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Unified Model for Light- and Heavy-Flavor Baryon Resonances ⋆

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Abstract. We report on the construction of a relativistic constituent-quark model capable of describing the spectroscopy of baryons with all flavors u, d, s, c, and b. Some selective spectra are presented, where comparisons to experimental data are yet possible.

1 Introduction

Over the decades the constituent-quark model (CQM) has ripened into a stage where its formulation and solution are well based on a relativistic (i.e. Poincaré-invariant) quantum theory (for a thorough review of relativistic Hamiltonian dynamics see ref. [1]). In such an approach one relies on an invariant mass operator $\hat{M}$, where the interactions are introduced according the so-called Bakamjian-Thomas construction [2]. If the conditions of the Poincaré algebra are fulfilled by $\hat{M}$, this leads to relativistically invariant mass spectra.

Relativistic constituent-quark models (RCQM) have been developed by several groups, however, with limited domains of validity. Of course, it is desirable to have a framework as universal as possible for the description of all kinds of hadron processes in the low- and intermediate-energy regions. This is especially true in view of the advent of ever more data on heavy-baryon spectroscopy from present and future experimental facilities.

We have developed a RCQM that comprises all known baryons with flavors u, d, s, c, and b within a single framework [3]. There have been only a few efforts so far to extend a CQM from light- to heavy-flavor baryons. We may mention, for example, the approach by the Bonn group who have developed a RCQM, based on the 't Hooft instanton interaction, along a microscopic theory solving the Salpeter equation [4] and extended their model to charmed baryons [5], still not yet covering bottom baryons. A further quark-model attempt has been undertaken by the Mons-Liège group relying on the large-N_c expansion [6,7], partially extended to heavy-flavor baryons [8]. Similarly, efforts are invested to expand other approaches to heavy baryons, such as the employment of Dyson-Schwinger equations together with either quark-diquark or three-quark calculations [9, 10].

⋆ Talk delivered by J. P. Day

Also an increased amount of more refined lattice-QCD results has by now become available on heavy-baryon spectra (see, e.g., the recent work by Liu et al. [11] and references cited therein).

2 The Model

Our RCQM is based on the invariant mass operator

$$\hat{M} = \hat{M}_{\text{free}} + \hat{M}_{\text{int}} \ , \tag{1}$$

where the free part corresponds to the total kinetic energy of the three-quark system and the interaction part contains the dynamics of the constituent quarks Q. In the rest frame of the baryon, where its three-momentum $\mathbf{P} = \sum_i^3 \mathbf{k}_i^2 = 0$, we may express the terms as

$$\hat{M}_{\text{free}} = \sum_{i=1}^{3} \sqrt{\hat{m}_i^2 + \hat{\mathbf{k}}_i^2} \ , \tag{2}$$

$$\hat{M}_{\text{int}} = \sum_{i<j}^{3} \hat{V}_{ij} = \sum_{i<j}^{3} (\hat{V}_{ij}^{\text{conf}} + \hat{V}_{ij}^{\text{hf}}) \ . \tag{3}$$

Here, the $\hat{\mathbf{k}}_i$ correspond to the three-momentum operators of the individual quarks with rest masses m_i and the Q-Q potentials $\hat{V}_{ij}$ are composed of confinement and hyperfine interactions. By employing such a mass operator $\hat{M}^2 = \hat{P}^\mu \hat{P}_\mu$, with baryon four-momentum $\hat{P}_\mu = (\hat{H}, \hat{P}_1, \hat{P}_2, \hat{P}_3)$, the Poincaré algebra involving all ten generators $\{\hat{H}, \hat{P}_i, \hat{J}_i, \hat{K}_i\}$, $(i = 1, 2, 3)$, or equivalently $\{\hat{P}_\mu, \hat{J}_{\mu\nu}\}$, $(\mu, \nu = 0, 1, 2, 3)$, of time and space translations, spatial rotations as well as Lorentz boosts, can be guaranteed. The solution of the eigenvalue problem of the mass operator $\hat{M}$ yields the relativistically invariant mass spectra as well as the baryon eigenstates (the latter, of course, initially in the standard rest frame).

We adopt the confinement depending linearly on the Q-Q distance r_{ij}

$$V_{ij}^{\text{conf}}(\mathbf{r}_{ij}) = V_0 + C r_{ij} \tag{4}$$

with the strength $C = 2.33$ fm^{-2}, corresponding to the string tension of QCD. The parameter $V_0 = -402$ MeV is only necessary to set the ground state of the whole baryon spectrum, i.e., the proton mass; it is irrelevant for level spacings.

The hyperfine interaction is most essential to describe all of the baryon excitation spectra. In a unified model the hyperfine potential must be explicitly flavor-dependent. Otherwise, e.g., the N and Λ spectra with their distinct level orderings could not be reproduced simultaneously. Therefore we have advocated for the hyperfine interaction of our universal RCQM the $SU(5)_F$ GBE potential

$$V_{\text{hf}}(\mathbf{r}_{ij}) = \left[V_{24}(\mathbf{r}_{ij}) \sum_{a=1}^{24} \lambda_i^a \lambda_j^a + V_0(\mathbf{r}_{ij}) \lambda_i^0 \lambda_j^0 \right] \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \ . \tag{5}$$

Here, we take into account only its spin-spin component, which produces the most important hyperfine forces for the baryon spectra; the other possible force components together play only a minor role for the excitation energies [16]. While $\boldsymbol{\sigma}_i$ represent the Pauli spin matrices of $SU(2)_S$, the λ_i^a are the generalized Gell-Mann flavor matrices of $SU(5)_F$ for quark i. In addition to the exchange of the pseudoscalar 24-plet also the flavor-singlet is included because of the $U(1)$ anomaly. The radial form of the GBE potential resembles the one of the pseudoscalar meson exchange

$$V_\beta(\mathbf{r}_{ij}) = \frac{g_\beta^2}{4\pi}\frac{1}{12m_i m_j}\left[\mu_\beta^2 \frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - 4\pi\delta(\mathbf{r}_{ij})\right] \tag{6}$$

for $\beta = 24$ and $\beta = 0$. Herein the δ-function must be smeared out leading to [13, 14]

$$V_\beta(\mathbf{r}_{ij}) = \frac{g_\beta^2}{4\pi}\frac{1}{12m_i m_j}\left[\mu_\beta^2 \frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - \Lambda_\beta^2 \frac{e^{-\Lambda_\beta r_{ij}}}{r_{ij}}\right]. \tag{7}$$

Contrary to the earlier GBE RCQM [13], which uses several different exchange masses μ_γ and different cut-offs Λ_γ, corresponding to $\gamma = \pi, K$, and η=η_8 mesons, we here managed to get along with a universal GBE mass μ_{24} and a single cut-off Λ_{24} for the 24-plet of $SU(5)_F$. Only the singlet exchange comes with another mass μ_0 and another cut-off Λ_0 with a separate coupling constant g_0. Consequently the number of open parameters in the hyperfine interaction could be kept as low as only three (see Tab. 1).

Table 1. Free parameters of the present GBE RCQM determined by a best fit to the baryon spectra.

Free Parameters		
$(g_0/g_{24})^2$	Λ_{24} [fm^{-1}]	Λ_0 [fm^{-1}]
1.5	3.55	7.52

Table 2. Fixed parameters of the present GBE RCQM predetermined from phenomenology and not varied in the fitting procedure.

Fixed Parameters						
Quark masses [MeV]				Exchange masses [MeV]		Coupling
$m_u = m_d$	m_s	m_c	m_b	μ_{24}	μ_0	$g_{24}^2/4\pi$
340	480	1675	5055	139	958	0.7

All other parameters entering the model have judiciously been predetermined by existing phenomenological insights. In this way the constituent quark masses have been set to the values as given in Tab. 2. The 24-plet Goldstone-boson (GB) mass has been assumed as the value of the π mass and similarly the singlet mass as the one of the η'. The universal coupling constant of the 24-plet has been chosen according to the value derived from the π-N coupling constant via the Goldberger-Treiman relation.

3 Results for Baryon Spectra

We have calculated the baryon spectra of the relativistically invariant mass operator $\hat{M}$ to a high accuracy both by the stochastic variational method [17] as well as the modified Faddeev integral equations [18, 19]. The present universal GBE RCQM produces the spectra in the light and strange sectors with similar or even better quality than the previous GBE RCQM [13]. In Figs. 1 and 2 we show the ground states and the first two excitations of $SU(3)_F$ singlet, octet, and decuplet baryons up to $J = \frac{7}{2}$, for which experimental data of at least three stars are quoted by the PDG [15] and J^P is known. Evidently a good overall description is achieved. Most importantly, the right level orderings specifically in the N, Δ, and Λ spectra as well as all other $SU(3)_F$ ground and excited states are reproduced in accordance with phenomenology. The reasons are exactly the same as for the previous GBE RCQM, what has already been extensively discussed in the literature [12–14]. Unfortunately, the case of the $\Lambda(1405)$ excitation could still not be resolved. It remains as an intriguing problem that can possibly not be solved by RCQMs relying on $\{QQQ\}$ configurations only; an explicit coupling to the K-N decay channel whose threshold lies nearby might be needed.

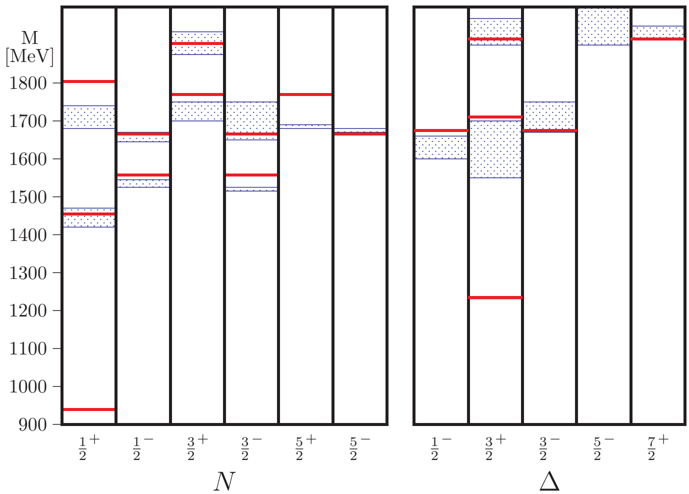

Fig. 1. Nucleon and Δ excitation spectra (solid/red levels) as produced by the universal GBE RCQM in comparison to phenomenological data [15] (the gray/blue lines and shadowed/blue boxes show the masses and their uncertainties).

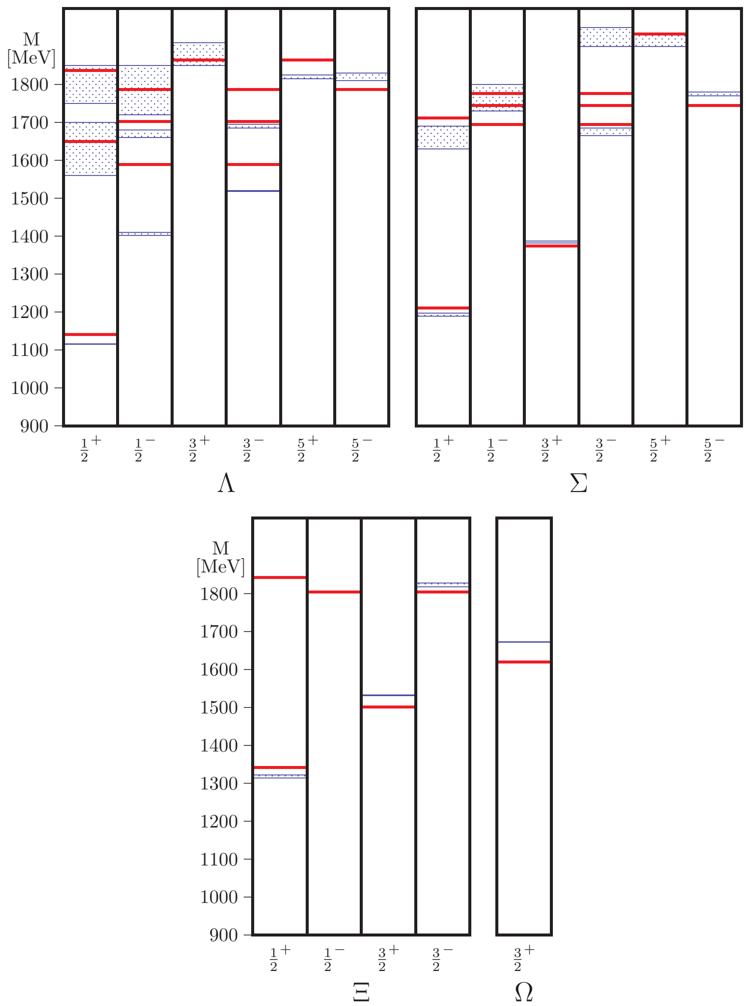

Fig. 2. Same as Fig. 1 but for the strange baryons.

What is most interesting in the context of the present work are the very properties of the light-heavy and heavy-heavy Q-Q hyperfine interactions. Can the GBE dynamics reasonably account for them? In Figs. 3 and 4 we show the spectra of all charm and bottom baryons that experimental data with at least three- or four-star status by the PDG exist for [15]. As is clearly seen, our universal GBE RCQM can reproduce all levels with respectable accuracy. In the Λ_c and Σ_c spectra some experimental levels are not known with regard to their spin and parity J^P. They are shown in the right-most columns of Fig. 3. Obviously they could easily be accommodated in accordance with the theoretical spectra, once their J^P's are determined. Furthermore the model predicts some additional excited states for charm and bottom baryons that are presently missing in the phenomenological data base.

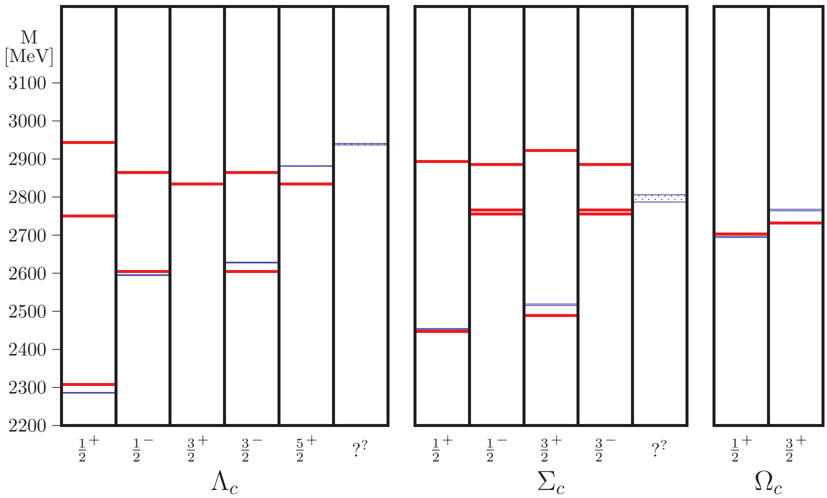

Fig. 3. Same as Fig. 1 but for charm baryons.

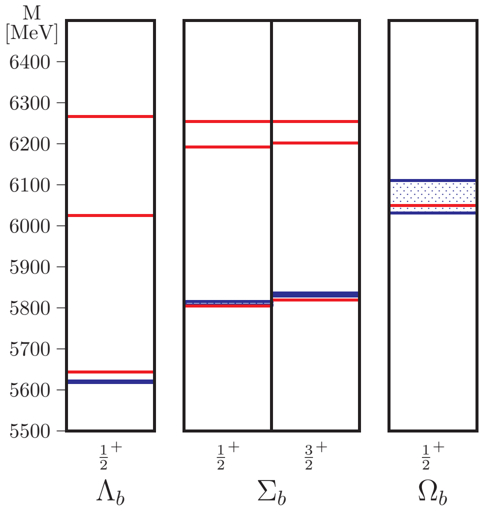

Fig. 4. Same as Fig. 1 but for bottom baryons.

Of course, the presently available data base on charm and bottom baryon states is not yet very rich and thus not particularly selective for tests of effective Q-Q hyperfine forces. The situation will certainly improve with the advent of further data from ongoing and planned experiments. Beyond the comparison to experimental data, we note that the theoretical spectra produced by our present GBE RCQM are also in good agreement with existing lattice-QCD results for heavy-flavor baryons. This is especially true for the charm baryons vis-à-vis the recent work by Liu et al. [11].

4 Discussion and Conclusion

We emphasize that the most important ingredients into the present RCQM are relativity, specifically Poincaré invariance, and a hyperfine interaction that is derived from an interaction Lagrangian built from effective fermion (constituent quark) and boson (Goldstone boson) fields connected by a pseudoscalar coupling [12]. It appears that such kind of dynamics is quite appropriate for constituent quarks of any flavor.

As a result we have demonstrated by the proposed GBE RCQM that a universal description of all known baryons is possible in a single model. Here, we have considered only the baryon masses (eigenvalues of the invariant mass operator $\hat{M}$). Beyond spectroscopy the present model will be subject to further tests with regard to the baryon eigenstates, which are simultaneously obtained from the solution of the eigenvalue problem of $\hat{M}$. They must prove reasonable in order to make the model a useful tool for the treatment of all kinds of baryons reactions within a universal framework.

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