.11}$$

and the eqations of motion are easily *solved* by the ansatz

$$X^{\mu}(\sigma, \tau) = X_R^{\mu}(\tau - \sigma) + X_L^{\mu}(\tau + \sigma), \tag{11.12}$$

which is importance for our novel string field theory in as far as it is actually the τ-derivatives of the 26-vectorial functions in the solution $X_R^{\mu}(\tau - \sigma)$ and $X_L^{\mu}(\tau + \sigma)$, which are going to be identified as we shall see soon by our "objects". Note immediately, that these right and left mover variables X_R^{μ} and X_L^{μ} only depend on *one* variable each, namely respectively on $\tau_R \stackrel{def}{=} \tau - \sigma$ and $\tau_L \stackrel{def}{=} \tau + \sigma$, so that the equations of motion with τ conceived of as the time have indeed been solved. The ansatz functions X_R^{μ} and X_L^{μ} are more like initial conditions for the solution.

In terms of these initial condition variables X_R^{μ} and X_L^{μ} the constraints take the very simple form

$$(\dot{X}_R^{\mu}(\tau_R))^2 = (\dot{X}_R^{\mu}(\tau - \sigma))^2 = 0(\text{constraint}) \tag{11.13}$$

$$(\dot{X}_R^{\mu}(\tau_L))^2 = (\dot{X}_R^{\mu}(\tau + \sigma))^2 = 0(\text{constraint}) \tag{11.14}$$

The overview of description of our object rewriting of the string theory is that we let there be an object for every point in (a period for) the coordinates τ_R and

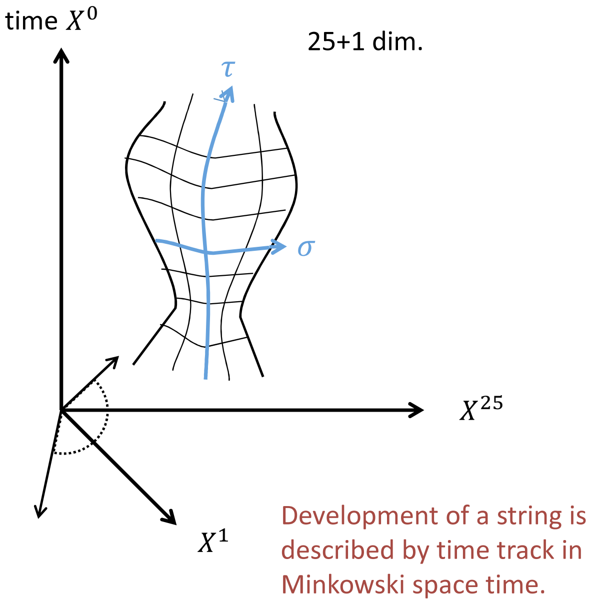


Fig. 11.2.

τ_L in the case of only closed strings, and that the objects are closely related to the variables $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$. For continuity of these variables as functions of respectively τ_R and τ_L the images of these functions $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$ are - except for fluctuations at least - smooth curves, because of the constraints (11.14) these curves must lie on the lightcone(s).

Since we can also consider the variables τ_R and τ_L as σ-variables for constant τ the periodicity w.r.t. σ of the position variables etc. - for in fact both open and closed strings- but at least clearly for the closed strings, comes to imply that the just mentioned images for $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$ become closed curves on the light cone.

Two points further are illustrated by the figure 11.3: 1) We "discretize", so that we replace the in principle continuum infinity of τ_R or τ_L values by a series of discrete points with a "distance between these points" being proportional to a small quantity a later taken to go to zero. 2) We treat the even numbered and the odd numbered "discretized points" differently, as is on the figure illustrated by them being denoted differently by dots and crosses.

11.2.1 Open String Case

The open string case has a tiny technical complication:

At the boundaries of the open string one has boundary conditions which are translated into our model favorite language of $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$ implies

$$\dot{X}^\mu_L(\tau) = \dot{X}^\mu_R(\tau) \text{from boundary at } \sigma = 0$$
$$\dot{X}^\mu_L(\tau) = \dot{X}^\mu_R(\tau - 2\pi) \text{from the } \sigma = \pi \text{ end boundary.,} \qquad (11.15)$$

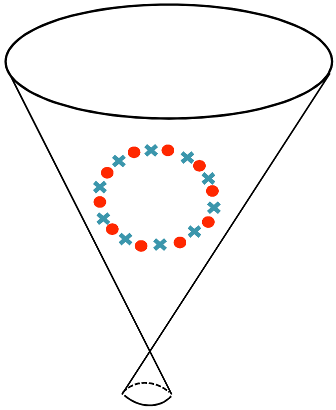


Fig. 11.3.

where we assumed the notation that the length of the string in σ-parameter language is π. These two boundary conditions (11.15) imply that we can identify, for the open string, the $\dot{X}_R = \dot{X}_L$ and have that this common (differentiated)"initial condition variable" must be periodic with the period 2π meaning twice the σ-variable range corresponding to the string length. Thus in the beginning announced we got the fact that while we for only closed string theories have to distinguish $\dot{X}_R$ and $\dot{X}_L$, this is no longer needed for an open string.

11.2.2 More Precise Correspondence between Strings and "even objects"

More precisely we shall divide up into "discretized" pieces the σ range around a closed string or the tour forward and backward along an open string into, let us say, N pieces. What we really want, is to divide up a period for say $\dot{X}_R^\mu$ in its argument τ_R (and in the closed case the same for $\dot{X}_L$, while in the open string case just identify the left and right mover variables because of boundary conditions). The precise way of dividing up could be thought of as dividing in equal steps in the variable, say τ_R, but there is still some coordinate specification/gauge choice left even after the conformal gauge choice. In fact one can still as such a rudimentary freedom of choosing coordinates select any (increasing) function $\tau_R'(\tau_R)$ and any (increasing) function $\tau_L'(\tau_L)$ as a new set of coordinates (having in the background of the mind the identifications $\tau' = \tau - \sigma'$ and $\tau_L = \tau + \sigma$). By discretizing we replace essentially a variable as τ_R by an integer valued variable - counted modulo N if N corresponds to the period -, so that the τ_R value corresponding to the integer I is denoted $\tau_R(I)$. Interpolating we can easily make an approximate sense of even τ_R defined for non-integer values of I. Thus we formally associate any string with a series something we call "objects"- and which is something only defined in our model -, which are characterized each by a set of degrees of freedom (as if it were particles): J^μ_R and the conjugate variables Π^μ or better only Π^i where the i only runs over the transverse coordinates $i = 1, 2, ..., 24$. The reader may crudely think of these objects as a kind of partons, but really we simply have to define them by their relation to the $\dot{X}^\mu_R$ (and for the closed string case also to the separate $\dot{X}^\mu_L$). To every discretization point on the τ_R axis say, let us say discretization point number I, we associate an "object" for which the dynamical variables $J^\mu_R(I)$ are given as

$$J^\mu_R(I) = X^\mu_R(\tau_R(I + 1/2)) - X^\mu_R(\tau_R(I - 1/2)). \quad (11.16)$$

Notice that since the difference between the two argument values $\tau_R(I - 1/2)$ and $\tau_R(I + 1/2)$ is small this definition of $J^\mu_R(I)$ for a discretization point on the τ_R-axis in reality means that

$$J^\mu_R(I) \approx \dot{X}^\mu_R \frac{d\tau_R}{dI}, \quad (11.17)$$

and so indeed as announced our variables J^μ_R assigned to the "objects" are "essentially" the τ_R-derivative $\dot{X}^\mu_R$ of the right mover X_R part of the solution.

11.2.3 The Even Odd Detail

Now there is an important technical detail in the setup of our model:

We have the problem that if one shall make creation and annihilation operators for some "objects" in a way analogous to how one in usual quantum field theory have creation and annihilation operators for particles, one shall describe these creation and annihilation operators by having an argument for a set of variable describing the "object", a set of variables which *commute with each other*. It is indeed well-known that one must in quantum field theory *either* take the creation and annihilation operators to be functions of the spatial momenta of the particles created/annihilated *or* one can use instead position variables, and that corresponds to working with the second quantized fields $\phi(x)$. But the usual simple mutual commutation rules for creation and annihilation operators could not be obtained if one would attempt to construct them to correspond to a combination of dynamical variables for the particles that did not commute with each other. What could also a creation operator depending on mutually complementary variables for a single particle correspond to creating ? It could not create a particle with the specified quantum numbers in such a case because that would be against the Heisenberg uncertainty relation. In the corresponding way we must choose whatever variables we let our "objects", to be associated with creation, and annihilation operators depend on be arranged so as to commute. But then we have problem, because does our $\dot{X}^\mu_R$'s which are proportional to the object-variables J^μ_R commuting? No, they do not commute in as far as the theory of a single starting from the Nambu Lagrancian e.g. in the conformal gauge leads to

$$[\dot{X}^\mu_R(\tau'_R), \dot{X}^\nu_R(\tau_R)] \propto g^{\mu\nu}\delta'(\tau'_R - \tau_R). \quad (11.18)$$

Thinking discretizing, such a derivative of a delta-function commutator means that in the discretized chain the $\dot{X}_R$ or equivalently J_R 's which are *next neighbors do NOT commute*.

So we had to invent a trick to avoid to have to make creation and annihilation operators for "objects" sitting in the chains of "objects" along the variable τ_R as neighbors in the discretization.

The trick which we have chosen consists in *only using in the creation and annihilation operators every second of the by discretization by (11.16) defined "object"-variables* J_R. That is to say we choose to *only construct creation and annihilation operators* for those "objects", which in the discretized series of objects for a given string have got an *even number* I. One could say that we in our model construct our Hilbert space only in terms of such "even objects", and one could almost say only consider these "even objects" as "really existing" in our basic Hilbert space description.

But then we are coming to the problem that we need to a full description of the string states also the "odd objects": what to do about them? We say that when you have a series of the "even objects" on a string, we make the rule to construct in between any two next to neighboring "even objects" (i.e. two "even objects " deviating in number by just 2) an "odd object" from the conjugate momenta Π^i say of the neighboring "even objects". (There is another technical detail connected with the + and - components in the infinite momentum frame we have chosen to work with, so we shall avoid discussing conjugate momenta to other than the transverse components J_R^i - the first 24 components -. Therefore we only consider these first 24 components of conjugate momenta to the J_R's). In fact we have to take the following rule for constructing the "odd object" J_R^i components for the "odd object" number I (where I then is an odd integer (modulo the even number N)),

$$J_R^i = -\pi\alpha'(\Pi_R^i(I+1) - \Pi_R(I-1)), \tag{11.19}$$

in order to obtain the commutation rule corresponding to the derivative of delta function commutation rule (11.18) discretized.

The reader should check and understand that with this construction of the "odd objects" any quantum state of the string expressed as a state of the variables $\dot{X}_R^i$ (and for the closed string also $\dot{X}_R^i$) can be expressed as a corresponding quantum state of a set of N/2 (N must be even) "even objects", because the even object commutation rules

$$[J_R^i(I), \Pi_R^k(K)] = i\delta^{ik}\delta_{IK} \tag{11.20}$$

corresponds just to the commutation rules for the $\dot{X}_R^i$ (and $\dot{X}_L^i$). There is though one little technical detail to be studied in later works: The absolute position of the string were differentiated away from our discussion by dotting the X_R and $X_L$and correspondingly the formula for the "odd objects" does *not* make use of the sum over all the "even object" conjugate variables Π_R^i around the closed chain. So there is suggestively the possibility of identifying the average position of the string proportional to this sum over all the conjugate to even object variables.

11.2.4 Several Strings

So far we should have now given the prescription for constructing a cyclically ordered chain of "even objects" corresponding to a given quantum state of a single open string. (If one wants a closed string one shall construct two cyclically ordered chains of "even objects" one for right movers consisting of J_R "even objects" and consisting J_L left mover "even objects"). Since the commutators were arranged to isomorphic to the discretized $\dot{X}_R$ (and $\dot{X}_L$) it should be possible to construct such an "even object" state. It is then of course also trivial and completely analogous to usual quantum field theory construction of a state with $N/2$ particles to construct a Hilbert space (Fock space) state for $N/2$ of our "even objects". Corresponding to a single open string we thus simply have Hilbert space state with the large (divergent in the limit of $a \to 0$) infinitely many ($= N/2$) "even objects" sitting approximately in a cyclic chain on the light cone in a 25 +1 dimensional (Minkowskian) J^μ_R-space.

But now it is the main point of a model being a string field theory (SFT), that such a model *can describe several strings in one Hilbert space state*. Once we have made our formulation of one string in our "even object" formulation it is, however, rather trivial to construct states with an arbitrary number of strings. One can just act on the "zero even object" with all the product of creation operators corresponding to the various strings - we want to have in the state to be described - each creating the "even objects" associated with the string in question. So to speak if string number 1 is described by the product

$$
\begin{aligned}
C_1|0> = \int & \Psi_1(J^I_R(0), J^i_R(2), ..., J^i_R(N-2)) \cdot \\
& \Pi_{I=0,2,4,\ldots,N-2}\, a^\dagger(J^i_R(I)) \\
& \cdot \Pi_{I=0,2,\ldots,N-2} \Pi_{i=1,2,\ldots,24}\, dJ^i_R(I)|0>,
\end{aligned} \tag{11.21}
$$

where $\Psi_1(J^I_R(0), J^i_R(2), ..., J^i_R(N-2))$ is the wave function for the state of the single string 1 described in terms of "even objects", and the string number 2 by an analogous expression, then a state with both string 1 and string 2 (say they are open) is given as

$$
\begin{aligned}
C_1 C_2|0> = \int & \Psi_1(J^I_R(0), J^i_R(2), ..., J^i_R(N-2)) \\
& \Pi_{I=0,2,4,\ldots,N-2}\, a^\dagger(J^i_R(I)) \Pi_{I=0,2,\ldots,N-2} \Pi_{i=1,2,\ldots,24}\, dJ^i_R(I) \\
& \cdot \int \Psi_2(J^I_R(0), J^i_R(2), ..., J^i_R(N-2)) \Pi_{I=0,2,4,\ldots,N-2}\, a^\dagger(J^i_R(I)) \\
& \Pi_{I=0,2,\ldots,N-2} \Pi_{i=1,2,\ldots,24}\, dJ^i_R(I)|0> .
\end{aligned} \tag{11.22}
$$

Luckily the commutation of the creation operators for "even objects" makes it unnecessary to specify any order in which the creation operator products corresponding to different strings have to be written.

We thus have a scheme for constructing Hilbert space states - in the Hilbert space which is really that of massless scalar "even objects" conceived of as particles in an ordinary Fock space - corresponding to any number of strings wanted. In this sense we have a string field theory.

11.2.5 Final Bit of Gauge Choice

As already mentioned the choice of parameterization (often called gauge choice) were not finished by the conformal gauge, since we could still transform the variables τ_R and τ_L by replacing them by some increasing function of themselves. In the "infinite momentum frame gauge" the choice is to fix this freedom by requiring the density of $P^+ = P^0 + P^{25}$ (longitudinal momentum we can say) momentum is constantly measured in say $\tau_R = \tau - \sigma$ or say in σ. When we discretize as described above and take it that the τ_R distance between neighboring "objects" along the τ_R-axis should be the same all along, then this gauge choice comes to mean that each object gets the same P^+ momentum. We can therefore describe this gauge choice - which is essentially the usual one in infinite momentum frame, just discretized our way - by saying that we impose some fixed small value for the $+$ component of our "object"(-variables)

$$J_R^+ = \frac{a\alpha'}{2} \text{ in our first attempt (later problem comes).} \tag{11.23}$$

With this gauge choice we have made the number of objects N and thus of "even objects" N/2 proportional to the $P^+ = P^0 + P^{25}$ component of the 26-momentum of the string in question. So e.g. the conservation of this component of momentum corresponds to the conservation of the number of say "even objects". After this choice of gauge extremely little is left to be chosen for the reparameterization: you can still for the closed string shift the starting point called $\sigma = 0$, but that is all. Corresponding to this extremely little reparameterization left unfixed you can still cyclically shift along the topological circles on which the objects of a string sits, and that turn out due to the possibility for adding a constant to τ also to be true for the open string. The objects corresponding to a cycle for a string are cyclically order but the starting point is still an unchosen gauge ambiguity. To an open string we have one such loop or cycle, and to a closed one we have two.

11.3 Comparing Our String Field Theory to Other Ones

It should be stressed that our "novel" string field theory really is novel/new, since it deviates from earlier ones like Kakku and Kikkawa or Wittens string field theories in important ways even if some calculations should soon turn out similar:

- 1. The information kept in our formalism is *not* the full one kept by the theories by Kaku Kikkawa or by Witten, but deviates by having relative to these other string field theories thrown out - actually only a null set of - information. It is the information about how the strings hang together, that is thrown out. We could say that we - Ninomiya and Nielsen - only in our rewritten string states keep track of where in the space time you may see a piece of string, but not of how one piece hangs together with another piece. If a couple of strings cross each other there is a point in target space wherein four pieces of string meet, two belonging to each of the crossing strings. In usual string field theories, such as Kaku and Kikkawa [10] and Witten's[11], it is part of the information kept in the Hilbert space vector describing the state of the universe which of these 4 pieces are connected to which. In our formulation, however, *this information has been dropped*.

- 2. A further consequence of this drop of information is that if two strings scattering by just exchanging tails - as one must think scattering should typically happen classically - then really nothing have to happen at all in our formalism. Indeed it is a second characteristic property of our string field theory model, that in the scattering counted in terms of our "even objects" (which are the ones truly represented in the Hilbert space; the odd ones are just mathematical constructions from the conjugate variables for the "even" ones) *nothing happens!* The scattering process is not represented in the Hilbert space formalism of ours.
- 3. A consequence of item2. is that the S-matrix gets calculated formally as an *overlap* of the initial with the final state.
- 4. And this fact is also connected with that the Hilbert space or Fock space of our formulation is the extremely simple free massless scalar Fock space. Actually though there is gauge fixing, that makes the states of the "even objects" even have their J^+ components fixed by (11.1). This is contrary to the other string field theories which have much more complicated structures.
- 5. But perhaps the most important distinction for the other string field theories is that *we use a description in terms of something quite different from the strings themselves, namely our 'even objects"*, while the other string field theories have quite clearly all through their formalism the strings one started from. In ours the string has been hit to the extent that we at the end must ask: What happened to the string? The answer is roughly that there is no string sign left in the Hilbert space structure of being only that of free massless scalars. Rather *the string in our formalism only finds way into the calculations via the initial and final states put in!* That is to say that in our formalism it looks that the whole story of the strings only will appear because there is an extra "stringy" assumption put in about the initial state - and presumably it is necessary even to put it in for the final state - so that the whole string story is not part of the structure of the theory nor of the equation of motion, but rather on an equal level with the cosmological start of the Universe, or the initial conditions of low entropy allowing there to be a second law of thermodynamics. If it should turn out that indeed even extra assumptions about *the final states* are needed to make our formalism function as a string theory, then one could say that in our formalism an influence from future is required.

With all these deviations from the usual string field theories, one may worry whether our rewriting truly is a rewriting and thus can count as a true string field theory, because does it indeed describe the conventional string theory, or could it be that we had thrown away too much (even though only a null set)?

Because of this possibility that our model does not truly represent string theory at the end it becomes important - also for the purpose of testing if our model is string theory - to check the various wellknown features of string theory. We have not long ago published an article [3] in which we showed that the mass spectrum of the strings in our string field theory became the usual one. This is one such little check that our model/string field theory is on the right track. In the succession of this article we shall concentrate on sketching the calculation of the scattering amplitude for two ground state strings (tachyons) scattering elastically into two also tachyonic ground state open strings.

Actually it turned out that we were not quite right in the first run, because we only get one term out of three terms that should be present in the correct Veneziano model. This little shock we sought to repair by modifying our gauge fixing condition and allowing "even objects" also with negative J^+. As we shall see later we think it reflects a more general problem with infinite momentum frame.

11.4 Yet More Technical Details

11.4.1 The + and − components of J_R

Especially if one wants to get an idea about our work [3] checking the spectrum of our strings it is necessary to keep in mind that it is only the components J_R^i for $i = 1, 2, ..., 24$, which are simply independent dynamical variables for the "even object". The remaining two components are not independent. Rather:

- +: The J_R^+ components of actually both even and odd objects are fixed to $\pm\frac{a\alpha'}{2}$ as a remaining gauge choice after the conformal gauge has been used to gauge fix to the largest extend. This would have been the infinite momentum frame choice basically, once we assumed that the distances in say σ-variable per object were (put) equal for all the objects. It really means that number of "objects" represent the P^+-momentum of the string associated with those objects.
- −: Next the components J_R^- are fixed from the requirement gotten from the constraints in string theory, namely that $(J_R^\mu)^2 = 0$. This condition fixes the −component (essentially energy) in terms of the 24 transverse components J_R^i and the gauge fixed J_R^+. Remembering that the "odd objects" are constructed from the even ones by means of (11.19) we can write the −components as :

For even objects: (11.24)

$$J_R^-(even\ I) = \frac{\sum_{i=1,2,\ldots,24}(J_R^i)^2}{2\cdot a\alpha'/2} \quad (11.25)$$

For odd I object(constructed): (11.26)

$$J_R^-(odd\ I) = \frac{\pi^2\alpha'\sum_{i=1,2,\ldots,24}(\Pi_R^i(I+1)-\Pi_R^i(I-1))^2}{a} \quad (11.27)$$

It may be interesting to have in mind that from the point of view of our Hilbert space description with a Fock space only having "even objects", and even those only with their transverse - the 24 components - the odd objects as well as both the + and the - components are just "mathematical constructions" simply put up as mathematicians definitions. In this manner the two of the 26 dimensions are pure "construction"! as well as half the number of objects.

It were basically by means of these "constructions" for a cyclical chain of first even, then filled out by odd ones in between, that we in our previous article[3] checked the spectrum of masses. We ran, however, into a slight species doubler problem: Because of our discretization of the τ_R-variable we were seeking a spectrum of latticized theory (in one spatial dimension, the τ_R), and thus we got according to our theorem that one gets species doublers when seeking to make only right mover in fact a species doubler[23]. In order to get rid of that we propose to impose a continuity rule as a postulate.

11.4.2 The Non-Parity Invariant Continuity Rule

The continuity rule which we saw earlier we had to impose to avoid a doubling of the usual string spectrum in our model is actually just the continuity rule, which you would any expect. Crudely it just is that you require the variation of the object J_R^μ or J_R^i to vary slowly from object to the next object. So physically it is extremely reasonable to assume this continuity rule. But we assume it - and have to assume it so - for both *even* and *odd* "objects", and then because of the antisymmetry of the definition of the odd J_R^i in terms of the conjugate of the even ones, we obtain a condition that *is not symmetric under the shift of sign of the object enumeration number* I. Intuitively you expect that if a chain of numbers J_R^i say, enumerated by I vary smoothly with I counted in positive direction, then it should also vary smoothly, if we count in the opposite direction. Because of our 'strange" definition of the odd object J_R-values, however, the continuity concept we are driven towards does *not* have this intuitive property of being inversion invariant. Let us in fact write our smooth variation or continuity requirement for three successive "objects" in the chain - with an odd one in the middle say -

$$J_R^i(I+1) \approx -\pi\alpha'(\Pi_R^i(I+1) - \Pi_R^i(I-1)) \approx J_R^i(I-1) \qquad (11.28)$$

Imposing this non-reflection invariant continuity rule not only is a way to at least assume away the species doubler from the lattice, but it also gives an orientation to the τ_R-variable. For instance when we below shall match wave functions for strings in initial and final states to calculate the overlap, this oriented continuity condition can let us ignore possible overlaps, if the two, to be matched, chains of "objects" are not oriented - in terms of the continuity condition - in the matching way. This rule reduces significantly the possibilities for forming overlap contributions. From a symmetry point of view it may be quite natural that working with only right mover say there should be some asymmetry under reflection.

Thinking, however, on our model as the fundamental theory representing a seeming world with a string theory, it means that this rather strange "continuity principle" not being reflection invariant has somehow to be imposed by the laws of nature. But now as already stated the Hilbert space structure and the dynamics in terms of "even objects" are just the free massless scalar theory, and there is no place for such a reflection non-invariant continuity condition, except in initial and "final state conditions" So in terms of our "even objects" we must have a truly rather funny initial state assumption: The "even objects" sit in chains that are continuous or smooth in our special sense in one direction, but therefore cannot be it in the opposite direction!

Of course in some way this continuity is a description of the continuity of the strings, their hanging together.

11.5 Sketch of Calculation

As one - and perhaps the most important - tests of whether our string field theory in fact leads to the Veneziano model scattering amplitude (at least up to some overall factor, which we shall leave for later works, and modulo a rather short treatment only of the rather important appearance of the Weyl anomaly in 2 dimensions, which happens to be where the dimension of 26 is needed in our calculation). We shall also reduce the troubles of calculation by choosing a very special Lorentz frame, something that would not in principle have mattered provided the theory of ours had been known to be Lorentz invariant. However, since we use infinite momentum frame - which is not manifestly Lorentz invariant - it is in principle dangerous to choose a special frame.

11.5.1 The Veneziano Model to be Derived

Let us shortly - and especially with also a purpose of the extra factor in the integrand, for which we shall need the anomaly for the Weyl symmetry - recall what Veneziano model amplitude we shall derive, if we shall claim that it is a success for supporting that our model/our string field theory is indeed describing string theory of the bosonic 25+1 dimensional type, the most simple string theory having though as a little problem, a tachyon. Since it is the simplest and historically the first to have a Veneziano amplitude for four external particles [20], firstly later we generalized to larger number of external particles [4], we shall start by deriving the Veneziano model for four external particles, although not in the phenomenologically supports case of Veneziano, $3\pi + \omega$. Rather we consider here just four external tachyons each having mass square

$$m^2 = -\frac{1}{\alpha'} \tag{11.29}$$

where α' is slope of the - before inclusion of loops - assumed "linear Regge trajectories", the leading one of which has the expression

$$\alpha(t) = \alpha(0) + \alpha' t, \tag{11.30}$$

where

$$\alpha(0) = -\alpha' m^2 = 1. \tag{11.31}$$

The four point Veneziano model is basically given by the Euler Beta function, which can be defined by the integral

$$B(x, y) = \int_0^1 z^{x-1}(1-z)^{y-1} dz \tag{11.32}$$

being used say for (11.33)

$$B(-\alpha(t), -\alpha(s)) = \int_0^1 z^{-\alpha(t)-1}(1-z)^{-\alpha(s)-1} dz. \tag{11.34}$$

In writing such four point amplitudes one uses normally the Mandelstam variables

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \quad (11.35)$$
$$t = (p_1 - p_4)^2 = (p_2 - p_3)^2 \quad (11.36)$$
$$u = (p_1 - p_3)^2 = (p_2 - p_4)^2 \quad (11.37)$$
$$\text{obeying the relation} \quad (11.38)$$
$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2 = 4m^2 = -4/\alpha'. \quad (11.39)$$

Here the four- or rather 26-momenta $p_i = p_i^\mu$ $(i = 1, 2, 3, 4)$ are the external momenta for the tachyonic string states we consider as scattering states in the simplest case considered here, all four counted the physical way, i.e. with positive energies p_i^0 for the process

$$1 + 2 -> 3 + 4, \quad (11.40)$$

being considered. Since we consider the case of pure strings without any Chan-Paton factor giving quarks at the ends, the full scattering amplitude becomes a sum over three terms of the betafunction form. In front there is a factor g^2 involving the coupling constant g for the string or Veneziano theory being its square g^2. We shall, however, postpone the presumably a bit complicated but very interesting question of the overall normalization in our theory to a later article. Thus the full amplitude expected is

$$A(s, t, u) = g^2\{B(-\alpha(s), -\alpha(t)) \quad (11.41)$$
$$+B(-\alpha(s), -\alpha(u)) + B(-\alpha(t), -\alpha(u))\} \quad (11.42)$$
$$= g^2(\int_0^1 z^{-2\alpha' p_1\cdot p_2 - 4}(1 - z)^{-\alpha' p_1\cdot(-p_4)-4} dz + \quad (11.43)$$
$$\int_0^1 z^{-2\alpha' p_1\cdot p_2 - 4}(1 - z)^{-\alpha' p_1\cdot(-p_3)-4} dz + \quad (11.44)$$
$$\int_0^1 z^{-2\alpha' p_1\cdot(-p_4) - 4}(1 - z)^{-\alpha' p_1\cdot(-p_3)-4} dz). \quad (11.45)$$

Since we have chosen to set up our model in what deserves to be called infinite momentum frame and to use the gauge that each object carries the same p^+ or rather having the fixed value $J^+ = a\alpha'/2$ according to (11.1), our formalism is *a priori highly non-Lorentz invariant*, and it almost requires a miracle for it to turn out at the end Lorentz invariant. It is therefore non-trivial and a priori dangerous only as we have chosen in the beginning to compared our model to the Veneziano model in the special case that the four external particles have the same p^+ components,

$$p_1^+ = p_2^+ = p_3^+ = p_4^+ \quad (11.46)$$
$$\text{and consequently} \quad (11.47)$$
$$N_1 = N_2 = N_3 = N_4, \quad (11.48)$$

where the (even) integers N_i (i=1,2,3,4) denote the numbers of “objects” attached to the four external particles. This choice of a special coordinate frame leads to a simplification of the term without poles in the s-channel:

$$g^2B(-\alpha(t), -\alpha(u)) = \tag{11.49}$$

$$g^2B(-1 - \alpha'(p_1 - p_4)^2, -1 - \alpha'(p_1 - p_3)^2) = \tag{11.50}$$

$$g^2B(-1 + \alpha'(\vec{p}_{T1} - \vec{p}_{T4})^2, -1 + \alpha'(\vec{p}_{T1} - \vec{p}_{T3})^2) = \tag{11.51}$$

$$g^2 \int_0^1 z^{-2+\alpha'(\vec{p}_{T1}-\vec{p}_{T4})^2}(1-z)^{-2+\alpha'(\vec{p}_{T1}-\vec{p}_{T3})^2} dz \quad . \tag{11.52}$$

Here we have denoted the "transverse" parts - meaning the first 24 components by

$$\vec{p}_T = \{p^i\}_{i=1,2,...,24}. \tag{11.53}$$

The simplification comes about because the +- term in the contraction with the metric in, say, $(p_1 - p_4)^2$ drops out because of our very special frame choice so that $(p_1 - p_4)^+ = 0$,and so the $(p_1 - p_4)^-$ does not matter, and

$$(p_1 - p_4)^2 = -(\vec{p}_{T1} - \vec{p}_{T4})^2. \tag{11.54}$$

11.5.2 Amplitude in Our Model, Principle of No Interaction!

Whereas in string theory there seems to be an interaction between the strings, it is rather surprising - and a hallmark for our theory - that in the formulation of ours in terms of the object states the S-matrix elements, that shall give the Veneziano amplitude as we shall show, is simply equal to the overlap! That is to say it is calculated as if the genuine S-matrix is just the unit operator. More precisely the S-matrix $< 1 + 2|S|3 + 4 >$, that shall describe the scattering of say, two incoming open strings 1+2 into two outgoing 3+4 is obtained by writing the states in our formalism - in terms of even "objects" - corresponding or representing the two string state 1+2, say $|1 + 2 >_{eo}$ and also to the two string state 3+4 corresponding state in even object space, say $|3 + 4 >_{eo}$, and then simply one takes the overlap of these incoming and outgoing states:

$$< 1 + 2|S|3 + 4 > = < 1 + 2|_{eo}|3 + 4 >_{eo} . \tag{11.55}$$

Here the subindex $_{eo}$ stands for "even objects" and means the state described in our even object notation. This means that in terms of our string field theory = "even object formulation" a scattering goes on without anything happening (whatever might happen in reality must have been thrown out in the construction of our string field theory model). Symbolically this formula for the S-matrix is shown on the figure 11.4:

11.5.3 Procedure

The main tasks in order to evaluate the scattering amplitude are

- A. First we must evaluate in some useful notation the wave functions for the incoming and outgoing strings - we shall in this article only consider scattering of two open strings coming in and two open strings coming-out, all in the tachyonic ground states.

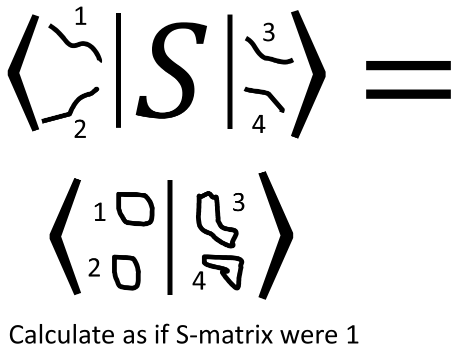


Fig. 11.4.

- B. We must figure out in how many different ways the "even objects" associated to the strings 1 and 2 in the initial state of the scattering can be *identified* with "even objects" associated with the final state strings.
- C. For each way of identification of every "even object" in the initial state(i.e. associated with one of the incoming strings/particles) with an "even object" in the final state (i.e. associated with one of these outgoing particles), we have in principle two wave functions for the "even objects" and shall compute the overlap of these two wave functions.
- D. Then we have to find the total overlap by summing over all the different ways of identifications, considered under B.
- E. This summation under D. will turn out to be approximated by an integral and we shall indeed see, that it becomes essentially the integration in the Euler Beta function definition thus providing the Veneziano model.

In performing this procedure we make some important approximations and simplifications:

- a. We shall assume that due to the continuity of the object series it is by far more profitable for obtaining a big overlap contribution to keep as many of the pairs of neighboring "objects" in the initial state, say strings 1 and 2, remain neighbors again in the final state. This means that we assume that the as contributions to the overlap dominating identification - in the sense of B. - are those in which the largest unbroken series of "even objects" go from one initial state string to one of the outgoing strings. This means the most connected or simplest pattern of identification.
  In fact the not yet quite confirmed though speculation is, that the successively more and more broken up pattern of identification of initial and final "even objects" will turn out to correspond to higher and higher (unitarity correction) loops in dual models(=Veneziano models). Thus we expect, that considering only the least broken transfer of the "even objects" from the initial to the final strings shall give us the lowest order Veneziano model (the original Veneziano model without unitarity corrections).

- b. We shall of course use, that we take the limit $a \to 0$ and correspondingly, that the numbers N_1, N_2, N_3, N_4 of "objects" associated with the various strings go to infinity. Thus we can integrate over the number of objects in a chain going some definite way, say from string 1 to string 4.
- c. To simplify our calculations we choose the very special case of the four strings - the two incoming 1 and 2 and the two outgoing 3 and 4 - all are associated with *same number of "even objects"* $N/2$ (and then of course N 'objects" altogether). This assumption is with our letting the number of "objects" be proportional to the P^+-component of the 26-momentum of the string in question, i.e. just the choice of Lorentz frame, so as to have all the four external strings/particles have the same P^+. So it looks like just being a coordinate choice, but there is the little problem strictly speaking: that our use of infinite momentum frame makes our theory not guaranteed to be Lorentz invariant. Anyway we do it only this non-invariant way in the present article and leave it for later, either to prove Lorentz invariance of our model, or to do it in a more general frame.
- d. As a further strengthening of point b. above about the chains coming in as long pieces as possible being dominant we remember, that our continuity condition (11.28) was *not reflection invariant*. It would therefore be extremely little overlap, if we should attempt to identify the "even objects" of a series in the initial state with a series in the final state in the opposite order. That is to say we require, that the orientation in the pieces of series going over as hanging together from initial to final state are kept. Otherwise the contribution is assumed negligible.
  Together b. and this item d. means that the dominant contributions come when possibly the longest connected pieces go over from one initial to one final without changing orientation of the piece.

We shall in the following seek a way to progress, that relatively quickly leads to string-theory-like expressions and thinking. But the reader shall have in mind that even, if we shall approach string-theory-like expressions, we have at the outset had a formulation - namely our string field theory - in which at first the stringyness is far from obvious. Rather it seemed that the stringy structure only comes in with the initial and final states, while the structure of our free massless scalar Hilbert space or Fock space is too trivial to contain any sign of being a string theory. It is therefore still interesting to calculate the results of our theory, even if it quickly should go into to run along lines extremely similar to usual string theory.

11.5.4 Construction of Wave Functions for Cyclically Ordered Chains Corresponding to Strings

The wave functions for open strings were in fact investigated in our previous article [3] in as far as the quantum system of N objects forming a cyclically ordered chain corresponding to an open string were resolved into harmonic oscillators and thus a Gaussian wave function were obtained in a high (of order N) dimensional space. The trick we shall use here is to write the wave function of this character by means of a functional integral so reminiscent of the Feynman-Dirac-Wentzel functional integral for a string propagation already put into the conformal gauge, that we can say that we already managed to "sneak in" the string by this technology.

In fact one considers in single string description functional integrals of the type:

$$\int \exp(-\int_A (\vec{\partial}\phi(\sigma^1, \sigma^2))^2 d\sigma^1 d\sigma^2)\mathcal{D}\phi, \tag{11.56}$$

with some boundary conditions along the edge of the region A say in (σ^1, σ^2) space, over which the integral in the exponent is performed. We shall for our purpose of making an expression for the wave function in terms of our "even objects" for a string state consider that the region A is taken to be a unit disk and at the edge we imagine putting a series of "even objects" each being assigned a small interval along the circular boundary. Then we identify for example the object J_R^i with the difference of the values of a ϕ^i taken at the two end points of the little interval on the circle surrounding A assigned to the object in question. That is to say for say object number I (here I is even) having as its interval, say, the little region between the points on the circle marked by the angles

$$\theta_{beg} = 2\pi * \frac{I-1}{N} \tag{11.57}$$

$$\theta_{end} = 2\pi * \frac{I+1}{N} \tag{11.58}$$

we identify (e.g.) the difference

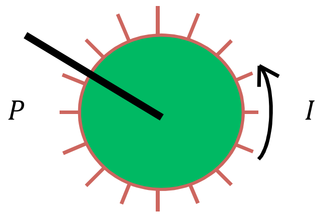
$$J_R^i(I) \overset{ident.}{=} \phi^i(\exp(i\theta_{end})) - \phi^i(\exp(i\theta_{beg})), \tag{11.59}$$

where we have of course taken a new ϕ^i for each of the 24 i-marked components of J_R and where we have identified the (σ^1, σ^2)- space with the complex plane by considering ϕ^i a function of $\sigma^1 + i\sigma^2$.

The idea, which we seek to use here is that - possibly by some minor modifications, which we must state - we should imagine, that we want to construct a prescription for obtaining a wave function of the type $\Psi(J_R^i(0), J_R^i(2), ..., J_R^i(N-2))$ as used in the expression (11.21), describing say the ground state of a string in our formalism by imposing a boundary condition - depending on a set of values for all the "even objects" in a chain - on the functional integral(s). With these boundary conditions imposed at the end the functional integral become the wave function value $\Psi(J_R^i(0), J_R^i(2), ..., J_R^i(N-2))$ for the in the boundary condition used $J_R^i(i)$ values.

Let us before fixing the details immediately reveal that we shall have an extra boundary condition in the center of the disk A at which point we shall cut off an infinitesimally little disk and use the thereby opened boundary conditions to "let in the (transverse components of) the 26-momentum of the string in question". This "letting in" means in principle that we put on the inside of the little circle a series of J_R arranged to correspond to string with the right 26-momentum, but due to the smallness of the little circle the details except for this total momentum does not matter. In the figure we illustrate this situation on which we think: The line ending at the center and ascribed a P symbolize the just mentioned "in-let" in this center. The small tags on the edge of the disk symbolize the attachments of the "even objects", the values of which are used to fix the boundary conditions for the functional integral. Crudely the idea behind this procedure could be considered

We imagine the "objects" sitting
along the edge of a disk, over which
is defined a $\varphi(\sigma^1, \sigma^2)$ to be
functionally integrated over:



The J's of the "objects" are related
to the derivatives $\partial_\mu \varphi$ at the edge.

Fig. 11.5.

to be that *we let a (here open) string propagate along during an imaginary time (say an imaginary τ), whereby only the lowest mass state survives.* The heavier eigenstates of mass decay in amplitude faster that the lightest state by such an imaginary being spent. Thus one gets after infinite imaginary "time" the ground state selected out. Thus investigating the wave function reached after such an infinite imaginary "time" it should turn out being the ground state wave function, and so we should be able to use it as the Ψ we want, if we want the wave function for a ground state string (the tachyon). Then the idea is of course to write the infinite imaginary "time" development by means of Wentzel-Dirac-Feynman path way integration.

Thus we get into our way of presenting the wavefunction

$$\Psi(J_R^i(0), J_R^i(2), ..., J_R^i(N-2))$$

a functional integral with at first having a region, like A, being an infinite half cylinder. The axis along the half infinite cylinder is the imaginary part of the infinite imaginary 'time", while the coordinate around the cylinder is rather the parameter, τ_R, enumerating the objects in the cyclically ordered chain of objects associated with an open string.

Then the type of functional integral here considered is "essentially" (meaning except for an anomaly becoming very important at the end) invariant under conformal transformations of the region A. Thus ignoring - or seeing that they are not there in the case considered - anomalies we can transform the infinite half cylinder into the unit disk with the little hole in the middle, through which we "let in" the momentum of the string.

Note how the string here comes in (only): We got to a functional integral strongly related to what one usually work with in string theory, *just with the purpose of constructing a wave function* $\Psi(J^i_R(0), J^i_R(2), ..., J^i_R(N-2))$ *describing the string state in terms of our objects.* But there is nothing "stringy" in our Hilbert space structure of our object-theory. The string only comes in via this wave function.

But of course it still means, that after we have got this wave function in, we get our calculations being so similar to usual string theory, that we can almost stop our article there, and the string theorist may have exercised the rest so often, that we do not need to repeat. But logically we have to repeat because we are logically doing something else:

There is in our formulation in terms of the 'objects" a priori no strings. We are on the way to see, that after all the strings must be there, because otherwise it would be strange, that we just get the Veneziano amplitude for scattering.

11.5.5 Adjustment of the Details of the Functional Integral

A few details about the functional integral may be good or even rather important to have in mind:

- I. As long we - as here - just seek to write the exponential for the wave function (which as we know for harmonic oscillators have the Gaussian form - of an exponential of a quadratic expression in the $J^i_R(I)$'s (even I) -) we could use the old proposal by David Fairlie and one of us (HBN) of evaluating the exponential as the heat production in a resistance constructed as the surface region A as a conducting sheet with specific resistance $\pi(2?)\alpha'$. Then one shall identify the boundary conditions by letting the current running out at the interval assigned to a certain "even object" be equal to the $J^i_R(I)$ for that "even object".
- II. There a is little problem, which we have to solve one way or the other with getting the "continuity condition" (11.28) discussed in 11.4.2. Having fixed only the boundary condition to the "*even* objects" through their $J^i_R(I)$ but not involving the conjugate variables $\Pi^i_R(I)$ there is of course no way in which the strange non-reflection symmetric continuity condition of our could be imposed. Concerning the classical approximation one may actually find out that one easily can find the classical ϕ^i solution over the complex plane introduced above after the formula (11.59) which reflects the continuity condition as well as you can require for a classical solution by *extracting only the analytical part of the saddle point for* $\phi^i(\sigma^1 + i\sigma^2)$.
  Indeed one might - and we probably ought to do it - construct a model, in which we use both even and odd J^i_R's on the boundary, in the sense that we assign only half as long intervals on the border for each object - meaning that we replace (11.58) by

$$\theta_{beg} = 2\pi * \frac{I - 1/2}{N} \tag{11.60}$$

$$\theta_{end} = 2\pi * \frac{I + 1/2}{N} \tag{11.61}$$

and use it for both even I and odd I.
But now what are we to impose for the odd object intervals on the disk border? We want to obtain a wave function $\Psi(J^i_R(0), J^i_R(2), ..., J^i_R(N-2))$ expressed *only* as a function of the "*even* object" J^i_R's, while *no* Π^i_R *are accessible among the variables, on which the wave function depends*.
However, in functional integrals one can easily extract what corresponds to the conjugate variable; they are so to speak related to the time derivatives, by relations of the type that the conjugate to a variable q in a general Lagrangian theory is given by

$$p = \frac{\partial L}{\partial \dot{q}}. \tag{11.62}$$

On the other hand the continuity condition tells us that approximately the odd J^i_R's can be replaced by their even neighbors. Thus the proposal is being pointed out that we identify the appropriate time derivatives with the values of the neighboring even $J^i_R(I)$'s. Putting up this proposal is rather easily seen to correspond to, that the boundary condition relating ϕ^i near the boundary to the even object J_R's, which we are allowed to use, get decoupled from say the anti-analytic component in ϕ^i. So with such a boundary inspired by the non-reflection invariant continuity condition would lead to an arbitrary solution for say the anti-analytical part, while the analytical part would get coupled. We should like to develop this approach in further paper(s), but it may not really be needed.
Instead of seeking to put our continuity condition (11.28) into the functional integral formalism, we here shall use it is as a rule for which pieces of cyclically ordered chains can be identified, and then we shall get only oriented two dimensional surfaces - looking formally like string-surfaces for closed oriented strings although what we are talking about are *open strings* (but remember that we get the diagrams for open look like the ones say Mandelstam have for closed) -.

- III. Although it is in fact functional integrals like (11.56), that we basically need, it is so that such a functional integral has divergences. These divergences must in principle be cut off. But now it turns out that the cut off necessarily comes to depend on a metric. Therefore we should rather write our functional integral (11.56) as if depending on a metric tensor $g_{\alpha\beta}(\sigma^1, \sigma^2)$ in the 2-dimensional space time, although it formally would look that there *is actually no such dependence on the metric, at least as long as we just scale it up or down by Weyl transformations*. This seemingly metric dependent functional integral looks like

$$\int \exp(\int g^{\alpha\beta}\partial_\alpha\phi(\sigma^1, \sigma^2)\sqrt{g}d\sigma^1 d\sigma^2)\mathcal{D}\phi, \tag{11.63}$$

where then boundary conditions and region of the (σ^1, σ^2)-parameterization must be further specified. The cut off procedure should also be specified; it could for instance be a lattice curt off, a lattice in the (σ^1, σ^2) variables, say. Then the importance of the metric is that you need the metric to describe the lattice spacing. Note though also that formally a scaling of the metric/Weyl transformation

$$g_{\alpha\beta} \to \exp 2\omega g_{\alpha\beta}, \tag{11.64}$$

even when the scaling function $\exp 2\omega$ depends on the $(\sigma 1, \sigma^2)$ does *seemingly* not change anything, because the determinant g of the two by two matrix $g_{\alpha\beta}$ just scales with $\exp 4\omega$ so that the square root just compensates for the scaling of the upper index $g^{\alpha\beta}$ metric. The Weyl transformation symmetry is *only* broken by the cut off (the lattice) depending on $g_{\alpha\beta}$. It is via this cut off the anomaly can come in.

11.5.6 Overlap Contributions

The crucial step in calculating the Veneziano model amplitude in our model/string field theory is to see what are the possibilities for identifying all the even objects associated with the initial strings/particles 1 and 2 to the ones associated with the final state strings/particles 3 and 4 in a way that to the largest extend keep neighboring (or better next to neighboring, since we only consider the "*even* objects") "even objects" going into neighboring ones in the same order(same succession).

To simplify the possibilities, we have to consider what we have chosen to assume - basically by appropriate choice of coordinate system - namely that each of the four strings or particles are associated with the same number of "objects". We may remember that by our gauge choice the number of "objects" N associated with say an open string is proportional to the P^+ component of its momentum, so that choosing a frame, wherein all the four external particles have equal P^+ implies that they have an equal number of associated "objects" also.

Now to keep the "objects" most in the succession they already have in the initial state also in the final state we must let connected pieces of "even objects" pass from say string 1 to string 4. Then the rest of the "even objects" associated with string 1 must go to string 3. Now the "even object" numbers on string 2 that must go to respectively to 3 and to 4 is already fixed for what happened for string 1. Since they have to sit in succession and a cyclic rotation of the cyclically ordered chains is the very last rudiment of gauge choice, there is no physically significant freedom in the identification except for the starting number of how many objects go from 1 to 4.

On the figure it is illustrated how different series of "even objects" from 1 or 2 marked with some signature are refound - with same signature - in 3 or 4. The idea of course is that each of the four series marked by the four different signatures are refound in both initial (1+2) and final (3+4) states, and really are the same. It is understood that the series of "even objects" identified to be in both initial and final states are identified "even object" for "even object" in same succession.

To get the contributions to the overlap - and thereby amplitude - from all the physically different "identification ways" one shall sum over the various values, a non-negative integer number, of "even objects" from 1 that are refound in 4. Since such numbers are of order N - which means it goes to infinity as our cut off parameter $a \to 0$ - the actual overlap contribution from each separate value of the number summed over varies slowly and smoothly (we may check by our calculation) and we can replace it by an integral over say the fraction of the "even objects" in 1 (i.e. associated with 1) that are identified with 'even objects"

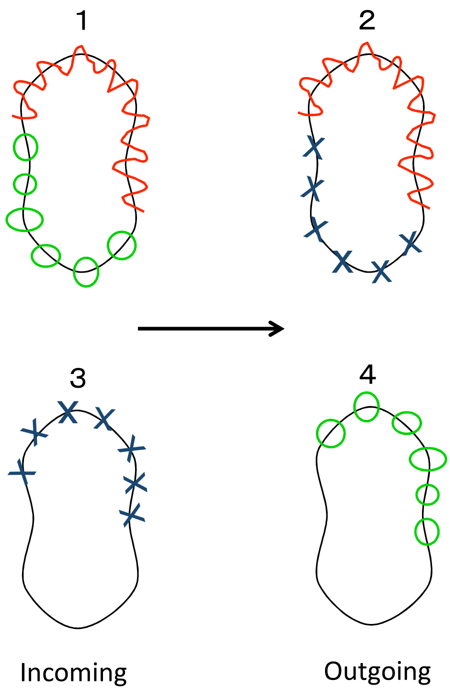


Fig. 11.6.

associated with 4. It is this integration that shall turn out to be the integration in the integration in the Euler Beta function making up the Veneziano model.

11.5.7 The Overlap for One Identification

But before integrating or summing we have to write down the overlap as obtained, if one only includes the possibility of one single "identification" (correspondence between the "even objects" in the initial state 1+2 with them in the final 3+4). This overlap of two states 1+2 and of 3+4 with a fixed "identification" is of course simply the Hilbert product of the two states of the set of $N/2 + N/2$ "even objects" - at least if one ignores the low probability of two "even objects" in say 1 and 2 being in the same state - so that we calculate it as an inner product in an $N/2+N/2$ particle/"even object" system. It becomes an inner product of the form

$$\int \Psi^*_{\substack{3+4,\\ \text{with}\\ \text{identification} I}} ((J^i_R(0), J^i_R(2), ..., J^i_R(N-2))|_1, (J^i_R(0), J^i_R(2), ..., J^i_R(N-2))|_2)$$
$$\times \Psi_{1+2}((J^i_R(0), J^i_R(2), ..., J^i_R(N-2))|_1, (J^i_R(0), J^i_R(2), ..., J^i_R(N-2))|_2)$$
$$\times \Pi_{i,I=0,2,...,N-2,k=1,2} dJ^i_R(I)|_k. \quad (11.65)$$

Now the crucial point of our technique is that this inner product integration over the J^i_R-values for all the "even objects" associated with 1 or 2 (and identified with "even objects" in 3 or 4) when the wave functions are written as the functional integrals, we use, can be interpreted as just gluing functional integral regions together. The point is that the functional integrals basically are just - when cut off - integrals over ϕ^i values in all the different "lattice" points along the region A boarder say. At the boarders there is specified linear relations of the ϕ^i values there - or rather the derivatives, but they are also linear relations - to the $J^i_R(i)$'s assigned places on this border. One now has to argue that apart from an overall constant factor we can consider the integration over the $J^i_R(I)$'s in (11.65) going in as part of the functional integration in a functional integral in which the regions A for the two sides (initial and final) are glued together to one big functional integral. Since the integration over the "even object"-variables $J^i_R(I)|_k$ have now been interpreted as part of the functional integration, the new resulting functional integral has no longer any boundary conditions associated with such $J^i_R(I)|_k$'s. Rather the 'big' functional resulting - and expressing the overlap for a specific "identification" - only has as boundary conditions the inlets of the external 26-momenta(or rather their transverse components only),P_1, P_2, P_3, P_4.

One should notice how this picturing by functional integrals come to look really extremely analogous to gluing together strings. There is though a little deviation from the usual open string theory at first, because we have cyclically ordered chains topologically of form as circles as would be closed strings to represent the open strings. Corresponding to this little deviation we get at first that final contribution from "identification" of "even objects" between final and initial states, becomes conformally equivalent to a Riemann sphere with the four inlets from the four external strings being attached to this Rieman sphere. This is what you would expect for closed string scattering in the usual string theory, but *we* obtain this for *open* strings! It turns, however, out that all our four "inlets" - where the momentum boundary conditions are imposed - come by a calculation we shall sketch - to sit on a circle on the Riemann sphere. Thus there is "reflection" symmetry between the two sides of this sphere and mathematically our overlap for the fixed identification come to be equal to a functional integral as usually used for open strings. In this way our model has the possibility of agreeing exactly with usual string theory.

11.5.8 Seeing the Hope

A bit of imagination of how our topologically infinite half cylinders can glue together would reveal, that we could arrange to get them pressed down in a plane but with - we must stress though - in two layers. In such a form we could have arranged that the result would look like a *double* layer four string bands meeting along intervals with their neighbors but only in one point with their opposite string. In order that we could bring it to look like this, we should put the two incoming strings opposite and the two outgoing strings opposite to each other. This would be the usual string gluing picture for the open strings - just doubled, but that essentially does not matter - for the $B(-\alpha(t), -\alpha(u))$ term. This means that it is extremely promising that we should obtain this term of the Veneziano model.

But !:

- 1. What happens to the other two terms $B(-\alpha(s), -\alpha(t))$ and $B(-\alpha(s), -\alpha(u))$, which we should also have gotten, to get the full Veneziano model?
- 2. We have in principle to check that our model predicts the correct weight factor on the integrand in the Veneziano model. We mean that the integrand, which we obtain does not only have the right dependence on the external momenta, but also the right dependence as a function of the integration variable - which in our model comes from the summation over the different "identifications".
  Since in our model this integration comes from the simple summation over "identifications" our model has a very clear rule for what weighting to obtain and one just has to calculate carefully not remembering the anomaly in the functional integral evaluation etc.
- 3. So far we were sloppy about the + and - components, or rather we only started calculating the factors in the integrand coming from the transverse momenta or transverse J^i_R components so far.

11.5.9 Integrand weighting Calculation

If we want to evaluate the integrand meaning the contribution from one specific identification more carefully, we have to be specific about how we for such an "identification" make the construction of the full surface on which the functional integration ϕ^i at the end gets defined. We obtain - using the idea that the overlap integration can be absorbed into the functional integration - that we must glue together four (either infinite half cylinders or) disks corresponding to the four external strings. To be concrete it is easiest to represent the two final state strings 3 and 4 by *exteriors* of a disk rather than by a disk as we represent the initial strings 1 and 2. The inlets for the final state strings are then at infinity of the Riemann surface, while those of the initial strings are at zero.

Now however, we have two incoming and two outgoing, and so we are forced to work with a complex plane with *two* layers.

We take say one layer where the complement of the unit disk is put to be the essential disk for string 3, while the other layer belongs then to string 4. Similarly we must have two layers for the initial strings, but now the important point is that the initial and the final ones are to be glued together in a slightly complicated way, depending also on the integration variable, which is essentially given by the number of "even objects" going from 1 to 4 say.

Having settled on giving 3 and 4 each their layer in the complex plane in the complement of the unit disk, we have let these outside unit disk layers be glued to the inside the unit disk ones associated with the two incoming particles 1 and 2 along the unit circle of course. But now the length measured say in angle - or in number of "even objects" proportional to the angle - along which say the layer in the inside assigned to string 1 has to be glued together with complement of disk region layer assigned to string 4 along a piece of circle proportional to the amount/number of "even objects" passing from string 1 to string 4 in the "identification" we consider. along the rest of the unit circle then of course the disk assigned to string 1 is identified with the outside disk layer connected to string 3. Similarly along the first piece of circle - where 1 is connected to 4 - of course the layer of string 2 shall be connected along the circle to the layer of string 3 (in the outside). Correspondingly along the "rest" of the unit circle the layer assigned to string 2 (inside) is attached (identified with) the layer of string 4. In the figure 11.7 you may see an attempt to give an idea of what to do before having put on the final state strings associated with the complements of the unit disk in their two layers. But on the figure the inner layers are prepared for the gluing together. Now modulo the anomaly - i.e. naively - the functional integrals

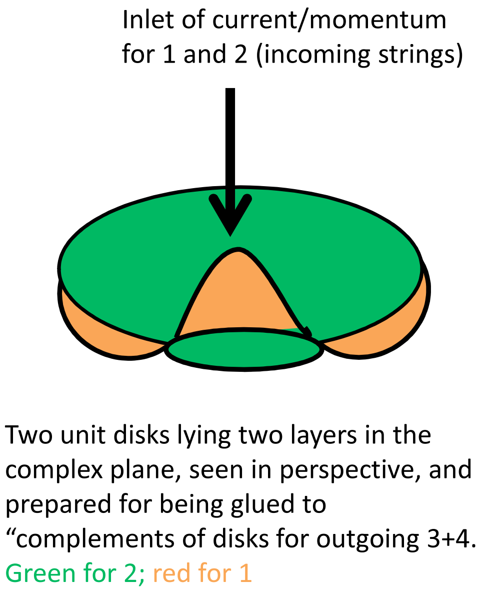


Fig. 11.7.

considered are conformally invariant, so that we are postponing the anomaly allowed to perform a conformal transformation of the combined region - now lying in two layers - of the four disks or complements of disks associated with the four external strings, and the result of the functional integral being proportional to the overlap contribution from the "identification" in question should not be changed.

Since the angle θ (circle piece length) along which say layer of string 1 is identified with the layer of string 4 is proportional to the *number* of "even objects" we shall simply integrate to get the an expression proportional to the full overlap (and thus to the Veneziano amplitude hopefully) integrate simply with the measure $d\theta$.

At first it looks that we have a little problem by only having wave functions as functions of the 24 transverse coordinates so that seemingly the + and - components of the 26-momenta cannot appear in our hoped for Veneziano model integral. However, luckily for the term we actually obtain $B(-\alpha(u), \alpha(t))$ we found above in equation (11.52) that in fact all the terms in the exponents for z and $(1-z)$ that depend on the external 26-momenta could be arranged *to come only from the transverse momenta* provided we have made the very special frame choice that the four external particles have the same P^+ components. So in this our simplifying the contributions from the + and - components turn out not needed. The point really was that just having the gauge choice and the frame choice arranging the + component for the $p_1 - p_4$ and for the $p_1 - p_3$ become zero the character of the u and t of having a + component multiplied with a - one made it enough to ignore but the transverse contributions.

11.5.10 The Conformal Transformation

The way to evaluate the contribution to the overlap of $|1+2>$ with $|3+4>$ from one "identification" is to rewrite it into a functional integral the region of which is composed from the four disks or disk complements corresponding to the four external particles/strings. We obtain at first a manifold described as double layered in the Riemann sphere. It has two branch points on the unit circle corresponding to the points where the "even objects" on say 1 shifts from going to 3 to going to 4 (or opposite). Basically we choose to map this doubled layered region by a map with two square root singularities at the two branch points.

11.5.11 Anomaly

The anomaly that gives us an extra factor mutiplying the contribution from a single "identification" is usually written formulated as the trace anomaly

$$< T^{\alpha}_{\alpha} >= -\#\mathtt{fields} * \frac{1}{48\pi}\sqrt{g} * R, \tag{11.66}$$

(in the notation of our article with Habara wherein $\sqrt{g}R = -2\partial_{\alpha}\partial^{\alpha}\Omega$ for the metric tensor of the form $g_{\alpha\beta} = \exp(2\Omega)\eta_{\alpha\beta}$,and) where R is the scalar curvature of the metrical space given by the metric tensor (in two dimensions enumerated by $\alpha = 1, 2$;). Here the energy momentum tensor is denoted $T^{\alpha\beta}$ is indeed for the theory of the field(s) ϕ^i which had been Weyl invariant as it formally looks like, and so the trace $T^{\alpha}_{\alpha} = 0$ would be zero. The symbol $\#\mathtt{fields}$ denotes the number of fields ϕ_i; it would in the 26=25+1 theory be 24.

This anomaly can be seen to come in by having in mind that we want to perform a conformal transformation - in fact the one corresponding to the analytical function

$$f(z) = \sqrt{\frac{z - \exp(i\delta)}{z - \exp(-i\delta)}} - \tag{11.67}$$

(here we used the notation that the end points of the cut along the unit circle seperating where sheet 1 connects to 4 from where it connects to the sheet associated with particle 3 were arranged to be $\exp(i\delta)$ and $\exp(-i\delta)$.) and then the anomaly gives rise to corrections to the "naive" result that the functional integral is invariant under a conformal transformation. In fact we may first have in mind that we shall evaluate the functional integral (11.56) with a lattice or other cut off only depending on the internal geometry so that it only can give variations depending on the metric tensor $g_{\alpha\beta}$, which under a conformal transformation only changes its scale locally as under a Weyl transformation (11.64). So what we only need to calculate to obtain the effect of the anomaly is how the overall factor on the metric tensor varies under the conformal transformation, we shall use (11.67). Such a scaling is given by the numerical value of the derivative of the function (here f) representing the conformal transformation,

$$g_{\alpha\beta} \to \Omega g_{\alpha\beta} \text{ where then } \Omega = |\frac{\partial f}{\partial z}|^2. \tag{11.68}$$

This is to be understood, that the metric tensor describing the complex plane metric in the f-plane is $\exp 2\Omega g_{\alpha\beta}$ when the metric induced from the z-plane usual metric is $g_{\alpha\beta}$.

It is easy to see that scaling the metric tensor (locally) with an infinitesimal scaling factor $\exp 2\Omega$ with $\Omega << 1$ leads to a correction to the logarithm of the functional integral by $\int \Delta\omega T^{\alpha}_{\alpha} d^2\sigma$. Since the trace T^{α}_{α} of the energy momentum tensor is only non-zero according to (11.66) where there is a non-zero curvature, and our two layered surface lies mostly in the flat complex plane, we only get contributions to this Weyl transformation local change of scale from the two (singular) branche points $\exp(i\delta)$ and $\exp(-i\delta)$, where the curvature R has delta-function contributions.

We can without any change in value of the functional integral make a formal reparametrization from say the double sheeted complex plane to the single layered one by means of $f = \sqrt{\frac{z-\exp(i\delta)}{z-\exp(-i\delta)}}$ provided one then use after the transformation the *transformed metric tensor*. With a conformal transformation the transformed metric inherited from the z-plane into the f-plane will only deviate from the flat metric $\eta_{\alpha\beta}$ in the f-plane by a Weyl transformation. We know that there only shall be curvature - of delta function type - at those points in the f-plane that are the images of the branch points $z = \exp(i\delta)$ and $z = \exp(-i\delta)$, and so the (Weyl transformed) metric reflecting the metric space from the z-plane into the f-plane, $\exp(2\Omega)\eta_{\alpha\beta}$, i.e. $R = 0$ outside these two points $f = 0 \text{and} \infty$.

This means that the Ω outside those two points in the f-plane must be a harmonic function of f, meaning the real part of an anlytical function. This outside the two points harmonic function shall though have *singularities at the two points* on the f-plane (or the corresponding Riemann sphere rather) delivering the delta-function contributions,

$$R = 4\pi\delta(Re(f))\delta(Im(f)) \text{at f=0 say.}, \tag{11.69}$$

At the branch points, we have points with the property that going around one of them in the z-plane or system of sheets one get a return angle θ being 2π more than after the mapping into the f-plane (or Riemann space). Thus the integral over the curvature delivering this extra amount of parallel transport extra shift angle should in an infinitesimal region around the image of a branch point - say the point $f = 0$ - be 2π. So with a notation with the rule of such an (excess) angle being given as

$$\int_{area} R\sqrt{g} d^2\sigma = 2\theta \tag{11.70}$$

with θ the extra angle of rotation on return, [1] there will in the metric inherited from the z-plane in the f-plane be delta function contribution to the curvature scalar R at the points corresponding to the branch points $z = \exp(i\delta)$ and $z = \exp(-i\delta)$ in the z-plane, and thus $f = 0$ and $f = \infty$ in the f-plane. One can easily see that because there is just 2π extra angle to go around such a branch point in the sheeted z-plane the delta-function contribution becomes e.g. for the $f = 0$ point

$$R = 4\pi\delta^2(f). \tag{11.72}$$

If r is the distance to the image of the branch point, say the $f = 0$ point, so that $r = |f|$, the solution to $R = -2\partial_\alpha\partial^\alpha\Omega$ for this delta function R is a logarithm of the form

$$\Omega(r) = \ln(r/K). \tag{11.73}$$

(Here K is some constant in the sense of not depending on r) That implies that taken at the point $r = 0$ the $\Omega(0)$ is logarithmically divergent so that the integral to which the anomaly of the logarithm of the correction to the integrand is proportioanal becomes divergent. However, we have anyway given up caculating in this article the overall normalization of the Veneziano model, we hope to derive. We shall therefore be satisfied with only calculating the contribution in Ω *that varies with the angle* δ over which we (finally) integrate. Now the conformal transformation mapping the two-sheeted z-plane into the one-sheeted f-plane is

$$f(z) = \sqrt{\frac{z - \exp(i\delta)}{z - \exp(-i\delta)}}, \tag{11.74}$$

and so its logarithmic derivative

$$\frac{df}{fdz} = \frac{1}{2}\left(\frac{1}{z - \exp(i\delta)} - \frac{1}{z - \exp(-i\delta)}\right), \tag{11.75}$$

and the derivative proper

$$\frac{df}{dz} = \frac{1}{2} * \sqrt{\frac{z - \exp(i\delta)}{z - \exp(-i\delta)}} * \left(\frac{1}{z - \exp(i\delta)} - \frac{1}{z - \exp(-i\delta)}\right). \tag{11.76}$$

We are interested in a hopefully finite term in the change in going from the z-plane simple metric to the one in the f-plane, which is the part of

$$\Omega_{z \text{ to } f} = \ln\left(\left|\frac{df}{dz}\right|\right) \tag{11.77}$$

depending on the "integration variable" δ. This means that make precise the cut off by saying that we must make the cut off by somehow smoothing out the branch

[1] In our notation we have the rule that going around an area and thereby obtaining for for a parallel transported vector on return a rotation by an angle θ, that the integral over this area

$$\int_{area} R\sqrt{g}d^2\sigma = 2\theta \tag{11.71}$$

point singularity in a fixed way in the *z-plane*. This means that we perform a regularization by putting into our transformation a fixed distance ϵ in the z-plane marking the distance of z to one of the branch points. That is to say we consider a little circle say of points around the exact branch point counted in the z-plane

$$z_{\text{on little circle}} = \epsilon \exp(i\chi) + \exp(i\delta) \tag{11.78}$$

(or analogously using $\exp(-i\delta)$ instead of $\exp(i\delta)$.) On this little circle we find that the scaling - Weyl transformation - going from the z-plane to the f-plane using (11.76, 11.77)

$$\Omega_{z \text{ to } f} = \ln(|df/dz|_{circle}) \approx \ln(\frac{1}{2\sqrt{\epsilon}\sqrt{2\sin(\delta)}}) \tag{11.79}$$

for the $z \approx \exp(i\delta)$ case. The δ-dependent part is of course $-\frac{1}{2} * \ln(\sin(\delta))$. This is the δ-dependent part of $\Omega_{z \text{ to } f}$ which comes into the anomaly correction for the logarithm of the full (product over the 24 values of i of the) functional integral, according to (11.66) and (11.72) of course with a coefficient proportional to the number of truly present dimensions in the functional integral - which is only the transverse dimensions 24 -.

Thus the δ dependent part of the anomaly ends up being in the logarithm of the contribution to the overlap from one value of δ (meaning one "identification"):

$$\begin{aligned}\Delta_{anomaly} \ln \text{integrand} &= \delta \text{ independent } + (d-2) * \frac{1}{48\pi} * \frac{1}{2} \ln \sin \delta * (4\pi + 4\pi) \\ &= \frac{d-2}{6} * \ln \sin \delta + ...\end{aligned} \tag{11.80}$$

Now we should remember that we have decided in this article to go for the form of the amplitude but have left for further studies the over all normalization of the amplitude. This means that the terms in the logarithm of the integrand of the hopefully to appear Veneziano amplitude which do not depend on the integration variable δ (which is proportional to the number of (even) objects from string 1 that goes into string 4) but only so on the cut off parameter ϵ are neglected.

Now the conformal mapping (11.74) brings the inlet points for external momenta for the four external particles into the positions sketched on the figure 11.8: Imagining on this figure that one varies the integration variable δ, then the "inlet" points for the two incomming strings 1 and 2 will remain sitting opposite to each other one the unit circle and analogously the two final state string inlet points 3 and 4. So the distances between 1 and 2 or between 3 and 4 are constant as function of δ and so we can ignore the terms comming proportional to in fact $s = -(p_1^i + p_2^i)^2 + ... = -(p_3^i + p_4^i)^2 + ...$ from the heat production in the analogue model from the current running co as to depend on the distance between 1 and 2 or analogously between 3 and 4. In a similar way we are allowed with our decission to only keep the δ dependent terms to ignore terms involving only one of the four inlet points. There are such contributions but they depend on the inlet momentum squared (with a divergent coefficient), but since only on one point the δ-dependence is not there provided we cut off in a δ independent way of course.

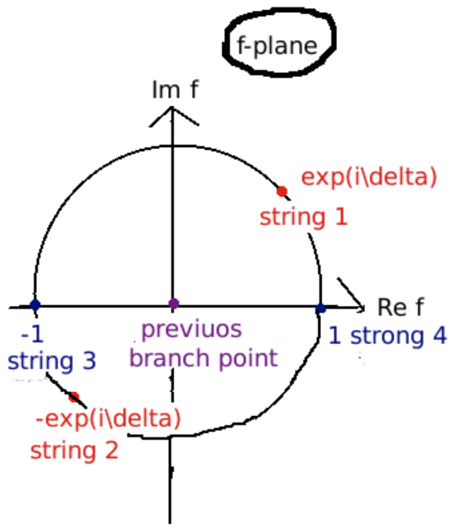


Fig. 11.8.

The divergences form cut off of the anomaly correction connected with originallly - in z-plane - branch points come only in to the extend that they get their cut off small circles scaled to a differnt degree depending on δ.

It is not difficult to see that to seek identification of the δ dependent terms with the integration variable z (not to be confused with our complex plane z which is of course something different) in the Veneziano model we must identify

$$z = \sin^2(\frac{1}{2} * \delta) \tag{11.81}$$

$$1 - z = \cos^2(\frac{1}{2} * \delta). \tag{11.82}$$

Very important for stressing how successful our model/rewritting is to reproduce the Veneziano model integration measure in the z-integration correctly. In our model this integration correponds to the summation over the discrete variable being the number of (even) objects going from string 1 to string 4 and it is proportional to δ, and thus we at first simply the measure of integration is $d\delta$. But now to compare with the usual Veneziano formula expressions or our slight rewrittings of it we must of course relate $d\delta$ to dz:

$$dz = \cos(\frac{1}{2}\delta)\sin(\frac{1}{2}\delta)d\delta \propto \sin(\delta)d\delta \propto \sqrt{z(1-z)}d\delta. \tag{11.83}$$

This happens to show that the correction factor from the anomaly needed to just compensate the factor comming from

$$d\delta = \frac{dz}{\sin(\delta)} \tag{11.84}$$

would be just cancelled by the anomaly provided

$$\frac{d-2}{6} = 1. \tag{11.85}$$

(Apart from a little calculational mistake above by a factor 4) this means that we get precisely the right Veneziano model when the number of transverse dimensions $d - 2 = 6 * 1 = shouldbe24$. The famous result that the bosonic string must exist in d=26 space time dimesions.

11.6 Our Shock; Only One Term

When we went through the above sketched calculation and arrived at *only* the one term proportional to $B(-\alpha(t), -\alpha(u))$, which is the term without poles in the s-channel, it were somewhat unexpected at first. After all we had made up a model essentially written out so as to *make it* the string theory and thereby the Veneziano model. Then it gave only one out of the three terms it should have given. May be even more strangely, if we imagine investigating crossing symmetry it looks we would get a different term after what particles are incoming and which outgoing. So the term we got is not even properly crossing symmetry invariant. Nevertheless it were very encouraging that we got something so reminiscent of the Veneziano model as simply one of the terms.

We believe we have found a way to get the two missing terms also come out:

In fact we think that it is in a way the infinite momentum frame gauge, which we used, that is the reason for the surprising problem for our model: Really one may say that the infinite momentum frame is a method for avoiding having to think about the vacuum, which in quantum field theories is usually an enormously complicated state. In the infinite momentum frame type calculations you imagine an approximation in which the particles have so high energy that they manage not to "feel" the vacuum. But such an approximation may not be a good one. So we thought it might be best somehow to introduce at least some rudimentary effects of a vacuum even though we want to continue to work with an infinite momentum frame formalism, especially an infinite momentum frame gauge/parameterization choice.

The idea, which we here propose, and which actually seems to help to obtain the lacking two terms in the full Veneziano model amplitude, is to allow not only as we did at first for "objects" with the +components $J_R^+ = a\alpha'/2$ (a positive number), *but also allow "negative objects"* having rather their $J_R^+ = -a\alpha'/2$. At least with inclusion of such negative "objects" you make it at least a possibility to have not totally trivial state with the property of the vacuum of having the "longitudinal" momentum $P^+ = 0$. The vacuum could so to speak consist of a compensating number of usual positive say "even objects" and corresponding number of "negative even objects".

In fact it looks that we with such "negative" "objects" can imagine some of our strings represented by an "extended" cyclically ordered chain(replacement). Hereby we mean that it contains in the "extended" cyclically ordered chain not only usual positive J_R^+ objects, but also one or more series of negative J_R^+ objects, arranged so that the excess of positive ones over negative ones is proportional to the total P^+ component of the 26-momentum for the string in question. With such "extended" cyclically ordered chains representing some open string we obtain the possibility of the negative part of say string 2 annihilating with part of the cyclically ordered chain of string 1. Similarly one of the final state strings could be produced with content in its cyclically ordered chain of some series of negative objects having been produced together with some positive ones in another final state string.

By very similar procedure to the one used above to the term $B(-\alpha(u), -\alpha(t))$, but now including the negative objects we seem to be able to produce the two missing terms. The detail of the calculation to obtain the full Veneziano amplitude/model will appear soon by the authors [21].

11.7 Conclusion and Outlook

We have in the present article sketched how using our string field theory formalism in which strings are rewritten into be described by states of "even objects" we can obtain the scattering amplitude to be the usual Veneziano model amplitude. It must though be immediately admitted that we at first got *only one out of the three terms expected*. However, introducing "objects" that can have negative J^+_R-components and can function as a kind of holes for objects, we though believe, that it is promising to obtain the whole Veneziano amplitude. Our model or string field theory has previously been shown [3] to lead to the usual mass square spectrum for strings. In this way we collect increasing evidence that our formalism is indeed another representation of all of string theory.

The way we constructed our formalism working from string theory and only throwing away though a null set of information, it is of course a priori expected, that our formalism should be string theory. In so far there sufficient holes in the "derivation" of our formalism from string theory to be equivalent to the latter, that we still need the more indirect support from rederiving features of string theory such as the Veneziano amplitude from our model.

Our model is a formulation in terms of what we called "objects", and they "sit" in circular "cyclically ordered chains", to an open string is assigned one such circular chain of objects, to a closed string two. The "objects" are supposed to "sit" as smoothly as they can from quantum fluctuations - which put severe constraints though, since *the odd numbered "objects" in cyclically ordered chain are not independent dynamical variables, but rather given in terms of (the conjugate variables Π^i_R for) the neighboring "even numbered objects" by equation (11.19).*

Actually we even stressed that the smoothness or continuity condition because of the dependence of the odd objects on *differences* of the conjugate momenta of neighboring even ones become non-reflection invariant. That is to say that a cyclically ordered chain being smooth would not remain smooth, if one puts the objects in the opposite order! The crux of the matter is that we have a genuine string field theory in the sense that we construct a state space of Hilbert vectors describing a whole universe in a string theory governed world. Then of course there can in the various states of this Hilbert space exist different numbers of strings, well this is not hundred percent true, because contrary to other string field theories: our Hilbert space is described in terms of the "even objects" and the number of strings perfectly accurately given once you have a Hilbert space state. The "even objects" can namely be associated to strings in slightly different ways, so that the number of strings *only approximately* can be derived from a given state; even there are no exact eigenstates for the number of strings. But in practice we believe the approximate access to the number of strings in our description is sufficient. But that the number of strings is *not* cleanly defined feature of a state in our Hilbert space, is clear from the fact that we have scattering even scattering that change the number of strings, such as if two strings scattered and became three, but that nothing happen in our formalism under a scattering. We just obtained the Veneziano model scattering amplitude as an *overlap* of initial and final state just corresponding to that nothing happens in the object formulation. In this sense the strings resulting from the scattering must have been there all the time.

One may look at our model as *solution* of string theory in the sense that we have "even object " description that does not even develop with time so that the "even object" state is more like a system of initial data to a solution of string theory.

11.7.1 Outlook

We foresee that there must be really very much it would be reasonable to do in our formalism, which is in many ways simpler than usual string theory especially than usual string field theory.

Presumably it will be very easy to make the superstring version; if nothing else should work one could in principle bosonize the fermionic modes and then treat the resulting bosons similar to the way we treated in our model of the bosonic modes.

Of course we should really also properly finish getting the Veneziano model calculation remaining details. A special interest might be connected with the overall normalization, which we left completely out here, since our formalism has no obvious candidate for the string coupling g, so the latter should come out from whatever parameters such as our cut off parameter a and α' and possible vacuum characteristic, but we did not use openly vacuum properties in the calculation sketched.

Most interesting might be to use our formalism to obtain a better understanding of the Maldacena conjecture by developing our formalism for the Ads space and then see that the corresponding CFT can also be written by our formalism.

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