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Quadrupole polarizabilities of the pion in the Nambu–Jona-Lasinio model*

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Abstract. We present the results for the pion electromagnetic quadrupole polarizabilities, calculated within the Nambu–Jona-Lasinio model. We obtain the sign and magnitude in agreement with the respective experimental analysis based on the Dispersion Sum Rules. At the same time the dipole polarizabilities are well reproduced. Comparison is also made with the results from the Chiral Perturbation Theory.

The neutral and charged pion dipole and quadrupole polarizabilities have been recently analyzed using the Dispersion Relations (DR) and the Dispersion Sum Rules (DSR) [1,2], as displayed in Tables 2 and 3, together with the results of the Chiral Perturbation Theory (χPT) [3–5]. The first row shows our results based on the Nambu–Jona-Lasinio model (NJL) [6] model; for that purpose we have extended [7] the study of Ref. [8], where the dipole polarizabilities have been calculated, to the quadrupole case. We refer to these papers for details.

Our leading-N_c calculations are done according to the Feynman diagrams of Fig. 1. The amplitude is a function of the Mandelstam variables related to the $\gamma(p_1,\epsilon_1)+\gamma(p_2,\epsilon_2)\to\pi^a(p_3)+\pi^b(p_4)$ reaction for the on-shell pions and photons,

$$T(p_1,p_2,p_3)=e^2\epsilon_1^\mu\epsilon_2^\nu T_{\mu\nu},\quad T_{\mu\nu}=A(s,t,u)\mathcal{L}_1^{\mu\nu}+B(s,t,u)\mathcal{L}_2^{\mu\nu},$$

with the Lorentz tensors

$$\mathcal{L}_1^{\mu\nu}=p_2^\mu p_1^\nu-\frac{1}{2}sg^{\mu\nu},\qquad \mathcal{L}_2^{\mu\nu}=-\left(\frac{1}{2}u_1t_1g^{\mu\nu}+t_1p_2^\mu p_3^\nu+u_1p_3^\mu p_1^\nu+sp_3^\mu p_3^\nu\right).$$

Terms that vanish upon the conditions $\epsilon_1\cdot p_1=\epsilon_2\cdot p_2=0$ are omitted and the notation $\xi_1=\xi-m_\pi^2$, $\xi=s,t,u$ is used. The scalar quantities A and B enter the amplitudes $H_{++}=-(A+m_\pi^2B)$ and $H_{+-}=(u_1t_1/s-m_\pi^2)B$ for the equal-helicity and helicity-flipped photons. The dipole, α_1^i,β_1^i, and quadrupole,

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α_2^i, β_2^i, polarizabilities are obtained in the t-channel and extracted from the first two coefficients of the Taylor expansion of the amplitudes $\alpha A^i(s,t,u)/(2m_\pi)$ and $-\alpha m_\pi B^i(s,t,u)$ around $s = 0$ with $u = t = m_\pi^2$ [9],

$$\frac{\alpha}{2m_\pi}\left(A^i(0,m_\pi^2,m_\pi^2) + s\frac{d}{ds}A^i(0,m_\pi^2,m_\pi^2)\right) = \beta_1^i + \frac{s}{12}\beta_2^i,$$
$$-\alpha m_\pi\left(B^i(0,m_\pi^2,m_\pi^2) + s\frac{d}{ds}B^i(0,m_\pi^2,m_\pi^2)\right) = (\alpha_1+\beta_1)^i + \frac{s}{12}(\alpha_2+\beta_2)^i,$$

where the superscripts $i = N, C$ denote the neutral and charged pions, respectively, and $\alpha \simeq 1/137$ is the fine structure constant. The Born term, arising in the case of the charged pion, is removed from the amplitudes.

The NJL Lagrangian used in this work contains pseudoscalar isovector and scalar isoscalar four-quark interactions and is minimally coupled to the electromagnetic field.

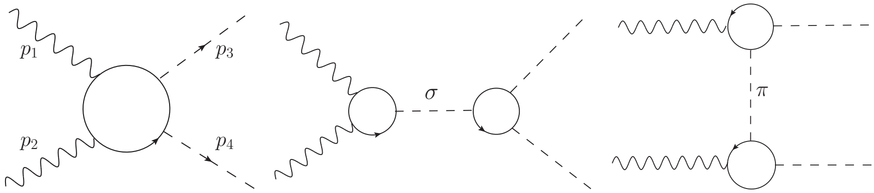

Fig. 1. Leading-N_c quark-loop diagrams for the $\gamma\gamma \to \pi\pi$ amplitude. The crossed terms are not displayed.

The diagrams of Fig. 1 provide polarizabilities which scale as N_c^0. Besides the quark one-loop diagrams, it is expected that the pion loops yield important contributions, mainly in the case where the tree-level results are absent (in the NJL model the chiral counting of meson tree-level results are classified in Refs. [8,10]). We include the lowest model-independent pion-loop diagram at the p^4 order, calculated within χPT in Refs. [11–13], and known to be the only non-vanishing contribution to the amplitude A at this order in the neutral channel. The pion loop in the charged mode contributes only to the quadrupole polarizabilities, with half the strength of the neutral quadrupole case. The pion loop as well as the σ-exchange diagram of Fig. 1 enter only the amplitude A.

The amplitude B for the neutral (charged) mode is completely determined by the quark box (quark box + pion exchange) diagrams, starting from the p^6 order for the dipole and from the p^8 order for the quadrupole polarizabilities. Thus the combinations $(\alpha_j + \beta_j)^i$, $(j = 1, 2)$, to which the B amplitude leads, provide a genuine test of the dynamical predictions of the NJL model at the leading order in $1/N_c$, as they are insensitive to the lowest-order χPT corrections.

All quark one-loop integrals are regularized using the Pauli-Villars prescription with one regulator Λ and two subtractions, which is consistent with the requirements of gauge invariance.

Table 1. The NJL model parameters for the charged and neutral channels, with the input marked by * and $f_\pi^* = 93.1$ MeV.

M^* [MeV]	m_π^* [MeV]	m [MeV]	G [GeV^{-2}]	Λ [MeV]
300	139	7.5	13.1	827
300	136	7.2	13.1	827

Table 2. The dipole (in units of $10^{-4}fm^3$) and quadrupole (in units of $10^{-4}fm^5$) neutral pion polarizabilities. The first row shows our model prediction at $M = 300$ MeV.

	$(\alpha_1+\beta_1)_{\pi^0}$	$(\alpha_1-\beta_1)_{\pi^0}$	$(\alpha_2+\beta_2)_{\pi^0}$	$(\alpha_2-\beta_2)_{\pi^0}$
$M = 300$ MeV	0.73	-1.56	-0.14	36.1
DR fit [1]	0.98 ± 0.03	-1.6 ± 2.2	-0.181 ± 0.004	39.70 ± 0.02
DSR [1]	0.802 ± 0.035	-3.49 ± 2.13	-0.171 ± 0.067	39.72 ± 8.01
χPT [5,3]	1.1 ± 0.3	-1.9 ± 0.2	0.037 ± 0.003	37.6 ± 3.3

Table 3. Same as in Table 2 for the dipole and quadrupole charged pion polarizabilities.

	$(\alpha_1+\beta_1)_{\pi^\pm}$	$(\alpha_1-\beta_1)_{\pi^\pm}$	$(\alpha_2+\beta_2)_{\pi^\pm}$	$(\alpha_2-\beta_2)_{\pi^\pm}$
$M = 300$ MeV	0.19	9.4	0.20	17.5
DR fit [2]	$0.18^{+0.11}_{-0.02}$	$13.0^{+2.6}_{-1.9}$	0.133 ± 0.015	$25.0^{+0.8}_{-0.3}$
DSR [2]	0.166 ± 0.024	13.60 ± 2.15	0.121 ± 0.064	25.75 ± 7.03
χPT [5,4]	0.16	5.7 ± 1.0	-0.001	16.2

In Table 1 we collect the model parameters, obtained by fitting the physical pion mass and the weak decay constant. The parameters of the model are the four-quark coupling constant G, the cutoff Λ, and the current quark mass m. These are determined by the choice of $f_\pi = 93.1$ MeV, $m_\pi = 139$ MeV (charged mode) or $m_\pi = 136$ MeV (neutral mode), and the constituent quark mass, M. In Table 4 we present the anatomy of our result for the case $M = 300$ MeV. We display separately all the gauge invariant contributions to the polarizabilities: the box (for neutral polarizabilities), box + pion exchange diagram (for the charged polarizabilities), the σ exchange, and the pion loop. The pion exchange diagram arises only for the charged channel and builds together with the box a gauge invariant amplitude. Let us first comment on the channels involving the difference of the electric and magnetic polarizabilities:

• $(\alpha_1-\beta_1)_{\pi^0}$: here the box contribution is largely canceled by the scalar exchange. At the p^4-order of the chiral counting they cancel exactly [8]. The higher-order contributions have a slow convergence rate, at p^8-order one reaches only about 50% of the full sum.

• $(\alpha_1-\beta_1)_{\pi^\pm}$: contrary to the neutral channel, the size of the σ-exchange diagram for this combination is about an order of magnitude larger than the box + pion exchange diagram, and it becomes the most important contribution. The pion loops are absent.

Table 4. Contribution of various diagrams for $M = 300$ MeV. Units as in Table 2.

	box + π-exchange	σ-exchange	pion-loop	total
$(\alpha_1+\beta_1)_{\pi^0}$	0.73	0	0	0.73
$(\alpha_1-\beta_1)_{\pi^0}$	-11.13	10.57	-1.0	-1.56
$(\alpha_2+\beta_2)_{\pi^0}$	-0.144	0	0	-0.144
$(\alpha_2-\beta_2)_{\pi^0}$	5.09	9.07	21.97	36.13
$(\alpha_1+\beta_1)_{\pi^\pm}$	0.189	0	0	0.189
$(\alpha_1-\beta_1)_{\pi^\pm}$	-0.977	10.36	0	9.39
$(\alpha_2+\beta_2)_{\pi^\pm}$	0.198	0	0	0.198
$(\alpha_2-\beta_2)_{\pi^\pm}$	-1.63	8.87	10.29	17.54

• $(\alpha_2-\beta_2)_{\pi^\pm}$: the pattern observed for the charged dipole difference repeats itself for the quadrupole polarizabilities. However in this case the subleading in the $1/N_c$ counting pion-loop diagram has the same magnitude as the σ-exchange term.

• $(\alpha_2-\beta_2)_{\pi^0}$: the pion loop dominates, $\sim 2\times\sigma$-exchange.

• Finally, regarding the sums of polarizabilities, we stress that the values (including the overall signs) of $(\alpha_2+\beta_2)_{\pi^0,\pi^\pm}$ are totally determined by the gauge-invariant quark box or the box + pion exchange contribution. This is a key result of the presented calculation. The sign is stable when the model parameters are changed. The magnitude depends on the value chosen for the constituent quark mass, but the best overall fit to the other empirical data, typically yielding $M \sim 300$ MeV, also yields the optimum values for the polarizabilities [7]. Moreover, the main part of the box contribution comes from the first non-vanishing p^8-order term in the chiral expansion. Based on this fact we expect that the contact term of the p^8 3-loop calculation in χPT may also play an important role in reversing the signs of the 2-loop order results for these quantities.

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