INFORMATIONAL BEING-IN

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In this paper the phenomenon of informational Being-in, that is, includedness is studied in a formally recursive (informational) way, dealing with basic definitions of includedness (informational in-volvement, em-bedding) and their consequences. It seems that the informational includedness is a phenomenon of informational entities, which involves them in a perplexedly recursive way and offers the richness of the informationally spontaneous parallelism, serialism, and circularity. In this respect, together with its informational openness and recursiveness, informational Being-in can come semantically as close as possible to its philosophical notion (concept) [2, 1]. Some includable structured phenomena of inference or reasoning (deduction, induction, abduction, modus ponens, tollens, rectus, and obliquus) are shown in a formal manner. The disposed formal apparatus enables an unbounded and even deepened philosophical investigation of the phenomenon of Being-in and its consequences. So, a formalistic investigation of informational Being-in can enrich its philosophical understanding.

1 Introduction

Being-in is the original term coined by *Heidegger* (in German, In-Sein or In-sein, in English, inhood) [2]. Informational Being-in[1] belongs to the most significant existentiales of the informational. "When someone calls our attention to the fact that 'in' also has an existential sense which expresses *involvement*, ...we tend to think of this as a metaphorical deviation from physical inclusion." (Dreyfus in [1], p. 41.) Informational 'in' means informationally involved, distributed and, for example, being dual in the sense of energy and information (Šlechta in [5], the Hamiltonians H_{EI} and H_{IE} in equations 14 and 15, respectively). "Grimm goes on to argue that the preposition 'in' is derived from the verb, rather than the verb from the preposition." ([2], p. 80.) This conclusion is essential, for Being-in in the informational has an active (verb-like, operational) role.

Being-in belongs to primordial situations concerning informational entities. It is a consequence of informing of entities and vice versa. In this paper, the basic informational properties of the phenomenon, state, or process termed Being-in will be studied. Instead of the philosophical term Being-in, the term *includedness* (or informational includedness) will be frequently used, which comes closer to the formal terminology in traditional mathematics (e.g., the notion of a subset), but is essentially different in its existential (informingly arising) nature if compared with the categorical (reductionistic) relation of inclusion. *Informational includedness* is a new term, determined in an informationally recursive way (circularly) and, in this respect, extending the structure of informational includedness boundlessly in an includable way.

Being-in is an informational existentiale, a for-

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mal existential expression, which concerns Being-in-the-world (informational realm of the exterior) as its essential state. An informational entity (thing, matter) informs as the entity in the world (its environment, informational region, within itself). The world is the synonym of that in which an entity informs, that is, the informational entity embracing informational realm. The question is, how can this general view (an entity's informational openness, interweavement, or connectedness) be considered in its informational entirety.

When an informational formula occurs as part of a larger formula, it is said to be in *included position* (see [13], included ppl. a.); otherwise it is said to be in absolute position and to constitute an informational entity. Some morphemes occur in included position, either partial or complete. In some sentences there are devices that signal the inclusion of two or more separate sentences. The included position is that had by a word, phrase or other linguistic form when it is part of a larger form. All of these forms of includedness are classical and do not embrace conceptualism of the informational includedness as, for instance, a distributed involvement of an entity within an informational realm.

The task of the present study is to develop and formalize a general concept of the Being-in of an informational entity, where this concept can be particularized according to the specific informational needs and occurring circumstances.

2 Being-in qua Informing

A philosophy of informational includedness roots in the philosophical notion of Being-in (for example, [2], ¶12) as an informational (philosophical) phenomenon which concerns the entity's Being. As we shall see, Being-in of something can meaningly never be exhausted, it simply does not come to an end because of its recursively open informational nature. So we have to present this informational virtue, faculty, or property of something in a strict formal way, that is, by systems of informational formulas describing the phenomenon of Being-in. The informational includedness means something essentially different in respect to the set-theoretical inclusion in mathematics, although the symbols $\subset$ and $\supset$ (the alternative to $\subset$) are used to mark both phenomena.

How does Being-in, that is, informational includedness inform? As a property of something which informs in a broader realm, it must be expressed as includedness, that is, as something concerning the informational operator (for example, $\models_{\text{in}}$, $\dashv_{\text{include}}$ or, simply, $\subset$, which read 'is in', 'is included in' or 'is an informational part of', respectively). Informational includedness means functional involvement of an entity into the informing of the other entity and itself.

The Being-in as such always concerns an entity, that is, something, marked by α. As a phenomenon, the Being-in is involved in something in an informational way. According to [9], we introduce the following four modes of the informational existentiale concerning something α in an includable way:

$\alpha \subset$	reads as: α *informs includingly;* α's *externalism* of including; α *is/are included* (in);
$\subset \alpha$	reads as: α *is informed includingly;* α's *internalism* of including; α *include(s);*
$\alpha \subset \alpha$	reads as: α *informs includingly itself* and *is informed includingly by itself;* α's *metaphysicalism* of including; α *includes and is included in itself;*
$\begin{pmatrix} \alpha \subset; \\ \subset \alpha \end{pmatrix}$	reads as: α *informs includingly* and *is informed includingly;* α's *phenomenalism* of including; α *includes and is included*

To fulfill the existential criteria of includedness, evidently, entity α informationally includes (involves) some informing entities and is informationally included in (involved by) some informing entities.

In this point of the study, the question arises, what could the meaning (interpretation) of the formalized forms of informational includedness of something be? The formalized externalistic interpretation of includedness could be the following:

$$(\alpha \subset) \rightleftharpoons \begin{pmatrix} \alpha \models; \\ \models \alpha; \\ \Xi(\alpha \subset) \end{pmatrix}$$

where $\Xi(\alpha \subset)$ is an element of the informational power set (symbol $\mathcal{P}$) with 16 elements (including the empty set $\emptyset$), that is,

$$\Xi(\alpha \subset) \in \mathcal{P}\left(\left\{\begin{array}{l} (\models \alpha) \subset, \\ (\alpha \models) \subset, \\ (\models \alpha) \subset \alpha, \\ (\alpha \models) \subset \alpha \end{array}\right\}\right)$$

In general, an informational set $\{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ is interpreted as the parallel system (array) of entities, that is,

$$\{\alpha_1, \alpha_2, \cdots, \alpha_n\} \rightleftharpoons (\alpha_1; \alpha_2; \cdots; \alpha_n)$$

The formalized internalistic interpretation of includedness is, for example,

$$(\subset \alpha) \rightleftharpoons \begin{pmatrix} \alpha \models; \\ \models \alpha; \\ \Xi(\subset \alpha) \end{pmatrix}$$

where $\Xi(\subset \alpha)$ is an element of the power set, that is,

$$\Xi(\subset \alpha) \in \mathcal{P}\left(\left\{\begin{array}{l} (\alpha \models) \subset \alpha, \\ (\models \alpha) \subset \alpha, \\ (\alpha \models) \subset, \\ (\models \alpha) \subset \end{array}\right\}\right)$$

The metaphysicalistic case of includedness interpretation could be

$$(\alpha \subset \alpha) \rightleftharpoons \begin{pmatrix} \alpha \models \alpha; \\ \Xi(\alpha \subset \alpha) \end{pmatrix}$$

where

$$\Xi(\alpha \subset \alpha) \in \mathcal{P}(\{\alpha \models \alpha\})$$

and the phenomenalistic case

$$(\alpha \subset; \subset \alpha) \rightleftharpoons \begin{pmatrix} \alpha \models; \\ \models \alpha; \\ \Xi(\alpha \subset); \\ \Xi(\subset \alpha) \end{pmatrix}$$

These are initial cases of includedness and each of them speaks in its own way, so various interpretations are possible.

3 A Definition and Consequences of Informational Includedness

Let us introduce the basic definition of informational includedness which will cover also the four previously shown examples (includable externalism, internalism, metaphysicalism, and phenomenalism).

Definition 1 [Informational Includedness] *Let entity α inform within entity β, that is, $\alpha \subset \beta$. This expression reads:* α informs within (is an informational component or constituent of) β. *Let the following parallel system of includedness (Being-in) be defined recursively:*

$$(\alpha \subset \beta) \rightleftharpoons_{\text{Def}} \begin{pmatrix} \beta \models \alpha; \\ \alpha \models \beta; \\ \Xi(\alpha \subset \beta) \end{pmatrix}$$

where for the extensional part $\Xi(\alpha \subset \beta)$ of the includedness $\alpha \subset \beta$, there is,

$$\Xi(\alpha \subset \beta) \in \mathcal{P}\left(\left\{\begin{array}{l} (\beta \models \alpha) \subset \beta, \\ (\alpha \models \beta) \subset \beta, \\ (\beta \models \alpha) \subset \alpha, \\ (\alpha \models \beta) \subset \alpha \end{array}\right\}\right)$$

The most complex element of this power set is denoted by

$$\Xi^{\beta,\alpha}_{\beta,\alpha}(\alpha \subset \beta) \rightleftharpoons \begin{pmatrix} (\beta \models \alpha) \subset \beta, \alpha; \\ (\alpha \models \beta) \subset \beta, \alpha \end{pmatrix}$$

Cases, where $\Xi(\alpha \subset \beta) \rightleftharpoons \emptyset$ and $\emptyset$ denotes an empty entity (informational nothing), are exceptional (reductionistic). □

Consequence 1 [An Extension of Informational Includedness] *Let us introduce the following markers:*

$$\begin{aligned} \theta(\xi) &= \begin{pmatrix} \xi \models (\beta \models \alpha); \\ (\beta \models \alpha) \models \xi \end{pmatrix}; \\ \vartheta(\xi) &= \begin{pmatrix} \xi \models (\alpha \models \beta); \\ (\alpha \models \beta) \models \xi \end{pmatrix}; \\ \xi &\in \{\beta, \alpha\} \end{aligned}$$

Then, the cases of includedness within Ξ-elements in Definition 1 induce, evidently,

$$
\begin{aligned}
((\beta \models \alpha) \subset \xi) &\rightleftharpoons \begin{pmatrix} \theta(\xi); \\ \Xi((\beta \models \alpha) \subset \xi) \end{pmatrix}; \\
((\alpha \models \beta) \subset \xi) &\rightleftharpoons \begin{pmatrix} \vartheta(\xi); \\ \Xi((\alpha \models \beta) \subset \xi) \end{pmatrix}; \\
((\beta \models \alpha) \subset \beta, \alpha) &\rightleftharpoons \begin{pmatrix} \theta(\beta); \\ \theta(\alpha); \\ \Xi((\beta \models \alpha) \subset \beta, \alpha) \end{pmatrix}; \\
((\alpha \models \beta) \subset \beta, \alpha) &\rightleftharpoons \begin{pmatrix} \vartheta(\beta); \\ \vartheta(\alpha); \\ \Xi((\alpha \models \beta) \subset \beta, \alpha) \end{pmatrix}
\end{aligned}
$$

where

$$
\begin{aligned}
\Xi((\beta \models \alpha) \subset \beta, \alpha) &\rightleftharpoons \begin{pmatrix} \Xi((\beta \models \alpha) \subset \beta); \\ \Xi((\beta \models \alpha) \subset \alpha) \end{pmatrix}; \\
\Xi((\alpha \models \beta) \subset \beta, \alpha) &\rightleftharpoons \begin{pmatrix} \Xi((\alpha \models \beta) \subset \beta); \\ \Xi((\beta \models \alpha) \subset \alpha) \end{pmatrix}
\end{aligned}
$$

□

Instead of proving this consequence, let us extend recursively Definition 1 one step deeper.

Consequence 2 [A Further Extension of Informational Includedness] *According to the basic definition of includedness, there is recursively,*

$$
(\alpha \subset \beta) \rightleftharpoons_{\text{Def}} \begin{pmatrix}
\beta \models \alpha; \\
\alpha \models \beta; \\
\Xi^{\beta,\alpha}_{\beta,\alpha}(\alpha \subset \beta) \rightleftharpoons \\
\begin{pmatrix}
\beta \models (\beta \models \alpha); \\
(\beta \models \alpha) \models \beta; \\
\Xi((\beta \models \alpha) \subset \beta); \\
\beta \models (\alpha \models \beta); \\
(\alpha \models \beta) \models \beta; \\
\Xi((\alpha \models \beta) \subset \beta); \\
\alpha \models (\beta \models \alpha); \\
(\beta \models \alpha) \models \alpha; \\
\Xi((\beta \models \alpha) \subset \alpha); \\
\alpha \models (\alpha \models \beta); \\
(\alpha \models \beta) \models \alpha; \\
\Xi((\alpha \models \beta) \subset \alpha)
\end{pmatrix}
\end{pmatrix}.
$$

where, for instance, the first extensional part is

$$
\Xi((\beta \models \alpha) \subset \beta) \in \mathcal{P} \left(\left\{ \begin{matrix} (\beta \models (\beta \models \alpha)) \subset \beta, \\ ((\beta \models \alpha) \models \beta) \subset \beta, \\ (\beta \models (\beta \models \alpha)) \subset \alpha, \\ ((\beta \models \alpha) \models \beta) \subset \alpha \end{matrix} \right\} \right)
$$

the second one

$$
\Xi((\alpha \models \beta) \subset \beta) \in \mathcal{P} \left(\left\{ \begin{matrix} (\beta \models (\alpha \models \beta)) \subset \beta, \\ ((\alpha \models \beta) \models \beta) \subset \beta, \\ (\beta \models (\alpha \models \beta)) \subset \alpha, \\ ((\alpha \models \beta) \models \beta) \subset \alpha \end{matrix} \right\} \right)
$$

the third one

$$
\Xi((\beta \models \alpha) \subset \alpha) \in \mathcal{P} \left(\left\{ \begin{matrix} (\alpha \models (\beta \models \alpha)) \subset \beta, \\ ((\beta \models \alpha) \models \alpha) \subset \beta, \\ (\alpha \models (\beta \models \alpha)) \subset \alpha, \\ ((\beta \models \alpha) \models \alpha) \subset \alpha \end{matrix} \right\} \right)
$$

and the fourth one

$$
\Xi((\alpha \models \beta) \subset \alpha) \in \mathcal{P} \left(\left\{ \begin{matrix} (\alpha \models (\alpha \models \beta)) \subset \beta, \\ ((\alpha \models \beta) \models \alpha) \subset \beta, \\ (\alpha \models (\alpha \models \beta)) \subset \alpha, \\ ((\alpha \models \beta) \models \alpha) \subset \alpha \end{matrix} \right\} \right)
$$

□

Within this consequence, the circular structures of the form

$$
\begin{aligned}
&((\beta \models \alpha) \models \beta); \\
&(\beta \models (\alpha \models \beta)); \\
&((\alpha \models \beta) \models \alpha); \\
&(\alpha \models (\beta \models \alpha))
\end{aligned}
$$

belonging to the first, second, third, and fourth extension will become significant in the context of entity metaphysicalism.

Let us explain in short the meaning of informational operators $\rightleftharpoons$, $\rightleftharpoons_{\text{Def}}$, $\subset$, $\in$, $=$ and, through this explanation, point out the difference regarding the equally marked mathematical operators and relations. Let have the following interpretation:

$\rightleftharpoons$	mean(s), informs meaningly;
$\rightleftharpoons_{\text{Def}}$	mean(s) by definition;
$\subset$	informs within (includingly);
$\in$	is an element of informational set of entities, informational lumps;
$=$	is a marker for, is the same as;

The meaning of an informational operator correlates with the meaning of the meaningly adequate verbal phrase which expresses an informational activity, happening, occurring, state, position, attitude, etc. The meaning of a mathematical operator concerns solely the mathematically well-defined abstract objects.

4 Informational Consequences of Includedness

The most characteristic consequences of informational includedness are circular forms of parallelism and serialism.

4.1 Parallelism and Serialism of Includedness

The includedness of an informational entity in concern to an informational entity induces a certain phenomenon of parallelism and serialism. More precisely, includedness generates informational circularity in the form of parallel and serial cycles which are of essential significance in emerging of the so-called metaphysicalism. Metaphysicalism shapes the background for the arising of cognitive and intelligent information, by mixing of intelligent informational lumps and composing them by intelligent selection into informational structures performing understanding, generating meaning, that is, cognizing.

The parallelism of $\alpha \subset \beta$ is already observed in Definition 1, where $\alpha \subset \beta$ is a parallel structure of transitions $\beta \models \alpha$, $\alpha \models \beta$, and extension $\Xi(\alpha \subset \beta)$. This structure is circularly-parallel in components $\beta \models \alpha; \alpha \models \beta$ in respect to β via (implicitly) α, that is, in a parallel transitive way. On the other hand, the extensional part of informational includedness $\Xi(\alpha \subset \beta)$ in Definition 1, can be chosen, for instance, as

$$\Xi_{\beta}^{\beta}(\alpha \subset \beta) \rightleftharpoons \begin{pmatrix} (\beta \models \alpha) \subset \beta; \\ (\alpha \models \beta) \subset \beta \end{pmatrix}$$

This structure is circularly serial in respect to β via α, etc. [e.g., $\Xi_{\alpha}^{\beta}(\alpha \subset \beta)$ is circularly serial also in respect to α via β].

4.2 Parallelism of Includedness

In some respect, the parallelism of includedness is straightforward, that is, informationally transparent. As we shall see, the parallel includedness can be defined in a common way (a mathematical fashion), moving from one 'relation' of includedness to the other.

Consequence 3 [Transitivity of Parallel Informational Includedness] *Let for informational entities α_i, α_j, and α_k be $\alpha_i \subset \alpha_j; \alpha_j \subset \alpha_k$. Then, the implication pertaining to the includable transitivity,*

$$(\alpha_i \subset \alpha_j; \alpha_j \subset \alpha_k) \Longrightarrow (\alpha_i \subset \alpha_k)$$

is informationally righteous. □

Proof. The intuitive proof of the consequence belongs to the semantics of a language (speech). If α_i informationally involves α_j and if α_j informationally involves α_k, then, in a language-logical sense, α_i involves α_k informationally via entity α_j.

Another, more formalistic proof of the consequence follows from the axiomatic concept of the informing of entities. Informational operator $\subset$ has to be comprehended as a particular case of operator $\models$. In case of parallel informing $\alpha \models \beta; \beta \models \gamma$, there follows $\alpha \models \gamma$. Q.E.D.

Consequence 4 [Parallelism of Informational Includedness] *Let for informational entities $\alpha_1, \alpha_2, \cdots, \alpha_n$ be*

$$\alpha_i \subset \alpha_{i+1};\ i = 1, 2, \cdots, n-1.$$

This formula depicts a parallel system of includedness, that is,

$$\begin{array}{l} \alpha_1 \subset \alpha_2; \\ \alpha_2 \subset \alpha_3; \\ \vdots \\ \alpha_{n-1} \subset \alpha_n \end{array}$$

which is transitively includable and parallel straightforward. There is,

$$\left.\begin{array}{l} \alpha_{i+1} \models \alpha_i; \\ \alpha_i \models \alpha_{i+1}; \\ \Xi(\alpha_i \subset \alpha_{i+1}) \end{array}\right\} i = 1, 2, \cdots, n-1$$

This parallel system implies the parallelism of different elementary informational forms of entities $\alpha_1, \alpha_2, \cdots, \alpha_n$ in a parallel descending, ascending, and also circular order regarding index i and the system's structural dependence on the includable extensions $\Xi(\alpha_i \subset \alpha_{i+1})$ $(i = 1, 2, \cdots, n-1)$. □

Proof. Let us look the parallel elementary structures, that is, parallel components of the parallel included system. There is

$$\begin{pmatrix} \alpha_1 \subset \alpha_2; \\ \alpha_2 \subset \alpha_3; \\ \vdots \\ \alpha_{n-1} \subset \alpha_n \end{pmatrix} \rightleftharpoons$$

$$\begin{pmatrix} \alpha_n \models \alpha_{n-1}; & \alpha_1 \models \alpha_2; & \Xi(\alpha_1 \subset \alpha_2); \\ \vdots & \alpha_2 \models \alpha_3; & \Xi(\alpha_2 \subset \alpha_3); \\ \alpha_3 \models \alpha_2; & \vdots & \vdots \\ \alpha_2 \models \alpha_1; & \alpha_{n-1} \models \alpha_n; & \Xi(\alpha_{n-1} \subset \alpha_n) \end{pmatrix}$$

Different forms of Ξ's are possible, conditioning the nature of the elementary circularity between α-entities. Different types of circularity will be shown in the subsequent consequences. Q.E.D.

Consequence 5 [Manifold Parallelism of Informational Includedness] *The implication concerning the manifoldness of parallel structured includedness follows directly from Consequence 3 in the form*

$$(\alpha_i \subset \alpha_{i+1}; i = 1, 2, \cdots, n-1) \Longrightarrow$$
$$\begin{pmatrix} \alpha_j \subset \alpha_k; j < k; \\ j \in \{1, 2, \cdots, n-1\}; k \in \{2, 3, \cdots, n\} \end{pmatrix}$$

The parallel manifoldness of includedness will become the basis for the manifoldness of the circular parallelism. □

The last consequence means that there are parallel groups of parallel includable cases of length ℓ $(2 \leq \ell \leq n-1)$ of the form

$$\left.\begin{matrix} \alpha_{i_1} \subset \alpha_{i_2}; \\ \alpha_{i_2} \subset \alpha_{i_3} \end{matrix}\right\} 1 \leq i_1 \leq i_2 \leq i_3 \leq n; \ell = 2;$$

$$\left.\begin{matrix} \alpha_{i_1} \subset \alpha_{i_2}; \\ \alpha_{i_2} \subset \alpha_{i_3}; \\ \alpha_{i_3} \subset \alpha_{i_4} \end{matrix}\right\} 1 \leq i_1 \leq i_2 \leq i_3 \leq i_4 \leq n; \ell = 3;$$

$$\vdots$$

$$\left.\begin{matrix} \alpha_1 \subset \alpha_2; \\ \alpha_2 \subset \alpha_3; \\ \vdots \\ \alpha_{n-1} \subset \alpha_n \end{matrix}\right\} \ell = n-1$$

According to Consequence 4, we can construct the ascending and descending sequences of parallel formulas $\alpha_{i_j} \models \alpha_{i_k}$ in regard to subscripts i_j and i_k.

4.3 Circular Parallelism of Includedness

Let us define a complete form of circular parallelism in the following form.

Definition 2 [Circular Parallelism of an Informational System]. *An informational system of entities $\alpha_1, \alpha_2, \cdots, \alpha_n$ is called circularly parallel, if*

$$\alpha_i \models \alpha_{i+1}; \ i = 1, 2, \cdots, n-1;$$
$$\alpha_n \models \alpha_1$$

where $n = 2, 3, \cdots$. □

Definition 3 [Completely Circular Parallelism of an Entity System]. *A system of (parallel) informational entities $\alpha_1, \alpha_2, \cdots, \alpha_n$ is called completely circularly parallel, if*

$$\alpha_i \models \alpha_j; \ i, j = 1, 2, \cdots, n$$

The circular completeness of parallelism enables the occurrence of all possible cycles of formulas $\alpha_i \models \alpha_j$ in which operands $\alpha_1, \alpha_2, \cdots, \alpha_n$ appear, in a recursive way. □

The circular parallelism follows from the transitive parallelism of informational includedness (Consequence 3), if to the parallel sequence $\alpha_1 \subset \alpha_2; \alpha_2 \subset \alpha_2; \cdots; \alpha_{n-1} \subset \alpha_n$ formula $\alpha_n \subset \alpha_1$ is added.

Consequence 6 [Circular Parallelism of Informational Includedness] *Let for informational entities $\alpha_1, \alpha_2, \cdots, \alpha_n$ be*

$$\alpha_i \subset \alpha_{i+1}; \ i = 1, 2, \cdots, n-1;$$
$$\alpha_n \subset \alpha_1$$

This system causes a circularly complete informational system of entities $\alpha_1, \alpha_2, \cdots, \alpha_n$ (Definition 3). □

Proof. According to the transitivity of informational includedness (operator $\subset$) (Consequence 3) there is, evidently,

$$\begin{pmatrix} \alpha_1 \subset \alpha_2; \\ \alpha_2 \subset \alpha_3; \\ \vdots \\ \alpha_{n-1} \subset \alpha_n; \\ \alpha_n \subset \alpha_1 \end{pmatrix} \Longrightarrow \begin{pmatrix} \alpha_n \subset \alpha_{n-1}; \\ \alpha_{n-1} \subset \alpha_{n-2}; \\ \vdots \\ \alpha_2 \subset \alpha_1; \\ \alpha_1 \subset \alpha_n \end{pmatrix}$$

The left sequence of formulas appears in an cyclic ascending order while the right sequence is cyclically descending (an opposite direction of the cycle). This obviously yields

$$\begin{pmatrix} \alpha_i \subset \alpha_{i+1}; \\ i = 1, 2, \cdots, n-1; \\ \alpha_n \subset \alpha_1 \end{pmatrix} \rightleftharpoons \begin{pmatrix} \alpha_i \subset \alpha_j; \\ i, j = 1, 2, \cdots, n \end{pmatrix}$$

The last formula shows the power of includable circularity, where

$$\begin{pmatrix} \alpha_i \subset \alpha_j; \\ i, j = 1, 2, \cdots, n \end{pmatrix} \Longrightarrow \begin{pmatrix} \alpha_i \models \alpha_j; \\ i, j = 1, 2, \cdots, n \end{pmatrix}$$

The power of includable circularity (operator $\subset$) is stronger than that of the direct circularity of informing (operator $\models$), because still the Ξ-extensional cases have to be considered. Q.E.D.

4.4 Serialism of Includedness

Let us study the possible cases of serialism of informational includedness and their consequences in regard to informing among entities. In the first step we study a straightforward serialism, and in the second step a circular one.

Definition 4 [Serialism of Informational Includedness] *Let us introduce the subscripted forms $\Phi_i^{\subset}$ of serialism concerning the informational includedness of entities $\alpha_1, \alpha_2, \cdots, \alpha_n$ and mark them in the following way:*

$$\Phi^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n) \Longrightarrow$$

$$\begin{pmatrix} \Phi_1^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n) \rightleftharpoons \\ (\alpha_1 \subset^* (\alpha_2 \subset (\alpha_3 \subset (\cdots (\alpha_{n-1} \subset \alpha_n) \cdots)))); \\ \Phi_2^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n) \rightleftharpoons \\ ((\alpha_1 \subset \alpha_2) \subset^* (\alpha_3 \subset (\cdots (\alpha_{n-1} \subset \alpha_n) \cdots))); \\ \vdots \\ \Phi_{n-1}^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n) \rightleftharpoons \\ (((\cdots ((\alpha_1 \subset \alpha_2) \subset \alpha_3) \cdots) \subset \alpha_{n-1}) \subset^* \alpha_n) \end{pmatrix}$$

By the asterisk marked operators $\subset$, that is, $\subset^*$, the main separators between the informer and the observer part of the expression are meant. Implicational operator $\Longrightarrow$ marks only specific includable serial cases on its right side (but not all possible cases). □

Example [Includedness of Includedness] Let us see what does the example of includedness of includedness, for example,

$$(\alpha \subset \beta) \subset \gamma$$

mean? Let us extendingly interpret this formula in a regular informational manner, when,

$$\left(\begin{pmatrix} \beta \models \alpha; \\ \alpha \models \beta; \\ \Xi(\alpha \subset \beta) \end{pmatrix} \subset \gamma \right) \rightleftharpoons$$

$$\begin{pmatrix} \gamma \models \begin{pmatrix} \beta \models \alpha; \\ \alpha \models \beta; \\ \Xi(\alpha \subset \beta) \end{pmatrix}; \\ \begin{pmatrix} \beta \models \alpha; \\ \alpha \models \beta; \\ \Xi(\alpha \subset \beta) \end{pmatrix} \models \gamma; \\ \Xi \left(\begin{pmatrix} \beta \models \alpha; \\ \alpha \models \beta; \\ \Xi(\alpha \subset \beta) \end{pmatrix} \subset \gamma \right) \end{pmatrix}$$

The right part of the formula can be linearly decomposed (informationally multiplied), e.g.,

$$((\alpha \subset \beta) \subset \gamma) \rightleftharpoons$$

$$\begin{pmatrix} \gamma \models (\beta \models \alpha); \gamma \models (\alpha \models \beta); \gamma \models \Xi(\alpha \subset \beta); \\ (\beta \models \alpha) \models \gamma; (\alpha \models \beta) \models \gamma; \Xi(\alpha \subset \beta) \models \gamma; \\ \Xi((\beta \models \alpha) \subset \gamma); \Xi((\alpha \models \beta) \subset \gamma); \\ \Xi((\Xi(\alpha \subset \beta) \subset \gamma) \end{pmatrix}$$

where for a maximal case of informationally (mutually) involved entities α, β, and γ, there is,

$$\Xi_{\beta,\alpha}^{\beta,\alpha}(\alpha \subset \beta) \rightleftharpoons \begin{pmatrix} (\beta \models \alpha) \subset \beta, \alpha; \\ (\alpha \models \beta) \subset \beta, \alpha \end{pmatrix};$$

$$\Xi_{\gamma,(\beta \models \alpha)}^{\gamma,(\beta \models \alpha)}((\beta \models \alpha) \subset \gamma) \rightleftharpoons$$

$$\begin{pmatrix} (\gamma \models (\beta \models \alpha)) \subset \gamma, (\beta \models \alpha); \\ ((\beta \models \alpha) \models \gamma) \subset \gamma, (\beta \models \alpha) \end{pmatrix};$$

$$\Xi_{\gamma,(\alpha \models \beta)}^{\gamma,(\alpha \models \beta)}((\alpha \models \beta) \subset \gamma) \rightleftharpoons$$

$$\begin{pmatrix} (\gamma \models (\alpha \models \beta)) \subset \gamma, (\alpha \models \beta); \\ ((\alpha \models \beta) \models \gamma) \subset \gamma, (\alpha \models \beta) \end{pmatrix};$$

$$\Xi_{\gamma,\Xi(\alpha \subset \beta)}^{\gamma,\Xi(\alpha \subset \beta)}(\Xi_{\beta,\alpha}^{\beta,\alpha}(\alpha \subset \beta) \subset \gamma) \rightleftharpoons$$

$$\begin{pmatrix} (\gamma \models \Xi_{\beta,\alpha}^{\beta,\alpha}(\alpha \subset \beta)) \subset \gamma, \Xi_{\beta,\alpha}^{\beta,\alpha}(\alpha \subset \beta); \\ (\Xi_{\beta,\alpha}^{\beta,\alpha}(\alpha \subset \beta) \models \gamma) \subset \gamma, \Xi_{\beta,\alpha}^{\beta,\alpha}(\alpha \subset \beta) \end{pmatrix}$$

We see how the complexity of informational includedness rapidly grows by the number of involved entities.

That which has to be clearly kept in mind is

$$((\alpha \subset \beta) \subset \gamma) \not\Leftrightarrow (\alpha, \beta \subset \gamma)$$

for only the process $\alpha \subset \beta$ is included in γ, but not α and β (a property of *non*-transitivity in case of informational includedness).

Another informational property which follows from extensions $\Xi^{\gamma,(\beta\models\alpha)}_{\gamma,(\beta\models\alpha)}((\beta \models \alpha) \subset \gamma)$ and $\Xi^{\gamma,(\alpha\models\beta)}_{\gamma,(\alpha\models\beta)}((\alpha \models \beta) \subset \gamma)$ is a consequent descending and ascending circularity in respect to γ, that is,

$$\begin{array}{l} (\gamma \models (\beta \models \alpha)) \models \gamma; \\ \gamma \models ((\beta \models \alpha) \models \gamma); \\ (\gamma \models (\alpha \models \beta)) \models \gamma; \\ \gamma \models ((\alpha \models \beta) \models \gamma) \end{array}$$

respectively. Other, mixed cycles, are also evident. □

Consequence 7 [A Consequence of Serialism of Informational Includedness Concerning the Informing] *A consequence of Definition 4 is simply the following:*

$$\Phi^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n) \Longrightarrow \begin{pmatrix} \Phi^{\models}(\alpha_1, \alpha_2, \cdots, \alpha_n); \\ \Phi^{\models}(\alpha_n, \alpha_{n-1}, \cdots, \alpha_1) \end{pmatrix}$$

where for a number $n \geq 2$ of involved entities

$$\Phi^{\models}(\alpha_1, \alpha_2, \cdots, \alpha_n) \rightleftharpoons$$

$$\begin{pmatrix} \Phi^{\models}_1(\alpha_1, \alpha_2, \cdots, \alpha_n) \rightleftharpoons \\ (\alpha_1 \models^* (\alpha_2 \models (\alpha_3 \models (\cdots(\alpha_{n-1} \models \\ \qquad \alpha_n)\cdots)))); \\ \Phi^{\models}_2(\alpha_1, \alpha_2, \cdots, \alpha_n) \rightleftharpoons \\ ((\alpha_1 \models \alpha_2) \models^* (\alpha_3 \models (\cdots(\alpha_{n-1} \models \\ \qquad \alpha_n)\cdots))); \\ \vdots \\ \Phi^{\models}_{n-1}(\alpha_1, \alpha_2, \cdots, \alpha_n) \rightleftharpoons \\ (((\cdots((\alpha_1 \models \alpha_2) \models \alpha_3)\cdots) \models \alpha_{n-1}) \models^* \\ \qquad \alpha_n) \end{pmatrix}$$

and adequately

$$\Phi^{\models}(\alpha_n, \alpha_{n-1}, \cdots, \alpha_1) \rightleftharpoons$$

$$\begin{pmatrix} \Phi^{\models}_1(\alpha_n, \alpha_{n-1}, \cdots, \alpha_1) \rightleftharpoons \\ (\alpha_n \models^* (\alpha_{n-1} \models (\alpha_{n-2} \models (\cdots(\alpha_2 \models \\ \qquad \alpha_1)\cdots)))); \\ \Phi^{\models}_2(\alpha_n, \alpha_{n-1}, \cdots, \alpha_1) \rightleftharpoons \\ ((\alpha_n \models \alpha_{n-1}) \models^* (\alpha_{n-2} \models (\cdots(\alpha_2 \models \\ \qquad \alpha_1)\cdots))); \\ \vdots \\ \Phi^{\models}_{n-1}(\alpha_n, \alpha_{n-1}, \cdots, \alpha_1) \rightleftharpoons \\ (((\cdots((\alpha_n \models \alpha_{n-1}) \models \alpha_{n-2})\cdots) \models \alpha_2) \models^* \\ \qquad \alpha_1) \end{pmatrix}$$

is the ascending and descending (counterascending) serial sequence of informing in respect to the greatest subscript n. □

Proof. The last consequence considers only the ascending and descending sequences of entities $\alpha_1, \alpha_2, \cdots, \alpha_n$ in respect to the subscript n. Informational entities $\Phi^{\models}(\alpha_1, \alpha_2, \cdots, \alpha_n)$ and $\Phi^{\models}(\alpha_n, \alpha_{n-1}, \cdots, \alpha_1)$ are evident consequences of entity $\Phi^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n)$. There exists even a stronger consequence of Definition 4, as presented by the next consequence. Q.E.D.

Consequence 8 [A General Consequence of an Ordered Serialism of Informational Includedness] *The following implication represents the most general system of the ordered serial includedness:*

$$\Phi^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n) \Longrightarrow$$

$$\begin{pmatrix} \Phi^{\models}_{ij}(\alpha_i, \alpha_{i+1}, \cdots, \alpha_j); \\ \Phi^{\models}_{ij}(\alpha_j, \alpha_{j-1}, \cdots, \alpha_i); \\ 1 \leq i;\ i < j;\ j \leq n; \\ i = 1, 2, \cdots, n-1; \\ j = 2, 3, \cdots, n \end{pmatrix}$$

where $(i, i+1, \cdots, j)$ and $(j, j-1, \cdots, i)$ is an ascending and descending interval (of natural numbers), respectively, and the length $\ell(\Phi^{\models}_{ij}(\alpha_i, \alpha_{i+1}, \cdots, \alpha_j))$ is between 2 and n. □

Proof. Except by the consequence determined serial sequences, there exist other, 'non-ordered' sequences as can be easily recognized from the previous example. That is, besides the (alphabetically, numerically) 'ordered' sequences, proceeding from $\Phi^{\subset}(\alpha, \beta, \gamma)$ for $\ell = 3$, that is,

$$(\alpha \models \beta) \models \gamma;\ \alpha \models (\beta \models \gamma);$$
$$\gamma \models (\beta \models \alpha);\ (\gamma \models \beta) \models \alpha$$

there are 'non-ordered' sequences

$$(\beta \models \gamma) \models \alpha;\ \beta \models (\gamma \models \alpha);$$
$$(\beta \models \alpha) \models \gamma;\ \beta \models (\alpha \models \gamma);$$
$$(\gamma \models \alpha) \models \beta;\ \gamma \models (\alpha \models \beta)$$

etc. and infinitely many others, recursively arising serial sequences. In this respect, $\Phi^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n)$ symbolizes the possible appearance of all $\models$-serial formulas concerning operands $\alpha_1, \alpha_2, \cdots, \alpha_n$. Therefore, symbol $\Longrightarrow$ is used instead of $\rightleftharpoons$ in the last definition and consequences. Q.E.D.

4.5 Circular Serialism of Includedness

Circularity belongs to the most significant faculties of informational serialism. By circular informational formulas the most complex and various phenomena concerning cognitive, intelligent, and understanding processes and entities can be not only conceptualized, but brought into positions and attitudes of informational arising (informational autopoiesis, self-reference, consciousness, etc.). This level of circularity, caused by cyclic informational includedness, reaches its highest point within the circular metaphysicalism.

Definition 5 [Circular Serialism of Informational Includedness] *Let us introduce the markers $\Phi_{\circlearrowleft}^{\subset}$ of circular serialism concerning the informational includedness of entities $\alpha_1, \alpha_2, \cdots, \alpha_n$ in cyclic respect to entity α_1 and mark them as follows:*

$$\Phi_{\circlearrowleft}^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1) \Longrightarrow$$
$$\begin{pmatrix} \Phi_{\circlearrowleft_1}^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1) \rightleftharpoons \\ (\alpha_1 \subset^* (\alpha_2 \subset (\alpha_3 \subset (\cdots(\alpha_n \subset \alpha_1)\cdots)))); \\ \Phi_{\circlearrowleft_2}^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1) \rightleftharpoons \\ ((\alpha_1 \subset \alpha_2) \subset^* (\alpha_3 \subset (\cdots(\alpha_n \subset \alpha_1)\cdots))); \\ \vdots \\ \Phi_{\circlearrowleft_n}^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1) \rightleftharpoons \\ (((\cdots((\alpha_1 \subset \alpha_2) \subset \alpha_3)\cdots) \subset \alpha_n) \subset^* \alpha_1) \end{pmatrix}$$

By the asterisk marked operators of includedness ($\subset^*$) the main separators between the informing (cyclic informer) and the observing (cyclic observer) part of cyclically structured expression are meant. □

Consequence 9 [A Consequence of Circular Serialism of Informational Includedness Concerning the Informing] *A consequence of Definition 5 is simply the following*

$$\Phi_{\circlearrowleft}^{\subset}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1) \Longrightarrow \begin{pmatrix} \Phi_{\circlearrowleft}^{\models}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1); \\ \Phi_{\circlearrowleft}^{\models}(\alpha_1, \alpha_n, \alpha_{n-1}, \cdots, \alpha_1) \end{pmatrix}$$

where for $n \geq 2$

$$\Phi_{\circlearrowleft}^{\models}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1) \rightleftharpoons$$
$$\begin{pmatrix} \Phi_{\circlearrowleft_1}^{\models}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1) \rightleftharpoons \\ (\alpha_1 \models^* (\alpha_2 \models (\alpha_3 \models (\cdots(\alpha_{n-1} \models \\ (\alpha_n \models \alpha_1))\cdots)))); \\ \Phi_{\circlearrowleft_2}^{\models}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1) \rightleftharpoons \\ ((\alpha_1 \models \alpha_2) \models^* (\alpha_3 \models (\cdots(\alpha_{n-1} \models \\ (\alpha_n \models \alpha_1))\cdots))); \\ \vdots \\ \Phi_{\circlearrowleft_n}^{\models}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1) \rightleftharpoons \\ (((\cdots((\alpha_1 \models \alpha_2) \models \alpha_3)\cdots) \models \alpha_{n-1}) \models \\ \alpha_n) \models^* \alpha_1) \end{pmatrix}$$

and adequately

$$\Phi_{\circlearrowleft}^{\models}(\alpha_1, \alpha_n, \alpha_{n-1}, \cdots, \alpha_1) \rightleftharpoons$$
$$\begin{pmatrix} \Phi_{\circlearrowleft_1}^{\models}(\alpha_1, \alpha_n, \alpha_{n-1}, \cdots, \alpha_1) \rightleftharpoons \\ (\alpha_1 \models^* (\alpha_n \models (\alpha_{n-1} \models (\alpha_{n-2} \models (\cdots(\alpha_2 \\ \models \alpha_1)\cdots))))); \\ \Phi_{\circlearrowleft_2}^{\models}(\alpha_1, \alpha_n, \alpha_{n-1}, \cdots, \alpha_1) \rightleftharpoons \\ ((\alpha_1 \models \alpha_n) \models^* (\alpha_{n-1} \models (\alpha_{n-2} \models (\cdots(\alpha_2 \\ \models \alpha_1)\cdots)))); \\ \vdots \\ \Phi_{\circlearrowleft_n}^{\models}(\alpha_1, \alpha_n, \alpha_{n-1}, \cdots, \alpha_1) \rightleftharpoons \\ (((\cdots(((\alpha_1 \models \alpha_n) \models \alpha_{n-1}) \models \alpha_{n-2})\cdots)) \\ \models \alpha_2) \models^* \alpha_1) \end{pmatrix}$$

is the ascending and descending (counterascending) circularly serial sequence of informing in respect to circular subscript 1. □

Proof. The cyclicity in respect to entity α is in the α_1's property to be in the position, together with other entities or alone, of the informer (left of operator $\models^*$) and the observer (right of operator $\models^*$), simultaneously. As we see, the cyclicity for other entities than α_1 can not be derived merely from the premise $\Phi^{\subseteq}_{\circlearrowright}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1)$.

The last consequence considers only the ascending and descending circular sequences of entities $\alpha_1, \alpha_2, \cdots, \alpha_n$ in respect to entity α_1. Entities $\Phi^{\models}_{\circlearrowright}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1)$ and $\Phi^{\models}_{\circlearrowleft}(\alpha_n, \alpha_{n-1}, \cdots, \alpha_1, \alpha_1)$ are evident consequences of entity $\Phi^{\subseteq}_{\circlearrowright}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1)$. But, there exists a stronger circular and non-circular (linear serial) consequence of Definition 5, as presented by the next consequence. Q.E.D.

According to Consequence 8 it is possible to deduce a similar consequence concerning the ordered cyclical serialism of informational includedness, where complex circular informational entities (operands), considering $\alpha_i \subset \alpha_j$,

$$\Phi^{\models}_{\circlearrowright_{ij}}(\alpha_i, \alpha_{i+1}, \cdots, \alpha_j, \alpha_i);$$
$$\Phi^{\models}_{\circlearrowleft_{ij}}(\alpha_i, \alpha_j, \alpha_{j-1}, \cdots, \alpha_i)$$

come to existence. The second formula can be viewed as a *countercycle* of the first formula, that is,

$$\Phi^{\models}_{\circlearrowleft_{ij}}(\alpha_i, \alpha_j, \alpha_{j-1}, \cdots, \alpha_i) \rightleftharpoons \Phi^{\models}_{\circlearrowright_{ij}}(\alpha_i, \alpha_{i+1}, \cdots, \alpha_j, \alpha_i)$$

where the difference is in $\circlearrowleft$ and $\circlearrowright$ subscript operator, respectively.

Consequence 10 [A General Consequence of an Ordered Circular Serialism of Informational Includedness] *The following implication represents the most general system of the ordered circular serial includedness:*

$$\Phi^{\subseteq}_{\circlearrowright}(\alpha_1, \alpha_2, \cdots, \alpha_n, \alpha_1) \Longrightarrow$$
$$\left(\begin{array}{l} \Phi^{\models}_{\circlearrowright_{ij}}(\alpha_i, \alpha_{i+1}, \cdots, \alpha_j, \alpha_i); \\ \Phi^{\models}_{\circlearrowleft_{ij}}(\alpha_i, \alpha_j, \alpha_{j-1}, \cdots, \alpha_i); \\ 1 \leq i;\ i < j;\ j \leq n; \\ i = 1, 2, \cdots, n-1; \\ j = 2, 3, \cdots, n \end{array}\right)$$

where $(i, i+1, \cdots, j, i)$ *and* $(j, j-1, \cdots, i, j)$ *is an ascending and descending circular interval (of natural numbers), respectively, and considering* $\alpha_i \subset \alpha_j$. □

A Comment. Circularly serial informational includedness causes an infinite number of possible cycles and subcycles of involved entities. This fact offers the possibilities of choice in a particular case and enables the syntactic and semantic diversity of arising informational cases.

4.6 Externalism, Internalism, and Phenomenalism of Includedness

Let us interpret additionally the appearance of includable externalism, internalism, and phenomenalism with the sense of their introduction into informational discourse.

Informational externalism of includedness concerning an entity says that the entity is a subunit of as yet undetermined informational unit ($\alpha \subset$). The question of the subunit embracing unit is left open and the identification of an adequate unit will appear as the consequence of the happening circumstances (e.g., within an arising formula system). Usually, on the most general level, we have the informational externalism (marked by $\alpha \models$). We arrive to the includable externalism through particularization of operator $\models$, replacing it by operator $\subset$. But, as we have recognized (Definition 1), the includedness (characterized by operator $\subset$) is a recursively and complexly determined form of informationalism (characterized by operator $\models$).

Informational internalism of includedness concerning an entity is a 'reverse' problem to informational externalism and says that the entity is a unit of as yet undetermined informational subunit(s) ($\subset \alpha$). The question of the unit including subunit(s) is left open and the identification (decomposition) of an adequate subunit will appear as the consequence of the happening circumstances (e.g., within an arising formula system). Usually, on the most general level, we have the informational internalism (marked by $\models \alpha$). We arrive to the includable internalism through particularization of operator $\models$, replacing it by operator $\subset$.

Informational phenomenalism of includedness concerning an entity is an informational system of includable externalism and internalism and says that the entity is simultaneously a subunit of as yet undetermined informational unit(s) and a unit of as yet undetermined informational subunit(s) ($\alpha \subset; \subset \alpha$). The questions of the subunit embracing unit(s) and the unit including subunit(s) are left open and the identification (composition and decomposition) of adequate unit(s) and subunit(s) will come to the surface as a consequence of the happening circumstances (e.g., within a complexly arising formula system). Usually, on the most general level, we have the informational complex of externalism and internalism (marked by $\models \alpha; \models \alpha$). We arrive to the includable phenomenalism through particularization of operator types $\models$, replacing them by operator types $\subset$.

4.7 Metaphysicalism of Includedness

Includable metaphysicalism concerns informational parallelism and serialism of several distinguished entities and is a consequence of the general metaphysical structure belonging to an informing entity. To understand the includable metaphysicalism, we have to show the circular parallel and serial schemes (metaphysical shells, structures) of very particular subentities behind (within) the informing entity.

The metaphysical informing of an entity—its metaphysicalism—is constituted by its subentities, which are pragmatically classified as informing, counterinforming, and informational embedding. It is understood that each of these entities has two components: an entity as an informational operand and its explicitly informing (acting) component. Thus, the entire entity α has its specific informing component $\mathcal{I}_\alpha$ in the sense of $\mathcal{I}_\alpha \subset \alpha$. Furthermore, informing of entity α informationally includes the so-called counterinforming of α, marked by $\mathcal{C}_\alpha$. This fact is expressed by the includable formula $\mathcal{C}_\alpha \subset \mathcal{I}_\alpha$. Counterinforming as an active component produces the counterinformational entity γ_α, which is through counterinforming arisen counterinformation. It has to be informationally connected (included) to the frame informational entity α through a distinguished component of informing, called informational embedding and marked by $\mathcal{E}_\alpha$. It is understood that this embedding component is a consequence of the counterinformational entity γ_α in the sense $\mathcal{E}_\alpha \subset \gamma_\alpha$. Informational embedding as an active component of α produces the γ_α-connective informational entity in respect to the frame entity α. This component is marked by ε_α and the corresponding formula of includedness is $\varepsilon_\alpha \subset \mathcal{E}_\alpha$. Last but not least, the embedding informational product ε_α includes α through $\alpha \subset \varepsilon$. By this, the includable cycle of α's metaphysical components comes into existence.

Metaphysicalism of includedness pertaining to an informing entity can unite the metaphysical parallelism and serialism within the entity. This metaphysical complexity of includedness ensures the most powerful and perplexed informational spontaneity (arising) and circularity. By a pragmatical way of filling the complex metaphysical shell, intelligent informational entities can come into appearance.

4.7.1 Metaphysical Parallelism of Includedness

That which we have intuitively described as a basic metaphysical system of an informing entity is, according to Consequence 6, a circular parallelism of informational includedness.

Definition 6 [Metaphysical Parallelism of Informational Includedness Pertaining to an Entity] *Let entities $\mathcal{I}_\alpha$, $\mathcal{C}_\alpha$, γ_α, $\mathcal{E}_\alpha$, and ε_α be metaphysical components of entity α, called α-s informing, counterinforming, counterinformational entity, informational embedding, and informational embedding entity, respectively. Then the following metaphysical, that is circular includable parallelism of the form*

$$\text{entity } \alpha \left\{ \begin{array}{l} \left.\begin{array}{l} \alpha \subset \varepsilon_\alpha; \\ \varepsilon_\alpha \subset \mathcal{E}_\alpha; \end{array}\right\} \alpha\text{'s embedding} \\ \left.\begin{array}{l} \mathcal{E}_\alpha \subset \gamma_\alpha; \\ \gamma_\alpha \subset \mathcal{C}_\alpha \end{array}\right\} \alpha\text{'s counterinforming} \\ \left.\begin{array}{l} \mathcal{C}_\alpha \subset \mathcal{I}_\alpha; \\ \mathcal{I}_\alpha \subset \alpha \end{array}\right\} \alpha\text{'s informing} \end{array} \right.$$

exists. This kind of includedness is called the complete includable parallelism. □

Consequence 11 [Metaphysical Parallelism of Informing Pertaining to an Entity] *A consequence of Definition 6 is the following:*

$$\begin{pmatrix} \alpha \subset \varepsilon_\alpha; \\ \varepsilon_\alpha \subset \mathcal{E}_\alpha; \\ \mathcal{E}_\alpha \subset \gamma_\alpha; \\ \gamma_\alpha \subset \mathcal{C}_\alpha; \\ \mathcal{C}_\alpha \subset \mathcal{I}_\alpha; \\ \mathcal{I}_\alpha \subset \alpha \end{pmatrix} \Longrightarrow \begin{pmatrix} \alpha \models \varepsilon_\alpha; & \alpha \models \mathcal{I}_\alpha; \\ \varepsilon_\alpha \models \mathcal{E}_\alpha; & \mathcal{I}_\alpha \models \mathcal{C}_\alpha; \\ \mathcal{E}_\alpha \models \gamma_\alpha; & \mathcal{C}_\alpha \models \gamma_\alpha; \\ \gamma_\alpha \models \mathcal{C}_\alpha; & \gamma_\alpha \models \mathcal{E}_\alpha; \\ \mathcal{C}_\alpha \models \mathcal{I}_\alpha; & \mathcal{E}_\alpha \models \varepsilon_\alpha; \\ \mathcal{I}_\alpha \models \alpha; & \varepsilon_\alpha \models \alpha \end{pmatrix}$$

The columns right of $\Longrightarrow$ are parallel metaphysical cycles and, simultaneously, they constitute a so-called double metaphysical cycle with its first (left column) and second (right column) transition. These cycles are countercyclical to each other. □

The validity of the last consequence is evident and can be derived from Definition 1 and Consequence 3.

Consequence 12 [A Weak Metaphysical Parallelism of Informing Pertaining to an Entity] *For the consequent of Consequence 11 even a weaker (more natural) condition suffices, that is,*

$$\begin{pmatrix} \varepsilon_\alpha \subset \mathcal{E}_\alpha; \\ \mathcal{E}_\alpha \subset \gamma_\alpha; \\ \gamma_\alpha \subset \mathcal{C}_\alpha; \\ \mathcal{C}_\alpha \subset \mathcal{I}_\alpha; \\ \mathcal{I}_\alpha \subset \alpha \end{pmatrix} \Longrightarrow \begin{pmatrix} \alpha \models \varepsilon_\alpha; & \alpha \models \mathcal{I}_\alpha; \\ \varepsilon_\alpha \models \mathcal{E}_\alpha; & \mathcal{I}_\alpha \models \mathcal{C}_\alpha; \\ \mathcal{E}_\alpha \models \gamma_\alpha; & \mathcal{C}_\alpha \models \gamma_\alpha; \\ \gamma_\alpha \models \mathcal{C}_\alpha; & \gamma_\alpha \models \mathcal{E}_\alpha; \\ \mathcal{C}_\alpha \models \mathcal{I}_\alpha; & \mathcal{E}_\alpha \models \varepsilon_\alpha; \\ \mathcal{I}_\alpha \models \alpha; & \varepsilon_\alpha \models \alpha \end{pmatrix}$$

This consequence does not require the explicit condition $\alpha \subset \varepsilon_\alpha$ which closes the includable metaphysicalism in a circular manner. □

Proof. Because of the transitivity of informational includedness (Consequence 3), there is

$$\begin{pmatrix} \varepsilon_\alpha \subset \mathcal{E}_\alpha; \\ \mathcal{E}_\alpha \subset \gamma_\alpha; \\ \gamma_\alpha \subset \mathcal{C}_\alpha; \\ \mathcal{C}_\alpha \subset \mathcal{I}_\alpha; \\ \mathcal{I}_\alpha \subset \alpha \end{pmatrix} \Longrightarrow (\varepsilon_\alpha \subset \alpha)$$

This consequence yields (Definition 1)

$$(\varepsilon_\alpha \subset \alpha) \rightleftharpoons \begin{pmatrix} \alpha \models \varepsilon_\alpha; \\ \varepsilon_\alpha \models \alpha; \\ \Xi(\varepsilon_\alpha \subset \alpha) \end{pmatrix}$$

Thus, the necessary transitions $\alpha \models \varepsilon_\alpha$ and $\varepsilon_\alpha \models \alpha$ exist. Q.E.D.

Consequence 13 [A Further Metaphysical Parallelism Pertaining to an Entity] *A further useful consequence of the parallel metaphysical includedness is*

$$\begin{pmatrix} \varepsilon_\alpha \subset \mathcal{E}_\alpha; \\ \mathcal{E}_\alpha \subset \gamma_\alpha; \\ \gamma_\alpha \subset \mathcal{C}_\alpha; \\ \mathcal{C}_\alpha \subset \mathcal{I}_\alpha; \\ \mathcal{I}_\alpha \subset \alpha \end{pmatrix} \Longrightarrow \begin{pmatrix} \varepsilon_\alpha, \mathcal{E}_\alpha, \gamma_\alpha, \mathcal{C}_\alpha \mathcal{I}_\alpha \subset \alpha; \\ \varepsilon_\alpha, \mathcal{E}_\alpha, \gamma_\alpha, \mathcal{C}_\alpha \subset \mathcal{I}_\alpha; \\ \varepsilon_\alpha, \mathcal{E}_\alpha, \gamma_\alpha \subset \mathcal{C}_\alpha; \\ \varepsilon_\alpha, \mathcal{E}_\alpha \subset \gamma_\alpha; \\ \varepsilon_\alpha \subset \mathcal{E}_\alpha \end{pmatrix}$$

which follows directly from Consequence 3. □

4.7.2 Metaphysical Serialism of Includedness

In parallel to metaphysical parallelism of informational includedness there exists the metaphysical serialism of informational includedness which can offer the cyclically most perplexed, interwoven, and involved possibilities, by which intelligent, understanding, or cognitive scenarios (processes, entities) can be constructed.

Definition 7 [Partial Metaphysical Serialism of Informational Includedness Pertaining to an Entity] *Let entities $\mathcal{I}_\alpha$, $\mathcal{C}_\alpha$, γ_α, $\mathcal{E}_\alpha$, and ε_α be metaphysical components of entity α, called α's informing, counterinforming, counterinformational entity, informational embedding, and informational embedding entity, respectively. Then, for example, the following metaphysical and reverse metaphysical, that is circular includable serialisms of the form*

$$\overbrace{\Bigg(\bigg(\Big(\big(\underbrace{(\alpha \subset \mathcal{I}_\alpha) \subset}_{\text{informing}} \underbrace{\mathcal{C}_\alpha) \subset \gamma_\alpha\big) \subset}_{\text{counterinforming}} \underbrace{\mathcal{E}_\alpha\Big) \subset \varepsilon_\alpha\bigg) \subset \alpha}_{\text{embedding}}}^{\text{metaphysical informing of } \alpha \text{ as a whole}}$$

and

$$\overbrace{\Bigg(\bigg(\Big(\big(\underbrace{(\alpha \subset \varepsilon_\alpha) \subset \mathcal{E}_\alpha\big)}_{\text{r-embedding}} \underbrace{\subset \gamma_\alpha\Big) \subset \mathcal{C}_\alpha\bigg)}_{\text{r-counterinforming}} \underbrace{\subset \mathcal{I}_\alpha\Bigg) \subset \alpha}_{\text{r-informing}}}^{\text{reverse metaphysical informing of } \alpha \text{ as a whole}}$$

can exist, respectively. This kind of includedness is called the partial includable serialism. *In the second formula, r-embedding, r-counterinforming, and r-informing mark reverse embedding, reverse counterinforming, and reverse informing, respectively.* □

Definition 8 [Multiform Metaphysical Serialism of Informational Includedness Pertaining to an Entity] *According to Definition 7 for two basic forms (ascending, marked by $\Upsilon_{\underset{\circ}{\subset}}(\alpha)$, and descending or reverse includable metaphysical cycle, marked by $\Upsilon_{\underset{\circ}{\supset}}$, of an entity α), the multiform metaphysical serialism is obtained by considering of all possible positions of the parenthesis pairs, that is,*

$$\Upsilon^{\subseteq}_{\mho}(\alpha) \rightleftharpoons$$

$$\begin{pmatrix} (((((\alpha \subset \mathcal{I}_\alpha) \subset \mathcal{C}_\alpha) \subset \gamma_\alpha) \subset \mathcal{E}_\alpha) \subset \varepsilon_\alpha) \subset^* \alpha; \\ ((((\alpha \subset \mathcal{I}_\alpha) \subset \mathcal{C}_\alpha) \subset \gamma_\alpha) \subset \mathcal{E}_\alpha) \subset^* (\varepsilon_\alpha \subset \alpha); \\ (((\alpha \subset \mathcal{I}_\alpha) \subset \mathcal{C}_\alpha) \subset \gamma_\alpha) \subset^* (\mathcal{E}_\alpha \subset (\varepsilon_\alpha \subset \alpha)); \\ ((\alpha \subset \mathcal{I}_\alpha) \subset \mathcal{C}_\alpha) \subset^* (\gamma_\alpha \subset (\mathcal{E}_\alpha \subset (\varepsilon_\alpha \subset \alpha))); \\ (\alpha \subset \mathcal{I}_\alpha) \subset^* (\mathcal{C}_\alpha \subset (\gamma_\alpha \subset (\mathcal{E}_\alpha \subset (\varepsilon_\alpha \subset \alpha)))); \\ \alpha \subset^* (\mathcal{I}_\alpha \subset (\mathcal{C}_\alpha \subset (\gamma_\alpha \subset (\mathcal{E}_\alpha \subset (\varepsilon_\alpha \subset \alpha))))) \end{pmatrix}$$

and

$$\Upsilon^{\subseteq}_{\mho}(\alpha) \rightleftharpoons$$

$$\begin{pmatrix} (((((\alpha \subset \varepsilon_\alpha) \subset \mathcal{E}_\alpha) \subset \gamma_\alpha) \subset \mathcal{C}_\alpha) \subset \mathcal{I}_\alpha) \subset^* \alpha; \\ ((((\alpha \subset \varepsilon_\alpha) \subset \mathcal{E}_\alpha) \subset \gamma_\alpha) \subset \mathcal{C}_\alpha) \subset^* (\mathcal{I}_\alpha \subset \alpha); \\ (((\alpha \subset \varepsilon_\alpha) \subset \mathcal{E}_\alpha) \subset \gamma_\alpha) \subset^* (\mathcal{C}_\alpha \subset (\mathcal{I}_\alpha \subset \alpha)); \\ ((\alpha \subset \varepsilon_\alpha) \subset \mathcal{E}_\alpha) \subset^* (\gamma_\alpha \subset (\mathcal{C}_\alpha \subset (\mathcal{I}_\alpha \subset \alpha))); \\ (\alpha \subset \varepsilon_\alpha) \subset^* (\mathcal{E}_\alpha \subset (\gamma_\alpha \subset (\mathcal{C}_\alpha \subset (\mathcal{I}_\alpha \subset \alpha)))); \\ \alpha \subset^* (\varepsilon_\alpha \subset (\mathcal{E}_\alpha \subset (\gamma_\alpha \subset (\mathcal{C}_\alpha \subset (\mathcal{I}_\alpha \subset \alpha))))) \end{pmatrix}$$

Such an includable system enables that all possible serial metaphysical cycles of both informing (informer) and observing (observer) come into existence. Thus, in the first formula, α is the main (operator $\subset^*$) metaphysical observer, while in the last formula it is the main metaphysical informer. □

Which are the consequences of includably embedded entities (operands) in a metaphysical case? The reader can construct the answers to this question taking into account the consequences pertaining to circular serialism of includedness (Consequence 9 and 10). There are infinitely many serial and circular-serial metaphysical consequences originating in entities $\Upsilon^{\subseteq}_{\mho}(\alpha)$ and $\Upsilon^{\subseteq}_{\mho}(\alpha)$ of the last definition. Let us see only some of the most interesting.

Consequence 14 [A Short-sized Metaphysical Serialism Pertaining to an Entity] *By introspection of Definition 8, one can prove the following implications concerning the short-sized forms of inclusiveness and informing in a metaphysical case:*

$$\Upsilon^{\subseteq}_{\mho}(\alpha) \Longrightarrow \begin{pmatrix} \alpha \subset \mathcal{I}_\alpha; \\ \varepsilon_\alpha \subset \alpha \end{pmatrix}$$

and

$$\Upsilon^{\subseteq}_{\mho}(\alpha) \Longrightarrow \begin{pmatrix} \alpha \subset \varepsilon_\alpha; \\ \mathcal{I}_\alpha \subset \alpha \end{pmatrix}$$

cause implications

$$\Upsilon^{\subseteq}_{\mho}(\alpha) \Longrightarrow \begin{pmatrix} \mathcal{I}_\alpha \models \alpha; \\ \alpha \models \mathcal{I}_\alpha; \\ \alpha \models \varepsilon_\alpha; \\ \varepsilon_\alpha \models \alpha \end{pmatrix}$$

and

$$\Upsilon^{\subseteq}_{\mho}(\alpha) \Longrightarrow \begin{pmatrix} \varepsilon_\alpha \models \alpha; \\ \alpha \models \varepsilon_\alpha; \\ \alpha \models \mathcal{I}_\alpha; \\ \mathcal{I}_\alpha \models \alpha \end{pmatrix}$$

The last two consequences of short-sized informing pertaining to $\Upsilon^{\subseteq}_{\mho}(\alpha)$ and $\Upsilon^{\subseteq}_{\mho}(\alpha)$ have equal consequents, evidently. □

Of course, the so-called includable extensions have been not considered.

Consequence 15 [A Medium-sized Metaphysical Serialism Pertaining to an Entity] *By introspection of Definition 8, we can prove the following implications concerning the medium-sized forms of inclusiveness and informing in a metaphysical case:*

$$\Upsilon^{\subseteq}_{\mho}(\alpha) \Longrightarrow$$

$$\begin{pmatrix} (\alpha \subset \mathcal{I}_\alpha) \subset \mathcal{C}_\alpha; \\ ((\alpha \subset \mathcal{I}_\alpha) \subset \mathcal{C}_\alpha) \subset \gamma_\alpha; \\ (((\alpha \subset \mathcal{I}_\alpha) \subset \mathcal{C}_\alpha) \subset \gamma_\alpha) \subset \mathcal{E}_\alpha; \\ ((((\alpha \subset \mathcal{I}_\alpha) \subset \mathcal{C}_\alpha) \subset \gamma_\alpha) \subset \mathcal{E}_\alpha) \subset \varepsilon_\alpha; \\ \mathcal{E}_\alpha \subset (\varepsilon_\alpha \subset \alpha); \\ \gamma_\alpha \subset (\mathcal{E}_\alpha \subset (\varepsilon_\alpha \subset \alpha)); \\ \mathcal{C}_\alpha \subset (\gamma_\alpha \subset (\mathcal{E}_\alpha \subset (\varepsilon_\alpha \subset \alpha))); \\ \mathcal{I}_\alpha \subset (\mathcal{C}_\alpha \subset (\gamma_\alpha \subset (\mathcal{E}_\alpha \subset (\varepsilon_\alpha \subset \alpha)))) \end{pmatrix}$$

and

$$\Upsilon^{\subseteq}_{\mho}(\alpha) \Longrightarrow$$

$$\begin{pmatrix} (\alpha \subset \varepsilon_\alpha) \subset \mathcal{E}_\alpha; \\ ((\alpha \subset \varepsilon_\alpha) \subset \mathcal{E}_\alpha) \subset \gamma_\alpha; \\ (((\alpha \subset \varepsilon_\alpha) \subset \mathcal{E}_\alpha) \subset \gamma_\alpha) \subset \mathcal{C}_\alpha; \\ ((((\alpha \subset \varepsilon_\alpha) \subset \mathcal{E}_\alpha) \subset \gamma_\alpha) \subset \mathcal{C}_\alpha) \subset \mathcal{I}_\alpha; \\ \mathcal{C}_\alpha \subset (\mathcal{I}_\alpha \subset \alpha); \\ \gamma_\alpha \subset (\mathcal{C}_\alpha \subset (\mathcal{I}_\alpha \subset \alpha)); \\ \mathcal{E}_\alpha \subset (\gamma_\alpha \subset (\mathcal{C}_\alpha \subset (\mathcal{I}_\alpha \subset \alpha))); \\ \varepsilon_\alpha \subset (\mathcal{E}_\alpha \subset (\gamma_\alpha \subset (\mathcal{C}_\alpha \subset (\mathcal{I}_\alpha \subset \alpha)))) \end{pmatrix}$$

cause implication

$$\Upsilon_{\circlearrowright}^{\subsetneq}(\alpha), \Upsilon_{\circlearrowright}^{\subsetneq}(\alpha) \Longrightarrow$$

$$\begin{pmatrix} (\alpha \models \mathcal{I}_\alpha) \models \mathcal{C}_\alpha; \\ \mathcal{C}_\alpha \models (\mathcal{I}_\alpha \models \alpha); \\ ((\alpha \models \mathcal{I}_\alpha) \models \mathcal{C}_\alpha) \models \gamma_\alpha; \\ \gamma_\alpha \models (\mathcal{C}_\alpha \models (\mathcal{I}_\alpha \models \alpha)); \\ (((\alpha \models \mathcal{I}_\alpha) \models \mathcal{C}_\alpha) \models \gamma_\alpha) \models \mathcal{E}_\alpha; \\ \mathcal{E}_\alpha \models (\gamma_\alpha \models (\mathcal{C}_\alpha \models (\mathcal{I}_\alpha \models \alpha))); \\ ((((\alpha \models \mathcal{I}_\alpha) \models \mathcal{C}_\alpha) \models \gamma_\alpha) \models \mathcal{E}_\alpha) \models \varepsilon_\alpha; \\ \varepsilon_\alpha \models (\mathcal{E}_\alpha \models (\gamma_\alpha \models (\mathcal{C}_\alpha \models (\mathcal{I}_\alpha \models \alpha)))); \\ \mathcal{E}_\alpha \models (\varepsilon_\alpha \models \alpha); \\ (\alpha \models \varepsilon_\alpha) \models \mathcal{E}_\alpha; \\ \gamma_\alpha \models (\mathcal{E}_\alpha \models (\varepsilon_\alpha \models \alpha)); \\ ((\alpha \models \varepsilon_\alpha) \models \mathcal{E}_\alpha) \models \gamma_\alpha; \\ \mathcal{C}_\alpha \models (\gamma_\alpha \models (\mathcal{E}_\alpha \models (\varepsilon_\alpha \models \alpha))); \\ (((\alpha \models \varepsilon_\alpha) \models \mathcal{E}_\alpha) \models \gamma_\alpha) \models \mathcal{C}_\alpha; \\ \mathcal{I}_\alpha \models (\mathcal{C}_\alpha \models (\gamma_\alpha \models (\mathcal{E}_\alpha \models (\varepsilon_\alpha \models \alpha)))); \\ ((((\alpha \models \varepsilon_\alpha) \models \mathcal{E}_\alpha) \models \gamma_\alpha) \models \mathcal{C}_\alpha) \models \mathcal{I}_\alpha \end{pmatrix}$$

Consequents of $\Upsilon_{\circlearrowright}^{\subsetneq}(\alpha)$ and $\Upsilon_{\circlearrowright}^{\subsetneq}(\alpha)$ coincide, evidently. □

Consequence 16 [A Long-sized Metaphysical Serialism Pertaining to an Entity] *By introspection of Definition 8, we can prove the following implication concerning the long-sized form of inclusiveness and informing in a metaphysical case:*

$$\Upsilon_{\circlearrowright}^{\subsetneq}(\alpha), \Upsilon_{\circlearrowright}^{\subsetneq}(\alpha) \Longrightarrow$$

$$\begin{pmatrix} (((((\alpha \models \mathcal{I}_\alpha) \models \mathcal{C}_\alpha) \models \gamma_\alpha) \models \mathcal{E}_\alpha) \models \varepsilon_\alpha) \models^* \alpha; \\ \alpha \models^* (\varepsilon_\alpha \models (\mathcal{E}_\alpha \models (\gamma_\alpha \models (\mathcal{C}_\alpha \models (\mathcal{I}_\alpha \models \alpha))))); \\ ((((\alpha \models \mathcal{I}_\alpha) \models \mathcal{C}_\alpha) \models \gamma_\alpha) \models \mathcal{E}_\alpha) \models^* (\varepsilon_\alpha \models \alpha); \\ (\alpha \models \varepsilon_\alpha) \models^* (\mathcal{E}_\alpha \models (\gamma_\alpha \models (\mathcal{C}_\alpha \models (\mathcal{I}_\alpha \models \alpha)))); \\ (((\alpha \models \mathcal{I}_\alpha) \models \mathcal{C}_\alpha) \models \gamma_\alpha) \models^* (\mathcal{E}_\alpha \models (\varepsilon_\alpha \models \alpha)); \\ ((\alpha \models \varepsilon_\alpha) \models \mathcal{E}_\alpha) \models^* (\gamma_\alpha \models (\mathcal{C}_\alpha \models (\mathcal{I}_\alpha \models \alpha))); \\ ((\alpha \models \mathcal{I}_\alpha) \models \mathcal{C}_\alpha) \models^* (\gamma_\alpha \models (\mathcal{E}_\alpha \models (\varepsilon_\alpha \models \alpha))); \\ (((\alpha \models \varepsilon_\alpha) \models \mathcal{E}_\alpha) \models \gamma_\alpha) \models^* (\mathcal{C}_\alpha \models (\mathcal{I}_\alpha \models \alpha)); \\ (\alpha \models \mathcal{I}_\alpha) \models^* (\mathcal{C}_\alpha \models (\gamma_\alpha \models (\mathcal{E}_\alpha \models (\varepsilon_\alpha \models \alpha)))); \\ ((((\alpha \models \varepsilon_\alpha) \models \mathcal{E}_\alpha) \models \gamma_\alpha) \models \mathcal{C}_\alpha) \models^* (\mathcal{I}_\alpha \models \alpha); \\ \alpha \models^* (\mathcal{I}_\alpha \models (\mathcal{C}_\alpha \models (\gamma_\alpha \models (\mathcal{E}_\alpha \models (\varepsilon_\alpha \models \alpha))))); \\ (((((\alpha \models \varepsilon_\alpha) \models \mathcal{E}_\alpha) \models \gamma_\alpha) \models \mathcal{C}_\alpha) \models \mathcal{I}_\alpha) \models^* \alpha \end{pmatrix}$$

For the listed long-sized metaphysical forms of informing only one of entities $\Upsilon_{\circlearrowright}^{\subsetneq}(\alpha)$ and $\Upsilon_{\circlearrowright}^{\subsetneq}(\alpha)$ must be given. □

4.7.3 A Mixed Parallel-serial Metaphysical Case

The most complex case of an entity metaphysicalism can be achieved by the mixture of both principles, the parallel and the serial one, at any point of the metaphysical informing, as a consequence of the parallel, serial, and metaphysical informational includedness. One can easily and in an arbitrary manner construct such cases.

4.7.4 A Pragmatic Filling of the Parallel and Serial Metaphysical Shells

The basic question is, how can a metaphysical shell be filled to achieve, for example, intelligent functions of an informational entity. Candidates for such a filling of the metaphysical shells are principles of reasoning and understanding, by which reason and meaning are informed, respectively. A mode of reasoning producing reason can be seen as a counterinforming component, while a mode of understanding producing meaning can be seen as an embedding component for that which has arisen by reasoning. Thus reason is embedded into the existing informational entity by meaning, which is the connecting information between the arisen reason and the informational content of the informational entity.

Definition 9 [Reasoning and Understanding Components as an Informational Entity Includedness] *Reasoning $\mathcal{R}$ and understanding $\mathcal{U}$ are attributes of an intelligently informing entity ι, which can demonstrate its reasonable informing through an adequate filling of its metaphysical shells by intelligently informing, reasoning, and understanding components as counterparts to informing, counterinforming, and embedding, respectively. Let us define the following parallel arrays:*

$$\iota(\xi) \rightleftharpoons \begin{pmatrix} \iota^1(\xi); \\ \iota^2(\xi); \\ \vdots \\ \iota^{i_\iota}(\xi) \end{pmatrix}; \; \mathcal{I}_\iota(\xi) \rightleftharpoons \begin{pmatrix} \mathcal{I}_\iota^1(\xi); \\ \mathcal{I}_\iota^2(\xi); \\ \vdots \\ \mathcal{I}_\iota^{i_\mathcal{I}}(\xi) \end{pmatrix}$$

are the parallel arrays of intelligent intelligent entity components $\iota^m(\xi)$ and its informing components of $\mathcal{I}_\iota^n$ concerning an exterior or interior (also complex) entity, marked by ξ. A pair of reasoning components of the form

$$\mathcal{R}_\iota(\xi) \rightleftharpoons \begin{pmatrix} \mathcal{R}_\iota^1(\xi); \\ \mathcal{R}_\iota^2(\xi); \\ \vdots \\ \mathcal{R}_\iota^{i_\mathcal{R}}(\xi) \end{pmatrix}; \; \rho_\iota(\xi) \rightleftharpoons \begin{pmatrix} \rho_\iota^1(\xi); \\ \rho_\iota^2(\xi); \\ \vdots \\ \rho_\iota^{i_\rho}(\xi) \end{pmatrix}$$

depicts the reasonably informing components $\mathcal{R}_\iota^i(\xi)$ together with the reason components $\rho_\iota^j(\xi)$. At last,

$$\mathcal{U}_\iota(\xi) \rightleftharpoons \begin{pmatrix} \mathcal{U}_\iota^1(\xi); \\ \mathcal{U}_\iota^2(\xi); \\ \vdots \\ \mathcal{U}_\iota^{i_\mathcal{U}}(\xi) \end{pmatrix}; \; \mu_\iota(\xi) \rightleftharpoons \begin{pmatrix} \mu_\iota^1(\xi); \\ \mu_\iota^2(\xi); \\ \vdots \\ \mu_\iota^{i_\mu}(\xi) \end{pmatrix}$$

are understanding components $\mathcal{U}_\iota^k(\xi)$ and meaning components $\mu_\iota^\ell(\xi)$ concerning entity ξ. □

Let us show the filling of metaphysical shells in Consequence 16, representing the long-sized, that is, serially the most complex loops of informing, however expressed in the informationally includable form and, thus, giving the implemented filling of shells a choice of an infinite number of all possible extensions (symbols of indexed Ξ).

The filling of the shells can be understood as a particularization (decomposition) process by which several substitutions of symbols take place. These substitutions are as follows:

(1) Shell entity α is replaced by intelligent parallel array $\iota(\xi)$;

(2) shell's informing $\mathcal{I}_\alpha$ is replaced by intelligent parallel array $\mathcal{I}_\iota(\xi)$;

(3) shell's counterinforming $\mathcal{C}_\alpha$ is replaced by reasoning parallel array $\mathcal{R}_\iota(\xi)$;

(4) shell's counterinformational entity γ_α is replaced by reason parallel array $\rho_\iota(\xi)$;

(5) shell's embedding $\mathcal{E}_\alpha$ is replaced by understanding parallel array $\mathcal{U}_\iota(\xi)$;

(6) shell's embedding informational entity ε_α is replaced by meaning parallel array $\mu_\iota(\xi)$; and

(7) shell's operators $\models$ are replaced by specifically complex (universal) operators $\subset$.

Consequence 17 [Filling the Shell of a Long-sized Metaphysical Includedness] *Let us have the following metaphysical shell filling when entity ι observes entity ξ:*

$$\xi \models$$

$$\left(\begin{array}{c}
(((((\iota(\xi) \subset \mathcal{I}_\iota(\xi)) \subset \mathcal{R}_\iota(\xi)) \subset \rho_\iota(\xi)) \subset \\
\mathcal{U}_\iota(\xi)) \subset \mu_\iota(\xi)) \subset^* \iota(\xi); \\
\iota(\xi) \subset^* (\mu_\iota(\xi) \subset (\mathcal{U}_\iota(\xi) \subset (\rho_\iota(\xi) \subset \\
(\mathcal{R}_\iota(\xi) \subset (\mathcal{I}_\iota(\xi) \subset \iota(\xi))))); \\
((((\iota(\xi) \subset \mathcal{I}_\iota(\xi)) \subset \mathcal{R}_\iota(\xi)) \subset \rho_\iota(\xi)) \subset \\
\mathcal{U}_\iota(\xi)) \subset^* (\mu_\iota(\xi) \subset \iota(\xi)); \\
(\iota(\xi) \subset \mu_\iota(\xi)) \subset^* (\mathcal{U}_\iota(\xi) \subset (\rho_\iota(\xi) \subset \\
(\mathcal{R}_\iota(\xi) \subset (\mathcal{I}_\iota(\xi) \subset \iota(\xi)))); \\
(((\iota(\xi) \subset \mathcal{I}_\iota(\xi)) \subset \mathcal{R}_\iota(\xi)) \subset \rho_\iota(\xi)) \subset^* \\
(\mathcal{U}_\iota(\xi) \subset (\mu_\iota(\xi) \subset \iota(\xi))); \\
((\iota(\xi) \subset \mu_\iota(\xi)) \subset \mathcal{U}_\iota(\xi)) \subset^* (\rho_\iota(\xi) \subset \\
(\mathcal{R}_\iota(\xi) \subset (\mathcal{I}_\iota(\xi) \subset \iota(\xi)))); \\
((\iota(\xi) \subset \mathcal{I}_\iota(\xi)) \subset \mathcal{R}_\iota(\xi)) \subset^* (\rho_\iota(\xi) \subset \\
(\mathcal{U}_\iota(\xi) \subset (\mu_\iota(\xi) \subset \iota(\xi)))); \\
(((\iota(\xi) \subset \mu_\iota(\xi)) \subset \mathcal{U}_\iota(\xi)) \subset \rho_\iota(\xi)) \subset^* \\
(\mathcal{R}_\iota(\xi) \subset (\mathcal{I}_\iota(\xi) \subset \iota(\xi))); \\
(\iota(\xi) \subset \mathcal{I}_\iota(\xi)) \subset^* (\mathcal{R}_\iota(\xi) \subset (\rho_\iota(\xi) \subset \\
(\mathcal{U}_\iota(\xi) \subset (\mu_\iota(\xi) \subset \iota(\xi))))); \\
((((\iota(\xi) \subset \mu_\iota(\xi)) \subset \mathcal{U}_\iota(\xi)) \subset \rho_\iota(\xi)) \subset \\
\mathcal{R}_\iota(\xi)) \subset^* (\mathcal{I}_\iota(\xi) \subset \iota(\xi)); \\
\iota(\xi) \subset^* (\mathcal{I}_\iota(\xi) \subset (\mathcal{R}_\iota(\xi) \subset (\rho_\iota(\xi) \subset \\
(\mathcal{U}_\iota(\xi) \subset (\mu_\iota(\xi) \subset \iota(\xi)))))); \\
(((((\iota(\xi) \subset \mu_\iota(\xi)) \subset \mathcal{U}_\iota(\xi)) \subset \rho_\iota(\xi)) \subset \\
\mathcal{R}_\iota(\xi)) \subset \mathcal{I}_\iota(\xi)) \subset^* \iota(\xi)
\end{array}\right)$$

The listed long-sized metaphysical forms of intelligent informing, reasoning, and understanding constitute a parallel system of serial parallel formulas for ι's observing of ξ. □

5 Includedness as a Logical Contradiction

The traditional (mathematical, logical) understanding of includedness (inclusion, inclusiveness) may seriously contradict the understanding of informational includedness (Being-in, involvement, interweavement, interrelation, interconnection of informational components, etc.). In the first case, the accent is given to the word *in*, while in the second case, the word *inter* (as interiority) is emphasized. Informational includedness expresses the inner character or the inward nature of informational something within informational something. For example, the so-called problem of *internal representation* [3, 11] concerns the problem of includedness. On the other hand, the philosophical Being-in seems to cover the substantial part of the broadened realm of informational includedness. *Subjectivity and interiority are the notions acquired by the human mind* (W. James, 1890 [13]).

Let us see the controversial notions which may substantially touch the first and the second understanding of includedness. The difference between the traditional and informational understanding of includedness comes to the surface in case

$$(\alpha \subset \beta) \rightleftharpoons \begin{pmatrix} \beta \models \alpha; \\ \alpha \models \beta; \\ \Xi(\alpha \subset \beta \end{pmatrix}; \ \beta \subset \alpha \rightleftharpoons \begin{pmatrix} \alpha \models \beta; \\ \beta \models \alpha; \\ \Xi(\beta \subset \alpha \end{pmatrix}$$

where for includable extensions $\Xi(\alpha \subset \beta)$ and $\Xi(\beta \subset \alpha)$, there is

$$\Xi(\alpha \subset \beta),\ \Xi(\beta \subset \alpha) \in \mathcal{P}\left(\left\{\begin{matrix} (\beta \models \alpha) \subset \beta, \\ (\alpha \models \beta) \subset \beta, \\ (\beta \models \alpha) \subset \alpha, \\ (\alpha \models \beta) \subset \alpha \end{matrix}\right\}\right)$$

and, thus,

$$\Xi^{\beta,\alpha}_{\beta,\alpha}(\alpha \subset \beta) = \Xi^{\alpha,\beta}_{\alpha,\beta}(\beta \subset \alpha)$$

This case does not deliver a difference between includednesses $\alpha \subset \beta$ and $\beta \subset \alpha$ and is to this extent contradictory. But, the difference becomes quite senseful in case of

$$\Xi^{\beta}_{\beta}(\alpha \subset \beta) \rightleftharpoons \begin{pmatrix} (\beta \models \alpha) \subset \beta; \\ (\alpha \models \beta) \subset \beta \end{pmatrix};$$

$$\Xi^{\alpha}_{\alpha}(\beta \subset \alpha) \rightleftharpoons \begin{pmatrix} (\alpha \models \beta) \subset \alpha; \\ (\beta \models \alpha) \subset \alpha \end{pmatrix}$$

where, in the first case, β is the dominant entity possessing the informing control over the transitions $\beta \models \alpha$ and $\alpha \models \beta$, while in the second case this role belongs to α.

The contradictory case to the traditional understanding concerns the extensional example $\Xi^{\beta,\alpha}_{\beta,\alpha}(\alpha \subset \beta)$ since simultaneously the control of the dominant, β, and the inclusively subordinated component, α, is requested. But, this contradiction may appear as a (clear) prejudice in the realm of informational.

Similar situation appears at the serially circular (e.g., metaphysical) includedness, which enables the serially circular informing in one and the other direction (e.g.,

$$(((((\iota \subset \mathcal{I}_\iota) \subset \mathcal{R}_\iota) \subset \rho_\iota) \subset \mathcal{U}_\iota) \subset \mu_\iota) \subset^* \iota$$

as an intelligently, that is reasonedly and understandingly structured informational shell). It is clear that in this case a serially embedded includedness and, at the end, circularly closed includedness must take place. The traditional doubts of this sort show how a spontaneous and circular informational phenomenalism can not only surpass but can also make the notional obstacles of such kind informationally (intelligently) productive and senseful.

6 Includedness and Reasoning (Inference)

Includedness (informational Being-in) and reasoning are essentially related informational entities. Reasoning (or inference) is possible only within a context of relatedness between a reason (cause, informational motive) and a certain informational consequence pertaining to the reason as a legalized fact. Historically, three basic ways of inference can be distinguished: deduction, induction, and abduction. All of them are philosophically and scientifically well-determined in respect to their formalistic power and practical disadvantages (reductionism, simplification, scientific straightforwardness, etc.). Within the discussion, their connectedness with the concept of informational includedness, that is, as an includable inference processibility, cannot be denied.

The includable informational modi presented are more concretized formulas of reasoning in one or another way (deductively, inductively, and/or abductively). For example, modus ponens is usually meant as a strict deductive principle, while modus tollens inclines to be inductive and modus obliquus abductive. Includable informational modi are typical scenarios (formulas) of informational inferring.

6.1 Includable Deduction

How does the informational Being-in concern the so-called deduction and what does the includable deduction mean? Does there exist a substantial connection between the Being-in as an informational phenomenon on one side and the deduction as a logical (inferential, derivative, conclusive) principle on the other side? Includable deduction seems to be our everyday principle of 'common-sense' inference of which we are not being always sufficiently aware.

Let us keep in mind the following facts concerning the processes of deduction: deduction means inference by reasoning from generals (universals) to particulars. E.g., particularizing informational operands and especially operators is a kind of 'hidden' (unconscious) deduction. Deducing (or deriving) theorems (conclusions, consequences, lemmas, etc.) from systems of axioms (definitions, hypotheses, etc.) is a characteristic deductive procedure in abstract theories (systems, mathematics, abstract sciences). Deduction opposes induction by reduction, if reduction is meant to be particularization (derivation from universals). The principle of conditionalization (known as 'deduction theorem') was already taken for granted by Aristotle.

At the beginning, let us list three 'deductive' informational operators:

- $\Longrightarrow$ is the most common deductive operator and its meaning is the following: $\alpha \Longrightarrow \beta$ means *if entity (operand) α is given (informationally existent), then it is permitted to transit to entity (operand) β.*

- $\longrightarrow$ (or $\supset$) is a narrower deductive operator and its meaning is: $\alpha \longrightarrow \beta$ (or $\alpha \supset \beta$) means *if entity (operand) α, then β.* We rarely use this type of deductive operator.

- $\frac{\cdots}{\cdots}$ (or $\prec$) denotes a complex and to some extent informationally precise deductive operator, which meaning is: $\frac{\alpha}{\beta}$ (or $\alpha \prec \beta$) means *from α there follows β (or α precedes β, also α implies β.)* Entity (operand) α usually denotes a complex (parallel) informational system.

In which way do the listed deductive operators concern informational includedness (informational operator $\subset$)?

In an experiential situation, deduction does not already concern the truth, but proceeds from a *hypothetical* informational entity (situation) to the *prognostic* informational entity. Also, weaker logical deduction rules can exist, for instance, those of the form $\alpha \Longrightarrow (\alpha \vee \beta)$ or $(\alpha, \neg\alpha) \Longrightarrow \gamma$, where γ is an arbitrary entity (from the false, $\neg\alpha$, an arbitrary formula can be logically deduced).

By Definition 1, the Being-in operator $\subset$ is defined complexly in regard to the general (yet non-particularized) operator $\models$ and to the operands α and β. Thus, we have the following consequence when particularizing of operator $\models$ to operator $\Longrightarrow$ is taking place.

Consequence 18 [Deduction Concerning Informational Includedness] *According to Definition 1, when implicatively particularizing operator $\models$, that is, $\models_{\Rightarrow}$ is equivalent to $\Longrightarrow$, there is*

$$(\alpha \subset_{\Rightarrow} \beta) \rightleftharpoons_{\mathrm{Def}} \begin{pmatrix} \beta \Longrightarrow \alpha; \\ \alpha \Longrightarrow \beta; \\ \Xi(\alpha \subset_{\Rightarrow} \beta) \end{pmatrix}$$

where for the extensional part $\Xi(\alpha \subset_{\Rightarrow} \beta)$ of the includable deduction $\alpha \subset_{\Rightarrow} \beta$, there is,

$$\Xi(\alpha \subset_{\Rightarrow} \beta) \in \mathcal{P}\left(\left\{ \begin{matrix} (\beta \Longrightarrow \alpha) \subset_{\Rightarrow} \beta, \\ (\alpha \Longrightarrow \beta) \subset_{\Rightarrow} \beta, \\ (\beta \Longrightarrow \alpha) \subset_{\Rightarrow} \alpha, \\ (\alpha \Longrightarrow \beta) \subset_{\Rightarrow} \alpha \end{matrix} \right\}\right)$$

The most complex element of this power set is denoted by

$$\Xi^{\beta,\alpha}_{\beta,\alpha}(\alpha \subset_{\Rightarrow} \beta) \rightleftharpoons \begin{pmatrix} (\beta \Longrightarrow \alpha) \subset_{\Rightarrow} \beta, \alpha; \\ (\alpha \Longrightarrow \beta) \subset_{\Rightarrow} \beta, \alpha \end{pmatrix}$$

Cases, where $\Xi(\alpha \subset_{\Rightarrow} \beta) \rightleftharpoons \emptyset$ and $\emptyset$ denotes an empty entity (informational nothing), are exceptional (reductionistic). □

To get even a more transparent impression what is going on with the last consequence, we can write definiens of definiendum in Consequence 18 by sample formulas

$$\begin{pmatrix} \beta \longrightarrow \alpha; \\ \alpha \longrightarrow \beta; \\ \Xi(\alpha \subset_{\rightarrow} \beta) \end{pmatrix} \text{ and } \begin{pmatrix} \frac{\beta}{\alpha}; \\ \frac{\alpha}{\beta}; \\ \Xi(\alpha \subset_{\frac{\cdots}{\cdots}} \beta) \end{pmatrix}$$

We can now discuss the deductive character of the operation of informational includedness (operator $\subset$ in a universal or particular form) and vice versa. Where lies the deductive point of informational includedness?

Deduction by itself is nothing else than a kind of informational involvement. Otherwise, the concept of deduction (coming from the Latin 'deducere', the German 'herabführen', and the English 'lead away' or 'trace the course of') would not be possible. To deduce something informationally means to extract it informationally (in German, abtrennen) out of something, certainly not in an informationally total (strictly including), but also in an initializing or initially arising (involving) informational way. Includable deduction is an arising informational phenomenon, emerging out of a deductively happening (intentional) situation. Within this illumination we have to explain the connection existing between the definition of informational includedness and the principles of deduction, expressed in the form of the so-called deductive rules. We should also make a clear distinction between deductive and inductive nature of the inference rules. The deductive always roots in previously strict causes and arises as a clear and constructively structured consequence. The inductive makes an intuitive jump from the particular to general and afterwards proceeds deductively. The question is if this jump is deductively legal.

Consequence 19 [A Primitive Circular Structure of Deduction Concerning Informational Includedness] *Considering the most complex extensional element in Consequence 18, $\Xi_{\beta,\alpha}^{\beta,\alpha}(\alpha \subset_{\Rightarrow} \beta)$, there is,*

$$\begin{pmatrix} (\beta \Longrightarrow \alpha) \subset_{\Rightarrow} \beta, \alpha; \\ (\alpha \Longrightarrow \beta) \subset_{\Rightarrow} \beta, \alpha \end{pmatrix} \rightleftharpoons$$

$$\begin{pmatrix} \beta \Longrightarrow (\beta \Longrightarrow \alpha); (\beta \Longrightarrow \alpha) \Longrightarrow \beta; \\ \Xi_{\beta,(\beta \Longrightarrow \alpha)}^{\beta,(\beta \Longrightarrow \alpha)}((\beta \Longrightarrow \alpha) \subset_{\Rightarrow} \beta); \\ \alpha \Longrightarrow (\beta \Longrightarrow \alpha); (\beta \Longrightarrow \alpha) \Longrightarrow \alpha; \\ \Xi_{\alpha,(\beta \Longrightarrow \alpha)}^{\alpha,(\beta \Longrightarrow \alpha)}((\beta \Longrightarrow \alpha) \subset_{\Rightarrow} \alpha); \\ \beta \Longrightarrow (\alpha \Longrightarrow \beta); (\alpha \Longrightarrow \beta) \Longrightarrow \beta; \\ \Xi_{\beta,(\alpha \Longrightarrow \beta)}^{\beta,(\alpha \Longrightarrow \beta)}((\alpha \Longrightarrow \beta) \subset_{\Rightarrow} \beta); \\ \alpha \Longrightarrow (\alpha \Longrightarrow \beta); (\alpha \Longrightarrow \beta) \Longrightarrow \alpha; \\ \Xi_{\alpha,(\alpha \Longrightarrow \beta)}^{\alpha,(\alpha \Longrightarrow \beta)}((\alpha \Longrightarrow \beta) \subset_{\Rightarrow} \alpha) \end{pmatrix}$$

The most interesting cases of circular implication are

$$(\beta \Longrightarrow \alpha) \Longrightarrow \beta;\ \alpha \Longrightarrow (\beta \Longrightarrow \alpha);$$
$$\beta \Longrightarrow (\alpha \Longrightarrow \beta);\ (\alpha \Longrightarrow \beta) \Longrightarrow \alpha$$

where, in the first case, β involves α and, then, this involvement involves β again, etc. In this way the process of involvement (informational includedness) proceeds (e.g. deductively improves in an implicative manner) circularly. □

6.2 Includable Induction

A strict separation between deduction and induction seems to be probably impossible. For instance, induction concerns derivation from something similarly as deduction. Induction does not mean bringing something into existence from nothing—at least not in the traditional sciences. Within informational theory, induction (as well as deduction) concerns informational arising, for example, counterinforming and informational embedding of the arisen informational entities.

Our question remains, how does the informational Being-in concern the so-called induction and what does the includable induction mean? We have to repeat the following questions: Does there exist a substantial connection between the Being-in as an informational phenomenon on one side and the induction as a logical (intuitive, inferential, derivative, conclusive) principle on the other side? Includable induction is a deeply implanted everyday intuitive principle of common sense and of the informational nature of things (discourses, speech acts, behaviors).

Let us keep in mind the following facts concerning the processes of induction: induction is the informational action of introducing and initiating in (arising, counterinforming). It is, for example, introduction and initiation of knowledge of something, that which leads to something (new). It is the initial step in logical (informational, also intuitive) understanding (undertaking). In this sense, induction is a process of inferring a general law (principles, axioms, hypotheses) from the observation of particular instances (e.g., ἐπαγωγή in Greek, means a bringing on, an advancing). Induction is a wider (transitive) sense of inference. If a theorem is true in one case, it is true in another case which may be called the *next* case. The prove is made by trial [13].

Induction also means inference by reasoning from particulars to generals (universals). E.g., universalizing (generalizing) informational operands and especially operators is a kind of 'hidden' (unconscious) induction. Inducing (or intuitively deriving, introducing) systems of axioms (definitions, hypotheses, etc.) with the intention to deduce theorems (conclusions, consequences, lemmas, etc.) is a characteristic inductive procedure in abstract theories (systems, mathematics, abstract sciences). Induction opposes deduction by generalization, if generalization is meant to be universalization (intuitive derivation from particulars).

Thus, three 'deductive' informational operators, $\Longrightarrow$, $\longrightarrow$, and $\stackrel{\cdots}{\underset{\cdots}{-}}$, can function also inductively because of the informational-arising nature of informational entities. Circular and particularly metaphysical scenarios of informing are inductive in the sense of introducing new entities into informational cycles and, in parallel, initiating the interpreting formulas for already existing entities. In this sense, induction is a substantial phenomenon of informational decomposition and composition [9].

Informational Being-in offers opportunities which can be taken into consideration. These opportunities have their roots in the recursive character of informational includedness and in the additional possibilities of pragmatic nature of decomposition and composition of informational entities and systems (formulas). The concept of informational inclusivism conditions the concept of inductivism, being an informationally inclusive phenomenon, proceeding, for instance, from granted particularities to certain universalities, being informationally involved by the first ones. A radical initial intuitive step is the introduction of the so-called informatio prima (the first of informational axioms) [10].

6.3 Includable Abduction

Abduction is a special (in traditional science, illegal) way of deduction which may include elementary induction too. In the similar way as deduction and induction, abduction as such concerns informational includedness. It means a leading away in the informational sense. For instance, it can represent a syllogism, of which the major premise (antecedent) is certain, and the minor only probable, so that the conclusion has only the probability of the minor [13]. But, this view of abduction does not embrace its entire informational realm, which can consider an initial (introductory) entity and then proceed away (e.g., counterinformationally) to another possible (probable) entity by a degree of similarity, sporadicalness, relatedness, etc. This is a characteristic phenomenon of abduction, a progress from one informational situation (attitude) to another, when the first being once informationally legalized (demonstrated, approved) and afterwards employed to the proving of other situations (attitudes).

Abduction may represent an indirect proof, like the apagoge, which means syllogistic reasoning, by which a thing is not directly proved, but shows, for example, the absurdity or impossibility of denying the thing in a certain, particularly informational way. Sometimes, it is called *reductio ad absurdum*. A good example of abductive reasoning is perhaps the so-called informational modus obliquus (see later).

6.4 Includable MODUS PONENS

Includable modus ponens is an informational inference rule constructed in the sense as it is known in symbolic logic. This rule uses a true conjunction of an affirmative (true) statement and a true implication of the affirmative and some other statement. In this situation the truth can be decided for the other statement. In our case, instead of truth, we have a certain value of including informing, conjunction is replaced by an informational operator of parallelism (e.g., semicolon ';', symbol $\|$, or a proper parallel informational operator $\|\!\models$). We also introduce the informational operator of implication $\Longrightarrow$ with the meaning 'implies/imply'.

Inference Rule 1 [Includable MODUS PONENS] *Informational modus ponens can be expressed in terms of informational externalism, internalism, metaphysicalism, and phenomenalism giving 16 basic inference rules concerning an entity α's includedness. We list only four characteristic cases.*

The rule for an externalistic inference *on including externalism $\beta \subset$ from externalisms $\alpha \subset$ and $\beta \models$ is*

$$\frac{\alpha \subset;\ ((\alpha \subset) \Longrightarrow (\beta \models)) \subset}{\beta \subset}$$

A similar rule for an internalistic inference *on including internalism $\subset \beta$ from internalisms $\subset \alpha$ and $\Longrightarrow \beta$ is*

$$\frac{\subset \alpha;\ \subset ((\subset \alpha) \Longrightarrow (\models \beta))}{\subset \beta}$$

Trivially seems to be the rule for a metaphysicalistic inference on including metaphysicalism $\beta \subset \beta$ *from metaphysicalisms* $\alpha \subset \alpha$ *and* $\beta \models \beta$, *where*

$$\frac{\begin{pmatrix} \alpha \subset \alpha; \\ \big((\alpha \subset \alpha) \Longrightarrow (\beta \models \beta)\big) \subset \\ \big((\alpha \subset \alpha) \Longrightarrow (\beta \models \beta)\big) \end{pmatrix}}{\beta \subset \beta}$$

At last we have a case of the rule for a phenomenalistic inference *on including phenomenalism* $(\beta \subset; \subset \beta)$ *from phenomenalisms* $(\alpha \subset; \subset \alpha)$ *and* $(\beta \models; \models \beta)$, *where*

$$\frac{\begin{pmatrix} (\alpha \subset; \subset \alpha); \\ \begin{pmatrix} \big((\alpha \subset; \subset \alpha) \Longrightarrow (\beta \models; \models \beta)\big) \subset; \\ \subset \big((\alpha \subset; \subset \alpha) \Longrightarrow (\beta \models; \models \beta)\big) \end{pmatrix} \end{pmatrix}}{(\beta \subset; \subset \beta)}$$

The last rule means to infer phenomenalistically by modus tollens in the sense of includedness upon a phenomenalistic case of informing. □

The listed informational rules of modus ponens are only the most characteristic ones. We did not present any of the possible cross-modal rules, that is from externalistic-internalistic to phenomenalistic-metaphysicalistic ones (additionally, 12 possible cases).

The rules of modus ponens belong to the most obvious (normal, generally agreed) rules of inference primarily because of their categorical value. However, within the informational logical realm, the rule of modus ponens is certainly only one of possible rules of inference. Applying only this kind of rules would mean to infer in a particularly reductionistic and informationally unidirectional way.

6.5 Includable MODUS TOLLENS

The informational modus tollens pertaining to informational includedness is a good example of the difference arising from the positions of categorical reasoning on one side and the informationally phenomenological reasoning—for example, includably-as-in-the-informationally-involved-way—on the other side. Includedness as an informational in-volvement must not be comprehended categorically, since reasoning in this way would lead to the categorical nonsense, traditional-logic controversy, and 'common-sense' (say, occurrent, in German, vorhanden) absurdity. From another point of view, the informational Being-in represents the most general term concerning the 'In', which at a given situation or attitude speaks for a particular situation or attitude. In this sense, informational Being-in is always particularizes and if not, the empty place in its whole meaning only waits to be complemented.

Inference Rule 2 [Includable MODUS TOLLENS] *Some cases of informational modus tollens can again be expressed in terms of the pure informational externalism, internalism, metaphysicalism, and phenomenalism concerning an entity* α*'s includedness. The modus tollens rule for an* externalistic inference *on non-including externalism* $\alpha \not\subset$ *from externalisms* $((\alpha \models) \Longrightarrow (\beta \models)) \subset$ *and* $\beta \not\subset$ *is*

$$\frac{((\alpha \models) \Longrightarrow (\beta \models)) \subset;\ \beta \not\subset}{\alpha \not\subset}$$

A similar rule for an internalistic inference *on non-including internalism* $\not\subset \alpha$ *from internalisms* $\subset ((\models \alpha) \Longrightarrow (\models \beta))$ *and* $\not\subset \beta$ *is*

$$\frac{\subset ((\models \alpha) \Longrightarrow (\models \beta));\ \not\subset \beta}{\not\subset \alpha}$$

Trivially seems to be the rule for a metaphysicalistic inference on non-including metaphysicalism $(\alpha \models \alpha) \not\subset (\alpha \models \alpha)$ *from metaphysicalisms* $((\alpha \models \alpha) \Longrightarrow (\beta \models \beta)) \subset ((\alpha \models \alpha) \Longrightarrow (\beta \models \beta))$ *and* $(\beta \models \beta) \not\subset (\beta \models \beta)$, *thus,*

$$\frac{\begin{pmatrix} \begin{pmatrix} (\alpha \models \alpha) \Longrightarrow \\ (\beta \models \beta) \end{pmatrix} \subset \begin{pmatrix} (\alpha \models \alpha) \Longrightarrow \\ (\beta \models \beta) \end{pmatrix}; \\ (\beta \models \beta) \not\subset (\beta \models \beta) \end{pmatrix}}{(\alpha \models \alpha) \not\subset (\alpha \models \alpha)}$$

At last we have a case of the rule for a phenomenalistic inference *on non-including phenomenalism* $(\alpha \not\subset; \not\subset \alpha)$ *from phenomenalisms* $(((\alpha \models; \models \alpha) \Longrightarrow (\beta \models; \models \beta)) \subset; \subset ((\alpha \models; \models \alpha) \Longrightarrow (\beta \models; \models \beta)))$ *and* $(\beta \not\subset; \not\subset \beta)$, *so,*

$$\frac{\left(\begin{pmatrix}((\alpha \models; \models \alpha) \Longrightarrow (\beta \models; \models \beta)) \subset; \\ \subset ((\alpha \models; \models \alpha) \Longrightarrow (\beta \models; \models \beta))\end{pmatrix}; \\ (\beta \not\subset; \not\subset \beta)\right)}{(\alpha \not\subset; \not\subset \alpha)}$$

The last case means the inferring by modus tollens in the sense of includedness upon a phenomenalistic case of informing. □

The rule of informational modus tollens shows clearly that operators $\subset$ and $\not\subset$ must be comprehended differently from the adequate categorical relational symbols in logic. It is to emphasize that if operator $\subset$ is concretely particularized in the upper rules then operator $\not\subset$ must receive the same concrete particularization. It is to say in general that operators $\subset$ and $\not\subset$ are of the same kind (meaning) in the given context.

Further on, operators $\subset$ and $\not\subset$ express the activity of informational in-volvement and non-involvement (embedding and non-embedding), respectively. In this respect, an informational entity α may *particularly* be involved (informationally embedded) in itself ($\alpha \subset \alpha$) or not ($\alpha \not\subset \alpha$). From this non-categorical point of view, informational operator $\subset$, expressing the informational Being-in, behaves as a regular informational operator.

6.6 Includable MODUS RECTUS

The intentional of an informational entity is that which informs actively and participates in the informational arising and constitution of the entity. As soon as we say that informational acts are intentional, the question arises, how the extraction (bringing to the surface) of intentional information, hidden *in* the background of an informing entity, would be possible. Intention is nothing else than an informational phenomenon of informing, pertaining to the question "Why does an entity inform in just a particular way and does not inform in another one?" In this context, intention appears as a reason (motive, cause, hidden explanation) of an informing entity.

Informational modus rectus is a rule for the inference which concerns the intentional informing of an informational entity. Includable modus rectus reduces this inference to the includable informing of an entity, which means that informational involvement, embedding, and connectedness pertaining to the intention as a ruling (motivating) informational phenomenon is being searched.

Let us construct a case of includable modus rectus (which predicts the intention of an informing entity) as a conclusion of particular conclusions, that is, as a modal inference of modal inferences.

Inference Rule 3 [Includable MODUS RECTUS] *The basic scheme of the includable modus rectus concerning entity α and its intention ι_α could be the following basic phenomenalistic form:*

$$\frac{\alpha; (\alpha \Longrightarrow (\alpha \models_{\iota_\alpha}; \models_{\iota_\alpha} \alpha))}{\iota_\alpha \subset \alpha}$$

Taking into account the entity α's externalism, internalism, metaphysicalism, and phenomenalism, there is,

$$\frac{\left(\begin{array}{l} \dfrac{\alpha \models; ((\alpha \models) \Longrightarrow (\alpha \models_{\iota_\alpha}))}{(\alpha \models_{\iota_\alpha}) \subset (\alpha \models)}; \\ \dfrac{\models \alpha; ((\models \alpha) \Longrightarrow (\models_{\iota_\alpha} \alpha))}{(\models_{\iota_\alpha} \alpha) \subset (\models \alpha)}; \\ \dfrac{\alpha \models \alpha; ((\alpha \models \alpha) \Longrightarrow (\alpha \models_{\iota_\alpha} \alpha))}{(\alpha \models_{\iota_\alpha} \alpha) \subset (\alpha \models \alpha)}; \\ \dfrac{(\alpha \models; \models \alpha); ((\alpha \models; \models \alpha) \Longrightarrow (\alpha \models_{\iota_\alpha}; \models_{\iota_\alpha} \alpha))}{(\alpha \models_{\iota_\alpha}; \models_{\iota_\alpha} \alpha) \subset (\alpha \models; \models \alpha)} \end{array}\right)}{\iota_\alpha \subset \alpha}$$

This inference rule of includable modus rectus expresses the common phenomenon of intentional informing (intention ι_α with intentional informing $\mathfrak{I}_{\iota_\alpha}$) of entity α. Thus, intention ι_α as a distinguished entity informs within α as

$$\begin{pmatrix}(\iota_\alpha \models \mathfrak{I}_{\iota_\alpha}) \models \iota_\alpha; \\ \iota_\alpha \models (\mathfrak{I}_{\iota_\alpha} \models \iota_\alpha)\end{pmatrix} \subset \alpha$$

in an intentional manner. □

6.7 Includable MODUS OBLIQUUS

The Latin *obliquus* concerns that which is slanting, sideways, oblique, indirect, covert, envious [8], or also out-of-the-way. For instance, the abduction [4] as a mode of logical inference could be characterized as oblique in comparison to the deduction. But, the oblique mode of inference could be that which becomes interwoven in the realm of the absurd, unbelievable, unforeseeable, contradictory, obscure, etc. In this respect, oblique conclusions may appear as the most surprising (e.g. counterinformational) cases of inference.

Inference Rule 4 [Includable MODUS OBLIQUUS] *Let o mark an oblique informational operand (entity) with oblique informing $\mathcal{O}_o$ and let α be a regular informational entity which informs includably, $\alpha \subset$, is informed includably, $\subset \alpha$, is metaphysically includable, $\alpha \subset \alpha$, and phenomenalistically includable, $\alpha \subset; \subset \alpha$, in the domain of belief β. Then we can obtain one of several possible formulas for the includable modus obliquus, for example:*

$$\frac{\left(\begin{array}{l}
\dfrac{(\alpha \models) \models_\beta;\ ((\alpha \not\models) \Longrightarrow (o \models)) \subset}{(o \models_{\mathcal{O}_o}) \subset_\beta (\alpha \models_{\mathcal{I}_\alpha})};\\
\dfrac{\models_\beta (\models \alpha);\ \subset ((\not\models \alpha) \Longrightarrow (\models o))}{(\models_{\mathcal{O}_o} o) \subset_\beta (\models_{\mathcal{I}_\alpha} \alpha)};\\
\dfrac{\left(\begin{array}{r}\alpha \models_\beta \alpha;\ (((\alpha \not\models \alpha) \Longrightarrow (o \models o)) \subset\\ ((\alpha \not\models \alpha) \Longrightarrow (o \models o)))\end{array}\right)}{(o \models_{\mathcal{O}_o} o) \subset_\beta (\alpha \models_{\mathcal{I}_\alpha} \alpha)};\\
\dfrac{\left(\begin{array}{r}(\alpha \models_\beta; \models_\beta \alpha);\ (((\alpha \not\models; \not\models \alpha) \Longrightarrow o) \subset;\\ \subset ((\alpha \not\models; \not\models \alpha) \Longrightarrow o))\end{array}\right)}{(o \models_{\mathcal{O}_o}; \models_{\mathcal{O}_o} o) \subset_\beta (\alpha \models_{\mathcal{I}_\alpha}; \models_{\mathcal{I}_\alpha} \alpha}
\end{array}\right)}{\left(\begin{array}{l}
(o \models) \subset (\alpha \models);\ (\models o) \subset (\models \alpha);\\
(o \models o) \subset (\alpha \models \alpha);\\
(o \models; \models o) \subset (\alpha \models; \models \alpha)
\end{array}\right)}$$

where $\models_\beta$ and $\subset_\beta$ are believable operators. □

Certainly, we must not forget that the last inferential scheme of modus obliquus is obtained on an intuitive basis and that many other senseful, obliquely structured inference schemes may exist.

7 Includedness as a Consequence of Informational Internalism

Informational internalism pertaining to an entity α was expressed formally by $\models \alpha$. Operator $\models$ was said to be the most general informational operator (an operational joker), which can be particularized according to the occurring informational circumstances. Informational includedness in the internalistic sense is a particularization, marked by $\subset \alpha$. The empty left side of operator $\subset$ points to the openness, to the question: *What can informationally be included in α?* or *What is the includable internalism of entity α?*

Includedness is a consequence of the particularization of informational internalism. This particularization was determined by Definition 1 as a recursive (recursively infinite) scheme with further 15 possibilities (if we exclude the empty case, as the 16$^{\text{th}}$ possibility). Thus, the particularization from general internalism of the form $\models \alpha$ to the includable internalism of the form $\subset \alpha$ brought a complex recursive scheme, in which, the initial form $\models \alpha$ there appears as a particular case. This does not mean an informational contradiction but circularity.

Let us show how even the extreme extensional case $\Xi^{\beta,\alpha}_{\beta,\alpha}(\alpha \subset \beta)$ obtains its full significance in the so-called everyday speech [6]. Let the role of α be assigned to words and that of β to their contexts in speech. In [6], the following dictum seems to be highly senseful (pp. 80–81):

- *Words and ideas are inseparable. ... Words and ideas hold together. ... Every word gets its meaning from some kind of context and we recognize it in that, or similar contexts. The context suggests the word, the word suggests the context. The context may be* physical. *... The context may be* psychological. *... The context may be* verbal. *Every word that you understand when you read or listen has meaning in that, and similar verbal contexts. The word belongs in the context. The word lives in the context. The two are inseparable.*

We can understand that in the cited case not only $\alpha \models \beta; \beta \models \alpha$ holds, but also $\alpha \subset \beta; \beta \subset \alpha$ is informationally senseful. The last citation helps us to understand that in the case of $\alpha \subset \beta; \beta \subset \alpha$ there is no a problem of contradiction in comprehension of the informational Being-in. As it was said, the word belongs to the context and vice versa, the context is includably impacted by the word. This statement holds especially for the process of informational arising of context (e.g., speech), where words intentionally influence the arising of context and context influences the choice of the words constituting the arising context.

Informational includedness of something is a consequence of the perceiving abilities of the something observing entity. An informational depiction of something in the observing entity concerns the problem of informing between entities [11], where the depiction of something is called the internal representation (or real presence [7]) of something. Such representation is always informationally included in the observing entity, while the vice versa case does not hold at all (there is, for example, no informational influence of the observing entity on the observed entity).

8 Being-in and Being-in-the-world

Being-in-the-world is a philosophical term, being coined by Heidegger [2]. "It is a general basic state of an entity and of informational entity in particular (in this case, $\beta_{\text{Being-in-the-world}} \subset \alpha$, where α marks an entity in question). It is sketched in terms of an orientation towards Being-in as such. Being-in-the-world stands for a unitary phenomenon and cannot be broken up into contents which may be pieced together. But, it has several constitutive items in its structure." As we will understand, Being-in-the-world informationally dwells in Being-in which always pertains to an informational entity (as a property, involvement, characteristics).

Being-in-the-world is informationally particularized Being-in, where the world is still comprehended in a universally open way. The world is also a specific category of thinking which must not be equalized with the physical (space-temporal) world in which phenomena appear and disappear. An informational entity informs in the world if the surrounding world (environment, its exterior) impacts the structure of the entity's informing and the entity perceives also the responses to its own informing to the world.

Being-in-the-world is a condition sine qua non for the arising of the so-called intelligent informing. Intelligent can mean to inform inventively, ingeniously, creatively also in the sense of the chaotic, unforeseeable, with the intention to adapt, reach a goal, survive, solve a problem, etc. Informational Being-in-the-world is more concretized informing of something than the informational Being-in, which is a general framework for further informational particularization or certain universalization. If we introduce markers $\beta_{\text{Being-in}}$ and $\beta_{\text{Being-in-the-world}}$ then one can express this relation by the operator of informational includedness $\subset$. At least, there must hold $\beta_{\text{Being-in-the-world}} \subset \beta_{\text{Being-in}}$. Because of an informational interaction, there can also exist $\beta_{\text{Being-in}} \subset \beta_{\text{Being-in-the-world}}$. Entity α informs as being in the world, that is, $\alpha \subset \beta_{\text{Being-in-the-world}}$ and arises informationally within this circumstances.

9 Conclusion

Informational Being-in comes not only very close to the philosophical Being-in [2, 1], but can surmount it by theoretical-formal expressions (arising formulas) of informational language, showing the decompositional power of the initially set includable problems. Through the discussion of informational includedness in this paper we have learned its complexity in parallel, serial, and parallel-serial structures. We chose (Definition 1) an informationally logical and flexible case of the includedness definition. Certainly, other cases of even a more complex definition of informational includedness are possible.

The most pretentious case of an includable structure seems the metaphysical case, where further and more detailed decomposition (interpretation) together with introduction of parallel formulas is possible and senseful. The case of a complex parallel structure of serial-parallel and parallel-serial formulas can be viewed as the most appropriate candidate in conceptualizing an informational system performing as an intelligent entity.

Metaphysical includedness was composed of several reasonable chosen informational entities, that is components, which have been staying for the so-called informing, counterinforming, and informational embedding of the entity in question. The informing component seems to be a necessity for the explication of the essential and detailed characteristics of the informing entity, its own intention (informational perseverance as a consequence of the existing informational structure) in spontaneity and circularity.

The counterinforming component (as a counterpart of informing, e.g. a form of its informational 'subconsciousness') was involved in production of new, also contradictory, and yet not informationally embedded (connected) information, which is nothing else than a kind of 'originally' arisen phenomenon. This component (counterinforming with counterinformational entity) seems to be the most problematic concept in concern to its informational implementation, for example, by a computer program and, lastly, by an informational machine as a substantially differently structured and conceptualized nowadays computer system. In general, we have presupposed that informational operators perform in an informational, counterinformational, and informationally embedding manner.

At last, the arisen and to the informational entity arrived (from its exterior perceived) information has to be informationally connected to that what already exists, that is, to the informing (body) of the entity itself. This phase of metaphysical phenomenalism we have called informational embedding.

An entity's metaphysical shell as described, is the most rationally imaginable (minimalist) structure of informing. Its particularization means the filling and extending (interpreting) of the three main components. The metaphysical shell conceptualizes unlimited possibilities in the informational arising of the entity α. It is the way to its concrete and artificial implementation, for instance, within an informational machine $\mathcal{M}$. Thus, $\alpha \subset \mathcal{M}$, where $\mathcal{M}$ systemically supports the informing of α. Counterinforming $\mathcal{C}_\alpha$ with counterinformational entity γ_α supports the arising of originality o within an entity's counterinforming, that is, $((o \subset \mathcal{C}_\alpha) \subset \mathcal{I}_\alpha) \subset \alpha$, etc. Originality as counterinforming can inform in different ways: every time new value to something is given, it can be seen as an original informing; familiar entities can be looked in different light; bringing together known entities and link them in new ways is an original approach. Originality can grow out of that which is already known (e.g., producing knowledge by knowledge from knowledge [12]).

Informational Being-in as developed in this paper is the beginning step in making informational theory axiomatic and constructive and, through this, tracing the way to an informational system implementation. The problem of knowledge as informational entity will show how the formalistic (axiomatic and inferring) power of informational theory can lead to new concepts, techniques, methods and theoretical approaches, which can absorb the today scientific methodologies and connect them into an informationally arising system. The biggest challenge on this way is the so-called informational machine, which will perform as a real informational accelerator, offering the widest possible framework for informational experiments and applications. In this sense, the future informational machine must be capable to mimic the most complex parallel-serial systems of informational arising, supporting systemically the informing, counterinforming, and embedding of any informational entity. Within this perspective, informational Being-in with its externalism, internalism, metaphysicalism, and phenomenalism seems to be the keystone of the arising informational theory and methodology.

Let us close our theoretical and formalistic discourse on informational Being-in with a dictum of George Steiner ([7], pp. 174–175), which approves the potentiality of informational approach (the theory in this essay) in its wholeness:

- *Though acts of reception and of understanding are in some measure fictions of ordered intuition, myths of reason, this truth does not justify the denial of intentional context. It is an absurd to discard as mendacious, as anarchically opaque, the bearing of contextual probability and suggestion, as it is to invest in such probability any blind trust. ... We advance step by step towards a delineation of the given space; our perceptions are more and more justly incident to the circumference of possible intent and meaning. The congruence is never complete. It is never uniform with its object. If it was, the act of reception would be wholly equivalent to that of original enunciation.*

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