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14 $K^o - \bar{K^o}$, $D^o - \bar{D^o}$ in a Local $SU(3)$ Family Symmetry

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Abstract. Within a broken $SU(3)$ gauged family symmetry, we report the analysis of $\Delta F = 2$ processes induced by the tree level exchange of the new massive horizontal gauge bosons, which introduce flavor-changing couplings. We provide a parameter space region where this framework can accommodate the hierarchical spectrum of quark masses and mixing and simultaneously suppress within current experimental limits the contributions to $K^o - \bar{K^o}$ and $D^o - \bar{D^o}$ mixing. In addition we find out that the mass of the $SU(2)_L$ weak singlet vector-like D quark introduced in this BSM, may be of the orden of 10 TeV.

Povzetek. Avtor v okviru svojega predloga teorije z zlomljeno družinsko simetrijo $SU(3)$ analizira procese tipa $\Delta F = 2$, ki jih inducira izmenjava novih masivnih horizontalnih umeritvenih bozonov na drevesnem nivoju, kar privede do sklopitev, ki spremenijo okus. Najde območje v prostoru parametrov, ki dovoljuje izmerjeni masni spekter kvarkov ter njihovo mešalno matriko, pri tem pa so prispevki mešanja $K^o - \bar{K^o}$ in $D^o - \bar{D^o}$ pod trenutnimi eksperimentalnimi mejami. Maso napovedanega kvarka D, ki je šibki singlet vektorskega tipa $SU(2)_L$, oceni na $\sim$ 10 TeV.

Keywords: Quark and lepton masses and mixing, Flavor symmetry,
$\Delta F = 2$ Processes.
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14.1 Introduction

Flavor physics and rare processes play an important role to test any Beyond Standard Model(BSM) physics proposal, and hence, it is crucial to explore the possibility to suppress properly these type of flavor violating processes.

Within the framework of a vector-like gauged $SU(3)$ family symmetry model[1,2], we study the contribution to $\Delta F = 2$ processes[3]-[6] in neutral mesons at tree level exchange diagrams mediated by the gauge bosons with masses of the order of some TeV's, corresponding to the lower scale of the $SU(3)$ family symmetry breaking.

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The reported analysis is performed in a scenario where light fermions obtain masses from radiative corrections mediated by the massive bosons associated to the broken $SU(3)$ family symmetry, while the heavy fermions; top and bottom quarks and tau lepton become massive from tree level See-saw mechanisms. Previous theories addressing the problem of quark and lepton masses and mixing with spontaneously broken $SU(3)$ gauge symmetry of generations include the ones with chiral local $SU(3)_H$ family symmetry as well as other $SU(3)$ family symmetries. See for instance [7]-[14] and references therein.

14.2 $SU(3)$ family symmetry model

The model is based on the gauge symmetry

$$G \equiv SU(3)_F \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \tag{14.1}$$

where $SU(3)$ is a completely vector-like and universal gauged family symmetry. That is, the corresponding gauge bosons couple equally to Left and Right Handed ordinary Quarks and Leptons, with g_H, g_s, g and g' the corresponding coupling constants. The content of fermions assumes the standard model quarks and leptons:

$$\Psi_q^o = (3,3,2,\frac{1}{3})_L \quad , \quad \Psi_l^o = (3,1,2,-1)_L \tag{14.2}$$

$$\Psi_u^o = (3,3,1,\frac{4}{3})_R \quad , \quad \Psi_d^o(3,3,1,-\frac{2}{3})_R \quad , \quad \Psi_e^o = (3,1,1,-2)_R \tag{14.3}$$

where the last entry is the hypercharge Y, with the electric charge defined by $Q = T_{3L} + \frac{1}{2}Y$.

The model includes two types of extra fermions: Right Handed Neutrinos: $\Psi_{\nu_R}^o = (3,1,1,0)_R$, introduced to cancel anomalies [7], and a new family of $SU(2)_L$ weak singlet vector-like fermions: Vector like quarks $U_L^o, U_R^o = (1,3,1,\frac{4}{3})$ and $D_L^o, D_R^o = (1,3,1,-\frac{2}{3})$, Vector Like electrons: $E_L^o, E_R^o = (1,1,1,-2)$, and New Sterile Neutrinos: $N_L^o, N_R^o = (1,1,1,0)$.

The particle content and gauge symmetry assignments are summarized in Table 14.1. Notice that all $SU(3)$ non-singlet fields transform as the fundamental representation under the $SU(3)$ symmetry.

14.3 $SU(3)$ family symmetry breaking

To implement the SSB of $SU(3)$, we introduce two flavon scalar fields:

$$\eta_i = (3,1,1,0) = \begin{pmatrix} \eta_{i1}^o \\ \eta_{i2}^o \\ \eta_{i3}^o \end{pmatrix} \quad , \quad i = 1,2 \tag{14.4}$$

	$SU(3)$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
ψ_q^o	3	3	2	$\frac{1}{3}$
ψ_{uR}^o	3	3	1	$\frac{4}{3}$
ψ_{dR}^o	3	3	1	$-\frac{2}{3}$
ψ_l^o	3	1	2	-1
ψ_{eR}^o	3	1	1	-2
$\psi_{\nu R}^o$	3	1	1	0
Φ^u	3	1	2	-1
Φ^d	3	1	2	+1
η_i	3	1	1	0
$U_{L,R}^o$	1	3	1	$\frac{4}{3}$
$D_{L,R}^o$	1	3	1	$-\frac{2}{3}$
$E_{L,R}^o$	1	1	1	-2
$N_{L,R}^o$	1	1	1	0

Table 14.1. Particle content and charges under the gauge symmetry

with the "Vacuum ExpectationValues" (VEV's):

$$\langle\eta_1\rangle^T = (\Lambda_1, 0, 0) \quad , \quad \langle\eta_2\rangle^T = (0, \Lambda_2, 0) \,. \tag{14.5}$$

It is worth to mention that these two scalars in the fundamental representation is the minimal set of scalars to break down completely the $SU(3)$ family symmetry. The interaction Lagrangian of the $SU(3)$ gauge bosons to the SM massless fermions is

$$i\mathcal{L}_{int,SU(3)_F} = g_H \; \left(\bar{f}_1^o \; \bar{f}_2^o \; \bar{f}_3^o\right) \gamma_\mu \begin{pmatrix} \frac{Z_1^\mu}{2} + \frac{Z_2^\mu}{2\sqrt{3}} & \frac{Y_1^{+\mu}}{\sqrt{2}} & \frac{Y_2^{+\mu}}{\sqrt{2}} \\ \frac{Y_1^{-\mu}}{\sqrt{2}} & -\frac{Z_2^\mu}{\sqrt{3}} & \frac{Y_3^{+\mu}}{\sqrt{2}} \\ \frac{Y_2^{-\mu}}{\sqrt{2}} & \frac{Y_3^{-\mu}}{\sqrt{2}} & -\frac{Z_1^\mu}{2} + \frac{Z_2^\mu}{2\sqrt{3}} \end{pmatrix} \begin{pmatrix} f_1^o \\ f_2^o \\ f_3^o \end{pmatrix} \tag{14.6}$$

where g_H is the $SU(3)$ coupling constant, Z_1, Z_2 and $Y_j^{\pm} = \frac{Y_j^1 \mp i Y_j^2}{\sqrt{2}}$, $j = 1, 2, 3$ are the eight gauge bosons.

Thus, the contribution to the horizontal gauge boson masses from the VEV's in Eq.(14.5) read

- $\langle\eta_1\rangle$: $\frac{g_H^2\Lambda_1^2}{2}(Y_1^+Y_1^- + Y_2^+Y_2^-) + \frac{g_H^2\Lambda_1^2}{4}(Z_1^2 + \frac{Z_2^2}{3} + 2Z_1\frac{Z_2}{\sqrt{3}})$
- $\langle\eta_2\rangle$: $\frac{g_H^2\Lambda_2^2}{2}(Y_1^+Y_1^- + Y_3^+Y_3^-) + g_H^2\Lambda_2^2\frac{Z_2^2}{3}$

The "Spontaneous Symmetry Breaking" (SSB) of $SU(3)$ occurs in two stages

$$SU(3) \times G_{SM} \xrightarrow{\langle\eta_2\rangle} SU(2)\ ? \times G_{SM} \xrightarrow{\langle\eta_1\rangle} G_{SM}$$

FCNC ?

Notice that the hierarchy of scales $\Lambda_2 > \Lambda_1$ yield an "approximate $SU(2)$ global symmetry" in the spectrum of $SU(2)$ gauge boson masses.

Therefore, neglecting tiny contributions from electroweak symmetry breaking, we obtain the gauge boson mass terms.

$$(M_1^2 + M_2^2)\, Y_1^+ Y_1^- + M_1^2\, Y_2^+ Y_2^- + M_2^2\, Y_3^+ Y_3^- + \frac{1}{2} M_1^2\, Z_1^2 + \frac{1}{2}\frac{M_1^2 + 4M_2^2}{3}\, Z_2^2 + \frac{1}{2}(M_1^2)\frac{2}{\sqrt{3}}\, Z_1\, Z_2 \quad (14.7)$$

$$M_1^2 = \frac{g_H^2 \Lambda_1^2}{2} \quad , \quad M_2^2 = \frac{g_H^2 \Lambda_2^2}{2} \quad (14.8)$$

	Z_1	Z_2
Z_1	M_1^2	$\frac{M_1^2}{\sqrt{3}}$
Z_2	$-\frac{M_1^2}{\sqrt{3}}$	$\frac{M_1^2+4M_2^2}{3}$

Table 14.2. $Z_1 - Z_2$ mixing mass matrix

Diagonalization of the $Z_1 - Z_2$ squared mass matrix yield the eigenvalues

$$M_-^2 = \frac{2}{3}\left(M_1^2 + M_2^2 - \sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2}\right) \quad (14.9)$$

$$M_+^2 = \frac{2}{3}\left(M_1^2 + M_2^2 + \sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2}\right) \quad (14.10)$$

and finally

$$(M_1^2 + M_2^2)\, Y_1^+ Y_1^- + M_1^2\, Y_2^+ Y_2^- + M_2^2\, Y_3^+ Y_3^- + M_-^2\, \frac{Z_-^2}{2} + M_+^2\, \frac{Z_+^2}{2}\,, \quad (14.11)$$

where

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} \quad (14.12)$$

$$\cos\phi\ \sin\phi = \frac{\sqrt{3}}{4}\, \frac{M_1^2}{\sqrt{M_1^4 + M_2^2(M_2^2 - M_1^2)}} \quad (14.13)$$

14.4 Electroweak symmetry breaking

For electroweak symmetry breaking we introduce two triplets of $SU(2)_L$ Higgs doublets, namely;

$$\Phi^u = (3,1,2,-1) \qquad , \qquad \Phi^d = (3,1,2,+1)\,, \tag{14.14}$$

with the VEV's

$$\langle \Phi^u \rangle = \begin{pmatrix} \langle \Phi^u_1 \rangle \\ \langle \Phi^u_2 \rangle \\ \langle \Phi^u_3 \rangle \end{pmatrix} \quad , \quad \langle \Phi^d \rangle = \begin{pmatrix} \langle \Phi^d_1 \rangle \\ \langle \Phi^d_2 \rangle \\ \langle \Phi^d_3 \rangle \end{pmatrix} \quad , \tag{14.15}$$

where

$$\langle \Phi^u_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{ui} \\ 0 \end{pmatrix} \quad , \quad \langle \Phi^d_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{di} \end{pmatrix} \, . \tag{14.16}$$

The contributions from $\langle \Phi^u \rangle$ and $\langle \Phi^d \rangle$ generate the W and Z_o SM gauge boson masses

$$\frac{g^2}{4}\,(v_u^2 + v_d^2)\,W^+W^- + \frac{(g^2 + g'^2)}{8}\,(v_u^2 + v_d^2)\,Z_o^2 \tag{14.17}$$

$$+ \text{ tiny contribution to the } SU(3) \text{ gauge boson masses and mixing} \tag{14.18}$$

$$\text{with } Z_o, \tag{14.19}$$

$v_u^2 = v_{1u}^2 + v_{2u}^2 + v_{3u}^2$, $v_d^2 = v_{1d}^2 + v_{2d}^2 + v_{3d}^2$. So, if $M_W \equiv \frac{1}{2} g v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

14.5 Fermion masses

14.5.1 Dirac See-saw mechanisms

The scalars and fermion content allow for quarks the gauge invariant Yukawa couplings

$$H_u\, \overline{\psi^o_q}\, \Phi^u\, U^o_R \;+\; h_{iu}\, \overline{\psi^o_{uR}}\, \eta_i\, U^o_L \;+\; M_U\, \overline{U^o_L}\, U^o_R \;+ h.c \tag{14.20}$$

$$H_d\, \overline{\psi^o_q}\, \Phi^d\, D^o_R \;+\; h_{id}\, \overline{\psi^o_{dR}}\, \eta_i\, D^o_L \;+\; M_D\, \overline{D^o_L}\, D^o_R \;+ h.c \tag{14.21}$$

M_U , M_D are free mass parameters and H_u , H_d , h_{iu} , h_{id} are Yukawa coupling constants. When the involved scalar fields acquire VEV's, we get in the gauge basis ${\psi^o_{L,R}}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms $\bar{\psi}^o_L \mathcal{M}^o \psi^o_R + h.c$, where

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & h\,v_1 \\ 0 & 0 & 0 & h\,v_2 \\ 0 & 0 & 0 & h\,v_3 \\ h_1\Lambda_1 & h_2\Lambda_2 & 0 & M \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ b1 & b_2 & 0 & M \end{pmatrix} . \tag{14.22}$$

$\mathcal{M}^o$ is diagonalized by applying a biunitary transformation $\psi^o_{L,R} = V^o_{L,R}\,\chi_{L,R}$.

$$V_L^{o\,T} \mathcal{M}^o\, V_R^o = Diag(0, 0, -\lambda_3, \lambda_4) \tag{14.23}$$

$$V_L^{o\,T} \mathcal{M}^o \mathcal{M}^{o\,T}\, V_L^o = V_R^{o\,T} \mathcal{M}^{o\,T} \mathcal{M}^o\, V_R^o = Diag(0, 0, \lambda_3^2, \lambda_4^2)\,, \tag{14.24}$$

where λ_3 and λ_4 are the nonzero eigenvalues, λ_4 being the fourth heavy fermion mass, and λ_3 of the order of the top, bottom and tau mass for u, d and e fermions, respectively. We see from Eqs.(14.23,14.24) that from tree level the See-saw mechanism yields two massless eigenvalues associated to the light fermions:

14.6 One loop contribution to fermion masses

The one loop diagram of Fig. 1 gives the generic contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$,

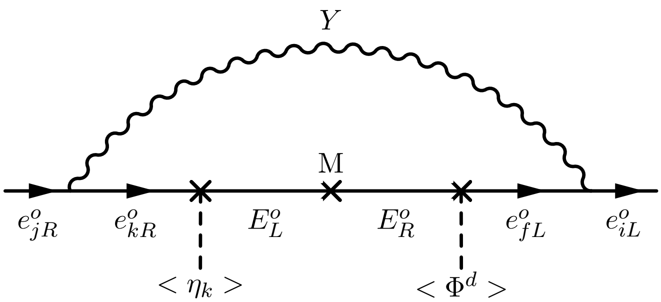

Fig. 14.1. Generic one loop diagram contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

$$m_{ij} = c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m^o_k\, (V^o_L)_{ik} (V^o_R)_{jk} f(M_Y, m^o_k) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi}\,, \tag{14.25}$$

M_Y being the mass of the gauge boson, c_Y is a factor coupling constant, Eq.(14.6), $m^o_3 = -\lambda_3$ and $m^o_4 = \lambda_4$, and $f(x, y) = \frac{x^2}{x^2 - y^2} \ln \frac{x^2}{y^2}$,

$$\sum_{k=3,4} m^o_k\, (V^o_L)_{ik} (V^o_R)_{jk} f(M_Y, m^o_k) = \frac{a_i\, b_j\, M}{\lambda_4^2 - \lambda_3^2}\, F(M_Y)\,, \tag{14.26}$$

$i = 1, 2, 3$, $j = 1, 2$, and $F(M_Y) \equiv \frac{M_Y^2}{M_Y^2 - \lambda_4^2} \ln \frac{M_Y^2}{\lambda_4^2} - \frac{M_Y^2}{M_Y^2 - \lambda_3^2} \ln \frac{M_Y^2}{\lambda_3^2}$. Adding up all possible the one loop diagramss, we get the contribution $\bar{\psi^o_L} \mathcal{M}^o_1\, \psi^o_R + h.c.$,

$$\mathcal{M}_1^o = \begin{pmatrix} D_{11} & D_{12} & 0 & 0 \\ D_{21} & D_{22} & 0 & 0 \\ D_{31} & D_{32} & D_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \frac{\alpha_H}{\pi} \,, \tag{14.27}$$

$$D_{11} = \mu_{11}(\tfrac{F_{Z_1}}{4} + \tfrac{F_{Z_2}}{12} + F_m) + \tfrac{1}{2}\mu_{22}F_1 \quad D_{12} = \mu_{12}(-\tfrac{F_{Z_2}}{6} - F_m)$$

$$D_{21} = \mu_{21}(-\tfrac{F_{Z_2}}{6} - F_m) \qquad D_{22} = \tfrac{1}{2}\mu_{11}F_1 + \tfrac{1}{3}\mu_{22}F_{Z_2}$$

$$D_{31} = \mu_{31}(-\tfrac{F_{Z_1}}{4} + \tfrac{F_{Z_2}}{12}) \qquad D_{32} = \mu_{32}(-\tfrac{F_{Z_2}}{6} + F_m)$$

$$D_{33} = \tfrac{1}{2}(\mu_{11}F_2 + \mu_{22}F_3)$$

$$\alpha_H = \frac{g_H^2}{4\pi} \quad , \quad F_1 \equiv F(M_{Y_1}) \quad , \quad F_2 \equiv F(M_{Y_2}) \quad , \quad F_3 \equiv F(M_{Y_3}) \tag{14.28}$$

$$F_{Z_1} = \cos^2\phi\, F(M_-) + \sin^2\phi\, F(M_+) \tag{14.29}$$

$$F_{Z_2} = \sin^2\phi\, F(M_-) + \cos^2\phi\, F(M_+) \tag{14.30}$$

$$F_m = \frac{\cos\phi \sin\phi}{2\sqrt{3}} \left[F(M_+) - F(M_-) \right] . \tag{14.31}$$

F_{Z_1}, F_{Z_2} are the contributions from the diagrams mediated by the Z_1, Z_2 gauge bosons, F_m comes from the $Z_1 - Z_2$ mixing diagrams, with M_1, M_2, M_-, M_+ the horizontal boson masses, Eqs.(7-11),

$$\mu_{ij} = \frac{a_i\, b_j\, M}{\lambda_4^2 - \lambda_3^2} = \frac{a_i\, b_j}{a\, b}\, \lambda_3\, c_\alpha\, c_\beta \,, \tag{14.32}$$

$c_\alpha = \cos\alpha, c_\beta = \cos\beta, s_\alpha = \sin\alpha, s_\beta = \sin\beta$ are the mixing angles from the diagonalization of $\mathcal{M}^o$. Therefore, up to one loop corrections the fermion masses are

$$\bar{\psi}_L^o \mathcal{M}^o\, \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o\, \psi_R^o = \bar{\chi}_L\, \mathcal{M}\, \chi_R \,, \tag{14.33}$$

where $\psi_{L,R}^o = V_{L,R}^o\, \chi_{L,R}$, and $\mathcal{M} \equiv \left[Diag(0,0,-\lambda_3,\lambda_4) + V_L^{o\,T} \mathcal{M}_1^o\, V_R^o \right]$ may be written as:

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta\, m_{13} & s_\beta\, m_{13} \\ m_{21} & m_{22} & c_\beta\, m_{23} & s_\beta\, m_{23} \\ c_\alpha\, m_{31} & c_\alpha\, m_{32} & (-\lambda_3 + c_\alpha c_\beta\, m_{33}) & c_\alpha s_\beta\, m_{33} \\ s_\alpha\, m_{31} & s_\alpha\, m_{32} & s_\alpha c_\beta\, m_{33} & (\lambda_4 + s_\alpha s_\beta\, m_{33}) \end{pmatrix} , \tag{14.34}$$

The diagonalization of $\mathcal{M}$, Eq.(14.34) gives the physical masses for u and d quarks, e charged leptons and ν Dirac neutrino masses.

Using a new biunitary transformation $\chi_{L,R} = V^{(1)}_{L,R}\, \Psi_{L,R};\ \bar{\chi}_L\, \mathcal{M}\, \chi_R = \bar{\Psi}_L {V^{(1)}_L}^T \mathcal{M}\, V^{(1)}_R \Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$$ {V^{(1)}_L}^T \mathcal{M}\, \mathcal{M}^T\, V^{(1)}_L = {V^{(1)}_R}^T \mathcal{M}^T\, \mathcal{M}\, V^{(1)}_R = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\,, \qquad (14.35) $$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. So, the rotations from massless to mass fermions eigenfields in this scenario reads

$$ \psi^o_L = V^o_L\, V^{(1)}_L\, \Psi_L \qquad \text{and} \qquad \psi^o_R = V^o_R\, V^{(1)}_R\, \Psi_R \qquad (14.36) $$

14.6.1 Quark Mixing Matrix V_{CKM}

We recall that vector like quarks are $SU(2)_L$ weak singlets, and hence the interaction of L-handed up and down quarks; ${f^o_{uL}}^T = (u^o, c^o, t^o)_L$ and ${f^o_{dL}}^T = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$ \frac{g}{\sqrt{2}}\, \bar{f^o}_{uL} \gamma_\mu f^o_{dL} W^{+\mu} = \frac{g}{\sqrt{2}}\, \bar{\Psi}_{uL}\, (V_{CKM})_{4\times 4}\, \gamma_\mu \Psi_{dL}\, W^{+\mu}\,, \qquad (14.37) $$

where the non-unitary quark mixing matrix V_{CKM} of dimension 4×4 is

$$ (V_{CKM})_{4\times 4} = [(V^o_{uL}\, V^{(1)}_{uL})_{3\times 4}]^T\, (V^o_{dL}\, V^{(1)}_{dL})_{3\times 4} \qquad (14.38) $$

14.7 Numerical results for quark masses and mixing

As an example of the possible spectrum of quark masses and mixing from this scenario, we show up the following fit of parameters at the M_Z scale [15]

Using the input values for the horizontal boson masses, Eq.(8), and the coupling constant of the $SU(3)$ symmetry:

$$ M_1 = 3.3 \times 10^3 \text{ TeV} \quad , \quad M_2 = 3.3 \times 10^5 \text{ TeV} \quad , \quad \frac{\alpha_H}{\pi} = 0.05\,, \qquad (14.39) $$

we write the tree level $\mathcal{M}^o_q$, and up to one loop corrections $\mathcal{M}^o_q$ quark mass matrices, as well as the corresponding mass eigenvalues and mixing:

d-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 906.643 \\ 0 & 0 & 0 & 5984.81 \\ 0 & 0 & 0 & 8139.76 \\ 3.00124 \times 10^6 & -670943. & 0 & 9.10502 \times 10^6 \end{pmatrix} \text{MeV}\,, \tag{14.40}$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_d = \begin{pmatrix} -5.64571 & -11.0583 & 46.8646 & 15.829 \\ -29.9051 & -39.4588 & -11.5894 & -3.91444 \\ 40.9245 & -30.3588 & -2859.86 & 130.424 \\ 0.0409246 & -0.0303588 & 0.386143 & 9.61036 \times 10^6 \end{pmatrix} \text{MeV}\,, \tag{14.41}$$

the d-quark mass eigenvalues

$$(\, m_d, m_s, m_b, M_D \,) = (\, 2.97549\,,\, 51.0\,,\, 2860.72\,,\, 9.61036 \times 10^6 \,)\, \text{MeV}\,, \tag{14.42}$$

and the product of mixing matrices:
$V_{d\,L} = V_{d\,L}^o\, V_{d\,L}^{(1)}$:

$$\begin{pmatrix} 0.981831 & 0.17522 & -0.0728363 & 0.0000922 \\ -0.183881 & 0.783786 & -0.593184 & 0.0005976 \\ 0.0468496 & -0.5958 & -0.801765 & 0.0008133 \\ -0.0000187 & -6.6982 \times 10^-10 & 0.0010134 & 0.999999 \end{pmatrix} \tag{14.43}$$

$V_{d\,R} = V_{d\,R}^o\, V_{d\,R}^{(1)}$:

$$\begin{pmatrix} 0.146421 & -0.175219 & -0.922135 & 0.312291. \\ 0.577678 & -0.783785 & 0.217014 & -0.0698145 \\ 0.803005 & 0.595801 & 0.0142936 & 4.3164 \times 10^{-9} \\ -0.0056951 & -9.0660 \times 10^{-8} & 0.319949 & 0.947418 \end{pmatrix} \tag{14.44}$$

u-quarks:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 673649. \\ 0 & 0 & 0 & 5.57857 \times 10^6 \\ 0 & 0 & 0 & 7.8041 \times 10^6 \\ 4.10528 \times 10^8 & -4.1775 \times 10^7 & 0 & 1.92243 \times 10^{10} \end{pmatrix} \text{MeV}\,, \tag{14.45}$$

$$\mathcal{M}_u = \begin{pmatrix} -0.47816 & -0.551837 & 5.4868 & 0.117774 \\ -3.21341 & 602.954 & 4467.75 & 95.9001 \\ 4.51209 & 1368.75 & -173107. & 714.009 \\ 0.00225605 & 0.684377 & 16.632 & 1.92287 \times 10^{10} \end{pmatrix} \text{MeV}\,, \tag{14.46}$$

the u-quark mass eigenvalues

$$(m_u, m_c, m_t, M_U) = (1.37677\,,\ 638.055\ ,\ 173170\,,\ 1.92287 \times 10^{10})\text{MeV} \quad (14.47)$$

and the product of mixing matrices:
$V_{u\,L} = V^o_{u\,L}\, V^{(1)}_{u\,L}$:

$$\begin{pmatrix} 0.996356 & 0.0468431 & -0.0712817 & 0.0000350 \\ -0.0010006 & -0.829224 & -0.558915 & 0.0002900 \\ -0.0852899 & 0.556949 & -0.826155 & 0.0004057 \\ 0 & 0.0000128 & 0.0004998 & 1. \end{pmatrix} \quad (14.48)$$

$V_{u\,R} = V^o_{u\,R}\, V^{(1)}_{u\,R}$:

$$\begin{pmatrix} 0.0003359 & 0.0934631 & -0.995394 & 0.0213497 \\ 0.0032952 & 0.995617 & 0.0934386 & -0.0021725 \\ 0.999995 & -0.0033122 & 0.0000265 & 0 \\ -1.4066 \times 10^{-8} & 0.0001676 & 0.0214593 & 0.99977 \end{pmatrix} \quad (14.49)$$

and the quark mixing matrix V_{CKM}:

$$\begin{pmatrix} 0.974441 & 0.224613 & -0.0035948 & 0.0000219 \\ 0.224564 & -0.973557 & 0.041928 & -0.0000382 \\ -0.0059177 & 0.0416636 & 0.999114 & -0.0010126 \\ 6.3092 \times 10^{-8} & -8.2754 \times 10^{-6} & -0.0004999 & 5.0666 \times 10^{-7} \end{pmatrix} \quad (14.50)$$

14.8 $\Delta F = 2$ Processes in Neutral Mesons

Here we study the tree level FCNC interactions that contribute to $K^o - \bar{K^o}$, $D^o - \bar{D^o}$ mixing via Z_1 , $Y_2^{\pm}$ exchange from the depicted diagram in Fig. 2.

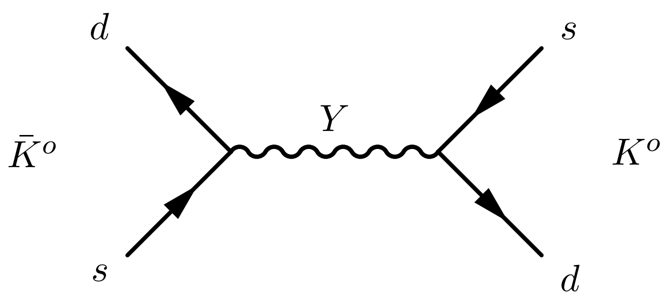

Fig. 14.2. Generic tree level exchange contribution to $K^o - \bar{K^o}$ from the $SU(3)$ horizontal gauge bosons.

The Z_1 , $Y_2^{\pm}$ ($Y_2^{\pm} = \frac{Y_2^1 \mp i Y_2^2}{\sqrt{2}}$) gauge bosons become massive at the second stage of the $SU(3)$ symmetry breaking, and have flavor changing couplings in both left- and right-handed fermions, and then contribute the $\Delta S = 2$ effective operators

$$\mathcal{O}_{LL} = (\bar{d}_L \gamma_\mu s_L)(\bar{d}_L \gamma^\mu s_L) \quad , \quad \mathcal{O}_{RR} = (\bar{d}_R \gamma_\mu s_R)(\bar{d}_R \gamma^\mu s_R) \tag{14.51}$$

$$\mathcal{O}_{LR} = (\bar{d}_L \gamma_\mu s_L)(\bar{d}_R \gamma^\mu s_R) \tag{14.52}$$

The $SU(3)$ couplings to fermions, Eq.(14.6), when written in the mass basis yield the gauge couplings

$$\mathcal{L}_{int,Z_1} = \frac{g_H}{2} \left(C_{L\,Z_1}\, \bar{d_L} \gamma_\mu s_L + C_{R\,Z_1}\, \bar{d_R} \gamma_\mu s_R \right) Z_1^\mu \tag{14.53}$$

$$\mathcal{L}_{int,Y_2^1} = \frac{g_H}{2} \left(C_{L\,Y_2^1}\, \bar{d_L} \gamma_\mu s_L + C_{R\,Y_2^1}\, \bar{d_R} \gamma_\mu s_R \right) Y_1^{1\,\mu} \tag{14.54}$$

$$\mathcal{L}_{int,Y_2^2} = \frac{g_H}{2} \left(C_{L\,Y_2^2}\, \bar{d_L} \gamma_\mu s_L + C_{R\,Y_2^2}\, \bar{d_R} \gamma_\mu s_R \right) i\, Y_1^{2\,\mu} \tag{14.55}$$

with the coefficients

$$\begin{aligned}
C_{L\,Z_1} &= L_{11}\, L_{12} - L_{31}\, L_{32} \quad , \quad C_{R\,Z_1} = R_{11}\, R_{12} - R_{31}\, R_{32} \\
C_{L\,Y_2^1} &= L_{12}\, L_{31} + L_{11}\, L_{32} \quad , \quad C_{R\,Y_2^1} = R_{12}\, R_{31} + R_{11}\, R_{32} \\
C_{L\,Y_2^2} &= (L_{12}\, L_{31} - L_{11}\, L_{32}) \, , \quad C_{R\,Y_2^2} = (R_{12}\, R_{31} - R_{11}\, R_{32})
\end{aligned} \tag{14.56}$$

where $V_{L,R} =\equiv V^o_{L,R}\, V^{(1)}_{L,R}$, and $L_{ij} \equiv V_{L\,ij}$, $R_{ij} \equiv V_{R\,ij}$. For each gauge boson, the effective four-fermion hamiltonian at the scale of the gauge boson mass is

$$\mathcal{H}_{Z_1} = \frac{g_H^2}{4M_{Z_1}^2} \left(C^2_{L\,Z_1}\, \mathcal{O}_{LL} + 2\, C_{L\,Z_1} C_{R\,Z_1} \mathcal{O}_{LR} + C^2_{R\,Z_1}\, \mathcal{O}_{RR} \right) \tag{14.57}$$

$$\mathcal{H}_{Y_2^1} = \frac{g_H^2}{4M_1^2} \left(C^2_{L\,Y_2^1}\, \mathcal{O}_{LL} + 2\, C_{L\,Y_2^1} C_{R\,Y_2^1} \mathcal{O}_{LR} + C^2_{R\,Y_2^1}\, \mathcal{O}_{RR} \right) \tag{14.58}$$

$$\mathcal{H}_{Y_2^2} = -\frac{g_H^2}{4M_1^2} \left(C^2_{L\,Y_2^2}\, \mathcal{O}_{LL} + 2\, C_{L\,Y_2^2} C_{R\,Y_2^1} \mathcal{O}_{LR} + C^2_{R\,Y_2^2}\, \mathcal{O}_{RR} \right) \tag{14.59}$$

with $M_{Y_2^1} = M_{Y_2^2} = M_1$. Therefore, the total four-fermion hamiltonian $\mathcal{H}_{SU(2)} = \mathcal{H}_{Z_1} + \mathcal{H}_{Y_2^1} + \mathcal{H}_{Y_2^2}$ can be written as

$$\mathcal{H}_{SU(2)} = \frac{g_H^2}{4M_1^2}\Big[(C^2_{L\,Z_1} + C^2_{L\,Y_2^1} - C^2_{L\,Y_2^2})\mathcal{O}_{LL} + (C^2_{R\,Z_1} + C^2_{R\,Y_2^1} + C^2_{R\,Y_2^2})\mathcal{O}_{RR}$$
$$+2(C_{L\,Z_1}C_{R\,Z_1} + C_{L\,Y_2^1}C_{R\,Y_2^1} - C_{L\,Y_2^2}C_{R\,Y_2^2})\mathcal{O}_{LR}\Big]$$

$$+\frac{g_H^2}{4}\left(\frac{1}{M^2_{Z_1}} - \frac{1}{M_1^2}\right)\left[C^2_{L\,Z_1}\mathcal{O}_{LL} + C^2_{R\,Z_1}\mathcal{O}_{RR} + 2C_{L\,Z_1}C_{R\,Z_1}\mathcal{O}_{LR})\right] \quad (14.60)$$

From the coefficients in eq.(14.56) we obtain:

$$C^2_{L\,Z_1} + C^2_{L\,Y_2^1} - C^2_{L\,Y_2^2} = \delta_L^2 \quad , \quad C^2_{R\,Z_1} + C^2_{R\,Y_2^1} - C^2_{R\,Y_2^2} = \delta_R^2 \quad , \quad , \quad (14.61)$$

$$C_{L,Z_1}\,C_{R,Z_1} + C_{L,Y_2^1}\,C_{R,Y_2^1} - C_{L,Y_2^2}\,C_{R,Y_2^2} = \delta_L\,\delta_R$$
$$+\,2(L_{11}\,R_{31}\, - L_{31}\,R_{11})(L_{32}\,R_{12}\, - L_{12}\,R_{32})\,, \quad (14.62)$$

and we can write

$$\mathcal{H}_{SU(2)} = \frac{g_H^2}{4M_1^2}\left[\delta_L^2\;\mathcal{O}_{LL} + \delta_R^2\;\mathcal{O}_{RR} + \delta_{LR}^2\;\mathcal{O}_{LR}\right] \quad (14.63)$$
$$+\frac{g_H^2}{4}\left(\frac{1}{M^2_{Z_1}} - \frac{1}{M_1^2}\right)\left[(L_{11}L_{12} - L_{31}L_{32})^2\;\mathcal{O}_{LL} + (R_{11}R_{12} - R_{31}R_{32})^2\;\mathcal{O}_{RR}\right.$$
$$\left.+2(L_{11}L_{12} - L_{31}L_{32})(R_{11}R_{12} - R_{31}R_{32})\;\mathcal{O}_{LR})\right]$$

with

$$\delta_L = L_{11}\,L_{12} + L_{31}\,L_{32} \quad , \quad \delta_R = R_{11}\,R_{12} + R_{31}\,R_{32} \quad (14.64)$$

$$\delta_{LR} = 2(\delta_L\,\delta_R + 2(L_{11}\,R_{31}\, - L_{31}\,R_{11})(L_{32}\,R_{12}\, - L_{12}\,R_{32})) \quad (14.65)$$

The reported parameter space region in section 7 generate $M_{Z_1} \approx M_1$ with quite good approximation, and then the dominant contribution to neutral meson mixing comes from the four-fermion Hamiltonian in eq.(14.63). The suppression of the generic meson mixing couplings $\frac{\zeta_{ij}}{\Lambda^2}(\bar{q}_{i\,L}\gamma^{\mu}P_{L,R}\,q_j)^2$ come out as follows

14.8.1 $K^o - \bar{K^o}$ meson mixing

$$\begin{aligned} \delta_L &= 0.144124 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_L|} = 32594.5\ \text{TeV} \\ \delta_R &= 0.452775 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_R|} = 10375.2\ \text{TeV} \\ \sqrt{|\delta_{LR}|} &= 0.361261 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,\sqrt{|\delta_{LR}|}} = 13003.4\ \text{TeV} \end{aligned} \quad (14.66)$$

14.8.2 $D^o - \bar{D^o}$ meson mixing

$$\begin{aligned} \delta_L &= -0.000829741 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_L|} = 5.66157 \times 10^6 \text{ TeV} \\ \delta_R &= -0.00328084 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,|\delta_R|} = 1.43184 \times 10^6 \text{ TeV} \\ \sqrt{|\delta_{LR}|} &= 0.456165 \quad , \quad \frac{M_1}{\frac{g_H}{2}\,\sqrt{|\delta_{LR}|}} = 10298.1 \text{ TeV} \end{aligned} \tag{14.67}$$

These numerical values are within the suppression required for BSM contributions reported for instance in the review ”CKM Quark - Mixing Matrix” in PDG2018[16].

14.9 Conclusions

Horizontal gauge bosons from the local $SU(3)$ family symmetry introduce flavor changing couplings, and in particular mediate $\Delta F = 2$ processes at tree level. We reported the analytic and numerical contribution to $K^o - \bar{K^o}$ and $D^o - \bar{D^o}$ meson mixing from tree level exchange diagrams mediated by the $SU(2)$ horizontal gauge bosons Z_1 , $Y_2^{\pm}$. We provided in section 7 a particular parameter space region where this scenario can accommodate the hierarchy spectrum of quark masses and mixing, and simultaneously suppress properly the $\Delta S = 2$ and $\Delta C = 2$ processes.

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14.10 Appendix: Diagonalization of the generic Dirac See-saw mass matrix

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & c \end{pmatrix} \tag{14.68}$$

The tree level $\mathcal{M}^o$ 4×4 See-saw mass matrix is diagonalized by a biunitary transformation $\psi_L^o = V_L^o \chi_L$ and $\psi_R^o = V_R^o \chi_R$. The diagonalization of $\mathcal{M}^o \mathcal{M}^{o\,T}$ ($\mathcal{M}^{o\,T} \mathcal{M}^o$) yield the nonzero eigenvalues

$$\lambda_3^2 = \frac{1}{2}\left(B - \sqrt{B^2 - 4D}\right) \quad , \quad \lambda_4^2 = \frac{1}{2}\left(B + \sqrt{B^2 - 4D}\right) \tag{14.69}$$

and rotation mixing angles

$$\cos\alpha = \sqrt{\frac{\lambda_4^2 - a^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\alpha = \sqrt{\frac{a^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \, ,$$
$$\cos\beta = \sqrt{\frac{\lambda_4^2 - b^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\beta = \sqrt{\frac{b^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \, . \tag{14.70}$$

$$B = a^2 + b^2 + c^2 = \lambda_3^2 + \lambda_4^2 \quad , \quad D = a^2 b^2 = \lambda_3^2 \lambda_4^2 \, , \tag{14.71}$$

$$a^2 = a_1^2 + a_2^2 + a_3^2 \quad , \quad b^2 = b_1^2 + b_2^2 + b_3^2 \tag{14.72}$$

The rotation matrices V_L^o, V_R^o admit several parametrizations related to the two zero mass eigenstates, for instance

$$V_L^o = \begin{pmatrix} c_1 & -s_1\, s_2 & s_1\, c_2\, c_\alpha & s_1\, c_2\, s_\alpha \\ 0 & c_2 & s_2\, c_\alpha & s_2\, s_\alpha \\ -s_1 & -c_1\, s_2 & c_1\, c_2\, c_\alpha & c_1\, c_2\, s_\alpha \\ 0 & 0 & -s_\alpha & c_\alpha \end{pmatrix} \quad , \quad V_R^o = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_r & s_r\, c_\beta & s_r\, s_\beta \\ 0 & -s_r & c_r\, c_\beta & c_r\, s_\beta \\ 0 & 0 & -s_\beta & c_\beta \end{pmatrix} \tag{14.73}$$

$$a_n = \sqrt{a_1^2 + a_3^2} \;,\; b_n = \sqrt{b_1^2 + b_3^2} \;,\; a = \sqrt{a_n^2 + a_2^2} \;,\; b = \sqrt{b_n^2 + b_2^2} \, , \tag{14.74}$$

$$s_1 = \frac{a_1}{a_n} \, , \, c_1 = \frac{a_3}{a_n} \, , \, s_2 = \frac{a_2}{a} \, , \, c_2 = \frac{a_n}{a} \, , \, s_r = \frac{b_2}{b} \, , \, c_r = \frac{b_3}{b} \tag{14.75}$$

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