BLED WORKSHOPS
IN PHYSICS
VOL. 10, NO. 1
p. 77

Proceedings of the Mini-Workshop
Problems in Multi-Quark States
Bled, Slovenia, June 29 - July 6, 2009

What have we learned from the Nambu–Jona-Lasinio model

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Abstract. The Nambu–Jona-Lasinio model has played an important conceptual and pedagogical role in hadronic physics to visualize the spontaneous chiral symmetry breaking, the formation of the massive constituent quark and the behaviour of pion and sigma meson as a chiral rotation and vibration. I shall give a brief review of three new developments, (i) some observables for pion, (ii)more consistent results in three-flavour systems after introducing three-body and four-body interactions, and (iii) additional perspectives offered by algebraic models, in particular the two-level quasispin model,

1 Introduction

The Nambu–Jona-Lasinio model (NJL) is still inspiring hadronic physicists to gain a deeper qualitative or even semiquantitative understanding of the spontaneous chiral symmetry breaking, the formation of the massive constituent quark and the properties of light mesons. Further encouragement is coming from the progress how to derive NJL from QCD in a reasonable approximation, for example the Bogolyubov compensation method which is presented by Boris Arbuzov in these Proceedings.

On one hand, one is interested in further simplifications of NJL in order to see the role of $1/N$ expansions, sum rules and the effective pion-pion interaction (Sect. 4), as well as the bosonization in momentum space (Sect. 2). On the other hand, the applicability of the model is largely extended by further “complications” such as the three-body and four-body forces (Sect. 3).

I apologize that the review of our work is much longer than that of our friends, but you can find their presentation in these Proceedings.

2 Electromagnetic polarizabilities of pion

The Coimbra group [1] presented the calculation of pion electromagnetic dipole and quadrupole polarizabilities. They obtain the sign and magnitude in agreement with the respective experimental analysis based on the dispersion sum rules. The result are consistent also with the chiral perturbation theory.

For the neutral pion, the difference of the electric and magnetic dipole polarizabilities shows that the box contribution is largely canceled by the scalar exchange. For the charged pion, however, the pion exchange diagram builds together with the box a gauge invariant amplitude which is an order of magnitude smaller than the sigma-exchange diagram, and the pion loops are absent.

In the quadrupole polarizability difference of the neutral pion, the pion loop is about twice the sigma-exchange and dominates. For the charged pion, the pion-loop diagram has the same magnitude as the sigma-exchange term.

3 The effect of three-body and four-body interactions

The NJL model has been consistently extended to three-flavour systems, and recently, electromagnetic and weak decays of scalar and vector mesons have been calculated in leading orders of Feynman graphs [2,3]. For a good description of vector mesons, a vector-vector and axial vector-axial vector interaction is needed in addition to the usual scalar-scalar and pseudoscaar-pseudoscalar interaction.

Long ago, a three-body interaction (also called the “six-quark” t’Hooft interaction) was introduced in order to split the singlet and octet mesons – the U(1) symmetry problem. However it destabilizes the vacuum. The introduction of the four-body force (also called the “eight-quark interaction”) not only stabilizes the vacuum, but also influences the phase transition in hot dense systems and in strong magnetic fields [4]. This is a promising research topic for NJL.

4 The two-level quasispin model

In the Mini-Workshop Bled 2006, 2007 and 2008 [5–8] Borut Oblak and I presented a soluble two-level quasispin model of spontaneous chiral symmetry breaking, inspired by the Nambu–Jona-Lasinio model. It is the hadronic analogue of the Lipkin model in nuclear physics.

The model is characterized by a finite number N of quarks occupying a finite number $N = N_c N_f \mathcal{V}\Lambda^3/3\pi^2$ of states in the Dirac sea as well as in the valence space due to a sharp momentum cutoff Λ, and a periodic boundary condition in a box $\mathcal{V}$. We further simplify the one-flavour Nambu – Jona-Lasinio Hamiltonian ($N_f = 1, N_c = 3$) by taking all quark kinetic energies equal to $\frac{3}{4}\Lambda$ and by neglecting the interaction terms which change the individual quark momenta:

$$
\begin{aligned}
H = &\sum_{k=1}^{N}\Big(\gamma_5(k)h(k)\,\tfrac{3}{4}\Lambda + m_0\beta(k)\Big)\\
&-\frac{2G}{\mathcal{V}}\Big(\sum_{k=1}^{N}\beta(k)\sum_{l=1}^{N}\beta(l) + \sum_{k=1}^{N} i\beta(k)\gamma_5(k)\sum_{l=1}^{N} i\beta(l)\gamma_5(l)\Big)\,.
\end{aligned}
$$

Here $h = \boldsymbol{\sigma}\cdot\mathbf{p}/p$ is helicity and γ_5 and β are Dirac matrices. In terms of quasispin operators which obey spin commutation relations ($\alpha = x,\, y,\, z$)

$$
R_\alpha = \sum_{k=1}^{N}\frac{1+h(k)}{2}j_\alpha(k)\;,\quad L_\alpha = \sum_{k=1}^{N}\frac{1-h(k)}{2}j_\alpha(k)\;,\quad J_\alpha = R_\alpha + L_\alpha = \sum_{k=1}^{N} j_\alpha(k)\;,
$$

the model Hamiltonian can be written as

$$H = 2P(R_z - L_z) + 2m_0 J_x - 2g(J_x^2 + J_y^2)\,.$$

It commutes with R^2 and L^2 but not with R_z and L_z. Nevertheless, it is convenient to work in the basis $|\, R, L, R_z, L_z\,\rangle$ and diagonalize the Hamiltonian for fixed R and L.

From the quasispin model of the Nambu–Jona-Lasinio type one can learn several lessons:

(i) We show that the popular model parameters [9,10], $\Lambda = 648$ MeV, $G = 40.6$ MeV fm^3, $m_0 = 4.58$ MeV, yield the phenomenological values of quark constituent mass, quark condensate and pion mass **both** in the full Nambu – Jona-Lasinio model as well as in our quasispin model (using in both cases the Hartree-Fock + RPA approximations).

(ii) In the large N limit the exact results of our quasispin model approach the HF+RPA values, thus giving credit to using HF+RPA in usual calculations.

(iii) In the quasispin model it is very instructive that the number of colours N_c and the number of spatial states $\mathcal{V}\Lambda^3/6\pi^2$ appear on equal footing in the product $N = 2N_c\mathcal{V}\Lambda^3/6\pi^2$. The colour and the momentum quantum number together are just the house number of the particle since the interaction does not depend on them. Therefore it is the same limit $N \to \infty$ whether we take the large N_c limit or a large block $\mathcal{V}$. This explains why even with 3 colours the quasispin model behaves similarly as the theorems regarding large N_c limit suggest (good HF approximation, suppression of off-diagonal terms and their effects, etc.).

(iv) Most low-lying states in the excitation spectrum can be interpreted as multi-pion states and one can deduce the effective pion-pion interaction and scattering length. Also, some intruder states can be recognized as sigma-meson excitations or their admixtures to multi-pion states.
Since we are working in a finite volume $\mathcal{V}$ with periodic boundary conditions we cannot impose scattering boundary conditions. It is instructive that one can nevertheless extract information on scattering from a discrete spectrum. Energy levels of n-pion states can be interpreted to contain the average effective pion-pion potential $\bar{V}$: $E_{n\pi} = n\,m_\pi + \frac{1}{2}n(n-1)\bar{V}$.
We calculate the s-state scattering length in the first-order Born approximation (also derived by M.Lüscher [11] in a much more "sophisticated" way)

$$a = \frac{m_\pi/2}{2\pi}\int V(\mathbf{r})\,d^3r = \frac{m_\pi}{4\pi}\bar{V}\mathcal{V}\,.$$

In our example for $N = 192$ we have $\bar{V} = -7.1\,\text{MeV}$ and $\mathcal{V} = \pi^2 N/\Lambda^3 = 53\,\text{fm}^3$ This gives $a\,m_\pi = (m_\pi^2/4\pi)\bar{V}\mathcal{V} = -0.0836$ not far from phenomenological value (see [5,8]).

References

1. B. Hiller, W. Broniowski, A. A. Osipov, A. H. Blin, these Proceedings (2009); also available at arXiv:0909.4867.
2. V. Bernard, A. H. Blin, B. Hiller, U-G. Meissner, and M. C. Ruivo, Phys. Lett. **B305**, 163 (1993); also hep-ph/9302245
3. Yu. L. Kalinovski and M. K. Volkov, hep-ph/08091795
4. A. A. Osipov, B. Hiller, A. H. Blin, J. Moreira, these Proceedings (2009); also available at arXiv:0910.0371.
5. M. Rosina and B. T. Oblak, Bled Workshops in Physics **7**, No.1, 92 (2006); also available at http://www-f1.ijs.si/BledPub.
6. M. Rosina and B. T. Oblak, Bled Workshops in Physics **8**, No.1, 66 (2007); also available at http://www-f1.ijs.si/BledPub.
7. M. Rosina and B. T. Oblak, Bled Workshops in Physics **9**, No.1, 98 (2008); also available at http://www-f1.ijs.si/BledPub.
8. M. Rosina, Few Body Systems (2009), in print
9. M. Fiolhais, J. da Providência, M. Rosina and C. A. de Sousa, Phys. Rev. C **56**, 3311 (1997).
10. M. Buballa, Phys. Reports **407**, 205 (2005).
11. M. Lüscher, Commun. Math. Phys. **104**, 177 (1986); **105**, 153 (1986); Nucl. Phys. **B354**, 531 (1991).