ANALYTICAL SOLUTIONS FOR ONE-DIMENSIONAL CONSOLIDATION IN UNSATURATED SOILS CONSIDERING THE NON-DARCY LAW OF WATER FLOW

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Abstract

Analytical solutions were derived for the non-linear, one-dimensional consolidation equations for unsaturated soils. The governing equations with a non-homogeneous mixed-boundary condition were presented, in which the water flow was assumed to be governed by a non-Darcy law, whereas the air flow followed the Darcy law. The non-Darcy law was actually the non-linear, flux-gradients relationship. The consolidation equations were thus present in a strong, non-linear way. In order to analytically solve the equation, a homotopy analysis method (HAM) was introduced in the study, which is an analytical technique for nonlinear problems. Firstly, a governing equation in a dimensionless form was derived for a one-dimensional consolidation under unsaturated soils. The method was then used for a mapping technique to transfer the original nonlinear differential equations to a number of linear differential equations. These differential equations were independent with respect to any small parameters, and were convenient for controlling the convergence region. After this transferring, a series solution to the equations was then obtained using the HAM by selecting the linear operator and the auxiliary parameters. Meanwhile, comparisons between the analytical solutions and the results of the finite-difference method indicate that the analytical solution is more efficient. Furthermore, our solutions indicate that the dissipation of air pressure is much faster than that of water pressure, and the values for the threshold gradient I have obvious effects on the dissipation values of the excess pore-water pressure, but no significant effect on that of the excess pore-air pressure.

keywords

unsaturated soil, homotopy analysis method, analytical solutions, non-Darcy law, initial and boundary conditions

INTRODUCTION

The consolidation of unsaturated soils is a subject of great interest in geotechnical engineering practice [1-3]. In fact, the excess pore pressures dissipate with time and eventually return to their initial values in unsaturated soils, generated by external loading. The dissipation processes of excess pore pressures are called consolidation and result in a volume decrease [1]. Indeed, it is important to describe the dissipation of the excess pore pressures in understanding the consolidation of unsaturated soils, and the identification of the influencing internal mechanisms also plays an important role.

Several consolidation theories in unsaturated soils have been proposed over the past few years. The notable contributions have included the work of Blight [4], Scott [5], Barden [6] and Fredlund [7]. Fredlund and Hasan [7] proposed a one-dimensional consolidation theory, the most popular in the geotechnical engineering community, in which two partial differential equations were employed to describe the dissipation processes of excess pore pressures in unsaturated soils. Meanwhile, Qin et al. [2] gave an analytical solution for Fredlund's one-dimensional consolidation equation by applying the Laplace transform and Cayley-Hamilton mathematical methods in unsaturated soil with a finite thickness. Their boundary conditions were the top surface being permeable and the bottom surface being impermeable to air and water. Subjecting to the load exponentially varying with time, and using the same method and employing the same boundary conditions, Qin et al. [8] presented an analytical solution to the one-dimensional consolidation in unsaturated soils. Subjected to an arbitrary load, Shan et al. [9] employed a segregation variable method to obtain some exact solutions with three basic boundary conditions for unsaturated single-layer soils.

However, the assumption focuses in the above contributions were based on Darcy's law, which is valid regardless of the magnitude of the hydraulic gradients. Indeed, some evidence shows that the flow of pore water in unsaturated soil may not obey Darcy's law. There are currently very few models of unsaturated soils that take into account the non-linear flux-gradient relationship. The non-Darcy law is actually a non-liner flux-gradients relationship. In particular, there are no analytical solutions of the consolidation in unsaturated soils that take into account the non-Darcy law. Cui et al. [10] reported non-Darcy behavior for a range of observed hydraulic gradients under unsaturated conditions. Considering water as a non-Newtonian fliud, Liu [11] deduced a constitutive model for unsaturated soils. Liu and Birkholzer [12] proposed a relationship by generalizing the existing non-Darcy law and Darcy law.

This paper aims to present a mathematical model of one-dimensional consolidation in unsaturated soils by considering the non-linear flux-gradient relationship, and derive its exact solution by a homotopy analysis method.

A homotopy analysis method (HAM) [13] was adapted to solve the non-linear model of one-dimensional consolidation under unsaturated conditions. The method was independent of any small or large parameters and was valid for most non-linear problems in science and engineering. The homotopy analysis method has been successfully applied to many non-linear problems [14-16]. Finally, in order to verify the analytical solution, a comparison was carried out between the analytical solutions and the results of the finite-difference method in some cases. The results indicated that the analytical solution in the present study was reasonable. Moreover, the analytical solutions given here were valuable for understanding consolidation in unsaturated soil.

2 GOVERNING EQUATIONS

2.1 ASSUMPTIONS

The main assumptions for the one-dimensional consolidation governing equations are listed as follows:

(1) the solid particles and water phase are incompressible.
(2) the water flow is governed by a non-Darcy-type law, but air flow is governed by a Darcy-type law.
(3) the effects of temperature change, air dissolved in water, air diffusion, and the generation and diffusion of vapor are ignored.

2.2 NON-DARCY LAW

A general relationship between water flux and hydraulic gradients under unsaturated conditions was proposed by Liu [12] based on Swartzendruber's work [13]. The relationship can be written as

$$v = -k(i - \int_0^i \exp\{-(\frac{x}{I})^\alpha\}dx) \tag{1}$$

where v is the water velocity, k is the water permeability coefficient, i is the hydraulic gradient, I is the threshold gradient, and α is a constant parameter.

The one-dimensional differential form of Eq. (1) can be written as

$$v = -k(\frac{\partial u_w}{\gamma_w \partial z} + 1 - \int_0^{\frac{\partial u_w}{\gamma_w \partial z}+1} \exp\{-(\frac{x}{I})^\alpha\}dx) \tag{2}$$

where u_w is the pore-water pressure, $\gamma_w = \rho g$, ρ is the water density, and g is the acceleration due to gravity.

The integrand of the third item in equation (2) can be expanded as a Taylor series

$$\exp\{-(\frac{x}{I})^\alpha\} = \sum_{n=0} (-1)^n \frac{x^{n\alpha}}{n! I^{n\alpha}} \tag{3}$$

Taking the first two of the Taylor series, integral terms with the condition of $v = 0$ at $\frac{\partial u_w}{\gamma_w \partial z} + 1 = 0$ are simplified as

$$\int_0^{\frac{\partial u_w}{\gamma_w \partial z}+1} (1 - (\frac{x}{I})^\alpha)dx = \frac{\partial u_w}{\gamma_w \partial z} + 1 - \frac{1}{(\alpha+1)I^\alpha}(\frac{\partial u_w}{\gamma_w \partial z} + 1)^{\alpha+1} \tag{4}$$

Substituting Eq. (4) into Eq. (2), the following equation can be obtained as

$$v = -k\frac{1}{(\alpha+1)I^\alpha}(\frac{\partial u_w}{\gamma_w \partial z} + 1)^{\alpha+1} \tag{5}$$

The equation reduces to Darcy's law when α becomes zero ($I \neq 0$), i.e.,

$$v = -k(\frac{\partial u_w}{\partial z} + 1) \tag{6}$$

2.3 CONSOLIDATION OF UNSATURATED SOILS

Following Fredlund and Morgenstern [18], the constitutive relation for the water and air phases are

$$\frac{\partial(V_w/V)}{\partial t}=m_{1k}^w\frac{\partial(\sigma-u_a)}{\partial t}+m_2^w\frac{\partial(u_a-u_w)}{\partial t} \quad (7)$$

$$\frac{\partial(V_a/V)}{\partial t}=m_{1k}^a\frac{\partial(\sigma-u_a)}{\partial t}+m_2^a\frac{\partial(u_a-u_w)}{\partial t} \quad (8)$$

where, $\frac{\partial(V_w/V)}{\partial t}$ is the volume change of water in the soil, m_{1k}^w is the coefficient of water volume change with respect to the change in the net normal stress $\sigma-u_a$, m_2^w is the coefficient of water volume change with respect to the change in the matrix suction u_a-u_w , $\frac{\partial(V_a/V)}{\partial t}$ is the volume change of the air in the soil, m_{1k}^a is the coefficient of air volume change with respect to the change in the net normal stress $\sigma-u_a$, and m_2^a is the coefficient of air volume change with respect to the change in the matrix suction u_a-u_w . The subscript k stands for the K_0-loading condition without lateral deformation.

The continuity requirement leads to the following relations [3,18]:

$$m_{1k}^s=m_{1k}^w+m_{1k}^a \quad (9)$$

$$m_2^s=m_2^w+m_2^a \quad (10)$$

where m_{1k}^s and m_2^s are the coefficients of volume change of the soil with respect to a change in the net normal stress $\frac{\sigma_z}{2}-u_a$, and the matrix suction u_a-u_w , respectively. σ_z is the total normal stress in the z direction.

According to the law of mass conservation for water, the change in water volume can be written as follows:

$$\frac{\partial(V_w/V_0)}{\partial t}=\frac{\partial v_w}{\partial z} \quad (11)$$

Substituting Eq.(5) and Eq.(7) into Eq.(11), the following equation can be obtained as

$$\frac{\partial u_w}{\partial t}+C_w\frac{\partial u_a}{\partial t}=-C_V^w\frac{\partial^2 u_w}{\partial z^2}\left(\frac{\partial u_w}{\gamma_w\partial z}+1\right)^\alpha \quad (12)$$

where, $C_w=\frac{1-m_2^w/m_{1k}^w}{m_2^w/m_{1k}^w}, C_V^w=\frac{k_w}{\gamma_w m_2^w I^\alpha}$.

The air is considered to behave as ideal air, and based on Boyle's law and Darcy's law. The governing equation for the air phase yields

$$\frac{\partial u_a}{\partial t}+C_a\frac{\partial u_w}{\partial t}=-C_V^a\frac{\partial^2 u_a}{\partial z^2} \quad (13)$$

where $C_a=\frac{m_2^a}{m_{1k}^a-m_2^a-(1-S)nu_{atm}/(\bar{u}_a^0)^2}$,

$C_V^a=k_a\frac{RT}{g\bar{u}_a^0 M(m_{1k}^a-m_2^a-(1-S)nu_{atm}/(\bar{u}_a^0)^2)}$,

$\bar{u}_a^0=u_a^0+u_{atm}$, k_a is the air conductivity, R is the universal air constant, T is the absolute temperature, M is the average molecular mass of the air phase, u_a^0 is the initial excess air pressure, and u_{atm} is the atmospheric pressure.

2.4 BOUNDARY AND INITIAL CONDITIONS

In the present study, an unsaturated soil layer was considered as an infinite horizontal extent and thickness H (as shown in Fig.1).The top surface is permeable to water and air, whereas the bottom is impermeable to water and air.

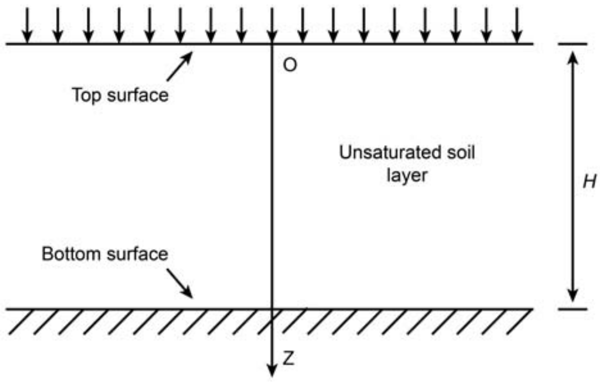

Figure 1. One-dimensional consolidation in unsaturated soils with a permeable top surface and an impermeable bottom base.

Hence, the initial conditions and boundary conditions are respectively expressed as

$$u_a(z,0)=u_a^0, u_w(z,0)=u_w^0 \quad (14)$$

$$\begin{cases} u_a(0,t)=u_a^0, u_w(0,t)=u_w^0 \\ \frac{\partial u_a(H,t)}{\partial z}=0, \frac{\partial u_w(H,t)}{\partial z}=0 \end{cases} \quad (15)$$

where u_a^0 and u_w^0 are the initial excess air and water pressures at t = 0, respectively. ($0\le z\le H$, $t\ge 0$)

3. ANALYTICAL SOLUTION

3.1 THE ANALYTICAL SOLUTION OF THE EQUATIONS

The Eqs.(12)- (15) can now be rewritten in the dimensionless form as

$$\frac{C_1}{I^\alpha}\frac{\partial^2\theta_w}{\partial\eta^2}(1+\alpha C_0\frac{\partial\theta_w}{\partial\eta})=\frac{\partial\theta_w}{\partial\tau}+C_wC_2\frac{\partial\theta_a}{\partial\tau} \quad (16)$$

$$C_3\frac{\partial^2\theta_a}{\partial\eta^2}=C_2\frac{\partial\theta_a}{\partial\tau}+C_a\frac{\partial\theta_w}{\partial\tau} \quad (17)$$

subject to the initial and boundary conditions

$$\theta_a(\eta,0)=1,\theta_w(\eta,0)=1 \text{ , in } 0\le\eta\le1 \quad (18)$$

$$\begin{cases}\theta_a(0,\tau)=1,\theta_w(0,\tau)=1\\ \dfrac{\partial\theta_a(1,\tau)}{\partial\eta}=0,\dfrac{\partial\theta_w(1,\tau)}{\partial\eta}=0\end{cases} \text{, in } \tau\ge0 \quad (19)$$

where, the dimensionless parameters are defined by

$$\theta_a=\frac{u_a}{u_a^0},\ \theta_w=\frac{u_w}{u_w^0},\ \eta=\frac{z}{H},\ \tau=-\frac{k_wt}{\gamma_wm_{1k}^sH^2},$$

$$C_0=\frac{u_w^0}{\gamma_wH},\ C_1=\frac{m_{1k}^s}{m_2^w},\ C_2=\frac{u_a^0}{u_w^0},$$

$$C_3=\frac{k_a}{k_w}\frac{u_a^0}{u_w^0}\frac{RT\gamma_wm_{1k}^s}{g\bar{u}_a^0M(m_{1k}^a-m_2^a-(1-S)nu_{atm}/(\bar{u}_a^0)^2)}.$$

3.2 SERIES SOLUTIONS GIVEN BY THE HAM

As a nonlinear analytical technique, the homotopy analysis method (HAM) is efficient in the selection of a series of basis functions and auxiliary linear operators, and easily makes the solution convergence. The technique is based on homotopy, which is an important part of topology. Using one interesting property of homotopy, we can transform any nonlinear problem into an infinite number of linear problems, no matter whether or not there exists a small or large parameter. These linear problems are not dependent on any small parameters, which is convenient for controlling the convergence region. After this transferring, a series solution to the nonlinear problem is then obtained by the HAM after the selection of auxiliary linear operator parameters. [13]

We chose the auxiliary linear operator,

$$L=\frac{\partial^2}{\partial\eta^2} \quad (20)$$

From Eq.(16), it is straightforward to define the nonlinear operator

$$N(\theta_w)=\frac{C_1}{I^\alpha}\frac{\partial^2\theta_w}{\partial\eta^2}-\frac{\partial\theta_w}{\partial\tau}-C_wC_2\frac{\partial\theta_a}{\partial\tau} \quad (21)$$

From Eq.(17), we define the operator

$$L_a(\theta_a)=C_3\frac{\partial^2\theta_a}{\partial\eta^2}-C_2\frac{\partial\theta_a}{\partial\tau}-C_a\frac{\partial\theta_w}{\partial\tau} \quad (22)$$

and the initial approximation

$$\theta_0^w=\theta_0^a=1+\eta(1-\eta)^2\exp(-0.4\tau)+\eta(1-\eta)^2\exp(-0.2\tau) \quad (23)$$

The zero-order deformation equation is constructed as (Liao 2004)

$$(1-p)L(\phi(\tau,\eta;p)-\theta_0^w)=phH_fN(\phi_w(\tau,\eta;p)) \quad (24)$$

$$(1-p)L(\phi_a(\tau,\eta;p)-\theta_0^a)=phH_fL_a(\phi_a(\tau,\eta;p)) \quad (25)$$

subject to the conditions

$$\phi_i(\eta,0)=0 \text{ , in } 0\le\eta\le1 \quad (26)$$

$$\begin{cases}\phi_i(0,\tau;p)=0\\ \dfrac{\partial\phi_i(1,\tau;p)}{\partial\eta}=0\end{cases} \text{, in } \tau\ge0 \quad (27)$$

where, $i = w,a$, $p\in(0,1)$ is the embedding parameter, h is a non-zero auxiliary parameter, H_f is a auxiliary function, and ϕ is an unknown function of τ, η, p respectively. It is obviously that when $p = 0$ and $p = 1$, it respectively holds that $\phi_i(\tau,\eta,0)=\theta_0^i,\phi(\tau,\eta,1)=\theta_i(\tau,\eta)$. Then when p increases from 0 to 1, $\phi_i(\tau, \eta, p)$ varies from θ_0^i to $\theta_i(\tau,\eta)$. With respect to p due to the Taylors' series, $\phi_i(\tau, \eta, p)$ can be expanded, i.e.

$$\phi_i(\tau,\eta,p)=\sum_{k=0}^{+\infty}\theta_k^i(\tau,\eta)p^k \quad (28)$$

where

$$\theta_k^i(\tau,\eta)=\frac{1}{k!}\frac{\partial^k\phi_i(\tau,\eta,p)}{\partial p^k}\bigg|_{p=0} \quad (29)$$

If h is chosen in such a way that this series is convergent at $p = 1$, so we have

$$\theta_i(\tau,\eta)=\sum_{m=0}^{+\infty}\theta_m^i(\tau,\eta) \quad (30)$$

Differentiating the zero-order deformation equations m times with respect to p, then dividing by $m!$, and finally setting $p = 0$, we have the m^{th}-order deformation equations, i.e.,

$$L(\theta_m^w(\tau,\eta)-\chi_m\theta_{m-1}^w(\tau,\eta))=hH_fR_m^w(\tau,\eta) \quad (31)$$

$$L(\theta_m^a(\tau,\eta)-\chi_m\theta_{m-1}^a(\tau,\eta))=hH_fR_m^a(\tau,\eta) \quad (32)$$

subject to the conditions

$$\theta_m^i(\eta,0)=0 \quad \text{in} \quad 0\le\eta\le 1 \tag{33}$$

$$\begin{cases}\theta_m^i(0,\tau)=0\\ \dfrac{\partial\theta_m^i(1,\tau)}{\partial\eta}=0\end{cases}, \text{ in } \tau\ge 0 \tag{34}$$

where

$$\chi_m=\begin{cases}0, m\le 1,\\ 1, m>1\end{cases} \tag{35}$$

and

$$R_m^w(\tau,\eta)=\frac{C_1}{I^\alpha}\frac{\partial^2\theta_{m-1}^w}{\partial\eta^2}+\frac{\alpha C_0 C_1}{I^\alpha}\sum_{j=0}^{m-1}\frac{\partial^2\theta_j^w}{\partial\eta^2}\frac{\partial\theta_{m-1-j}}{\partial\eta}- -\frac{\partial\theta_{m-1}^w}{\partial\tau}-C_w C_2\frac{\partial\theta_{m-1}^a}{\partial\tau} \tag{36}$$

$$R_m^a(\tau,\eta)=C_3\frac{\partial^2\theta_{m-1}^a}{\partial\eta^2}-C_2\frac{\partial\theta_{m-1}^a}{\partial\tau}-C_a\frac{\partial\theta_{m-1}^w}{\partial\tau} \tag{37}$$

To satisfy the initial condition Eq.(33), we chose the auxiliary function as: $H_f=\tau$.

The solution to the above equations was obtained using Maple software. It was seen that θ_m^w , θ_m^a can be respectively expressed by

$$\theta_m^w=\sum_{i=0}^{2m+2}\sum_{j=0}^{2m+3}\sum_{k=0}^{2m+1}a_{ijk}^m(h)e^{-\frac{1}{5}i\tau}\eta^j\tau^k \tag{38}$$

$$\theta_m^a=\sum_{i=0}^{2m}\sum_{j=0}^{2m+3}\sum_{k=0}^{2m}b_{ijk}^m(h)e^{-\frac{1}{5}i\tau}\eta^j\tau^k \tag{39}$$

where $a_{ijk}^m(h), b_{ijk}^m(h)$ are dependent upon h. And $a_{ijk}^m(h), b_{ijk}^m(h)$ can be easily obtained, substituting Eqs. (38)–(39) into the m-order deformation Eqs. (31)–(35) and all coefficients a_{ijk}^m , b_{ijk}^m (see appendix A) of the solution can be obtained one by one from the first coefficients. The first coefficients were given by the initial approximation Eq.(23), i.e.,

$$a_{000}^0(h)=1,\ a_{111}^0(h)=a_{131}^0(h)=1,\ a_{211}^0(h)=a_{231}^0(h)=1,\ a_{121}^0(h)=a_{221}^0(h)=-2 \tag{40}$$

So the solution can be given by

$$\theta_w(\tau,\eta)=\sum_{m=0}^{+\infty}\sum_{i=0}^{2m+2}\sum_{j=0}^{2m+3}\sum_{k=0}^{2m+1}a_{ijk}^m(h)e^{-\frac{1}{5}i\tau}\eta^j\tau^k \tag{41}$$

$$\theta_a(\tau,\eta)=\sum_{m=0}^{+\infty}\sum_{i=0}^{2m}\sum_{j=0}^{2m+3}\sum_{k=0}^{2m}b_{ijk}^m(h)e^{-\frac{1}{5}i\tau}\eta^j\tau^k \tag{42}$$

4 EXAMPLE AND VERIFICATION

It is important to ensure that the solution series can be converging. Fortunately, the convergence and the rate of approximation for the HAM solutions strongly depend on the values of the auxiliary parameter h. It is found that h must be negative ($h\in(-1,0)$) to ensure that the solution series converges.

In order to validate the analytical solution of one-dimensional consolidation for unsaturated soil, a typical example is computed by using the analytical solution and a comparison is made between the analytical solution of the mathematic model and the numerical results obtained by the finite-difference method using the typical example. The material parameters for the example are adopted as the same parameters in Table 1 and Table 2 [3].

Table 1. The material parameters for the solution of the examples.

Layer thickness (m)	H	10
Initial pore-air pressure (kPa)	u_a^0	20
Initial pore-water pressure (kPa)	u_w^0	40
Porosity	n	0.5
Saturation	S	0.8
Coefficient of volume change (kPa^{-1})	m_{1k}^s	-2.5×10^{-4}
Ratio of two coefficients of volume change	m_2^s/m_{1l}^s	0.4
Ratio of two coefficients of volume change	m_{1k}^w/m_{1k}^s	0.2
Ratio of two coefficients of volume change	m_2^w/m_{1k}^w	4

Table 2. The values of the parameters C_i (i = 0,1,2,3,w,a).

Parameter of equation	C_0	0.4
Parameter of equation	C_1	1.25
Parameter of equation	C_2	0.5
Parameter of equation	C_3	80.6
Interactive constant for water phase	C_w	-0.75
Interactive constant for air phase	C_a	-0.00775

Fig.2 and Fig.3 show that the change in the values of θ_w, θ_a at α = 2, I = 10, h = -0.001 with τ under different η. From the dimensionless definitions of the parameters, i.e., the change in the values of pore water pressure and pore air pressure at α = 2, I = 10, h = -0.001 with time under different depth z, we can find that the excess pore-water pressure is gradually increasing at beginning. It reaches the highest value at about $\tau = 5$ (i.e., $t = 1250/k_w$) and then begins to dissipate. But the excess pore-air pressure reaches the highest value at about $\tau = 1$ (i.e., $t = 250/k_w$) and then begins to dissipate.

Fig.4 and Fig.5 show that the change in the values of θ_w, θ_a at $\alpha = 2$, $I = 10$, $h = -0.001$ with η under different τ. That is the excess pore-water pressure and the excess pore-air pressure increase initially, and then decrease with the depth.

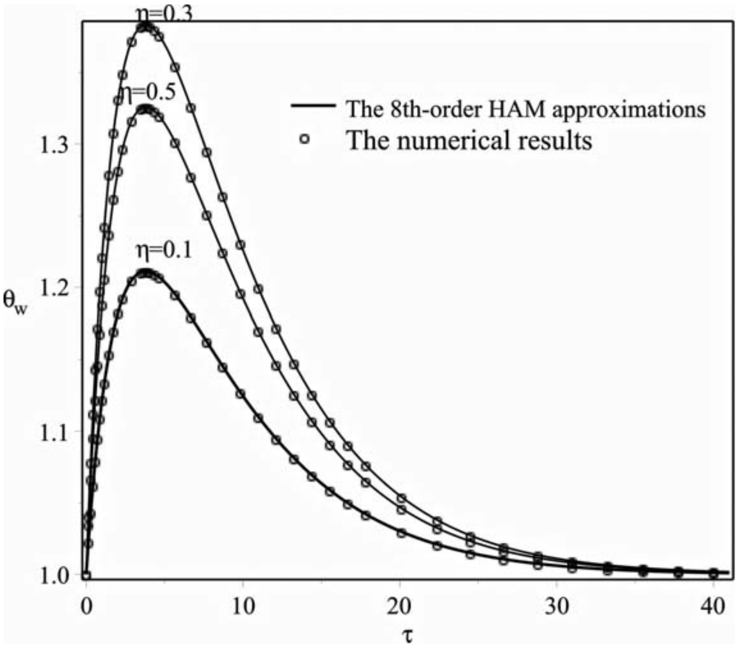

Figure 2. The 8th-order HAM approximations of θ_w at $\eta = 0.1, 0.3, 0.5$, $\alpha = 2$, $I = 10$, $h = -0.001$ open circle: the numerical results.

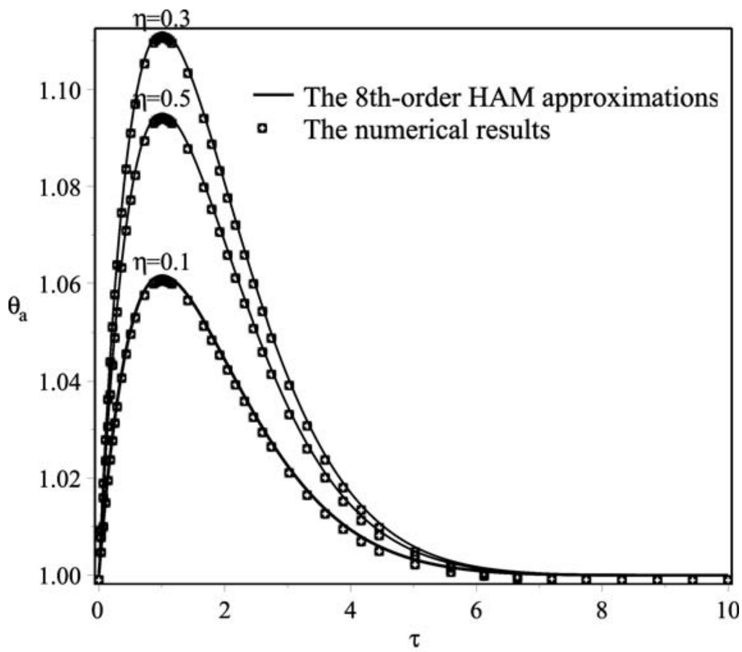

Figure 3. The 8th-order HAM approximations of θ_a at $\eta = 0.1, 0.3, 0.5$, $\alpha = 2$, $I = 10$, $h = -0.001$ open circle: the numerical results.

Figs. 2–5 show the comparison of the 8th-order HAM approximations and numerical solutions. It is clear that the computed results using the two methods are almost the same, which proves that the analytical solution proposed in this paper is credible.

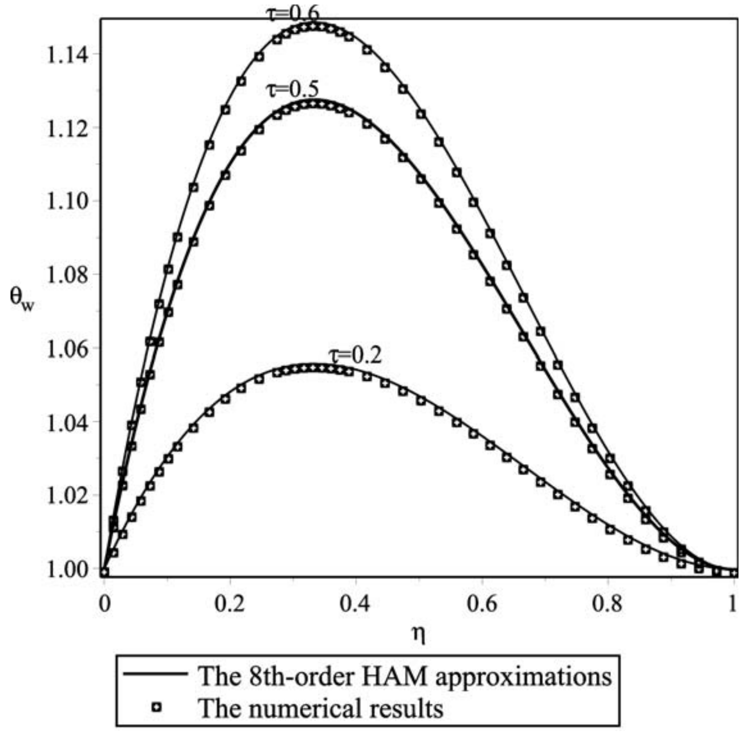

Figure 4. The 8th-order HAM approximations of θ_w at τ = 0.2, 0.5, 0.6, α = 2, I =10, h = -0.001 open circle: the numerical results.

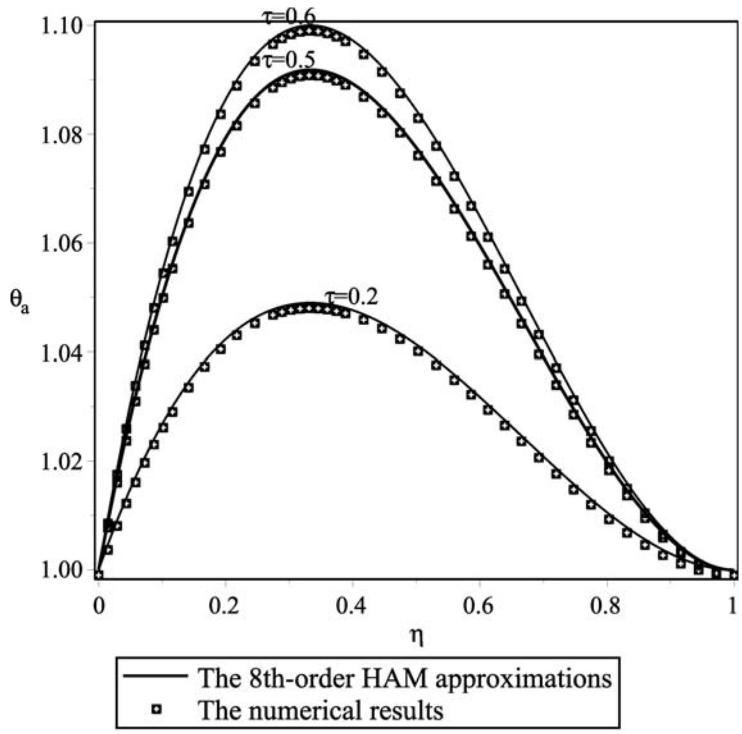

Figure 5. The 8th-order HAM approximations of θ_a at τ = 0.2, 0.5, 0.6, α = 2, I =10, h = -0.001 open circle: the numerical results.

Figs. 6-7 show that the values of *I* have obvious effects on the dissipation values of the excess pore-water pressure. It indicates that for smaller *I*, there is a smaller excess pore-water pressure. However, Fig. 6 shows that the values of *I* have no significant effect on the dissipation values of the excess pore-air pressure.

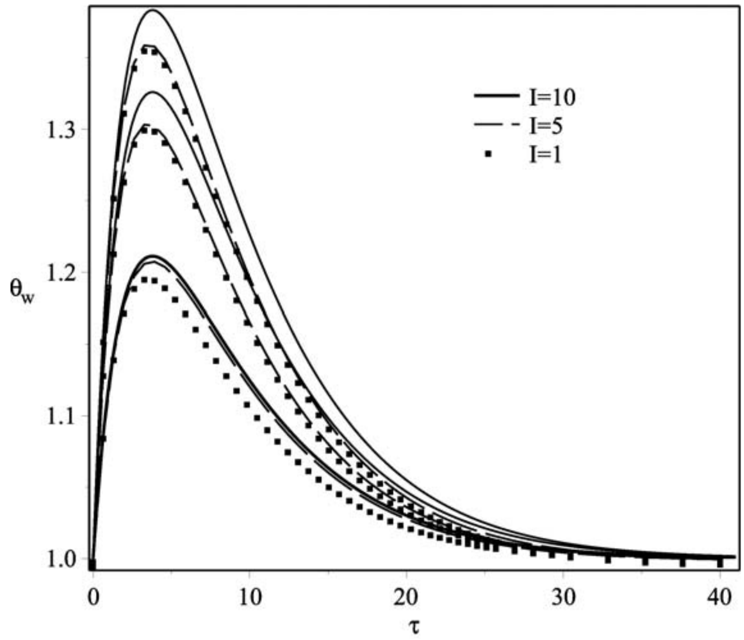

Figure 6. Change in θ_w with τ under different *I* at η = 0.1, 0.3, 0.6, respectively.

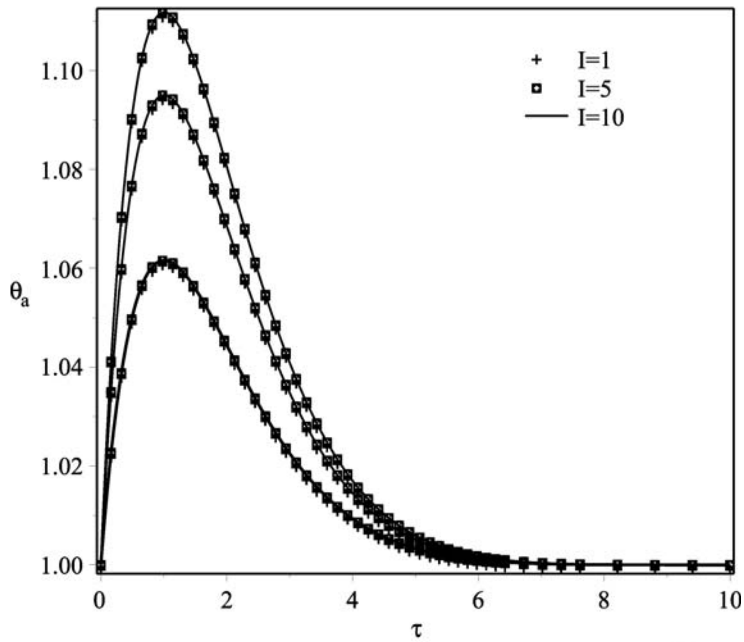

Figure 7. Change in θ_a with τ under different *I* at η = 0.1, 0.3, 0.6, respectively.

5 CONCLUSIONS

In this paper the water phase was assumed to obey the non-Darcy law. The simplified form of the governing equations for the one-dimensional consolidation of unsaturated soil was adopted. The series solutions based on the HAM for the excess pore-air pressure and the excess pore-water pressure were first obtained for the dimensionless consolidation in unsaturated soils. The solutions were applicable to the unsaturated soil layer with pore water and pore air pressure equivalent for the initial value at the top surface and the bottom impermeable to water and air. In addition, the validity of the present solutions was confirmed through a comparison with the numerical results.

Based on the solutions, the changes in the excess pore-air pressure and excess pore-water pressure with time were analyzed at different *I* and for different depths. In addition, it was found that the dissipation of air pressure was much faster than the dissipation of water pressure. Furthermore, the values of *I* had obvious effects on the dissipation values of the excess pore-water pressure but no significant effect on that of the excess pore-air pressure.

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APENDIX A

The values for $a_{ijk}^m(h), b_{ijk}^m(h)$ satisfy the recursive formula

$$L(\sum_{i=0}^{2m+2}\sum_{j=0}^{2m+3}\sum_{k=0}^{2m+1} a_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^j\tau^k - \chi_m\sum_{i=0}^{2m}\sum_{j=0}^{2m+1}\sum_{k=0}^{2m-1} a_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^j\tau^k) = hH_f\overline{R}_m^w(\tau,\eta)$$

$$L(\sum_{i=0}^{2m}\sum_{j=0}^{2m+3}\sum_{k=0}^{2m} b_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^j\tau^k - \chi_m\sum_{i=0}^{2m-2}\sum_{j=0}^{2m+1}\sum_{k=0}^{2m-2} b_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^j\tau^k) = hH_f\overline{R}_m^a(\tau,\eta)$$

where

$$\chi_m = \begin{cases} 0, m \le 1, \\ 1, m > 1 \end{cases}$$

and

$$\begin{aligned}\overline{R}_m^w(\tau,\eta) &= \frac{C_1}{I^\alpha}\frac{\partial^2}{\partial\eta^2}(\sum_{i=0}^{2m}\sum_{j=0}^{2m+1}\sum_{k=0}^{2m-1} a_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^j\tau^k) \\ &+ \frac{\alpha C_0 C_1}{I^\alpha}\sum_{j=0}^{m-1}\frac{\partial^2\theta_j^w}{\partial\eta^2}(\sum_{i=0}^{2j+2}\sum_{l=0}^{2j+3}\sum_{k=0}^{2j+1} a_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^l\tau^k)\frac{\partial\theta_{m-1-j}}{\partial\eta}(\sum_{i=0}^{2m-2j}\sum_{l=0}^{2m-2j+1}\sum_{k=0}^{2m-2j-1} a_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^l\tau^k) \\ &- \frac{\partial}{\partial\tau}(\sum_{i=0}^{2m}\sum_{j=0}^{2m+1}\sum_{k=0}^{2m-1} a_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^j\tau^k) - C_w C_2\frac{\partial}{\partial\tau}(\sum_{i=0}^{2m-2}\sum_{j=0}^{2m+1}\sum_{k=0}^{2m-2} b_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^j\tau^k)\end{aligned}$$

$$\begin{aligned}\overline{R}_m^a(\tau,\eta) &= C_3\frac{\partial^2}{\partial\eta^2}(\sum_{i=0}^{2m-2}\sum_{j=0}^{2m+1}\sum_{k=0}^{2m-2} b_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^j\tau^k) - C_2\frac{\partial}{\partial\tau}(\sum_{i=0}^{2m-2}\sum_{j=0}^{2m+1}\sum_{k=0}^{2m-2} b_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^j\tau^k) \\ &- C_a\frac{\partial}{\partial\tau}(\sum_{i=0}^{2m}\sum_{j=0}^{2m+1}\sum_{k=0}^{2m-1} a_{ijk}^m(h)\mathrm{e}^{-\frac{1}{5}i\tau}\eta^j\tau^k)\end{aligned}$$