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Boundary Layer of Dissociated Gas on Bodies of Revolution of a Porous Contour

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This paper studies the ideally dissociated gas (air) flow along a porous wall of the body of revolution within the fluid in the conditions of equilibrium dissociation. Using similarity transformations, the governing boundary layer equations are brought to a generalized form. The obtained equations are numerically solved in three-parametric twice localized approximation by finite differences method. Based on the obtained solutions, diagrams of distribution of physical quantities and characteristics of the boundary layer are drawn. General and some specific conclusions about behaviour of these quantities are also made for the studied compressible fluid flow.

Keywords: dissociated gas, boundary layer, porous countour, porosity parameter, fluid mechanics

0 INTRODUCTION

This paper investigates the dissociated gas (air) flow in the boundary layer on bodies of revolution. The contour of the body within the fluid is porous. The dissociated gas flows in the conditions of the so-called equilibrium dissociation [1] and [2].

The goal of our investigations is to apply the general similarity method to the considered flow problem. Naturally, the ultimate objective is to solve the obtained generalized equations in the appropriate approximation.

The general similarity method was first introduced by Loitsianskii [3] and later improved by Saljnikov [4] - Saljnikov's version. Investigators of the Belgrade School used this method to solve many boundary layer flow problems. The most significant results were accomplished in investigations of incompressible fluid flow and with the MHD boundary layer [5]. Solutions for some complicated flow problems [6] were also obtained, (e.g. for the case of the temperature calculating layer on a rotating surface [7]). Both versions of the general similarity method were successfully applied to planar boundary layers with homogenous compressive fluid flow, dissociated and ionized gas flow [8] and [9]. This paper presents the results obtained for the ideally dissociated gas (air) flow along a porous wall of the body of revolution within the fluid in the conditions of equilibrium dissociation. They were obtained using Saljnikov's version of the general similarity method.

1 MATHEMATICAL MODEL

Thermo-chemical equilibrium is assumed to be established in the whole area of the boundary layer. Therefore, a complete equation system for this case of axisymmetrical gas flow in the boundary layer (Fig. 1), with the corresponding boundary conditions [10] and , 11], is:

$$\frac{\partial}{\partial x}(\rho u r)+\frac{\partial}{\partial y}(\rho v r)=0,$$
$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\rho_e u_e\frac{du_e}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right),$$
$$\rho u\frac{\partial h}{\partial x}+\rho v\frac{\partial h}{\partial y}=-u\,\rho_e u_e\frac{du_e}{dx}+\mu\left(\frac{\partial u}{\partial y}\right)^2+$$
$$+\frac{\partial}{\partial y}\left[\frac{\mu}{\Pr}(1+l)\frac{\partial h}{\partial y}\right];$$
$$u=0,\quad v=v_w(x),\quad h=h_w\quad \text{for}\quad y=0,$$
$$u\to u_e(x),\quad h\to h_e(x)\quad \text{for}\quad y\to\infty. \qquad (1)$$

In the mathematical model (1), the first equation is a continuity equation of axisymmetrical compressible fluid flow on bodies of revolution, the second one is dynamic, and the third one is energy equation. In the energy equation, the function *l(p, h)* for the equilibrium two-componential mixture depends on Lewis number (Le) and on the enthalpy of the atomic h_A and molecular h_M components of the equilibrium

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dissociated gas (air). This function is determined with the expression [12]

$$l(p,\,h)=(\mathrm{Le}-1)(h_A-h_M)\left(\frac{\partial C_A}{\partial h}\right)_p, \qquad (2)$$

in which $C_A = \alpha$ stands for the concentration of the atomic component of the ideally dissociated gas.

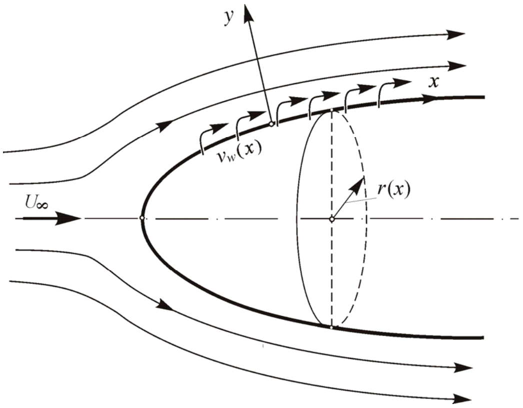

Fig. 1. *Gas flow around a body of revolution*

In the equation system and the boundary conditions, the usual notation is used. Thus, *u(x, y)* denotes the longitudinal projection of the velocity in the boundary layer, *v(x, y)* - transversal projection, ρ - density of the ideally dissociated gas, μ - dynamic viscosity, h - enthalpy. Here, x and y are longitudinal and transversal coordinates, respectively. Prandtl and Lewis numbers are defined with the expressions: $\mathrm{Pr} = \mu c_p/\lambda$, $\mathrm{Le} = \rho c_p/\lambda$ in which λ - stands for the thermal conductivity coefficient, c_p - specific heat of the dissociated gas at constant pressure and D - atomic component diffusion coefficient. The radius of the cross-section of the body of revolution, which is normal to axis of revolution, is denoted with *r(x)* The contour of the body, which figures in the continuity equation is given by the function *r(x)*. The subscript "*e*" denotes the physical quantities at the outer edge of the boundary layer, and the subscript "*w*" stands for the quantities at the wall of the body within the fluid. Here, $v_w(x)$ denotes the given velocity of the gas that flows through the solid porous wall ($v_w >$ 0 or $v_w > 0$).

Everywhere, the thickness of the boundary layer $\delta(x)$ is assumed to be significantly less than the radius of the body of revolution ($\delta(x) << r(x)$). Therefore, this thickness can be neglected compared to *r(x)* However, this assumption cannot be applied to long thin bodies [13].

Unlike other methods [14], the application of the general similarity method involves the usage of the momentum equation and the corresponding sets of similarity parameters. In order to obtain the momentum equation we start from the boundary layer continuity and dynamic equations.

The planar, steady flow of the equilibrium dissociated gas in the boundary layer is determined with the equations system [12] which differs from the system (1) only by the continuity equation. Therefore, for both flow types, the continuity equation can be written in a general form as

$$\frac{\partial}{\partial x}(\rho u r^j)+\frac{\partial}{\partial y}(\rho v r^j)=0,$$

where j = 0 for the planar, and j = 1 for axisymmetrical flow.

It has been shown [15] that a more suitable general form of the continuity equation should be used.

$$\frac{\partial}{\partial x}\left[\rho u\left(\frac{r}{L}\right)^j\right]+\frac{\partial}{\partial y}\left[\rho v\left(\frac{r}{L}\right)^j\right]=0. \qquad (3)$$

The equation for axisymmetrical flow (j = 1) and L = const. reduces to the first equation of the system (1). In the same equation, L is a characteristic constant length, and for the numerical calculation L = 1 [16].

2 TRANSFORMATION OF THE GOVERNING BOUNDARY LAYER EQUATIONS

As in already solved flow problems by means of the general similarity method, we introduce variables:

$$s(x)=\frac{1}{\rho_0\,\mu_0}\int_0^x \rho_w\,\mu_w\left(\frac{r}{L}\right)^{2j}dx,$$

$$z(x,y)=\frac{1}{\rho_0}\left(\frac{r}{L}\right)^j\int_0^y \rho\;dy. \qquad (4)$$

In the transformations (4) for the new longitudinal *s(x)* and transversal *z(x, y)* variables, the values ρ_0 and $\mu_0 = \rho_0 v_0$ denote the known values of the density and dynamic viscosity at a certain point of the boundary layer (ν_0 is kinematic viscosity). Here, ρ_w and μ_w are given values of these quantities at the inner edge of the boundary layer.

These transformations were also used in the papers [12] for $j = 0$ and [16] for $j = 1$, but for the case of a non-porous contour of the body of revolution within the fluid. It should be noted that the transformations (4), due to factors $(r/L)^2$ and r/L ($j = 1$), contain Mangler-Stepanov transformations [11].

When the continuity equation (3) is multiplied with the velocity $u_e(x)$ at the outer edge of the boundary layer, and when the dynamic equation of the system (1) is multiplied with$(r/L)^j$, by the usual procedure [15] we obtain the momentum equation from these equations. Namely, by integration transversally to the boundary layer (from $y = 0$ to $y \to \infty$) and the change of the variables, the momentum equation is obtained:

$$\frac{dZ^{**}}{ds} = \frac{F_{ot}}{u_e}. \tag{5}$$

While deriving the momentum equation, we defined the conditional displacement thickness $\Delta^*(s)$, conditional momentum loss thickness $\Delta^{**}(s)$ and nondimensional friction function $\zeta(s)$:

$$\Delta^*(s) = \int_0^\infty \left(\frac{\rho_e}{\rho} - \frac{u}{u_e} \right) dz,$$
$$\Delta^{**}(s) = \int_0^\infty \frac{u}{u_e} \left(1 - \frac{u}{u_e} \right) dz,$$
$$\zeta(s) = \left[\frac{\partial(u/u_e)}{\partial(z/\Delta^{**})} \right]_{z=0}. \tag{6}$$

The parameter of the form f is also defined:

$$f(s) = u_e' Z^{**} = f_1(s); \quad Z^{**} = \frac{\Delta^{**2}}{\nu_0},$$
$$(u_e' = du_e/ds). \tag{7}$$

The characteristic boundary layer function F_{ot} in the momentum equation (5) is determined with the expression

$$F_{ot} = 2\left[\zeta - (2+H) f \right] + \frac{2}{(r/L)^j} \frac{v_w \Delta^{**}}{\nu_0} \frac{\mu_0}{\mu_w},$$
$$H = \frac{\Delta^*}{\Delta^{**}}, \tag{8}$$

where the subscript *ot* denotes the body of revolution.

For the case of a non-porous wall of the body within the fluid (for which $v_w = 0$) the expression for the characteristic function F_{ot} comes down to the expression for the function F ($F_{ot} = F$) of the planar dissociated gas flow [12]. In this case, the expression for this function is formally the same as the one for incompressible fluid flow [3]. For $j = 0$ this function is completely the same as the function F_{dp} for the planar dissociated gas flow [17].

Based on the relations (8) for F_{ot}, the porosity parameter $\Lambda(s)$ can be defined as:

$$\Lambda(s) = -\frac{1}{(r/L)^j} \frac{\mu_0}{\mu_w} v_w \frac{\Delta^{**}}{\nu_0} =$$
$$= -V_w \frac{\Delta^{**}}{\nu_0} = \Lambda_1(s),$$
$$V_w = \frac{1}{(r/L)^j} \frac{\mu_0}{\mu_w} v_w \quad (j = 1), \tag{9}$$

where $V_w(s)$ represents the conditional fluid injection velocity. Therefore, the function F_{ot} of the boundary layer can be written in the form of

$$F_{ot} = 2\left[\zeta - (2+H) f \right] - 2\Lambda \tag{10}$$

and as such it will be used hereafter. It should be noted that because of the relationship between the quantities Z^{**}, Δ^{**} and f the momentum equation (5) can be written in two other forms [17].

In order to solve the governing equation system, we introduce a stream function $\psi(s, z)$:

$$u = \frac{\partial \psi}{\partial z},$$
$$\tilde{v} = \frac{1}{(r/L)^{2j}} \frac{\rho_0 \mu_0}{\rho_w \mu_w} \left[u \frac{\partial z}{\partial x} + \left(\frac{r}{L} \right)^j v \frac{\rho}{\rho_0} \right] =$$
$$= -\frac{\partial \psi}{\partial s} \qquad (j = 1) \tag{11}$$

in accordance with the relations that result from the continuity equation (3). For j = 0 these relations come down to the expressions used in the paper [12].

Applying transformations (4) and the stream function (11), the governing system takes the following form:

$$\frac{\partial \psi}{\partial z}\frac{\partial^2 \psi}{\partial s \partial z}-\frac{\partial \psi}{\partial s}\frac{\partial^2 \psi}{\partial z^2}=\frac{\rho_e}{\rho}u_e\frac{du_e}{ds}+ +\nu_0\frac{\partial}{\partial z}\left(Q\frac{\partial^2 \psi}{\partial z^2}\right),$$

$$\frac{\partial \psi}{\partial z}\frac{\partial h}{\partial s}-\frac{\partial \psi}{\partial s}\frac{\partial h}{\partial z}=-\frac{\rho_e}{\rho}u_e\frac{du_e}{ds}\frac{\partial \psi}{\partial z}+ +\nu_0 Q\left(\frac{\partial^2 \psi}{\partial z^2}\right)^2+\nu_0\frac{\partial}{\partial z}\left[\frac{Q}{\Pr}(1+l)\frac{\partial h}{\partial z}\right];$$

$$\frac{\partial \psi}{\partial z}=0,\quad \frac{\partial \psi}{\partial s}=-\frac{1}{(r/L)^j}\frac{\mu_0}{\mu_w}v_w= =-\tilde{v}_w=-V_w,\quad h=h_w \quad \text{for} \quad z=0,$$

$$\frac{\partial \psi}{\partial z}\to u_e(s),\quad h\to h_e(s) \quad \text{for} \quad z\to\infty. \qquad (12)$$

In the equation system (12), the nondimensional function Q is determined with the expression:

$$Q=\frac{\rho\,\mu}{\rho_w\,\mu_w};$$

$$Q=1 \quad \text{for} \quad z=0,$$

$$Q=\frac{\rho_e\,\mu_e}{\rho_w\,\mu_w}=Q(s) \quad \text{for} \quad z\to\infty. \qquad (13)$$

For further application of the general similarity method, the stream function $\psi(s, z)$ should be divided into two parts:

$$\psi(s,z)=\psi_w(s)+\bar{\psi}(s,z),\quad \bar{\psi}(s,0)=0. \qquad (14)$$

Here, $\psi_w(s)=\psi(s,0)$ denotes the stream function along the wall of the body within the fluid (z=0), and $\bar{\psi}(s,z)$ is now a new stream function.

Applying the relations (14), the equation system (12) is easily transformed into:

$$\frac{\partial \bar{\psi}}{\partial z}\frac{\partial^2 \bar{\psi}}{\partial s \partial z}-\frac{\partial \bar{\psi}}{\partial s}\frac{\partial^2 \bar{\psi}}{\partial z^2}-\frac{d\psi_w}{ds}\frac{\partial^2 \bar{\psi}}{\partial z^2}= =\frac{\rho_e}{\rho}u_e\frac{du_e}{ds}+\nu_0\frac{\partial}{\partial z}\left(Q\frac{\partial^2 \bar{\psi}}{\partial z^2}\right),$$

$$\frac{\partial \bar{\psi}}{\partial z}\frac{\partial h}{\partial s}-\frac{\partial \bar{\psi}}{\partial s}\frac{\partial h}{\partial z}-\frac{d\psi_w}{ds}\frac{\partial h}{\partial z}= =-\frac{\rho_e}{\rho}u_e\frac{du_e}{ds}\frac{\partial \bar{\psi}}{\partial z}+\nu_0 Q\left(\frac{\partial^2 \bar{\psi}}{\partial z^2}\right)^2+ +\nu_0\frac{\partial}{\partial z}\left[\frac{Q}{\Pr}(1+l)\frac{\partial h}{\partial z}\right];$$

$$\bar{\psi}(s,z)=0,\quad \frac{\partial \bar{\psi}}{\partial z}=0,\quad h=h_w \quad \text{for} \quad z=0,$$

$$\frac{\partial \bar{\psi}}{\partial z}\to u_e(s),\quad h\to h_e(s) \quad \text{for} \quad z\to\infty. \qquad (15)$$

Unlike the system (12), both equations of the system (15) contain a new term in which the derivative $d\psi_w/ds$ appears. This derivative is determined with:

$$\frac{d\psi_w}{ds}=\left(\frac{\partial \psi}{\partial s}\right)_{z=0}=-\frac{1}{(r/L)^j}\frac{\mu_0}{\mu_w}v_w=-V_w,\quad (j=1). \qquad (16)$$

The expression (16) for the derivative $d\psi_w/ds$ for j=0 comes down to the corresponding expression for the planar dissociated gas flow [17]. In the case of a nonporous wall of the body within the fluid, this derivative equals zero. Then, the terms in the equations (15) equal zero. In this case, the obtained equation system is completely identical with the one obtained in the paper [12] for the planar flow along a nonporous wall.

3 GENERALIZED BOUNDARY LAYER EQUATIONS ON BODIES OF REVOLUTION

For the application of the general similarity method, we used the procedure already used for both, incompressible and compressible fluid in [4]. We introduced new changes of the variables:

$$s=s,\quad \eta(s,z)=\frac{u_e^{b/2}}{S(s)}z,$$

$$S(s)=\left(a\nu_0\int_0^s u_e^{b-1}ds\right)^{1/2},\quad a,b=const.;$$

$$\bar{\psi}(s,z)=u_e^{1-b/2}S(s)\Phi(s,\eta),$$

$$h(s,z)=h_1\cdot\bar{h}(s,\eta);\quad h_1=const. \qquad (17)$$

Table 1. *List of variables*

Starting variables	x	y	$u_e(x)$	$u(x,y),\ v(x,y)$	$h(x,y)$	$y=0$	$y\to\infty$
New I	$s(x)$	$z(x,y)$	$u_e(s)$	$\psi(s,z),\ \overline{\psi}(s,z)$	$h(s,z)$	$z=0$	$z\to\infty$
New II	$s(x)$	$\eta(s,z)$	$u_e(s)$	$\Phi(s,\eta)$ $\Phi[\eta,\ (f_k),\ (\Lambda_k)]$	$\overline{h}(s,\eta)$ $\overline{h}[\eta,\ (f_k),\ (\Lambda_k)]$	$\eta=0$	$\eta\to\infty$

In expression (17), $\eta(s,\ z)$ is a newly introduces variable, $\Phi(s,\ \eta)$ is a new stream function, while $\overline{h}$ is a nondimensional enthalpy. Here, $h_1$denotes the enthalpy at the front stagnation point of the body within the fluid. Table 1 gives a list of the starting and the newly introduced variables.

Based on the variables (17), the important quantities and characteristics of the boundary layer (e.g. (6)) can be expressed in the form of:

$$\frac{u}{u_e}=\frac{\partial\Phi}{\partial\eta},\quad \Delta^{**}(s)=\frac{S(s)}{u_e^{b/2}}B(s),$$

$$\frac{\Delta^*}{\Delta^{**}}=H=\frac{A}{B},\quad \zeta=B\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)_{\eta=0},$$

$$A(s)=\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{\partial\Phi}{\partial\eta}\right)d\eta,$$

$$B(s)=\int_0^\infty\frac{\partial\Phi}{\partial\eta}\left(1-\frac{\partial\Phi}{\partial\eta}\right)d\eta. \tag{18}$$

The transversal variable and the stream function can also be written in a more suitable form as:

$$\eta(s,z)=\frac{B(s)}{\Delta^{**}(s)}z\ ,$$

$$\overline{\psi}(s,z)=\frac{u_e(s)\Delta^{**}(s)}{B(s)}\Phi(s,\eta). \tag{19}$$

After the calculation of the derivatives and after comprehensive transformations, the equation system (15) reduces to the form that is not generalized according to the general similarity method. Thus transformed dynamic and energy equations contain the factor u_e/u'_e in their terms. Therefore, the solution of the system will depend on the concrete form of the law of the given velocity at the outer edge of the boundary layer $u_e(s)$.

In order to bring the equation system (15) to a generalized form, it is necessary to introduce the transformations (17) and (19) and the corresponding sets of parameters. For application of Saljnikov's version of the general similarity method, we introduce a new stream function Φ and a new nondimensional enthalpy $\overline{h}$:

$$\overline{\psi}(s,z)=\frac{u_e\Delta^{**}}{B}\cdot\Phi[\eta,\ \kappa,\ (f_k),\ (\Lambda_k)],$$

$$h(s,z)=h_1\cdot\overline{h}\ [\eta,\ \kappa,\ (f_k),\ (\Lambda_k)]; \tag{20}$$

where κ is the local compressibility parameter [12].

In the generalized similarity transformations (20), with the newly introduced functions Φ and $\overline{h}$, the set of parameters of the form $f_k(s)$ of Loitsianskii type [3] and the set of parameters $\Lambda_k(s)$ of the porous wall [15] are defined as:

$$f_0(s)=\kappa=u_e^2/2h_1,\quad f_k(s)=u_e^{k-1}u_e^{(k)}Z^{**k},$$

$$\Lambda_k(s)=-u_e^{k-1}\left(\frac{V_w}{\sqrt{\nu_0}}\right)^{(k-1)}Z^{**k-1/2};$$

$$h_e+u_e^2/2=h_1,\quad \overline{h}_e=h_e/h_1,$$

$$\overline{h}_e(s)=1-\kappa,\quad k=1,\ 2,\ 3,\ ... \tag{21}$$

The introduced sets of parameters reflect the conditions of the outer flow, and in the transformations (20), they represent independent variables (instead of the variable s). Both sets of parameters (21) satisfy the corresponding recurrent simple differential equations:

$$\frac{u_e}{u'_e}f\frac{d\kappa}{ds}=2\,\kappa\,f=2\,\kappa\,f_1=\theta_0\,,$$

$$\frac{u_e}{u'_e}f_1\frac{df_k}{ds}=[(k-1)f_1+kF_{ot}]f_k+f_{k+1}=\theta_k$$

$$\frac{u_e}{u'_e} f_1 \frac{d\Lambda_k}{ds} = \\ = \left\{ (k-1) f_1 + \left[(2k-1)/2 \right] F_{ot} \right\} \Lambda_k + \\ + \Lambda_{k+1} = \chi_k \,. \tag{22}$$

These equations are formally the same as the ones for the case of incompressible fluid flow [3]. Furthermore, from (21) for $k=1$ it follows that $f_1(s) = u'_e Z^{**}$ is the already known parameter of the form (7), while from the set of the porosity parameters for the first parameter we get the expression

$$\Lambda_1 = -\frac{V_w}{\sqrt{\nu_0}} Z^{**1/2} = -\frac{V_w}{\sqrt{\nu_0}} \cdot \frac{\Delta^{**}}{\sqrt{\nu_0}} = -\frac{V_w \Delta^{**}}{\nu_0}$$

which is the same as the previously defined parameter (9).

Applying (17) and (20), the equation system (15) is brought to a generalized equation system of the dissociated gas boundary layer:

$$\frac{\partial}{\partial \eta}\left(Q \frac{\partial^2 \Phi}{\partial \eta^2} \right) + \frac{aB^2 + (2-b) f_1}{2B^2} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} + \\ + \frac{f_1}{B^2}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial \Phi}{\partial \eta} \right)^2 \right] + \frac{\Lambda_1}{B} \frac{\partial^2 \Phi}{\partial \eta^2} = \\ = \frac{1}{B^2}\left[\sum_{k=0}^{\infty} \theta_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial f_k \partial \eta} - \frac{\partial \Phi}{\partial f_k} \frac{\partial^2 \Phi}{\partial \eta^2} \right) + \\ + \sum_{k=1}^{\infty} \chi_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \Lambda_k \partial \eta} - \frac{\partial \Phi}{\partial \Lambda_k} \frac{\partial^2 \Phi}{\partial \eta^2} \right) \right],$$

$$\frac{\partial}{\partial \eta}\left[\frac{Q}{\Pr}(1+l) \frac{\partial \bar{h}}{\partial \eta} \right] + \frac{aB^2 + (2-b) f_1}{2B^2} \Phi \frac{\partial \bar{h}}{\partial \eta} - \\ - \frac{\rho_e}{\rho} \frac{2\kappa f_1}{B^2} \frac{\partial \Phi}{\partial \eta} + 2\kappa Q \left(\frac{\partial^2 \Phi}{\partial \eta^2} \right)^2 + \frac{\Lambda_1}{B} \frac{\partial \bar{h}}{\partial \eta} = \\ = \frac{1}{B^2}\left[\sum_{k=0}^{\infty} \theta_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial f_k} - \frac{\partial \Phi}{\partial f_k} \frac{\partial \bar{h}}{\partial \eta} \right) + \\ + \sum_{k=1}^{\infty} \chi_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial \Lambda_k} - \frac{\partial \Phi}{\partial \Lambda_k} \frac{\partial \bar{h}}{\partial \eta} \right) \right];$$

$$\Phi = 0,\ \frac{\partial \Phi}{\partial \eta} = 0,\ \bar{h} = \bar{h}_w = const. \ \text{for}\ \eta = 0, \\ \frac{\partial \Phi}{\partial \eta} \to 1,\ \ \bar{h} \to \bar{h}_e(s) = 1 - \kappa \ \ \text{for}\ \eta \to \infty. \tag{23}$$

The system of generalized equations (23) has the same form as the system [17] for planar flow of dissociated gas along a porous contour. For $j = 0$ these systems are completely identical because in that case the expressions for porosity parameters are also identical. For the case of a nonporous wall $(v_w = 0 \Rightarrow V_w = 0,\ \Lambda_1 = 0)$, the equations (23) formally come down to the corresponding equations obtained in the paper [12].

The equation system (23) is solved in the so-called n-parametric approximation. In three-parametric twice localized approximation ($f_0 = \kappa \neq 0$, $f_1 = f \neq 0$, $\Lambda_1 = \Lambda \neq 0$, $f_k = 0$, $\Lambda_k = 0$ for $k \geq 2$ and $\partial/\partial\kappa \approx 0,\ \partial/\partial\Lambda_1 \approx 0$) this system has the following form:

$$\frac{\partial}{\partial \eta}\left(Q \frac{\partial^2 \Phi}{\partial \eta^2} \right) + \frac{aB^2 + (2-b) f_1}{2B^2} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} + \\ + \frac{f_1}{B^2}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial \Phi}{\partial \eta} \right)^2 \right] + \frac{\Lambda_1}{B} \frac{\partial^2 \Phi}{\partial \eta^2} = \\ = \frac{F_{ot} f_1}{B^2}\left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial f_1} - \frac{\partial \Phi}{\partial f_1} \frac{\partial^2 \Phi}{\partial \eta^2} \right),$$

$$\frac{\partial}{\partial \eta}\left[\frac{Q}{\Pr}(1+l) \frac{\partial \bar{h}}{\partial \eta} \right] + \frac{aB^2 + (2-b) f_1}{2B^2} \Phi \frac{\partial \bar{h}}{\partial \eta} - \\ - \frac{2\kappa f_1}{B^2} \frac{\partial \Phi}{\partial \eta} \left[\frac{\rho_e}{\rho} - \left(\frac{\partial \Phi}{\partial \eta} \right)^2 \right] + 2\kappa Q \left(\frac{\partial^2 \Phi}{\partial \eta^2} \right)^2 + \\ + \frac{\Lambda_1}{B} \frac{\partial \bar{h}}{\partial \eta} = \frac{F_{ot} f_1}{B^2}\left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial f_1} - \frac{\partial \Phi}{\partial f_1} \frac{\partial \bar{h}}{\partial \eta} \right);$$

$$\Phi = 0,\ \frac{\partial \Phi}{\partial \eta} = 0,\ \bar{h} = \bar{h}_w = const. \ \text{for}\ \eta = 0, \\ \frac{\partial \Phi}{\partial \eta} \to 1,\ \ \bar{h} \to \bar{h}_e(s) = 1 - \kappa \ \ \text{for}\ \eta \to \infty. \tag{24}$$

Note that in the energy equation of the system (23), the localization per the parameter κ is performed in relation to the total nondimensional enthalpy $g = (h + u^2/2)/h_1$, when, according to [12], we can consider that $\partial g / \partial \kappa \approx 0$. The relation between the nondimensional $\bar{h}$ and the total enthalpy g is determined with the expression $\bar{h} = g - \kappa (\partial \Phi / \partial \eta)^2$.

The generalized equation system obtained here, represents a mathematical model of ideally dissociated gas (air) flow in the boundary layer along a body of revolution under the conditions of equilibrium dissociation. The terms that contain the porosity parameter Λ_1 are characteristic for the porous wall of the body of revolution. The equation system (24) is of the same form as the corresponding system for the planar flow problems [17].

For a numerical solution of the obtained equation system, the order of the dynamic equation is decreased:

$$\frac{u}{u_e}=\frac{\partial\Phi}{\partial\eta}=\varphi=\varphi(\eta,\,\kappa,\,f_1,\,\Lambda_1). \qquad (25)$$

Then, based on the results stated in [12], for the equilibrium dissociated air it is accepted that $\mathrm{Le}\approx 1$, and consequently $l=0$. According to the same author, the function Q (13) and the density ratio ρ_e/ρ, that figure in the equation system (24), can be expressed using the formulae:

$$Q=Q(\bar{h})=\left(\frac{\bar{h}_w}{\bar{h}}\right)^{1/3}, \qquad \frac{\rho_e}{\rho}\approx\frac{\bar{h}}{1-\kappa}. \qquad (26)$$

The formula for the function Q gives satisfactory results for a wide range of the pressure change. The formula for the density ratio that follows from the corresponding formulae stated in [12], gives a rather rough approximation.

Taking (25) and (26) into consideration, we come to a generalized equation system with four independent variables: η, κ, f_1 and Λ_1:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial\varphi}{\partial\eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial\varphi}{\partial\eta}+\frac{f_1}{B^2}\left[\frac{\bar{h}}{1-\kappa}-\varphi^2\right]+\frac{\Lambda_1}{B}\frac{\partial\varphi}{\partial\eta}=\frac{F_{ot}f_1}{B^2}\left(\varphi\frac{\partial\varphi}{\partial f_1}-\frac{\partial\Phi}{\partial f_1}\frac{\partial\varphi}{\partial\eta}\right),$$

$$\frac{\partial}{\partial\eta}\left[\frac{Q}{\mathrm{Pr}}\frac{\partial\bar{h}}{\partial\eta}\right]+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f_1}{B^2}\varphi\left[\frac{\bar{h}}{1-\kappa}-\varphi^2\right]+2\kappa Q\left(\frac{\partial\varphi}{\partial\eta}\right)^2+\frac{\Lambda_1}{B}\frac{\partial\bar{h}}{\partial\eta}=\frac{F_{ot}f_1}{B^2}\left(\varphi\frac{\partial\bar{h}}{\partial f_1}-\frac{\partial\Phi}{\partial f_1}\frac{\partial\bar{h}}{\partial\eta}\right);$$

$$\Phi=0,\ \varphi=0,\ \bar{h}=\bar{h}_w=const.\ \text{for}\ \eta=0,$$
$$\varphi\to 1,\quad \bar{h}\to\bar{h}_e=1-\kappa\quad \text{for}\ \eta\to\infty. \qquad (27)$$

4 NUMERICAL SOLUTION

Numerical solution of the obtained system of partial differential equations (27) is performed by the passage method, i.e., by the finite difference method. Here, the whole area of the boundary layer is changed with the planar integration grid with spaces Δf_1 and $\Delta\eta$ [4]. Some derivatives in the equations (27) are changed with the corresponding finite differences of the functions at discrete points of the grid: (M,K), $(M-1,K+1)$, $(M,K+1)$, $(M+1,K+1)$. The values of the functions φ,Φ and $\bar{h}$ are calculated at discrete points of each calculating layer $(K+1)$. Because of the complexity of the considered flow problem, the number of discrete points from $M=1$ to $M=N=401$ has been determined for each layer calculating.

For the numerical solution of the generalized equation system (27), i.e., of the corresponding equivalent system, a program in FORTRAN programming language has been written. This program was used in our paper [17], and is based on the one used in the paper [4]. The equations are solved for the following values of the parameters and coefficients: Pr = 0.712; a = 0.4408, b = 5.714 [4]. For the characteristic functions B and F_{ot} at a zero iteration, the following values are accepted: $B^0_{K+1}=0.449$ and $F^0_{ot,K+1}=0.4411$. They were also used in the investigations [4].

5 THE OBTAINED RESULTS

Numerical solutions of the generalized equations (27) are first obtained for each cross-section of the boundary layer in the form of tables. Then, based on the tables, diagrams of the nondimensional velocity, nondimensional enthalpy and characteristic quantities of the boundary layer are drawn. Again, note that due to transformations (4), the obtained generalized equations (27) are formally the same as the ones obtained in [17].

This paper gives only some of the most important diagrams obtained in the course of our investigations. Figure 2 is the diagram of the nondimensional velocity u/u_e for three cross-sections of the boundary layer. Figure 3 shows

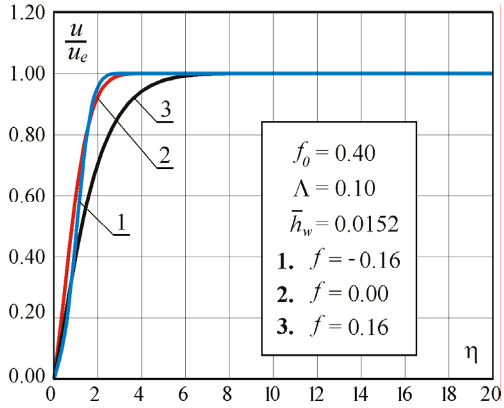

Fig. 2. *Diagram of the nondimensional velocity* u/u_e

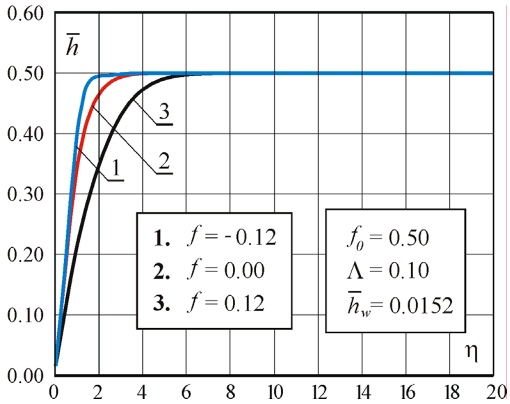

Fig. 3. *Diagram of the nondimensional enthalpy* $\overline{h}$

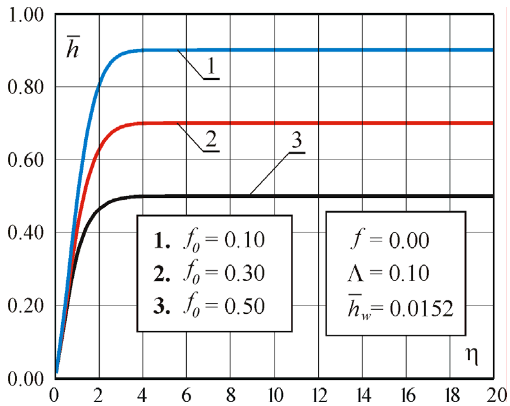

Fig. 4. *Diagram of the nondimensional enthalpy* $\overline{h}$ *for different values of the parameter* f_0

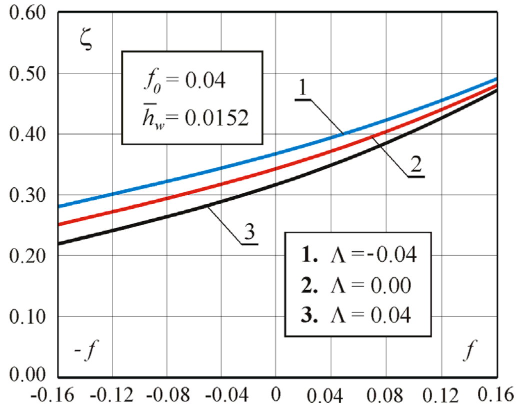

Fig. 5. *Distribution of the nondimensional friction function* $\zeta(f)$

the diagram of the nondimensional enthalpy $\overline{h}$ also at three cross-sections of the boundary layer when the compressibility parameter is $\kappa = f_0 = 0.5$ The diagrams on Fig. 4 represent the distribution of the nondimensional enthalpy for different values of the compressibility parameter. Finally, Fig. 5 shows the distribution of the nondimensional friction function $\zeta(f)$. Here, the subscript 1 in the parameter of the form $(f_1 = f)$ is left out.

6 CONCLUSIONS

The paper shows that the general similarity method can be applied in the studied case of the fluid flow. However, the application of this method to the problem of the ideally dissociated gas (air) flow in the boundary layer on bodies of revolution is associated with some complexities, which are primarily of mathematical nature. There are also some problems related to physical, i.e., thermochemical processes of gas flow (e.g. (26)). Nevertheless, we have obtained some important quality results that give us an insight into the behaviour of the distribution of physical and characteristic quantities at different cross-sections of the boundary layer.

It should be noted that the porosity parameter $\Lambda(s)$ (9) and a set of parameters $\Lambda_k(s)$ of the porous wall of the body of revolution (21), enabled the application of the general similarity method to this problem.

It is specifically pointed out that the system of generalized equations (23) has the same form as the corresponding system for planar flow of dissociated gas along a porous contour. But this is quite expected and it is due to the transformations (4) that contain the terms $(r/L)^2$ and r/L.

Based on the numerical results, i.e., on the presented and other diagrams, a general conclusion can be drawn: distributions of the solutions of the obtained boundary layer equations are of the same behaviour as with other dissociated gas flow problems.

For the considered case, the following concrete conclusions can be drawn:

- The nondimensional flow velocity $u/u_e = \partial\Phi/\partial\eta$ at different cross-sections of the boundary layer on the body of revolution (different f) converges very fast towards unity (Fig. 2).
- The influence of the compressibility parameter $\kappa = f_0$ on the distribution of the nondimensional enthalpy at the cross-section of the boundary layer is considerable (Fig. 4). This is due to the fact that the value of the nondimensional enthalpy is determined with this parameter at the outer edge of the boundary layer ($\bar{h} = \bar{h}_e = 1 - \kappa$ for $\eta \to \infty$).
- In order to obtain results that are more reliable, it is necessary to solve the system (23) in three-parametric approximation but without the localization per corresponding parameters. It is important that the solutions should be obtained without localization per the compressibility parameter. This parameter has a great influence on the change of the enthalpy in the boundary layer, and it even changes the general character of behaviour of the distribution of enthalpy. However, this solution would involve even more complexities of mathematical nature.
- From the diagrams in Figure 5 and others not shown here, it is obvious that the porosity parameter has a considerable influence on all the important characteristics of the boundary layer. This parameter influences the nondimensional friction function ζ, and, consequently, it influences the boundary layer separation point.

We should also note that there were some problems in numerical solution of the system (24). Namely, the program stopped working for some input values. This was the problem encountered in other cases of the boundary layer flow, as pointed out by some authors [7].

7 ACKNOWLEDGEMENT

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