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One can define the long-term forest exploitation process in different ways. Methods of operations research can be used successfully for this purpose. A group of experts at the University of Ljubljana, Yugoslavia developed a specific tree-growing function and, based on that, prescribed a set of procedures, relations and states which could represent mathematical model for optimizing the forest exploitation process. The method was defined as a discrete dynamic programming process, based upon Bellman's principle. In this paper we briefly describe the model and computer program prototype for solving it. They both can easily be extended by introducing more complexity into them. The program was written in FORTRAN and tested on DEC-10 computer.

1. INTRODUCTION

In the paper we describe a possible usage of dynamic programming, based upon Bellman's principle, in order to obtain the optimum solution of a predefined long-term forest exploitation process. In this process measures and activities are introduced at each step within the iterative solution process and their effects on the intermediate results and the final solution are stated in the form of mathematical relations. The whole forest area which is taken into consideration is divided into a number of homogeneous parts - segments. For each of these segments, five special functions are imposed by means of which we direct (guide) the process within a number of time intervals, either years or decades. The functions are: (F1) the maximum possible tree growth capacity, (F2) exploitation capacity, (F3) the quality of the existing tree specimens, (F4) level of administration - care for improved growing condition, and (F5) stage in segment development, based upon the age (oldness) of the tree specimens in it. The functions help to optimize exploitation endeavours. At each step of the iterative solution process four possible activities can be imposed: (A1) no activity at all, (A2) exchange of the existing tree specimen with a new one, (A3) rarefying, and (A4) restoration. The exploitation policy is defined by a sequence of chosen activities during the iterative process. The stated functions, activities and time intervals are part of the mathematical model and dynamic programming process respectively. The problem and the corresponding computer program can be extended by incorporating some additional functions and activities. Program prototype in FORTRAN was written and tested on DEC-10 computer at the University of Ljubljana, Yugoslavia. In the paper we also comment the problems which are associated with dynamic programming applied to such type of problem.

2. A PREDEFINED LONG-TERM FOREST EXPLOITATION PROCESS

We mean by that a stated sequence of policies and activities that should be pursued in forest exploitation within a life-time long period of some tree specimen in order to achieve optimum results in accordance with predefined goals and criterions. The possible and necessary actions and the most suitable time for these actions to be carried out depend on the values of some chosen and defined functions which describe sufficiently the tree-growing process. It is these functions, which must be predefined by a group of experts in forest exploitation process - both practitioners and theoreticians, by means of which more or less complexity and reality is introduced in the basic model that is to be solved. After several years of analytic and experimental studies of a group of experts in Ljubljana, Yugoslavia (VADNAL, KOTAR, ZADNIK, STIRN, GAŠPERŠIČ /2/, VADNAL, ZADNIK, STIRN /3/), the following suggestions and basic ideas have been made:

(i) The paradigm is bounded both geographically and in time. The whole area considered is divided into smaller regions which are further partitioned into smaller parts with specific environmental conditions and characteristics. Some parts within different regions can have similar or nearly equal environmental conditions and characteristics. They can or can not be exploited (treated) independently from each other. The area is exploited for a specific number of time periods, measured in years or decades.

(ii) In order to evaluate (quantify) observed characteristics of each part of the forest under consideration, time dependant analytical growth functions have been developed:

1) Function of growth

$$Y(t) = a(1 - (1+T)e^{-pT})^n,$$

where $T = (2n - 1) t^n / (np^n)$, for $a > 0$, $p > 0$, and $n > 1$. a defines the asymptotic value of $Y(t)$, $p = t_2$ is the value of t

from where on $Y(t)$ ' starts to decline, and n determines the speed of convergence $Y(t)$ towards value a . The value of $t = t_3$, when the forest restoration procedures start can be defined by solving the equation

$$1 + T + T^2 - e \times p(T) = 0$$

which is obtained from the relation

$t_3 Y'(t_3) = Y(t_3)$, or can be stated on the base of experiences.

- 2) Function of increase (by growth) $y(t) = Y(t)'$ where $y(t_2)' = y(p)' = 0$.
- 3) Function of average growth $f(t) = Y(t)/t$, where $f(t_3) = y(t_3)$, and therefore $t_3 Y(t_3) = Y(t_3)$.
- 4) Function of uniform growth $s(t) = mt$, where $s(t_3) = Y(t_3)$, and $s(t_3)' = Y(t_3)'$.

(iii) Five functions have been introduced which describe five possible states of forest exploitation process:

- 1) The maximum possible tree growth capacity - F1. F1 is constant during the exploitation process.
- 2) Exploitation capacity - F2. F2 is a continuous function, where $1 \leq F2 \leq 10$. Its value shows the degree in forest exploitation.
- 3) The quality of the existing tree specimens - F3. This function enables us to make an assessment about the quality of the trees and whether to begin the exploitation process of some particular part of the forest or not. The parts are divided into five qualitative groups, $1 \leq F3 \leq 5$.
- 4) Level of administration - care for better growing conditions - F4. The value of F4, where $1 \leq F4 \leq 5$, shows a degree of obstructing influence of unwanted tree species on growth of the wanted ones.
- 5) The segment development stage - F5, based upon the age (oldness) of the tree specimens in it. Tree species are divided into four stages, accordingly:
 $0 \leq F5 < t_1$, $t_1 \leq F5 < t_2$, $t_2 \leq F5 < t_3$, and $t_3 \leq F5 \leq t_4^*$
 Tree species of different stages differ both in size and quality and can therefore be used for different purposes.

(iv) The forest exploitation process is carried out by performing a sequence of prescribed activities. The sequence order of these activities helps to optimize the process. The following four activities have been taken into consideration:

- 1) No activity at all - A1.

In this case the forest develops in accordance with the laws of nature. The value of F2 changes only within the first two stages of segment development; it is diminished for an empirically defined quantity $pr(\text{stage}, F2, A1)$. Similarly, the values of F3 and F4 change within the first three stages of development. They too are diminished for some empirically defined quantities $pr(\text{stage}, F3,$

A1) and $pr(\text{stage}, F4, A1)$, respectively. Each time the value of the increment (1 for a year or 10 for a decade) is added to the previous value of F5. At the boundary points this causes a change of the stage of development.

- 2) Exchange of the existing tree specimen with a new one - A2. We can pursue this activity only within the first two segment development stages. The reason for doing that is the inadequate existing tree species. The exchange is carried out there - in those parts (segments) where the effects are most visible. A2 exercises the following influence upon F2, ..., F5: Their values are changed only within the first two stages of segment development. F2, F3 and F4 are increased by some empirically defined quantities $pr(\text{stage}, F2, A2)$, $pr(\text{stage}, F3, A2)$ and $pr(\text{stage}, F4, A2)$, respectively, while the value of F5 is reduced to zero and the process starts from the beginning.

- 3) Reforestation - A3.

The activity can only take place within the first three segment development stages. Within the first stage it is exercised by cutting down the unwanted species only what helps in quicker growth of the wanted ones. Here the activity imposes additional expenses. Within the second and third stage, in addition to the cutting down of the unwanted species, we also cut down some trees of the wanted species what helps in quicker growth of the most qualitative samples. Here the activity brings some profit. Due to A3 the values of F2, F3 and F4 are changed by some empirically obtained quantities $pr(\text{stage}, F2, A3)$, $pr(\text{stage}, F3, A3)$, and $pr(\text{stage}, F4, A3)$, respectively. A3 has no particular impact on F5. The time increment (1 or 10) is added to the value of F5 after the time period expires.

- 4) Restoration - A4.

Restoration starts at $t = t_3$, ends at $t = t_4$, and can take different length of time. It is characterized by wood-cutting on higher scale. After restoration is done, the forest segment under consideration passes over into the first stage. The way in which the stage of restoration is being accomplished has a great influence on the results of the forest exploitation process. The span of time $t_4 - t_3$ is generally divided into more steps. At each step the increments of F2 and F3 are computed and added to the previous values of F2 and F3, respectively. F4 does not change within A4, while the increment 1 (10) is added to F5 after each year (decade) that is passing by. F5 reduces to zero or some higher value after the restoration is over.

- (v) The outcomes - $R(\text{stage}, Ak)$. Each measure - activity undertaken at any step of the exploitation process, results and can be expressed in a form of costs and income. The resulting income of some particular step, say $I(\text{stage}, Ak)$, for $k = 1, 2, 3, 4$, depends upon F_i , for $i = 1, 2, \dots, 5$, and the size of the undertaken activity A_k . The entire costs can be divided into fixed costs $FIX(\text{stage})$ and variable costs $V(\text{stage}, Ak)$. Accordingly,

$$R(\text{stage}, Ak) = I(\text{stage}, Ak) - FIX(\text{stage}) - V(\text{stage}, Ak)$$

- 1) $R(\text{stage}, A1)$.

In this case we have no income and no variable costs. Therefore $R(\text{stage}, A1) = -FIX(\text{stage})$ where $FIX(\text{stage})$ is some empirically obtained

* where t_4 indicates the upper time limit of the stated tree growth cycle. t_1 is defined as $t_1 = a/10$.

quantity.

2) R(stage, A2).

Activity A2 is carried out only within the first two stages of the exploitation process. We have some possible income only within the second stage, which can be expressed as $q_i(\text{stage}, A2)Y(t)$ where $q_i(2, A2)$ is some empirically stated weight (factor), and $q_i(1, A2) = 0$. Fixed costs $\text{FIX}(\text{stage})$ are defined (empirically). Variable costs are obtained as a total of the costs of removal the unwanted species, say $q_c(\text{stage}, A2)Y(t)$, and the costs of planting new samples, say $\text{NT}(\text{stage})$, where q_c and NT are defined empirically. Accordingly,

$$R(\text{stage}, A2) = q_i(\text{stage}, A2)Y(t) - \text{FIX}(\text{stage}) - q_c(\text{stage}, A2)Y(t) - \text{NT}(\text{stage}) \text{ for stage} = 1, 2.$$

3) R(stage, A3).

Activity A3 is carried out within the first three stages. The outcome $R(\text{stage}, A3)$ can be expressed as

$$R(\text{stage}, A3) = q_i(\text{stage}, A3)Y(t) - \text{FIX}(\text{stage}) - q_c(\text{stage}, A3)Y(t) \text{ for stage} = 1, 2, 3, \text{ where } q_i, \text{FIX, and } q_c \text{ are obtained empirically and } q_i(1, A3) = 0. q_i \text{ and } q_c \text{ have a similar meaning as in case of } R(\text{stage}, A2).$$

4) R(4, A4).

Activity A4 is carried out only within the fourth stage. We differentiate two possibilities:

- I. $t_3 = t_4$, when all work is done in a very short time. In this case we deal with one outcome only, defined as

$$R(4, A4) = q_i(4, A4)Y(t_3) - \text{FIX}(4) - q_c(4, A4)Y(t_3) - \text{NT}(4)$$

- II. $t_3 \neq t_4$, and assume that there are d_4 steps of the exploitation process within the fourth step. At each step n_4 , where $n_4 = 1, 2, \dots, d_4$, we compute $R(4, A4, n_4)$ as

$$R(4, A4, n_4) = \frac{1}{d_4} q_i(4, A4, n_4)Y(t) - \text{FIX}(4) - \frac{1}{d_4} q_c(4, A4, n_4)Y(t) - \text{NT}(4, n_4)$$

where again q_i , q_c , and NT are defined empirically.

- (vi) Managing the exploitation process is possible before and after the process being in progress. By this we mean some further prescribed conditions and rules which should be or should not be taken into account within the problem solving process, in accordance with the type of optimization process that we pursue. We distinguish among the following possible alternatives:

- No alternations of the prescribed exploitation process are possible while the process being in progress.
- Some alternations of the originally prescribed exploitation process are possible while the process being in progress. We may interrupt the process, insert some new input data and proceed the process from this point on or start the program from the beginning.
- The final result of the process is prescribed in advance as well as the starting conditions.
- The final result is the optimum value that can be obtained by the prescribed starting conditions.

3. DYNAMIC PROGRAMMING PROCESS ALGORITHM PROTOTYPE

3.1. General description of the discrete dynamic programming

Discrete dynamic programming process is an iterative process (BELLMAN, DREYFUS /1/). At each step i , for $i = 0, 1, \dots, N$, we define a certain number of possible points x_{ij} , for $j = 1, 2, \dots, M_i$. For each point we compute the function value f_{ij} which is involved in the process of optimization. This value is the optimum value among function values $f_{i-1, k}$, for $k = 1, 2, \dots, M_{i-1}$, incremented by the computed outcomes between $x_{i-1, k}$ and this particular point x_{ij} . After computing f_{ij} for all $i = 0, 1, \dots, N$ and $j = 1, 2, \dots, M_i$ we define the optimum sequence of the operations and decisions that were made during the exploitation process for all steps $i = N, N-1, \dots, 0$. Sometimes two or more alternatives are possible which all give the same optimum solution.

3.2. Forest exploitation dynamic programming algorithm

In order to start the process we need the following input data:

- $a, p, n, t_1, t_2, t_3, t_4$; in the prototype we don't compute the values of t_1, t_2 and t_3 but we read them as input data instead.

For each segment, i.e. for each case of the exploitation process:

- all empirically defined values of $\text{FIX}(\text{stage})$, $\text{NT}(\text{stage})$, $q_i(\text{stage}, A_k)$, and $q_c(\text{stage}, A_k)$, for $\text{stage} = 1, 2, 3, 4$ and $k = 1, 2, 3, 4$.
- all empirically defined values of $\text{pr}(\text{stage}, F_i, A_k)$, for $\text{stage} = 1, \dots, 4$, $i = 2, 3, 4$ and $k = 1, \dots, 4$, that represent the increment of F_i due to the activity A_k .
- F_2, F_3, F_4, F_5 which define the starting conditions of the forest exploitation process. The whole experiment can be repeated several times for different starting values of F_2, \dots, F_5 .

The process is as follows:

I.

For each t , where $t = F_5 + 10, F_5 + 20, \dots$, in accordance with the stage to which t belongs ($\text{stage} = 1, 2, 3$ or 4), the possible activities (A_1, A_2, A_3 and/or A_4) at that stage take place for all existing (active) points $x_{t-10, s}$ (see 3.1.), where $s = 1, 2, 3, \dots$

For each activity that takes place at some $x_{t-10, s}$ the following happens:

- new values of F_2, F_3, F_4 and F_5 are computed, defining a new point $x_{t, j}$ and a new step.

There is: new value of $F_i = \text{old value of } F_i \mp \text{pr}(\text{stage}, F_i, A_k)$, for $i = 2, 3, 4$, where "-" sign appears for A_1 , and "+" sign appears for A_2, A_3 or A_4 , respectively. The value of F_5 is increased by 10 or reduced to zero (for A_2 or when $F_5 > t_4$).

- the outcomes $R(\text{stage}, A_k)$, see chapter 2.(v), are computed and added to $f_{t-10, s}$ (see 3.1) in order to obtain $f_{t, j}$.

Some of the 250 possible different points $x_{t, j}$, for $j = 1, 2, \dots, 250$, are encountered at each step t of the iterative process. They are defined by different values of F_2, F_3 and F_4 ($10 \times 5 \times 5 = 250$). The computer program builds two arrays $X(250, 6)$ and $FX(250)$ at each step of the iterative process in order to save all the the necessary intermediate data. The elements

in row j of array X contain the following data about the point $x_{t,j}$:

$x_{j1} = F2, x_{j2} = F3, x_{j3} = F4, x_{j4} = k, x_{j5} = s, x_{j6} = F5$

where

k ($= 1, 2, 3$ or 4) defines the activity A_k taken at step $t-10$ that caused a transition to $x_{t,j}$, and

s defines the point $x_{t-10,s}$ from which the transition to $x_{t,j}$ was made.

s stands for the row number of array X at previous step ($t-10$) in which the data about $x_{t-10,s}$ are stored.

The elements FX_j , for $j = 1, 2, \dots, 250$, represent the values of the computed f_{tj} .

II.

After completing the first part of the algorithm we locate the optimum value $f_{T,j}$ in the last step T of the iterative process, where

$f_{T,j} = \text{optimum}_s (f_{T,s}, \text{ for } s = 1, 2, \dots, 250)$

Afterwards we trace back to the beginning all actions that were carried out during the exploitation process. We do this by means of data stored in arrays X and FX .

If the number of steps in the iterative process is fixed and defined, let say by $t_4/10$, then $T = t_4$ in case when the starting value of $F5 = 0$, and $T < t_4$ otherwise.

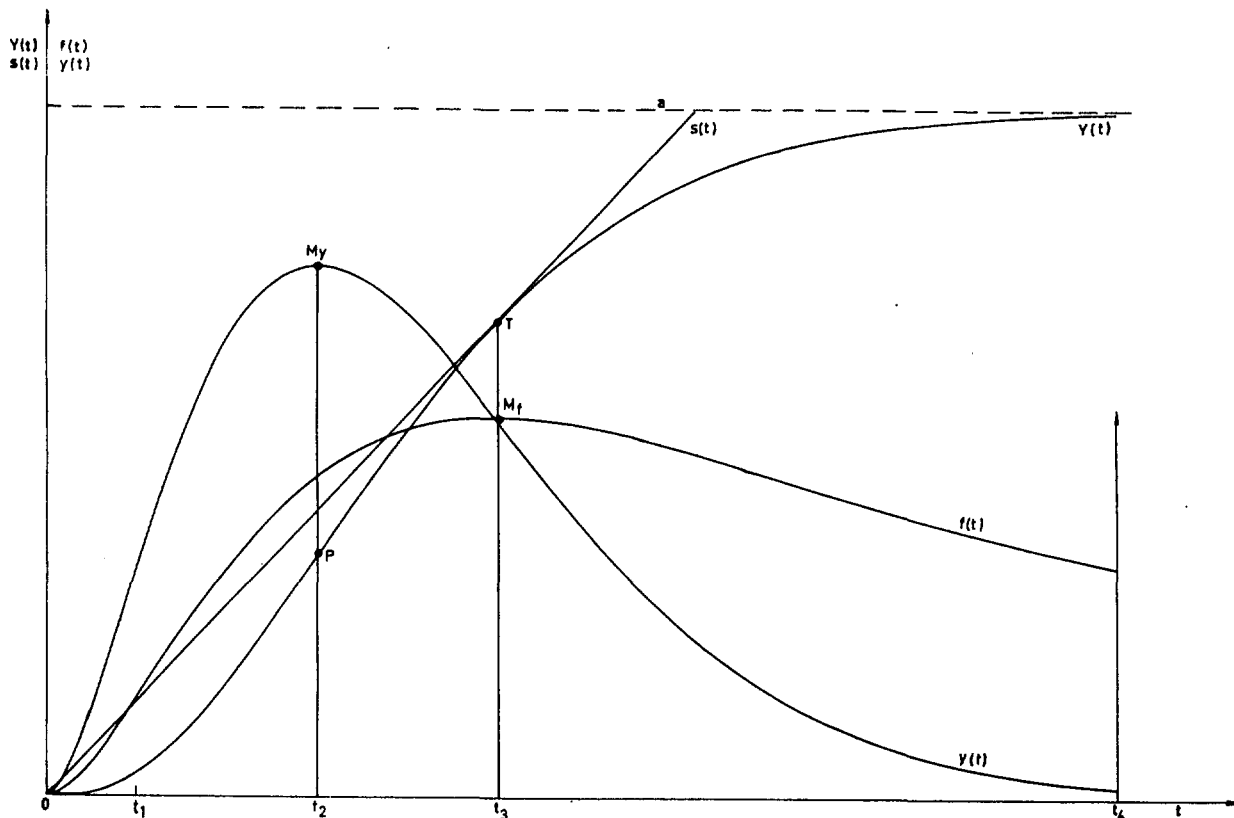
4. CONCLUSIONS ON THE APPROACH

Obtained experiences show that a method for optimising the forest exploitation process, based upon discrete dynamic programming is reasonable and adequate. More complexity and necessary modifications can easily be introduced after obtaining and analyzing some experimental results. Large number of input data and intermediate results which are storage demanding may be regarded as the only inconvenience. Different strategies may reduce this problem by applying the secondary disk storage at each step of the iterative process. The empirically obtained input data can also be stored in files in advance and kept there for as long as necessary.

REFERENCES

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Fig.1: Analytical growth functions[2,3]



RAČUNALNIŠKI PROGRAM ZA DOLOČITEV OPTIMALNE REŠITVE V DOLGOROČNEM IZKORIŠČANJU GOZDOV. Proces dolgoročnega izkoriščanja gozdov moremo opredeliti na več načinov. Primerne v ta namen so tudi metode operacijskega raziskovanja. Skupina strokovnjakov Univerze Edvarda Kardelja v Ljubljani je za določitev matematičnega modela optimizacije postopka izkoriščanja gozdov razvila posebno rastno funkcijo in na temelju le-te definirala zaporedje potrebnih postopkov, relacij in postulatov. Metoda je bila definirana kot postopek, ki temelji na diskretnem dinamičnem programiranju. V članku zgoščeno opišemo prototipa modela in računalniškega programa za njegovo rešitev. Model je možno razširiti z vgraditvijo nadaljnjih zahtev in pogojev. Program je napisan v programskem jeziku FORTRAN in je bil testiran na računalniku DEC-10.