Bled Workshops in Physics
Vol. 13, No. 1
p. 47

Proceedings of the Mini-Workshop
Hadronic Resonances
Bled, Slovenia, July 1 - 8, 2012

Highly excited states of baryons in large N_c QCD*

N. Matagne[a], Fl. Stancu[b]

[a] Service de Physique Nucléaire et Subnucléaire, University of Mons, Place du Parc, B-7000 Mons, Belgium
[b] Institute of Physics, B5, University of Liège, Sart Tilman, B-4000 Liège 1, Belgium

Abstract. The masses of highly excited negative parity baryons belonging to the $N = 3$ band are calculated in the $1/N_c$ expansion method of QCD. We use a procedure which allows to write the mass formula by using a small number of linearly independent operators. The numerical fit of the dynamical coefficients in the mass formula show that the pure spin and pure flavor terms are dominant in the expansion, like for the $N = 1$ band. We present the trend of some important dynamical coefficients as a function of the band number N or alternatively of the excitation energy.

1 The status of the $1/N_c$ expansion method

The large N_c QCD, or alternatively the $1/N_c$ expansion method, proposed by 't Hooft [1] in 1974 and implemented by Witten in 1979 [2] became a valuable tool to study baryon properties in terms of the parameter $1/N_c$ where N_c is the number of colors. According to Witten's intuitive picture, a baryon containing N_c quarks is seen as a bound state in an average self-consistent potential of a Hartree type and the corrections to the Hartree approximation are of order $1/N_c$. These corrections capture the key phenomenological features of the baryon structure.

Ten years after 't Hooft's work, Gervais and Sakita [3] and independently Dashen and Manohar in 1993 [4] derived a set of consistency conditions for the pion-baryon coupling constants which imply that the large N_c limit of QCD has an exact contracted $SU(2N_f)_c$ symmetry when $N_c \to \infty$, N_f being the number of flavors. For ground state baryons the $SU(2N_f)$ symmetry is broken by corrections proportional to $1/N_c$ [5,6].

Analogous to s-wave baryons, consistency conditions which constrain the strong couplings of excited baryons to pions were derived in Ref. [7]. These consistency conditions predict the equality between pion couplings to excited states and pion couplings to s-wave baryons. These predictions are consistent with the nonrelativistic quark model.

A few years later, in the spirit of the Hartree approximation a procedure for constructing large N_c baryon wave functions with mixed symmetric spin-flavor parts has been proposed [8] and an operator analysis was performed for $\ell = 1$ baryons [9]. It was proven that, for such states, the $SU(2N_f)$ breaking occurs at

* Talk delivered by Fl. Stancu

order N_c^0, instead of $1/N_c$, as it is the case for ground and also for symmetric excited states $[56, \ell^+]$ (for the latter see Refs. [10,11]). This procedure has been extended to positive parity nonstrange baryons belonging to the $[70, \ell^+]$ multiplets with $\ell = 0$ and 2 [12]. In addition, in Ref. [12], the dependence of the contribution of the linear term in N_c, of the spin-orbit and of the spin-spin terms in the mass formula was presented as a function of the excitation energy or alternatively in terms of the band number N. Based on this analysis an impressive global compatibility between the $1/N_c$ expansion and the quark model results for $N = 0, 1, 2$ and 4 was found [13] (for a review see Ref. [14]). More recently the $[70, 1^-]$ multiplet was reanalyzed by using an exact wave function, instead of the Hartree-type wave function, which allowed to keep control of the Pauli principle at any stage of the calculations [21]. The novelty was that the isospin term, neglected previously [9] becomes as dominant in Δ resonances as the spin term in N^* resonances.

The purpose of this work is mainly to complete the analysis of the excited states by including the $N = 3$ band for which results were missing in the systematic analysis of Ref. [12]. An incentive for studying highly excited states with $\ell = 3$ has been given by a recent paper [15] where the compatibility between the two alternative pictures for baryon resonances namely the *quark-shell picture* and the *meson-nucleon scattering* picture defined in the framework of chiral soliton models [16,17] has been proven explicitly. This work was an extension of the analysis made independently by Cohen and Lebed [18, 19] and Pirjol and Schat [20] for low excited states with $\ell = 1$.

As explained below, we shall analyze the resonances thought to belong to the $N = 3$ band by using the procedure we have proposed in Ref. [21] for the $N = 1$ band. Details can be found in Ref. [22].

2 Mixed symmetric baryon states

If an excited baryon belongs to a symmetric SU(6) multiplet the N_c-quark system can be treated similarly to the ground state in the flavour-spin degrees of freedom, but one has to take into account the presence of an orbital excitation in the space part of the wave function [10, 11]. If the baryon state is described by a mixed symmetric representation of SU(6) , the [**70**] at $N_c = 3$, the treatment becomes more complicated. In particular, the resonances up to about 2 GeV are thought to belong to $[\mathbf{70}, 1^-]$, $[\mathbf{70}, 0^+]$ or $[\mathbf{70}, 2^+]$ multiplets and beyond to 2 GeV to $[\mathbf{70}, 3^-]$, $[\mathbf{70}, 5^-]$, etc.

There are two ways of studying mixed symmetric multiplets. The standard one is inspired by the Hartree approximation [8] where an excited baryon is described by a symmetric core plus an excited quark coupled to this core, see *e.g.* [9, 12, 23, 24]. The core is treated in a way similar to that of the ground state. In this method each $SU(2N_f) \times O(3)$ generator is separated into two parts

$$S^i = s^i + S^i_c; \quad T^a = t^a + T^a_c; \quad G^{ia} = g^{ia} + G^{ia}_c; \quad \ell^i = \ell^i_q + \ell^i_c, \tag{1}$$

where s^i, t^a, g^{ia} and ℓ^i_q are the excited quark operators and S^i_c, T^a_c, G^{ia}_c and ℓ^i_c the corresponding core operators.

As an alternative, we have proposed a method where all identical quarks are treated on the same footing and we have an exact wave function in the orbital-flavor-spin space. The procedure has been successfully applied to the $N = 1$ band [21, 25, 26]. In the following we shall adopt this procedure to analyze the $N = 3$ band.

3 The mass operator

When hyperons are included in the analysis, the SU(3) symmetry must be broken and the mass operator takes the following general form [27]

$$M = \sum_i c_i O_i + \sum_i d_i B_i. \tag{2}$$

The formula contains two types of operators. The first type are the operators O_i, which are invariant under $SU(N_f)$ and are defined as

$$O_i = \frac{1}{N_c^{n-1}} O_\ell^{(k)} \cdot O_{SF}^{(k)}, \tag{3}$$

where $O_\ell^{(k)}$ is a k-rank tensor in SO(3) and $O_{SF}^{(k)}$ a k-rank tensor in SU(2)-spin. Thus O_i are rotational invariant. For the ground state one has $k = 0$. The excited states also require $k = 1$ and $k = 2$ terms. The rank $k = 2$ tensor operator of SO(3) is

$$L^{(2)ij} = \frac{1}{2}\left\{L^i, L^j\right\} - \frac{1}{3}\delta_{i,-j}\mathbf{L}\cdot\mathbf{L}, \tag{4}$$

which we choose to act on the orbital wave function $|\ell m_\ell\rangle$ of the whole system of N_c quarks (see Ref. [12] for the normalization of $L^{(2)ij}$). The second type are the operators B_i which are SU(3) breaking and are defined to have zero expectation values for non-strange baryons. Due to the scarcity of data in the $N = 3$ band hyperons, here we consider only one four-star hyperon $\Lambda(2100)7/2^-$ and accordingly include only one of these operators, namely $B_1 = -\mathcal{S}$ where $\mathcal{S}$ is the strangeness.

The values of the coefficients c_i and d_i which encode the QCD dynamics are determined from numerical fits to data. Table 1 gives the list of O_i and B_i operators together with their coefficients, which we believe to be the most relevant for the present study. The choice is based on our previous experience with the $N = 1$ band [26]. In this table the first nontrivial operator is the spin-orbit operator O_2. In the spirit of the Hartree picture [2] we identify the spin-orbit operator with the single-particle operator

$$\ell\cdot s = \sum_{i=1}^{N_c} \ell(i)\cdot s(i), \tag{5}$$

the matrix elements of which are of order N_c^0. For simplicity we ignore the two-body part of the spin-orbit operator, denoted by $1/N_c\,(\ell\cdot S_c)$ in Ref. [9], as being of a lower order (we remind that the lower case operators $\ell(i)$ act on the excited quark and S_c is the core spin operator).

Table 1. Operators and their coefficients in the mass formula obtained from numerical fits. The values of c_i and d_i are indicated under the heading Fit n ($n = 1, 2, 3, 4$) from Ref. [22].

Operator	Fit 1 (MeV)	Fit 2 (MeV)	Fit 3 (MeV)	Fit 4 (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 = 672 \pm 8$	$c_1 = 673 \pm 7$	$c_1 = 672 \pm 8$	$c_1 = 673 \pm 7$
$O_2 = \ell^i s^i$	$c_2 = 18 \pm 19$	$c_2 = 17 \pm 18$	$c_2 = 19 \pm 9$	$c_2 = 20 \pm 9$
$O_3 = \frac{1}{N_c} S^i S^i$	$c_3 = 121 \pm 59$	$c_3 = 115 \pm 46$	$c_3 = 120 \pm 58$	$c_3 = 112 \pm 42$
$O_4 = \frac{1}{N_c} [T^a T^a - \frac{1}{12} N_c (N_c + 6)]$	$c_4 = 202 \pm 41$	$c_4 = 200 \pm 40$	$c_4 = 205 \pm 27$	$c_4 = 205 \pm 27$
$O_5 = \frac{3}{N_c} L^i T^a G^{ia}$	$c_5 = 1 \pm 13$	$c_5 = 2 \pm 12$		
$O_6 = \frac{15}{N_c} L^{(2)ij} G^{ia} G^{ja}$	$c_6 = 1 \pm 6$		$c_6 = 1 \pm 5$	
$B_1 = -\mathcal{S}$	$d_1 = 108 \pm 93$	$d_1 = 108 \pm 92$	$d_1 = 109 \pm 93$	$d_1 = 108 \pm 92$
χ^2_{dof}	1.23	0.93	0.93	0.75

The spin operator O_3 and the flavor operator O_4 are two-body and linearly independent. The expectation values of O_3 are simply equal to $\frac{1}{N_c} S(S+1)$ where S is the spin of the whole system. For nonstrange baryons the eigenvalue of O_4 is $\frac{1}{N_c} I(I+1)$ where I is the isospin. For the flavor singlet Λ the eigenvalue is $-(2N_c+3)/4N_c$, favourably negative, as shown in Ref. [22].

Note that the definition of the operator O_4, indicated in Table 1, is such as to recover the matrix elements of the usual $1/N_c(T^aT^a)$ in SU(4), by subtracting $N_c(N_c+6)/12$. This is understood by using Eq. (30) of Ref. [25] for the matrix elements of $1/N_c(T^aT^a)$ extended to SU(6). Then, it turns out that the expectation values of O_4 are positive for octets and decuplets and of order N_c^{-1}, as in SU(4), and negative and of order N_c^0 for flavor singlets.

The operators O_5 and O_6 are also two-body, which means that they carry a factor $1/N_c$ in the definition. However, as G^{ia} sums coherently, it introduces an extra factor N_c and makes all the matrix elements of O_6 of order N_c^0 [25]. These matrix elements are obtained from the formulas (B2) and (B4) of Ref. [26] where the multiplet $[70, 1^-]$ has been discussed. Interestingly, when $N_c = 3$, the contribution of O_5 cancels out for flavor singlets, like for $\ell = 1$ [26]. This property follows from the analytic form of the isoscalar factors given in Ref. [26].

We remind that the SU(6) generators S^i, T^a and G^{ia} and the $O(3)$ generators L^i of Eq. (4) act on the total wave function of the N_c system of quarks as proposed in Refs. [21], [25] and [26]. The advantage of this procedure over the standard one, where the system is separated into a ground state core + an excited quark, is that the number of relevant operators needed in the fit is usually smaller than the number of data and it allows a better understanding of their role in the mass formula, in particular the role of the isospin operator O_4 which has always been omitted in the symmetric core + excited quark procedure. We should also mention that in our approach the permutation symmetry is exact [21].

Among the operators containing angular momentum components, besides the spin-orbit, we have included the operators O_5 and O_6, to check whether or not they bring feeble contributions, as it was the case in the $N = 1$ band. From Table 1 one can see that their coefficients are indeed negligible either included together as in Fit 1 or separately as in Fit 2 and 3. Thus in the expansion series, besides O_1, proportional to N_c, the most dominant operators are the pure spin O_3 and the pure isospin O_4.

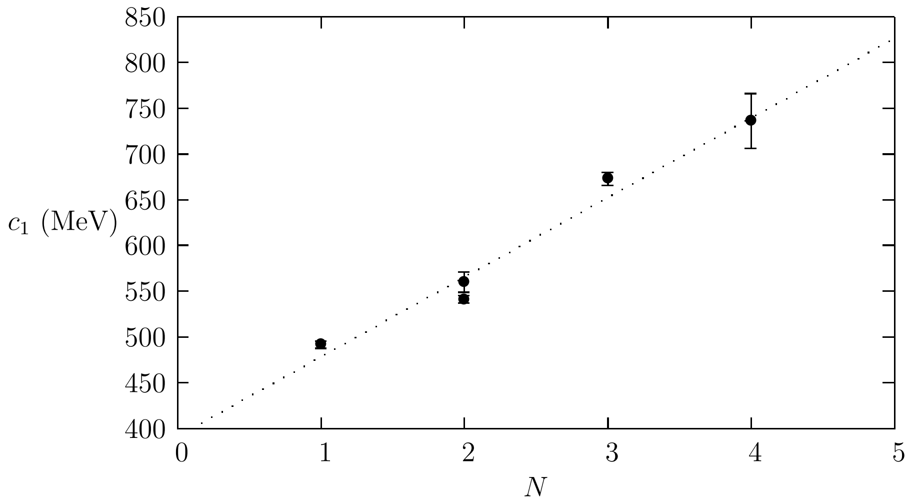

Fig. 1. The coefficient c_1 as a function of the band number N: N = 1 Ref. [26], N = 2 Ref. [10] for $[\mathbf{56}, 2^+]$ and Ref. [12] for $[\mathbf{70}, \ell^+]$, N = 3 Ref. [22], N = 4 Ref. [11]. The straight line is drawn to guide the eye.

4 Global results

The above analysis helps us to complete previous results for N = 1, 2 and 4 with the values of c_i obtained for N = 3. Therefore we can draw now a complete picture of the dependence of the coefficients c_1 and c_2 on N in analogy to Ref. [12] where results for N = 3 were missing. The new pictures are shown in Figs. 1 and 2. One can see that the values of c_1 follow nearly a straight line which can give rise to a Regge trajectory. Remember that c_1 describes the bulk content of the baryon mass, $c_1 N_c$ being the most dominant mass term. In a quark model language it represents the kinetic plus the confinement energy. As as discussed in Refs. [13, 14] the band number N also emerges from the spin independent part of a semi-relativistic quark model. If this part contributes to the total mass by a quantity denoted by M_0, then one can make the identification

$$c_1^2 = M_0^2/9 \tag{6}$$

when $N_c = 3$. In this way one can compare the Regge trajectory obtainable from the above results with that of a standard constituent quark model. It turns out

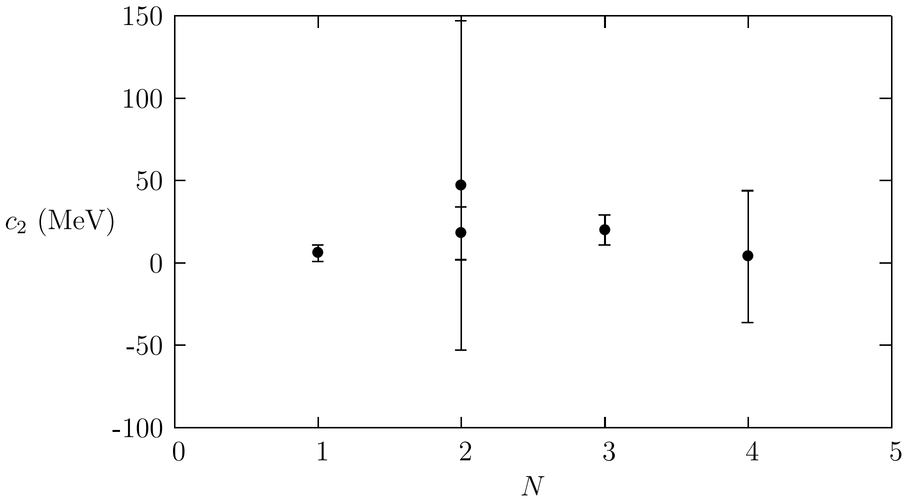

Fig. 2. Same as Figure 1 but for the coefficient c_2.

that they are close to each other [13,14]. and the value obtained here for c_1 at N = 3, missing in the previous work, is entirely compatible with the previous picture.

The behaviour of c_2 shows that the spin-orbit operator contributes very little to the mass, at all energies, in agreement to quark models, where it is usually neglected. Note that the behaviour of c_2 in Fig. 2 is slightly different from that of [12], because we presently take the value of c_2 at N = 1 from Ref. [26] (Fit 3 giving the lowest $\chi^2_{\rm dof}$) for consistency with our treatment, instead of that of Ref. [9], based on the ground state core + excited quark, the only available at the time the paper [12] was published.

We refrain ourselves from presenting the global picture of c_3, the spin term coefficient, because the results for positive parity mixed symmetric states are obtained on the one hand in the core + excited quark approach, where the isospin term is missing and on the other hand, for negative parity states where it is present, our approach is used. This term competes with the spin term. We plan to reanalyze the $[\mathbf{70}, \ell^+]$ multiplets before drawing a complete picture of c_3.

5 Conclusions

We have used a procedure which allows to write the mass formula by using a small number of linearly independent operators for spin-flavour mixed symmetric states of SU(6). The numerical fits of the dynamical coefficients in the mass formula for $N = 3$ band resonances show that the pure spin and pure flavor terms are dominant in the $1/N_c$ expansion, like for $N = 1$ resonances. This proves that the isospin term cannot be neglected, as it was the case in the ground state + excited quark procedure. We have shown the dependence of the dynamical coefficients c_1 and c_2 as a function of the band number N or alternatively of the excitation energy for $N = 1, 2, 3$ and 4 bands.

References

1. G. ’t Hooft, Nucl. Phys. **72** (1974) 461.
2. E. Witten, Nucl. Phys. **B160** (1979) 57.
3. J. L. Gervais and B. Sakita, Phys. Rev. Lett. **52** (1984) 87; Phys. Rev. **D30** (1984) 1795.
4. R. Dashen and A. V. Manohar, Phys. Lett. **B315** (1993) 425; ibid **B315** (1993) 438.
5. R. F. Dashen, E. Jenkins and A. V. Manohar, Phys. Rev. **D51** (1995) 3697.
6. E. Jenkins, Ann. Rev. Nucl. Part. Sci. **48** (1998) 81; AIP Conference Proceedings, Vol. 623 (2002) 36, arXiv:hep-ph/0111338; PoS E **FT09** (2009) 044 [arXiv:0905.1061 [hep-ph]].
7. D. Pirjol and T. M. Yan, Phys. Rev. D **57** (1998) 1449.
8. J. L. Goity, Phys. Lett. **B414** (1997) 140.
9. C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed, Phys. Rev. **D59** (1999) 114008.
10. J. L. Goity, C. Schat and N. N. Scoccola, Phys. Lett. **B564** (2003) 83.
11. N. Matagne and F. Stancu, Phys. Rev. **D71** (2005) 014010.
12. N. Matagne and F. Stancu, Phys. Lett. **B631** (2005) 7.
13. C. Semay, F. Buisseret, N. Matagne and F. Stancu, Phys. Rev. D **75** (2007) 096001.
14. F. Buisseret, C. Semay, F. Stancu and N. Matagne, Proceedings of the Mini-workshop Bled 2008, Few Quark States and the Continuum”, Bled Workshops in Physics, vol. 9, no. 1, eds. B. Golli, M. Rosina and S. Sirca. arXiv:0810.2905 [hep-ph].
15. N. Matagne and F. Stancu, Phys. Rev. **D84** (2011) 056013.
16. A. Hayashi, G. Eckart, G. Holzwart and H. Walliser, Phys. Lett. **147B** (1984) 5.
17. M. P. Mattis and M. E. Peskin, Phys. Rev. **D32** (1985) 58; M. P. Mattis, Phys. Rev. Lett. **56** (1986) 1103; Phys. Rev. **D39** (1989) 994; Phys. Rev. Lett. **63** (1989) 1455; M. P. Mattis and M. Mukerjee, Phys. Rev. Lett. **61** (1988) 1344.
18. T. D. Cohen and R. F. Lebed, Phys. Rev. Lett. **91**, 012001 (2003); Phys. Rev. **D67** (2003) 096008.
19. T. D. Cohen and R. F. Lebed, Phys. Rev. **D68** (2003) 056003.
20. D. Pirjol and C. Schat, Phys. Rev. **D67** (2003) 096009.
21. N. Matagne and F. Stancu, Nucl. Phys. A **811** (2008) 291.
22. N. Matagne and F. Stancu, Phys. Rev. D **85** (2012) 116003.
23. C. L. Schat, J. L. Goity and N. N. Scoccola, Phys. Rev. Lett. **88** (2002) 102002; J. L. Goity, C. L. Schat and N. N. Scoccola, Phys. Rev. **D66** (2002) 114014.
24. N. Matagne and F. Stancu, Phys. Rev. **D74** (2006) 034014; Nucl. Phys. Proc. Suppl. **174** (2007) 155.
25. N. Matagne and F. Stancu, Nucl. Phys. A **826** (2009) 161.
26. N. Matagne and F. Stancu, Phys. Rev. D **83** (2011) 056007.
27. E. Jenkins and R. F. Lebed, Phys. Rev. **D52** (1995) 282.