THE RESPONSE OF SATURATED SOILS TO A DYNAMIC LOAD

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abstract

This paper presents the two most significant types of deformation behavior for dynamically loaded, saturated soil. Flow liquefaction and cyclic mobility deserve special attention because of the large deformations that accompany these two phenomena. The submergence of a railway-line embankment due to the newly built Sava-river accumulation reservoir in Boštanj and the large landslide that occurred in the Stože area in the Julian Alps are case histories in Slovenia where flow liquefaction and cyclic mobility were analyzed. The dynamic loading caused by railway traffic and possible seismic activity were taken into account. Material from these two sites, silty sand and lacustrine carbonate silt, were used in extensive laboratory research, with the objective to define a procedure for excess pore-water pressure-generation modeling in dynamically loaded saturated soil.

It has been found recently that the change of the pore-water pressure is related to the dissipated energy density calculated from the hysteresis loops caused by dynamic loading. Based on the experimental results an empirical equation defining the generation of pore pressure during dynamic loading has been proposed. The equation is divided into two parts: the first part describing the residual pore-water pressure generation, and the second part describing the increment and decrement of pore-water pressure within the load cycle, the so-called temporary pore-water pressure change. The proper effective stresses and thus the stress path of the dynamically loaded soil can be defined by using the proposed energy-based numerical model.

The proposed pore-pressure model can also be used in deformation-behavior modeling. It was observed from the experimental results that after a few cycles of dynamic loading the saturated soil starts to exhibit a very low stiffness at the beginning of a load cycle, after which it begins to strengthen. The strain developed during this softening phase represents the main share of the total strain. The occurrence and duration of this phase are related to the energy dissipated during the cyclic loading as well, and the relation between the dissipated energy, the excess pore pressure and the short-term flow during cyclic mobility, give us an opportunity for a simple response modeling of the dynamically loaded saturated soils.

keywords

flow liquefaction, cyclic mobility, excess pore pressure, dissipated energy

1 INTRODUCTION

Due to the existing hydro-power potential, a chain of five new hydro-power plants is planned for the lower part of the Sava river. The construction of the first power plant started at the Boštanj site in November 2002 and it was completed in May 2006 [1].

The railway connection between Ljubljana and Zagreb runs along the Sava river. The construction of the accumulation reservoir caused the submergence of the railway-line embankment and raised questions about possible changes in the response of newly saturated soils resulting from the dynamic load caused by the railway. Therefore, the stability analyses considering a new ground-water level were required by the owner of the railway [2, 3]. Field and laboratory tests were carried out before the upheaval of the water in order to get the input parameters for the analyses. Two possible types of saturated soil behavior were in question: liquefaction and cyclic mobility. The following extensive research gives an opportunity for a detailed study of the liquefaction potential of silty sand from the lower Sava river at Boštanj.

The other case of material susceptible to liquefaction was found in the northern part of Slovenia – in the

Julian Alps. Lacustrine material of glacial origin has been recognized as very sensitive to various factors, such as water content and loading conditions, including seismic effects. A strong earthquake in April 1998, with its epicenter in the Krn mountains and a magnitude of M_{WA}=6.0, caused the collapse of an approximately 100-m-long section of the shore of the 20-km-distant Lake Bohinj. Saturated lacustrine soils were thought to be present. The same earthquake, in amplified form, caused serious damage to several buildings in the village of Mala vas, near Bovec, which were founded on saturated lacustrine soil [4]. Quite soon after the above-mentioned occurrences, in November 2000, a very severe landslide occurred in the area called Stože. It resulted in debris flow in its lower part. Layers of lacustrine carbonate silt of a relatively small thickness were observed in the material displaced during the landslide, and the question arose as to whether the presence of these layers was responsible for the landslide [5]. An investigation of the static and dynamic liquefaction potential of the lacustrine carbonate silt was initiated as a result [6, 7].

Several findings from the research mentioned above are presented in this paper. Saturated silty sand from the Sava river at Boštanj and the lacustrine carbonate silt from the Julian Alps are materials that forced the author of this paper as well as the geotechnical society in Slovenia to accept the danger of liquefaction as well as the occurrence of cyclic mobility in Slovenia as real possibilities. The findings from these two recent case histories have helped to introduce a more cautious treatment of dynamically loaded saturated soil response in daily practice. They are used in this paper as examples.

2 LIQUEFACTION OF SOIL

When dynamically loaded saturated soils are being considered, the term liquefaction is very important. Liquefaction is defined as the transformation of soil from the solid to the liquid state. It happens as a consequence of increased pore pressure and a reduced effective stress, mostly in saturated cohesionless soils. When such soil is subjected to rapid loading, e.g., earthquake loading or another kind of dynamic loading, the pore water is unable to drain in a very short time period. The loading conditions might be understood, therefore, as the undrained loading conditions. If it is not too dense, a cohesionless soil subjected to cyclic loading, especially cyclic loading in the shear mode, has a tendency to densify. As the pores between the soil grains are filled with water, which cannot drain sufficiently, the generation of excess pore pressure occurs. Figure 1 shows the changes in the soil skeleton caused by cyclic loading, which results in excess pore-water pressure being generated.

The term static liquefaction (flow failure) refers to the rapid increment of pore-water pressure followed by a sudden loss of strength after the peak value of the deviator stress is reached, until a residual/steady-state strength is reached. Flow liquefaction appears when the residual strength of the soil is smaller than the static shear stress required for the equilibrium of a soil mass. The liquefied stress state, in that case, is represented by the initial effective confining pressure, decreased by the excess pore pressure.

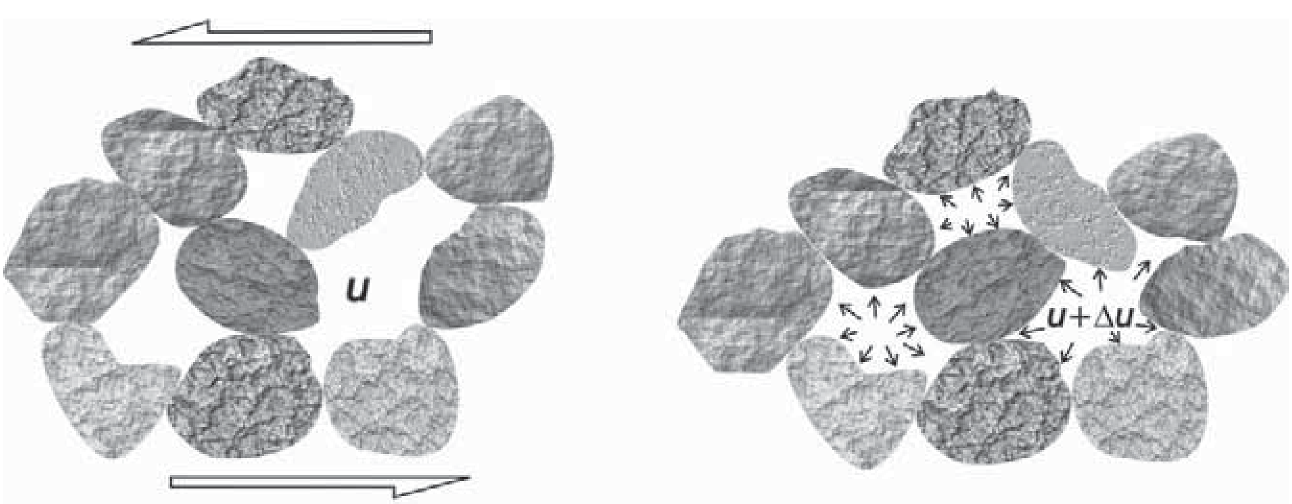

Figure 1. Cyclic loading in the shear mode causes grain movements and the generation of excess pore pressure.

2.1 SIMPLIFIED PROCEDURES FOR AN EVALUATION OF SOIL'S LIQUEFACTION POTENTIAL

Based on the findings of many previous case histories, with and without the occurrence of flow liquefaction, some simplified procedures were developed for evaluating the liquefaction potential in a specific case [8, 9, 10]. The procedures are based on different field measurements. The most widely used among them are SPT, CPT and shear-wave velocity measurements. The liquefaction potential for silty sand from the Sava river at Boštanj and lacustrine carbonate silt from the Julian Alps was evaluated in the manner of SPT and shear-wave velocity measurements. An earthquake with a magnitude of M=7.0 was proposed as the strongest possible type of dynamic load, and it can be seen from the results (Figure 2) that the estimated danger of the occurrence of liquefaction depends strongly on the procedure, and that the results differ. Thus, more detailed research was needed.

2.2 THE STATE-CRITERIA APPROACH

As described above, the occurrence of flow liquefaction depends upon the ability of a material to contract itself. This contractive behavior is the reason for the pore-pressure increase. Therefore, it is of interest to know where is the boundary, called the critical void ratio (CVR) line, between the contractive and dilative soil state. The states of the tested soil – silty sand from the Sava river and lacustrine carbonate silt from Julian Alps – were defined in terms of the void ratio. Tests at different effective confining pressures resulted in the CVR lines shown in Figure 3.

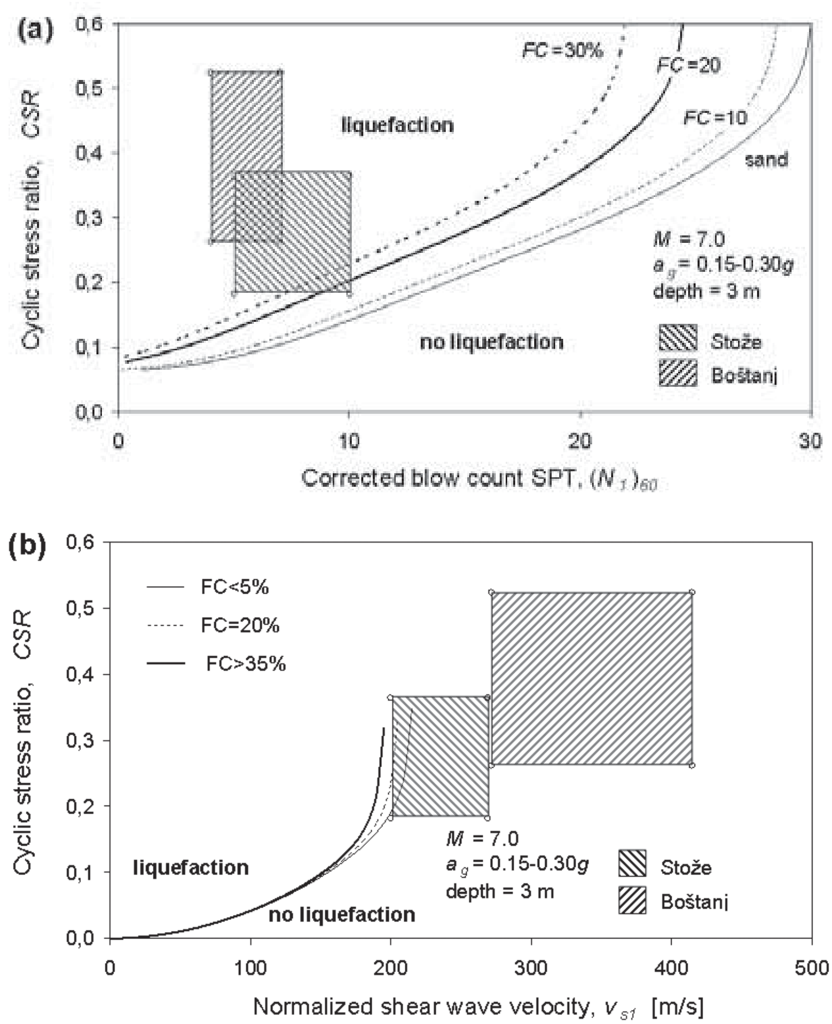

Figure 2. The danger of liquefaction occurring for silty sand from the Sava river and lacustrine carbonate silt from the Julian Alps. The CRR are based on (a) SPT measurements, (b) shear-wave velocity measurements.

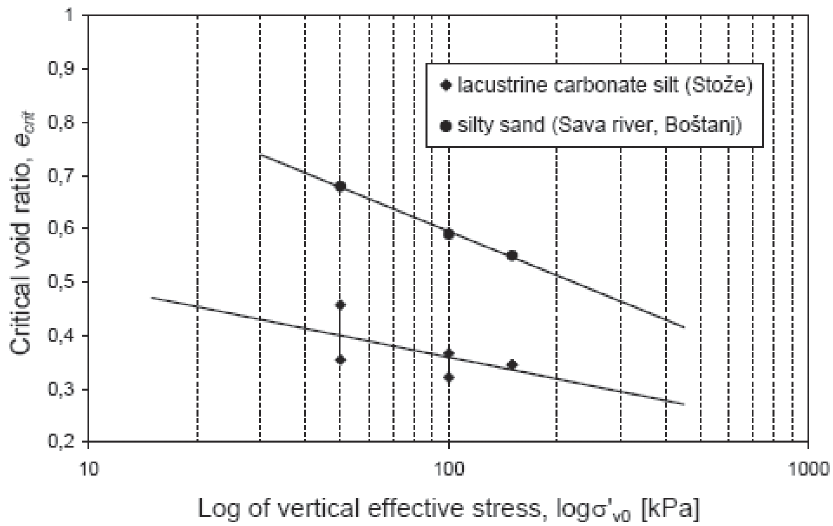

Figure 3. CVR line as a boundary between the loose contractive states and the dense dilative states of the two tested materials.

The generation of an excess pore-water pressure causes a decrease in the effective stresses. The effective-stress conditions leading to the occurrence of flow liquefaction can be most easily described in stress-path space. The response of five saturated cohesionless specimens isotropically consolidated to the same void ratio and different effective confining pressures, in undrained stress-controlled triaxial compression is shown in Figure 4 [11]. Regarding the initial states of the specimens according to the steady-state line, specimens 1 and 2 exhibit dilative behavior when the shearing starts, while specimens 3, 4 and 5 exhibit contractive behaviors, which is necessary for flow liquefaction.

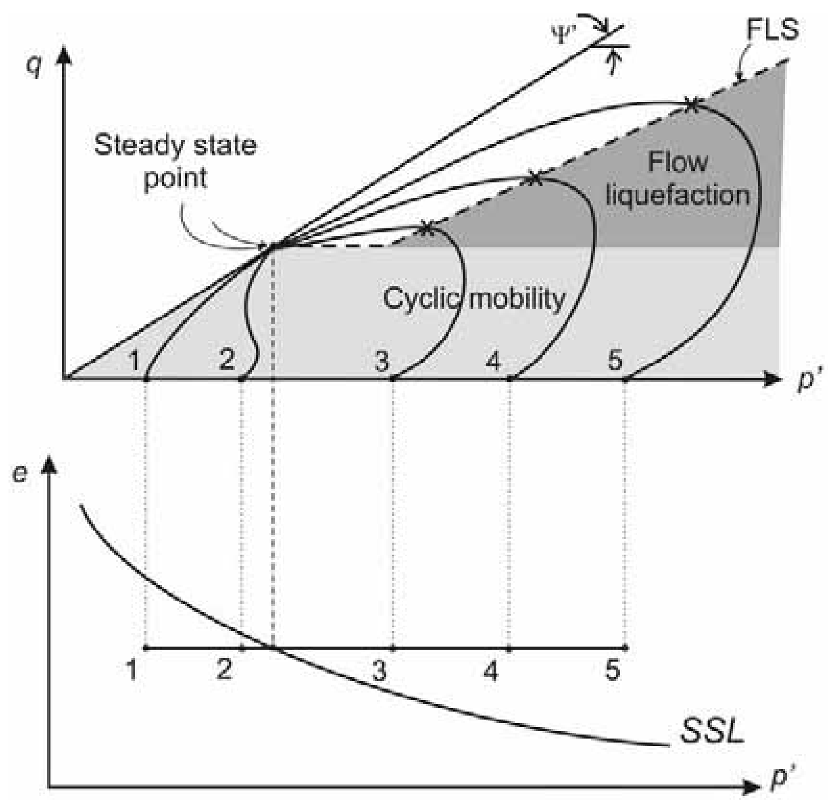

Figure 4. Initial conditions susceptible to either flow liquefaction or cyclic mobility (adapted from [11]).

The flow liquefaction is initiated at the peak of the stress path in the case of the latter three specimens. The locus of the points describing the effective stress conditions at the initiation of flow liquefaction is a straight line [11, 12] called the flow liquefaction surface (FLS). All the specimens reach the same steady-state point as they have the same void ratio. The FLS is truncated at the level of the steady-state point as flow liquefaction cannot occur if the stresses are below this point. The FLS therefore marks the boundary between the soil states at which either flow liquefaction or cyclic mobility can occur.

While cyclic mobility is described in more detail in the next section, the flow-liquefaction potential of two investigated materials, silty sand from the Sava river at Boštanj and lacustrine carbonate silt from Julian Alps, is estimated on the basis of triaxial test results. Samples of lacustrine carbonate silt (Figure 5) were reconstituted at different initial states, while the samples of silty sand (Figure 6) were intact. The tested state of the silty sand seems unsuitable for contractive behavior and thus flow liquefaction is also not expected. Loose samples of lacustrine carbonate silt contracts remarkably during shearing. To trigger the flow liquefaction the static shear stress should exceed the shear strength of a soil in the liquefied state.

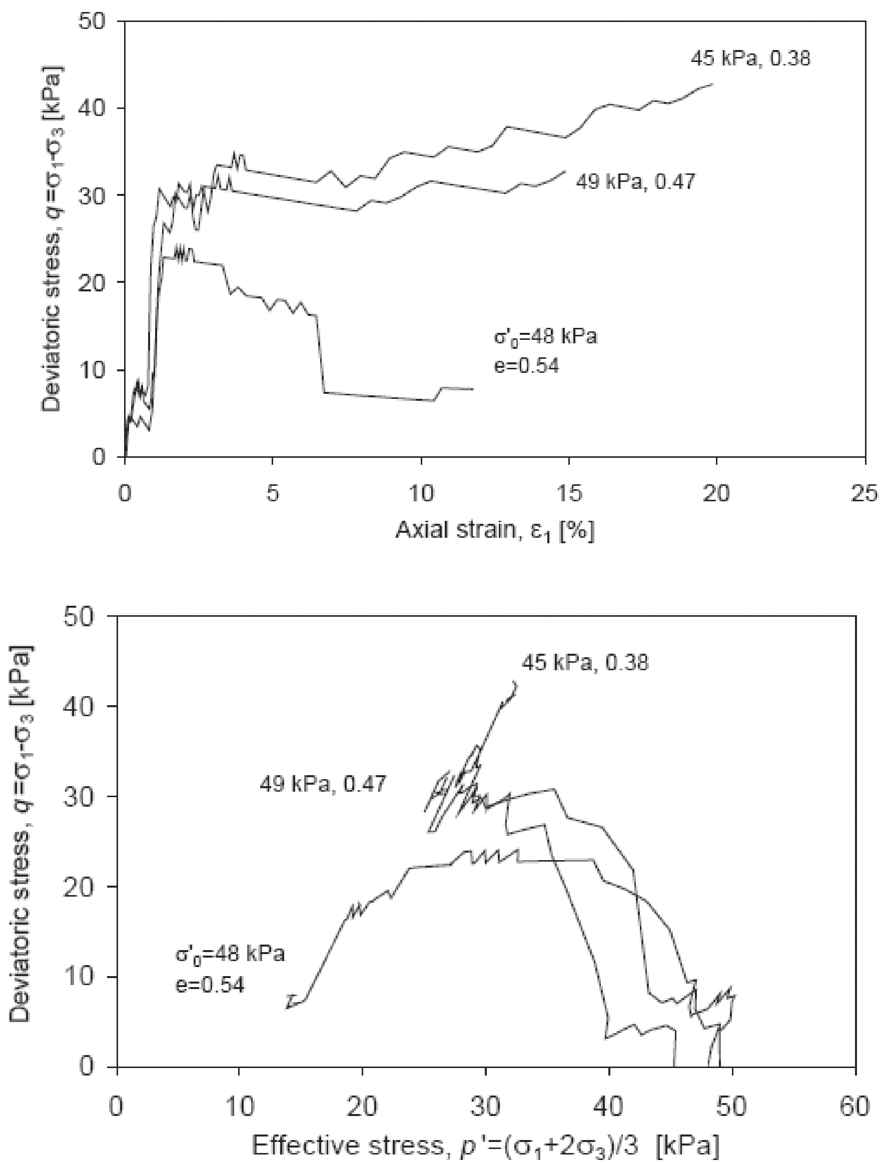

Figure 5. Undrained triaxial test of isotropic, consolidated, reconstituted samples of lacustrine carbonate silt from the Julian Alps.

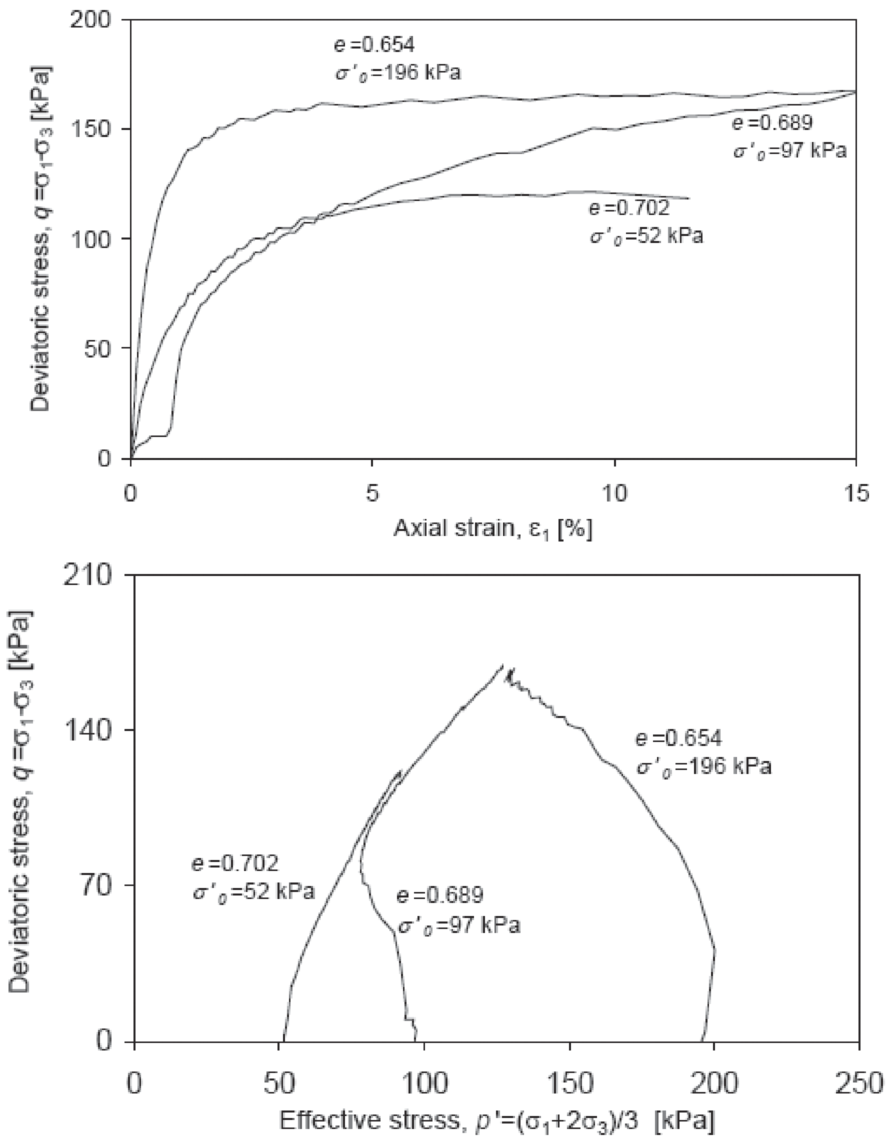

Figure 6. Undrained triaxial test of isotropic, consolidated, intact samples of silty sand from the Sava river at Boštanj.

3 CYCLIC MOBILITY

Cyclic mobility deformations are not of the flowing type and thus the damage would normally be smaller, but still severe, than in the case of flow liquefaction. Deformations due to cyclic mobility are developed incrementally during cyclic loading. The main reason for the dramatic increase of cyclic mobility deformations is the loss of stiffness caused by a decrease of the effective stresses.

An excess pore pressure generated during dynamic loading moves the stress path from its initial position in the direction of the failure envelope (Figure 7). If the cyclic stress is large enough, the steady-state strength might be exceeded during the cyclic loading. If this happens near the FLS, the effective stress path can touch the FLS. Momentary instability can occur therefore, leading to significant strain development. If the static shear stress is smaller than the steady-state strength, the strain generally ceases when the shear stress returns to the values below the steady-state strength.

If steady-state strength is not exceeded during the cyclic loading, the effective stress path approaches the so-called phase transformation surface (PTS) [13]. The PTS represents a kind of boundary between the dilative and the contractive behavior of loaded soil. Above the PTS a dilative tendency increases the effective confinement (and consequently the shear strength), while below the

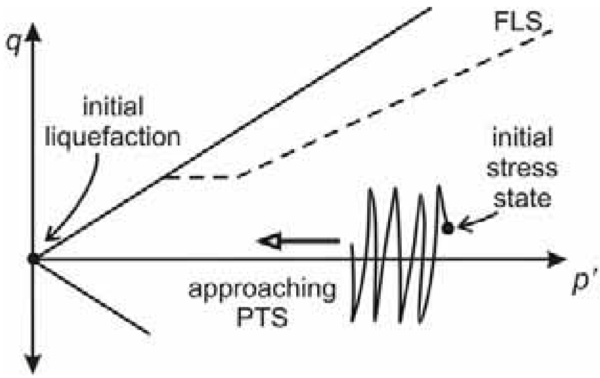

Figure 7. Generation of excess pore pressure due to the cyclic loading causes movement of the stress path from its initial position in the direction of the failure envelope.

PTS the soil exhibits a contractive behavior and thus a tendency to generate excess pore pressure. Youd [14] clearly described the rearrangements of soil grains that happen in cyclically loaded soil when the stress path approaches and crosses the PTS from one side or the other. A significant shear strain may develop without an appreciable shear stress at the moment when the PTS is crossed (Figure 8). This, almost flowing, behavior of the soil when the stress path meets the PTS causes serious problems in the numerical modeling of cyclic mobility phenomena [15].

When the cyclic stresses are larger than the static shear stresses, stress reversal occurs. Thus, each load cycle includes compression and extension loading. Any excess pore pressure generated during the cyclic loading causes the movement of the stress path in the direction of a zero effective stress (origin of *q*-*p* graph). This state is called the initial liquefaction [16]. When the stress path reaches it, only further oscillations along the compression and extension portions of the drained failure surface are possible due to the continuation of the cyclic or monotonic loading [11].

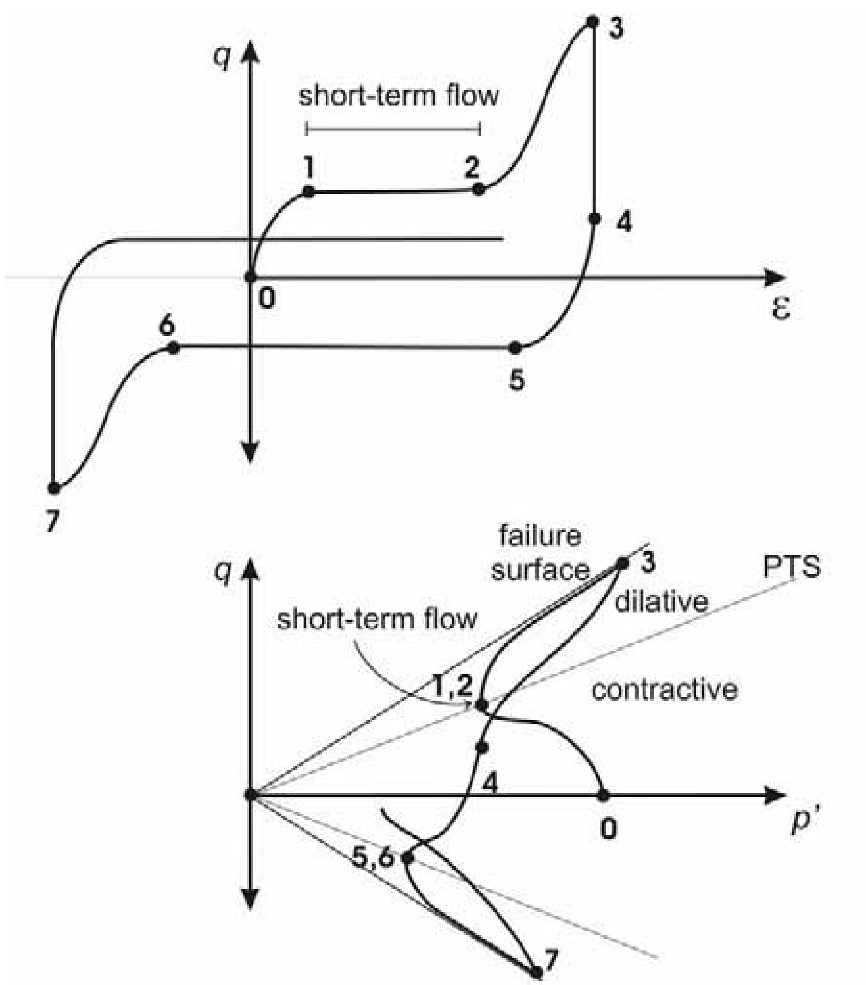

Figure 8. A typical stress path for cyclically loaded soil and the shear-strain relationship when it crosses the phase transformation surface (adapted from [15]).

It can be seen from the results of the two tested materials (Figure 5, Figure 6) that flow liquefaction is hardly likely to occur. There is a much higher risk of cyclic mobility, especially in the case of silty sand from the Sava river at Boštanj, which was found in the railway embankment. However, the dynamic load of a passing train might cause an increase in the pore pressure, leading to the occurrence of cyclic mobility. Two down-hole arrays with accelerometers and pore water-pressure sensors were established to enable monitoring of the dynamically loaded investigated soil response during and after the upheaval of the water and the saturation of the soil (Figure 9). The aim of the research was to define an effective procedure for an evaluation of the excess pore pressure. This would help to define a stress path during the dynamic loading and thus help with the prediction of any deformations.

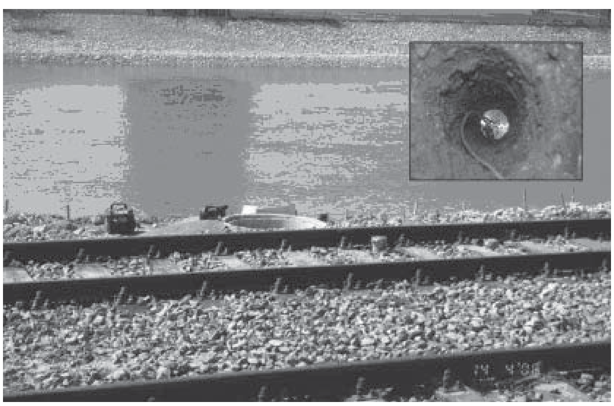

Figure 9. Railway embankment with a down-hole array and an accelerometer before the upheaval of the water.

4 AN ENERGY APPROACH TO EVALUATING THE EXCESS PORE PRESSURE

The stress path moves from its initial position due to changes in the shear stress, which are caused by loading, and due to the effective pressure decreasing during the cyclic loading (Figure 7). The effective pressure usually decreases due to the excess generation of pore pressure. We can be sure that an excess generation of pore pressure during the cyclic loading actually leads to stress-path movements and thus to changes in the soil strength and the stiffness.

Soil-grain rearrangements connected with soil-structure changes cause the pore pressure to increase when the soil contracts and decrease when it dilates. Changes from contractive to dilative behavior and vice versa happen when the stress path crosses the PTS. Cyclic loading causes the irrecoverable contraction of the soil skeleton (Figure 1), which in the case of an undrained loading condition is accompanied by the permanent generation of excess pore pressure.

It is obvious that if a proper model for pore-pressure changes during dynamic loading were to exist, it would enable simple access to the modeling of the response of dynamically loaded saturated soils. The idea for the solution was taken from metal-fatigue analyses, for which purpose a cumulative damage hypothesis was developed [17]. Using this hypothesis an irregular dynamic loading can be converted into an equivalent damaging quantity, which makes it possible to evaluate a stress path's movement from its initial position, approaching the phase-transformation surface and the failure envelope. The energy dissipated during dynamic loading is chosen as the equivalent damaging quantity.

Quantification by seismologists of the energy released during earthquakes and the use of the energy dissipation in performance-based design in structural earthquake engineering argues for the use of energy for an excess pore-pressure evaluation procedure. The first steps toward the energy approach to excess pore-pressure evaluation were made using the relationships between the energy released during earthquakes and the sites where liquefaction occurred [18, 19]. Nemat-Nasser and Shokooh [20] presented governing differential equations relating energy dissipation to the densification of dry samples and to the generation of excess pore-water pressure in saturated samples. The dissipated energy density W was defined generally at time t as

$$W(t)=\frac{1}{\sigma'_0}\int_0^t \sigma_{ij}d\varepsilon_{ij}\ , \qquad (1)$$

where σ_{ij} and ε_{ij} denote the stress and incremental strain tensors, respectively, t is the time in which the total dissipated energy is in question, and σ'_0 is an initial effective confining stress. In the case of the laboratory cyclic-loaded test results the dissipated energy density is defined as the area bounded by the hysteresis loops of the stress-strain curve (Figure 10).

Complementing the theoretical framework, several laboratory tests [21, 22, 23] as well as field measurements [24] have been performed to prove the relation. A typical general form, derived from the proposed expressions, could be written as

$$r_u = a\cdot W^b\ , \qquad (2)$$

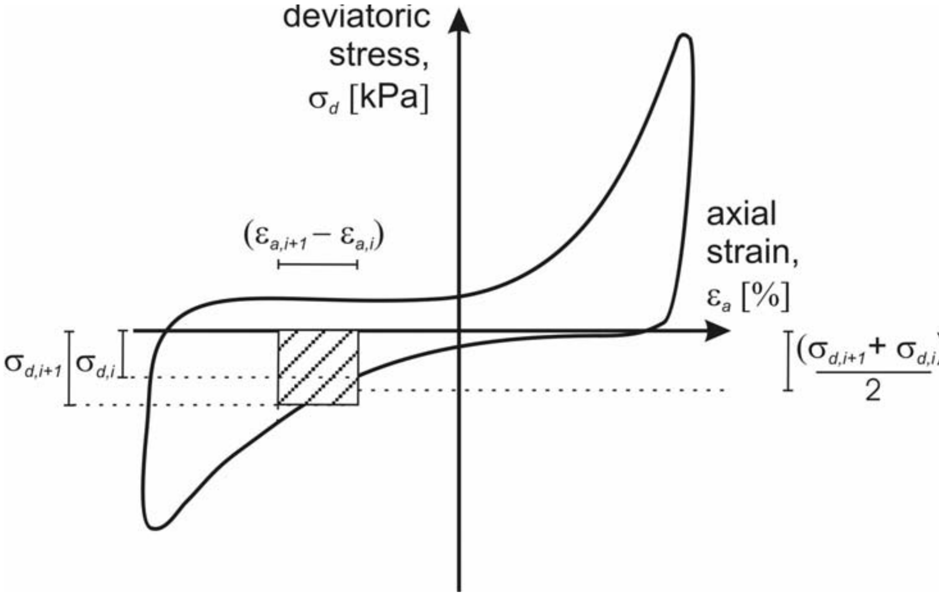

Figure 10. The dissipated energy per unit volume for a soil sample in case of cyclic triaxial test results.

where r_u denotes the pore-water pressure ratio ($=\Delta u/\sigma'_0$), Δu is the pore-water pressure change, while *a* and *b* are functions of the soil type, the relative density of the soil, the stress conditions, the initial soil-state parameters, etc. It should be mentioned again that the pore-water pressure change from Eq. 2 is caused only by the soil particles being rearranged and that this is a permanent change.

Lenart [25] divided the excess pore-water pressure generated during the dynamic loading into two parts: the temporary pore-pressure change and the residual pore-pressure change. Temporary pore-pressure changes can be observed as oscillations of the pore pressure in a normal pore-water pressure curve. They are caused by the transmission of compressive stresses onto the pore water. The origins of the pore-pressure oscillations and their effect upon the deformation behavior of the soil during dynamic loading are described in another paper [26].

Knowing the time function *F(t)* of a normal component of dynamic loading to which the soil is subjected, it is possible to write the equation for evaluating the pore-pressure ratio, r_u (Eq. 3). The parameters k_r and k_t are the residual and temporary pore-pressure parameters, which depend upon the type of soil and its state. Their evaluation procedure is described in more detail in [25]. Using the least-square method, the best agreement between the proposed relationship and the empirical results was found if a dissipated energy density, *W*, in Eq. 3 is raised to the power of *e*/10, where *e* means the base of the natural logarithm. As it is based on the dissipated energy density, the pore-pressure ratio calculated with Eq. 3 is independent of the loading frequency or the rate impacts in the case of the strain range typical for cyclic mobility or liquefaction [26].

$$r_u = (W)^{e/10} \cdot \left[k_r + k_t F(t)\right] \qquad (3)$$

The proposed equation was tested in the case of two investigated materials. Figure 11 shows the approximated pore-water pressure changes compared to the experimental results in the case of the undrained cyclic triaxial test of the lacustrine carbonate silt sample. The loading took the form of a sine wave with the frequency of the loading being 1 Hz. To prove the proposed equations' independence from the frequency and the rate of loading, an irregular loading test was performed on a sample of silty sand from the Sava river. The loading simulated a seismic load recorded during the Petrovac earthquake in Montenegro in 1979. Good agreement was obtained between the results of the numerical analysis and the results of the laboratory tests (Figure 12).

Similar pore water pressure response due to dynamic loading was observed also, when other kind of materials were tested. An interesting increase in share of temporary pore water pressure changes was noticed in the case of highly porous snail soil [27].

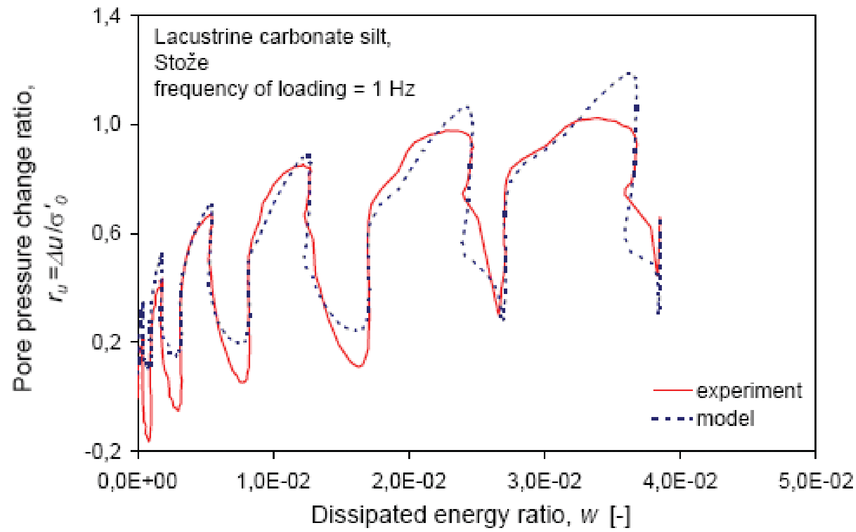

Figure 11. Pore-water pressure curve during an undrained cyclic triaxial test of lacustrine carbonate silt.

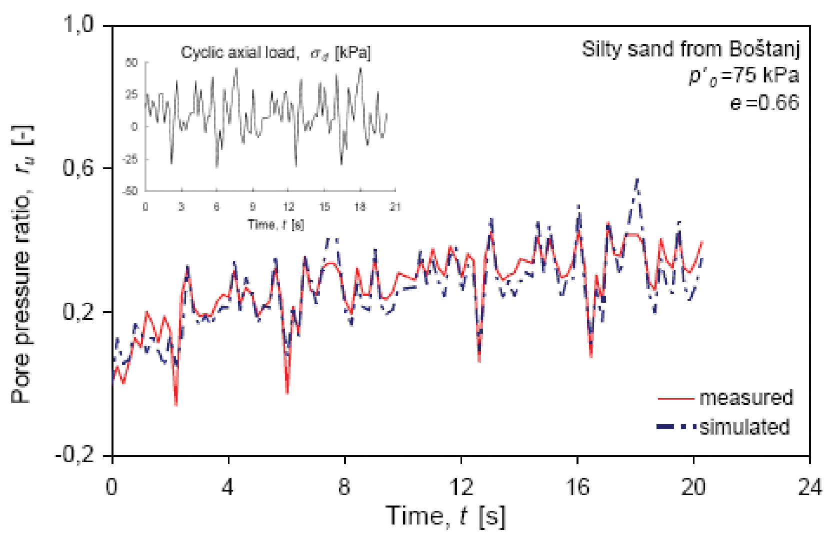

Figure 12. Results of an experiment compared to the modeling of an excess pore-water pressure generation.

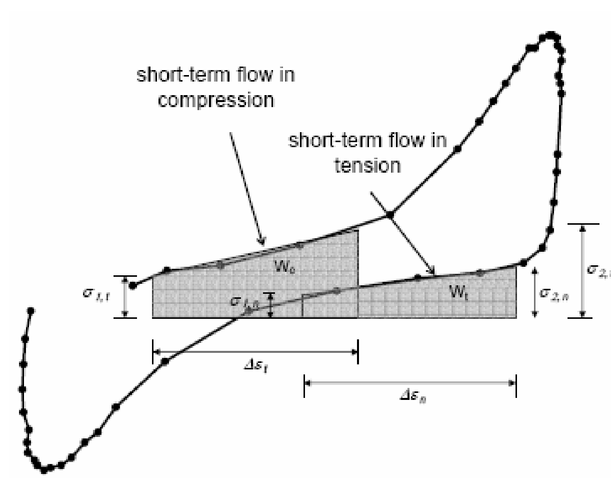

Figure 13. Evaluation of the energy dissipated during soil softening.

5 STRESS-STRAIN RELATION MODELING

If the generated excess pore-water pressure during the dynamic loading of saturated soil can be evaluated in the proper way, most of the work is done already. This makes it possible to evaluate the exact position of the stress path during the dynamic loading and thus to take into account the progressive degradation of the stiffness and the strength of the soil in an effective stress analysis.

The remaining problem is how to treat the phases of sudden soil softening and the significant regain in soil stiffness during cyclic loading at large deformations. Large deformations are limited by the soil hardening when the stress path crosses the PTS. A recent study [25] showed that this highly yielded segment can be limited by the amount of dissipated energy. Figure 13 presents an evaluation of the energy dissipated during the softening phase. It has been found through research [25] that the dissipated energy during the softening phase in a single load cycle is linearly related to the residual pore-pressure ratio. Using this finding one can define the residual pore-pressure ratio at which soil softening due to the cyclic mobility effect starts.

The pore-pressure model presented in the previous section and the relation between the pore pressure and the energy dissipated during short-term flows when the cyclic mobility occurs were used [28] for simple stress-strain relation modeling. The results in case of the tested lacustrine carbonate samples are presented below.

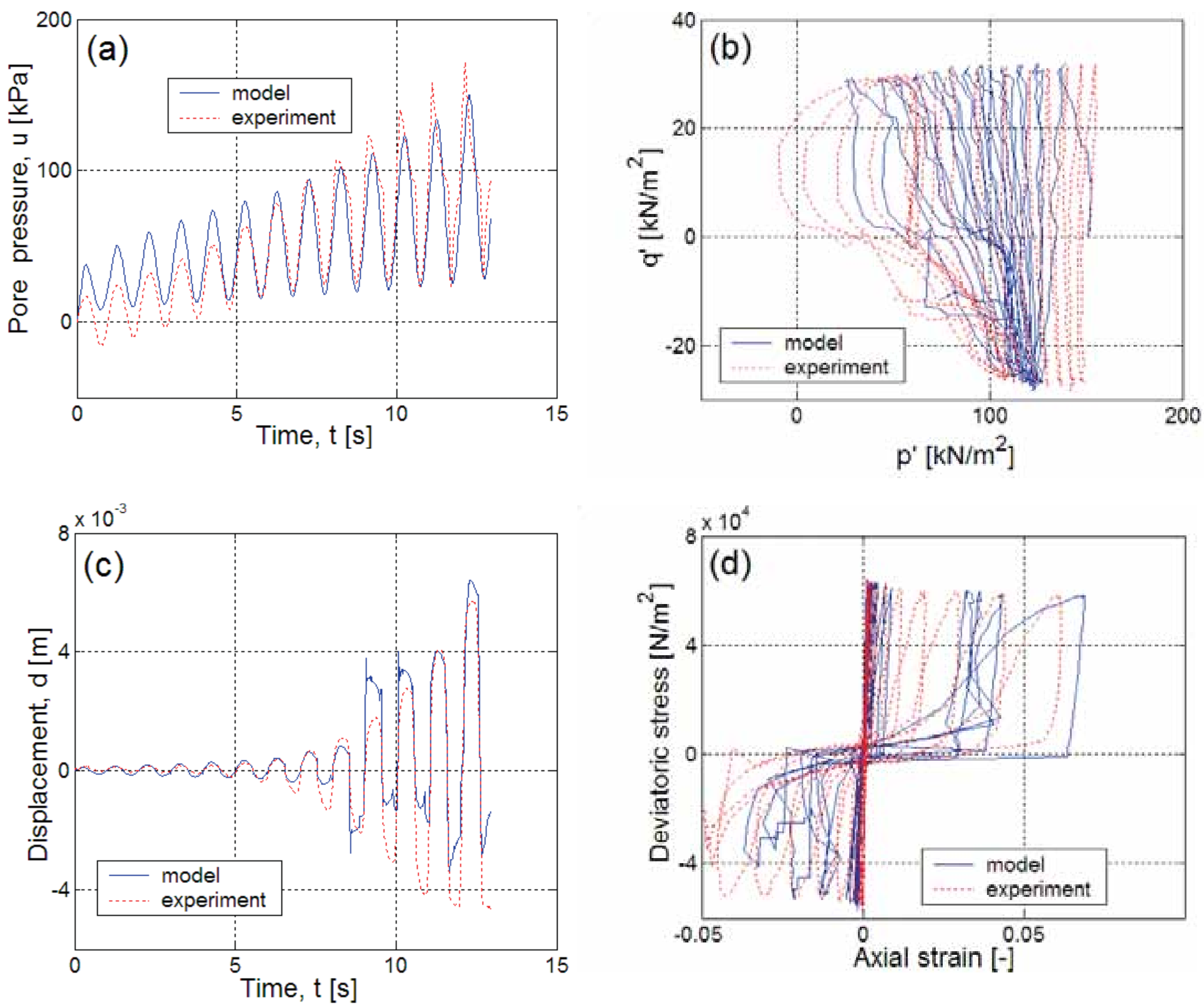

Figure 14. Typical simulation of the pore pressure (a), stress states (b), displacements (c) and stress-strain relation (d) for a cyclic triaxial test of a reconstituted sample of lacustrine carbonate silt.

6 CONCLUSION

Due to the large accompanying deformations and the possibility of severe damage, the response of dynamically loaded saturated soils has long attracted attention. Flow liquefaction and cyclic mobility are two phenomena that are often confused with each other. Their characteristics are described using two case histories from Slovenia: lacustrine carbonate silt from the Julian Alps and silty sand from the Sava river in Boštanj.

The effective stress decrease and the occurrence of large strain without any noticeable increase of the stress are common characteristics of flow liquefaction and cyclic mobility. An undrained loading condition, which is needed in both cases, is assumed to be present in saturated soil subjected to a rapid, dynamic load. Flow liquefaction appears suddenly when the residual strength of a soil is smaller than the static shear stress required for the equilibrium of a soil mass. On the other hand, cyclic mobility deformation develops incrementally during cyclic loading, mostly due to a decrease of the stiffness caused by a decrease of the effective stresses. It is important, therefore, to know the excess pore-water pressure generated during the cyclic loading of saturated soil, which impacts most upon the effective stresses in the soil.

An energy approach to saturated-soils response modeling during a dynamic load is presented in this paper. The energy concept is based on the idea that part of the energy of a dynamic load is dissipated into the soil. The density of the dissipated energy is represented by the area of the hysteretic strain-stress loop. The dissipated energy density is related to the generated excess pore-water pressure. The latter was divided into the temporary and residual generated excess pore-water pressure. Such a formulation helps to model very precisely the pore-water pressure oscillations during irregular dynamic loading.

The dissipated energy was evaluated during the soil-softening phase during cyclic mobility as well. Based on the observed linear relation between it and the residual excess pore-water pressure a promising attempt at modeling the response of dynamically loaded saturated soils was made.

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REFERENCES

[1] Official web page of Holding Slovenske elektrarne HSE, http://www.hse.si, 2006

[2] Lenart S, Petkovsek B (2006). Assessment of liquefaction potential of a loose soil in the base of a railroad, *Proceedings of the XIIIth Danube European Conference on Geotechnical Engineering*, Ljubljana, Vol. 2, 965-970

[3] Lenart S, Fifer Bizjak K, Petkovsek B (2005). Report on the dynamic analysis of the critical cross section of the railway line at Bostanj accumulation basin. ZAG, P 810/05-750-2 (in Slovene)

[4] Petkovšek, A. (2001). Geological-geotechnical Investigations of the Stože Landslide, *Ujma*, Ministrstvo za obrambo, Uprava RS za zaščito in reševanje, No. 14-15, 109-117

[5] Lenart S. (2006). Deformation characteristics of lacustrine carbonate silt in the Julian Alps. Soil *Dynamics and Earthquake Engineering*, 26, 131-142

[6] Lenart S., Žlender B. and Vilhar G. Liquefaction potential of lacustrine carbonate silt in the Julian Alps, *Soil Dynamics and Earthquake Engineering*, (to be published)

[7] Žlender B. and Lenart S. (2005). Cyclic liquefaction potential of lacustrine carbonate silt from Julian Alps, *Acta Geotechnica Slovenica*, 2005/1, 22-31

[8] Seed, H.B. and Idriss, I.M. (1971). Simplified procedure for evaluating soil liquefaction potential, *Jour. of the Soil Mechanics and Foundation Div.*, ASCE, Vol. 97, 1099-1119

[9] Andrus, R.D., Stokoe, K.H. (2000). Liquefaction Resistance of Soils from Shear-Wave Velocity, *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 126, No. 11, 1015-1025

[10] NCEER (1997). *Proceeding of the NCEER Workshop on Evaluation of Liquefaction Resistance of Soils*, Youd T.L. and Idriss I.M. (ed.), Technical Report NCEER-97-0022, National Center for Earthquake Engineering Research, State University of New York, Buffalo

[11] Kramer, S.L. (1996). *Geotechnical Earthquake Engineering*, Publ. Prentice Hall, 348-422

[12] Vaid, Y.P. and Chern, J.C. (1983) Effect of static shear on resistance of liquefaction, *Soils and Foundations*, Vol. 23, No. 1, 47-60

[13] Ishihara, K. (1985). Stability of natural deposits during earthquakes, *Proceedings, 11th International Conference on Soil Mechanics and Foundation Engineering*, San Francisco, Vol. 1, 321-376

[14] Youd, T.L. (1977). Discussion, *Soils and Foundations*, 17, No. 1, 82

[15] Elgamal, A., Yang, Z., Parra, E., and Ragheb, A. (2003). Modeling of Cyclic Mobility in Saturated Cohesionless Soils, *Int. J. Plasticity*, 19, 883-905

[16] Seed, H.B. and Lee, K.L. (1966). Liquefaction of saturated sands during cyclic loading, *J. Soil Mech. Found. Div.*, ASCE 92, SM6, 105-134

[17] Miner, M.A. (1945). Cumulative damage in fatigue, *Trans. ASME 67*, A159-A164

[18] Berrill, J.B. and Davis, R.O. (1985). Energy dissipation and seismic liquefaction of sands: Revised model, *Soils and Foundations*, Vol.25, No.2, 106-118

[19] Trifunac, M.D. (1995). Empirical criteria for liquefaction in sands via standard penetration tests and seismic wave energy, *Soil Dynamics and Earthquake Engineering*, Vol. 14, 419-426

[20] Nemat-Nasser, S. and Shokooh, A. (1979). A unified approach to densification and liquefaction of cohesionless sand in cyclic shearing, *Can. Geotech. J.*, 16(4), 659-678

[21] Law, K.T., Cao, Y.L. and He, G.N. (1990). An energy approach for assessing seismic liquefaction potential, *Can. Geotech. J.*, 27, 320-329

[22] Liang., L. (1995). *Development of an Energy Method for Evaluating the Liquefaction Potential of a Soil Deposit*, PhD Dissertation, Case Western Reserve University

[23] Figueroa, J.L., Saada, A.S., Liang, L., and Dahisaria, N.M. (1994). Evaluation of Soil Liquefaction by Energy Principles, *Journal of Geotechnical Engineering*, ASCE, Vol. 120, No. 9, 1554-1569

[24] Davis, R.O. and Berrill, J.B. (2001). Pore Pressure and Dissipated Energy in Earthquakes – Field Verification, *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 127, No. 3, 269-274

[25] Lenart, S. (2006). *Numerical model for pore pressure change evaluation in soils with high liquefaction potential*, PhD Thesis, University of Ljubljana

[26] Lenart, S. (2008). The use of dissipated energy at modeling of cyclic loaded saturated soils, *Earthquake Engineering: New Research*, Nova Science Publishers, Chapter 8

[27] Žlender, B. and Trauner, L. (2007). The dynamic properties of the snail soil from the Ljubljana marsh, *Acta Geotechnica Slovenica, 2007/2*, Vol.4, 48-61

[28] Lenart, S. and Logar, J. (2006). Numerical modelling of vertically loaded saturated soil, *Proc. of XIII. Danube-European Conference on Geotechnical Engineering*, 83-88