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12 A Democratic Suggestion

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Abstract. Within the framework of quark mass matrices with a democratic texture, the unitary rotation matrices that diagonalize the quark matrices are obtained by a specific parametrization of the Cabibbo-Kobayashi-Maskawa mixing matrix. Different forms of democratic quark mass matrices are derived from slightly different parametrizations.

Povzetek. Avtorica predstavi masne matrike kvarkov s skoraj demokratičnimi matrikami. Izbere različno parametrizacijo, ki preko unitarne transformacije vodijo do izmerjene mešalne matrike Cabibba-Kobayashija-Maskawe. Komentira sprejemljivost različnih parametrizacij.

12.1 Introduction

A main weakness of the Standard Model is the large number of free parameters. There is at present no explanation for their origin, and we don't know if there is some connection between them.

Most of the free parameters reside in "flavour space" - with six quark masses, six lepton masses, four quark mixing angles and ditto for the leptonic sector, as well as the strong CP-violating parameter $\bar{\Theta}$. The structure of flavour space is determined by the fermion mass matrices, i.e. by the form that the mass matrices take in the "weak interaction basis" where mixed fermion states interact weakly, in contrast to the "mass bases", where the mass matrices are diagonal.

One may wonder how one may ascribe such importance to the different bases in flavour space, considering that the information content of a matrix is contained in its matrix invariants, which in the case of a $N \times N$ matrix M are the N sums and products of the eigenvalues λ_j, such as $traceM$, $detM$,

$$\begin{aligned} I_1 &= \textstyle\sum_j \lambda_j = \lambda_1 + \lambda_2 + \lambda_3 ... \\ I_2 &= \textstyle\sum_{jk} \lambda_j\lambda_k = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + ... \\ I_3 &= \textstyle\sum_{jkl} \lambda_j\lambda_k\lambda_l = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + ... \\ &\vdots \\ I_N &= \lambda_1\lambda_2\cdots\lambda_N \end{aligned} \tag{12.1}$$

These expressions are invariant under permutations of the eigenvalues, which in the context of mass matrices means that they are flavour symmetric, and obviously independent of any choice of flavour space basis.

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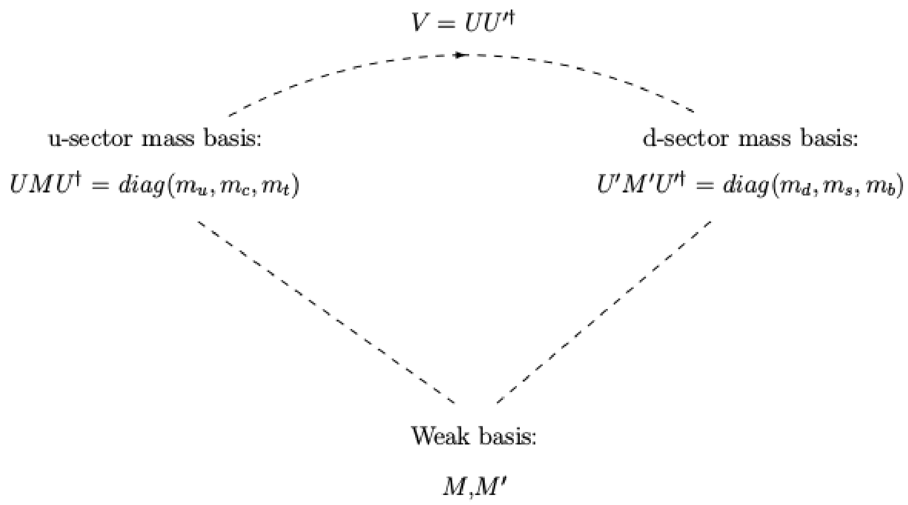
Even if the information content of a matrix is contained in its invariants, the form of a matrix may also carry information, albeit of another type. The idea - the hope - is that the form that the mass matrices have in the weak interaction basis can give some hint about the origin of the unruly masses. There is a certain circularity to this reasoning; to make a mass matrix ansatz is in fact to define what we take as the weak interaction basis in flavour space. We denote the quark mass matrices of the up- and down-sectors in the weak interaction basis by M and M', respectively. We go from the weak interaction basis to the mass bases by rotating the matrices by the unitary matrices U and U',

$$M \to UMU^\dagger = D = diag(m_u, m_c, m_t) \tag{12.2}$$

$$M' \to U'M'U'^\dagger = D' = diag(m_d, m_s, m_b)$$



The lodestar in the hunt for the right mass matrices is the family hierarchy, with two lighter particles in the first and second family, and a much heavier particle in the third family. This hierarchy is present in all the charged sectors, with fermions in different families exhibiting very different mass values, ranging from the electron mass to the about 10^5 times larger top mass. It is still an open question whether the neutrino masses also follow this pattern [1].

12.2 "Democratic" mass matrices

In the "democratic" approach [2], [3], [4] the hierarchical pattern is taken very seriously. The basic assumption is that in the weak interaction basis the fermion mass matrices are next to “democratic”, in the sense that they have a structure close to the $S(3)_L \times S(3)_R$ symmetric “democratic” matrix

$$\mathbf{N} = k \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{12.3}$$

The underlying philosophy is that in the Standard Model, where the fermions get their masses from the Yukawa couplings by the Higgs mechanism, there is no reason why there should be a different Yukawa coupling for each fermion. The couplings to the gauge bosons of the strong, weak and electromagnetic interactions are identical for all the fermions in a given charge sector, it thus seems like a natural assumption that they should also have identical Yukawa couplings. The difference is that the weak interactions take place in a specific flavour space basis, while the other interactions are flavour independent.

The democratic assumption is thus that the fermion fields of the same charge initially have the same Yukawa couplings. With three families, the quark mass matrices in the weak interaction basis then have the (zeroth order) form

$$M^{(0)} = k_u \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad M'^{(0)} = k_d \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{12.4}$$

where k_u and k_d have dimension mass. The corresponding mass spectra $(m_1, m_2, m_3) \sim (0, 0, 3k_j)$ reflect the family hierarchy with two light families and a third much heavier family, a mass hierarchy that can be interpreted as the representation $\mathbf{1} \oplus \mathbf{2}$ of $S(3)$. In order to obtain realistic mass spectra with non-zero masses, the $S(3)_L \times S(3)_R$ symmetry must obviously be broken, and the different democratic matrix ansätze correspond to different schemes for breaking the democratic symmetry.

12.2.1 The lepton sector

We can apply the democratic approach to the lepton sector as well, postulating democratic (zeroth order) mass matrices for the charged leptons and the neutrinos, whether they are Fermi-Dirac or Majorana states,

$$M_l^{(0)} = k_l \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad M_\nu^{(0)} = k_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{12.5}$$

Relative to the quark ratio $k_u/k_d \sim m_t/m_b \sim 40 - 60$, the leptonic ratio $k_\nu/k_l < 10^{-8}$ is so extremely small that it seems unnatural. One way out is to simply assume that k_ν vanishes, meaning that the neutrinos get no mass contribution in the democratic limit [5]. According to the democratic philosophy, then there would be no reason for a hierarchical pattern à la the one observed in the charged sectors; the neutrino masses could even be of the same order of magnitude.

Data are indeed compatible with a much weaker hierarchical structure for the neutrino masses than the hierarchy displayed by the charged quark fermion masses.

Unlike the situation for the quark mixing angles, in lepton flavour mixing there are two quite large mixing angles and a third much smaller mixing angle, these large mixing angles can be interpreted as indicating weak hierachy of the neutrino mass spectrum. The neutrino mass spectrum hierarchy could even be inverted; if the solar neutrino doublet (ν_1, ν_2) has a mean mass larger than the remaining atmospheric neutrino ν_3, the hierarchy is called inverted, otherwise it is called normal.

Supposing that the neutrino masses do not emerge from a democratic scheme, a (relatively) flat neutrino mass spectrum could be taken as a support for the idea that the masses in the charged sectors emerge from a democratic scheme.

12.3 The democratic basis

At the level of the zeroth order mass matrices the quark mixing matrix is $V = UU'^{\dagger} = U_{dem}U_{dem}^{\dagger} = \mathbf{1}$, where

$$U_{dem} = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & -2 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} \tag{12.6}$$

We use this to define the the democratic basis, meaning the flavour space basis where the mass matrices are diagonalized by (12.6) and the mass Lagrangian is symmetric under permutations of the fermion fields $(\varphi_1, \varphi_2, \varphi_3)$ of a given charge sector.

In the democratic basis the mass Lagrangian

$$\mathcal{L}_m = \bar{\varphi} M_{(democratic\ basis)} \varphi = k \sum_{jk=1}^{3} \bar{\varphi}_j \varphi_k$$

is symmetric under permutations of the fermion fields $(\varphi_1, \varphi_2, \varphi_3)$, while in the mass basis with

$$M_{(mass\ basis)} = \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix}$$

the mass Lagrangian has the form

$$\mathcal{L}_m = \lambda_1 \bar{\psi}_1 \psi_1 + \lambda_2 \bar{\psi}_2 \psi_2 + \lambda_3 \bar{\psi}_3 \psi_3 \tag{12.7}$$

which is clearly not invariant under permutations of the eigenvalues, nor under permutations of (ψ_1,ψ_3,ψ_3). We can perform a shift of the democratic matrix, by just adding a unit matrix $diag(a, a, a)$, so we get $M_0 \to M_1$,

$$M_1 = k \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} a & & \\ & a & \\ & & a \end{pmatrix} = \begin{pmatrix} k+a & k & k \\ k & k+a & k \\ k & k & k+a \end{pmatrix} \tag{12.8}$$

corresponding to the mass spectrum $(a, a, 3a + 3k)$. The matrix M_1 has a democratic texture, both because it is diagonalized by U_{dem}, and because the mass Lagrangian is invariant under permutations of the quark fields,

$$\mathcal{L}_{M_1} = (k + a) \sum \bar{\varphi}_j \varphi_j + k \sum_{j \neq k} \bar{\varphi}_j \varphi_j \tag{12.9}$$

.

If M_1 and M_1' both have a texture like (12.8), there is no CP-violation. This is independent of how many families there are, because of the degeneracy of the mass values. CP-violation only occurs once there are three or more non-degenerate families, because only then the phases can no longer be defined away.

We can repeat the democratic scheme with a number n of families, where the fermion mass matrices again are proportional to the $S(n)_L \times S(n)_R$ symmetric democratic matrix which is diagonalized by a unitary matrix analogous to U_{dem} in (12.6). To the $n \times n$-dimensional democratic matrix term, we can again add a $n \times n$-dimensional diagonal matrix $diag(a, a, ..., a)$, and get a $n \times n$-dimensional mass spectrum with n massive states, and $n - 1$ degenerate masses. The mass matrix still has a democratic texture, and there is still no CP-violation.

12.4 Breaking the democratic symmetry

In order to obtain non-degenerate, non-vanishing masses for the physical flavours (ψ_1, ψ_2, ψ_3), the permutation symmetry of the democratic fermion fields $(\varphi_1, \varphi_2, \varphi_3)$ must be broken. The proposal here is to derive the perturbed unitary rotation matrices U, U' for the up and down sectors from a specific parameterisation of the weak mixing matrix $V = UU'^{\dagger}$.

The idea is to embed the assumption of democratic symmetry into the Standard Model mixing matrix, by expressing the mixing matrix as a product

$$V = UU'^{\dagger} = (\tilde{U}U_{dem})(U_{dem}^{\dagger}\tilde{U}'^{\dagger}) \tag{12.10}$$

Since both the mixing matrix and its factors, according to the "standard" parameterisation [6], are so close to the unit matrix, the rotation matrices U, U' are effectively perturbations of the the democratic diagonalising matrix (12.6). In this way, the weak interaction basis remains close to the democratic basis.

12.4.1 Factorizing the mixing matrix

The Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix [7] can of course be parametrized - and factorized - in many different ways, and different factorizations correspond to different rotation matrices U and U'. The most obvious and "symmetric" factorization of the CKM mixing matrix is, following the "standard" parametrization [6] with three Euler angles α, β, 2θ,

$$V = \begin{pmatrix} c_\beta c_{2\theta} & s_\beta c_{2\theta} & s_{2\theta} e^{-i\delta} \\ -c_\beta s_\alpha s_{2\theta} e^{i\delta} - s_\beta c_\alpha & -s_\beta s_\alpha s_{2\theta} e^{i\delta} + c_\beta c_\alpha & s_\alpha c_{2\theta} \\ -c_\beta c_\alpha s_{2\theta} e^{i\delta} + s_\beta s_\alpha & -s_\beta c_\alpha s_{2\theta} e^{i\delta} - c_\beta s_\alpha & c_\alpha c_{2\theta} \end{pmatrix} = UU'^{\dagger} \tag{12.11}$$

with the diagonalizing rotation matrices for the up- and down-sectors

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \tag{12.12}$$

and

$$U' = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix},$$

respectively, where α, β, θ and γ correspond to the parameters in the standard parametrization of the CKM mixing matrix, in such a way that $\gamma = \delta/2$, $\delta = 1.2\pm0.08$ rad, and $2\theta = 0.201\pm0.011°$, while $\alpha = 2.38\pm0.06°$ and $\beta = 13.04\pm0.05°$.

¿From the rotation matrices U and U' we then obtain the mass matrices $M = U^\dagger diag(m_u, m_c, m_t)U$ and $M' = U'^\dagger diag(m_d, m_s, m_b)U'$, such that

$$M = \frac{1}{6} \begin{pmatrix} X+H & \hat{M}_{12} & Z+W \\ \hat{M}_{12}^* & X-H & Z-W \\ Z^*+W^* & Z^*-W^* & 6T-2X \end{pmatrix} \tag{12.13}$$

where T is the trace $T = m_u + m_c + m_t$, and with $D = \sqrt{3}s_\theta - \sqrt{2}c_\theta$, $C = \sqrt{3}s_\theta + \sqrt{2}c_\theta$, $F = c_\alpha s_\alpha(m_t - m_c)$,

$$X = \tfrac{1}{2}(m_c s_\alpha^2 + m_t c_\alpha^2 - m_u)(D^2 + C^2 - 2) + F(D - C)\cos\gamma + T + 3m_u$$
$$H = \tfrac{1}{2}(m_c s_\alpha^2 + m_t c_\alpha^2 - m_u)(D^2 - C^2) + F\cos\gamma(D + C)$$

$$W = \tfrac{1}{4}(m_c s_\alpha^2 + m_t c_\alpha^2 - m_u)\ (D^2 - C^2) - F\ (D + C)\ e^{-i\gamma}$$
$$Z = (m_c s_\alpha^2 + m_t c_\alpha^2 - m_u)\left[2 + \tfrac{1}{4}(D - C)^2\right] + \tfrac{F}{2}\ (D - C)\ (e^{i\gamma} - 2\ e^{-i\gamma}) - 2T + 6\ m_u$$
$$\hat{M}_{12} = -(m_c s_\alpha^2 + m_t c_\alpha^2 - m_u)\ (D\ C + 1) - F\ (C\ e^{i\gamma} - D\ e^{-i\gamma}) + T - 3\ m_u$$

Similarly for the down-sector,

$$M' = \frac{1}{6} \begin{pmatrix} X'+H' & \hat{M}'_{12} & Z'+W' \\ \hat{M}'^*_{12} & X'-H' & Z'-W' \\ Z'^*+W'^* & Z'^*-W'^* & 6T'-2X' \end{pmatrix} \tag{12.14}$$

with the parameters $T' = m_d + m_s + m_b$, $G = \sqrt{2}s_\theta - \sqrt{3}c_\theta$, $J = \sqrt{2}s_\theta + \sqrt{3}c_\theta$ and $F' = c_\beta s_\beta(m_b - m_s)$, and

$$X' = \tfrac{1}{2}(m_s s_\beta^2 + m_b c_\beta^2 - m_d)(G^2 + J^2 - 2) - F'(J + G)\cos\gamma + T' + 3m_b$$
$$H' = \tfrac{1}{2}(m_s s_\beta^2 + m_b c_\beta^2 - m_d)(G^2 - J^2) + F'(J - G)\cos\gamma$$
$$W' = \tfrac{1}{4}(m_s s_\beta^2 + m_b c_\beta^2 - m_d)(G^2 - J^2) + F'(G - J)e^{i\gamma}$$
$$Z' = (m_s s_\beta^2 + m_b c_\beta^2 - m_d)\left[2 + \tfrac{1}{4}(J + G)^2\right] + \tfrac{F'}{2}(J + G)(2e^{i\gamma} - e^{-i\gamma}) - 2T' + 6m_b$$
$$\hat{M}'_{12} = (m_s s_\beta^2 + m_b c_\beta^2 - m_d)\ (G\ J - 1) - F'\ (J\ e^{i\gamma} - G\ e^{-i\gamma}) + T' - 3\ m_b$$

In order to evaluate to what degree these rather opaque matrices are "democratic", we evaluate the matrix elements by inserting numerical mass values. For the up-sector we get the (nearly democratic) matrix texture

$$M = C_u \left[\begin{pmatrix} 1 & & \\ & k\,e^{-i(\alpha+\beta)} & \\ & & kp\,e^{-i\alpha} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & k\,e^{i(\alpha+\beta)} & \\ & & kp\,e^{i\alpha} \end{pmatrix} + \Lambda \right] \tag{12.15}$$

where the "small" matrix

$$\Lambda = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon & \varepsilon' e^{-i\beta} \\ 0 & \varepsilon' e^{i\beta} & \eta \end{pmatrix},$$

with $\varepsilon \sim \varepsilon' \ll \eta < k, p$, is what breaks the democratic symmetry, supplying the two lighter families with non-zero masses. With mass values calculated at $\mu = M_Z$ (Jamin 2014) [8],

$$(m_u(M_Z), m_c(M_Z), m_t(M_Z)) = (1.24, 624, 171550)\mathrm{MeV},$$

we get $\alpha \sim 2.7895^o$, $\beta \sim 2.7852^o$, $C_u = 54240.36$ MeV $\approx m_t/3$, and

$k \approx 1.00438$, $p \approx 1.06646$, $\varepsilon' \approx 0.0000505$,
$\varepsilon \approx 0.00004596 \approx 2m_u/C_u$, $\eta = 0.018154 \approx \frac{1}{2}\frac{m_t}{C_u}\frac{m_c}{C_u}$.

For the down-sector, with

$$(m_d(M_Z), m_s(M_Z), m_b(M_Z)) = (2.69, 53.8, 2850)\mathrm{MeV}$$

we get another democratic texture,

$$M' = C_d \begin{pmatrix} X + A & Ye^{-i\mu} & e^{-i\rho} \\ Ye^{i\mu} & X - A & (1+2A)e^{i\kappa} \\ e^{i\rho} & (1+2A)e^{-i\kappa} & X + Y - A - 1 \end{pmatrix} \tag{12.16}$$

where

$C_d = 966.5$MeV, $A = 0.0056$, $X = 1.0362$, $Y = 1.0305$ and
$\mu \le \kappa \sim 0.22^o < \rho \sim 0.226^o$.

Just like in the up-sector mass matrix, the matrix elements in M' display a nearly democratic texture. In both the up-sector and the down-sector the mass matrices are thus approximately democratic.

12.5 Calculability

In the mass matrix literature there is an emphasis on "calculability". The ideal is to obtain mass matrices that have a manageable form, but there is nothing that forces nature to serve us such user-friendly formalism. It is however tempting to speculate that there are relations between the elements that could make the democratic matrices more calculable, and in the search for matrices that are reasonably transparent and calculable, we look at a more radical factorization of the mixing matrix, viz.

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\omega & 0 & \sin\omega\, e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\omega\, e^{i\delta} & 0 & \cos\omega \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \quad (12.17)$$

and

$$U' = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

where, as before, $\delta = 1.2 \pm 0.08$ rad, and $\omega = 2\theta = 0.201 \pm 0.011°$, while $\alpha = 2.38 \pm 0.06°$, and $\beta = 13.04 \pm 0.05°$. These rotation matrices are still "perturbations" of the democratic diagonalizing matrix (12.6), and the up-sector mass matrix has a texture similar to (12.13),

$$M = \frac{1}{6} \begin{pmatrix} R+Q+S\ \cos\delta & R-Q-iS\ \sin\delta & A-Be^{-i\delta} \\ R-Q+iS\ \sin\delta & R+Q-S\ \cos\delta & A+Be^{-i\delta} \\ A-Be^{i\delta} & A+Be^{i\delta} & T-2(R+Q) \end{pmatrix} \quad (12.18)$$

where T is the trace, $T = m_u + m_c + m_t$, and

$$\begin{aligned} &R = N\ (2\, c_\omega\, c_\omega - 1) + T - 2\sqrt{2}\, c_\omega\ F, \quad Q = 3\, s_\omega\, s_\omega\ N + 3\, m_u, \\ &S = -2\sqrt{6}\, c_\omega\ s_\omega\ N + 2\sqrt{3} s_\omega\ F \\ &A = N\ (2\, c_\omega\, c_\omega + 2) - 2\, T + \sqrt{2}\, c_\omega\ F + 6\, m_u, \quad B = \sqrt{6}\, c_\omega\ s_\omega\ N + 2\sqrt{3}\ F\ s_\omega \end{aligned}$$

with $N = m_c\ s_\alpha\ s_\alpha + m_t\ c_\alpha\ c_\alpha - m_u,\ F = c_\alpha\ s_\alpha\ (m_t - m_c)$. This matrix can be reformulated in a form similar to (12.15),

$$M_u = C_u \left[\begin{pmatrix} 1 & & \\ & k\, e^{-i\alpha} & \\ & & kp\ e^{-i(\alpha-\beta)} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & k\, e^{i\alpha} & \\ & & kp\ e^{i(\alpha-\beta)} \end{pmatrix} + \Lambda \right]$$

where $\alpha = \arctan(S\sin\delta/(R-Q))$, $\beta = \arctan(B\sin\delta/(A+B\cos\delta))$, and

$$\Lambda = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon & \varepsilon' e^{-i\beta} \\ 0 & \varepsilon' e^{i\beta} & \eta \end{pmatrix}$$

with

$$\begin{aligned} &k = |M_{12}|/M_{11} = \tfrac{|R-Q-iS\sin\delta|}{R+Q+S\cos\delta}, \quad p = |M_{13}|/|M_{12}| = \tfrac{|A-Be^{-i\delta}|}{|R-Q-iS\sin\delta|}, \\ &\varepsilon = (|M_{22}||M_{11}| - |M_{12}|^2)/|M_{11}|^2 = \tfrac{4RQ-S^2}{|R+Q+S\cos\delta|^2} \approx 2m_1/A, \\ &\varepsilon' = (|M_{23}||M_{11}| - |M_{13}||M_{12}|)/|M_{11}|^2, \\ &\eta = (|M_{33}||M_{11}| - |M_{13}|^2)/|M_{11}|^2 \approx \tfrac{1}{2}\tfrac{m_c}{A}\tfrac{m_t}{A} \end{aligned}$$

Inserting the masses $(m_u(M_Z), m_c(M_Z), m_t(M_Z)) = (1.24, 624, 171550)$ MeV, we get $C_u = 53723.5$MeV, $k = 1.00318$, $p = 1.0828$, and

$\varepsilon = 0.00004646 \approx 2(m_u/C_u)$, $\varepsilon' = 0.0000444$, $\eta = 0.0185 \approx \frac{1}{2}(m_t/C_u)(m_c/C_u)$

For the down-sector, with

$$U' = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix},$$

the mass matrix $U'^{\dagger} \mathrm{diag}(m_d, m_s, m_b) U'$ reads

$$M' = C_d \begin{pmatrix} X+A & Y & 1 \\ Y & X-A & 1+2A \\ 1 & 1+2A & X+Y-A-1 \end{pmatrix}$$

where

$C_d = 2(m_d c_\beta^2 + m_s s_\beta^2) - 2\sqrt{3} c_\beta s_\beta (m_s - m_d)) + 2(m_b - m_s - m_d)$
$X = (2m_b + m_s + m_d + 2(m_d c_\beta^2 + m_s s_\beta^2) + 2\sqrt{3} c_\beta s_\beta (m_s - m_d))/C_d$
$Y = (2m_b + m_s + m_d - 4(m_d c_\beta^2 + m_s s_\beta^2))/C_d$,
$A = 2\sqrt{3} c_\beta s_\beta (m_s - m_d))/C_d$.

Inserting the masses $(m_d(M_Z), m_s(M_Z), m_b(M_Z)) = (2.69, 53.8, 2850)$ MeV, we moreover get the numerical values

$C_d = 926.448 MeV \approx m_b/3$, $X = 1.0375$, $A = 0.0070$, $Y = 1.0318$.

12.6 Conclusion

By including the democratic rotation matrix in the parametrization of the weak mixing matrix, we obtain mass matrices with specific democratic textures. In this way we make contact between the democratic hypothesis and the experimentally derived parameters of the CKM mixing matrix, avoiding the introduction of additional concepts.

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13 Discussion Section on LHC Data

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Abstract. We report on exchanges entertained and new developments reported and discussed at the workshop "What Comes Beyond the Standard Models" held in Bled, Slovenia, July 11th-19th 2015.

New LHC data, various unification schemes with and without gravity, the nature of fermions, flavor and the number of families, condensates, and other topics of current interest were all heatedly discussed.

Povzetek. Avtor poroča o diskusijah med predavanji in v diskusijskih sekcijah na izbrane teme, ter o napredku, ki ga je prinesla letošnja blejska delavnica "What Comes Beyond the Standard Models" . Posebej omenja zadnje analize meritev na LHC, o teorijah, ki prinašajo enotno sliko lastnosti fermionov, pomagajo razumeti pojav ustreznih bozonskih polj, vključno z gravitacijo, o napovedih o številu družin fermionov, o skalarnih poljih in lastnostih fermionov, o pojavu kondenzatov in ostalih temah na tem področju.

13.1 SM Electroweak Symmetry Breaking Sector

The LHC starts run II after having found a relatively light scalar particle that could be the predicted Standard Model Higgs at 125 GeV and not much more (to the disappointment of a part of the community that firmly expected TeV-scale supersymmetry). Still, this summer the ATLAS collaboration reported [1] a two-gauge boson spectrum in dijet searches (see talk by Llanes-Estrada in these proceedings) that shows an excess at 2 TeV not confirmed by CMS data.

We discussed whether this could be just a statistical fluctuation. Should increased data taking consolidate the excess, an interesting scenario to analyze was proposed, whether a top-ball [2] made of 6 top quarks and 6 top antiquarks all in an s-wave (with wavefunction antisymmetry allowed by the color and flavor degrees of freedom) might have been produced. The 2 TeV mass of the excess could be about right, since $12 \times m_t \simeq 2.1$ TeV which allows for some Higgs-exchange induced binding, though its production cross-section needed for the low-statistics LHC run-I would need to be very large. This cross-section needs to be estimated by theorists.

Independently of whether new resonances coupling to the Electroweak Symmetry Breaking Sector (EWSBS) are found, this can be studied by means of Effective Field Theory for the currently observed particles h (the new 125 GeV scalar) and $\omega^i \sim W_L,\ Z_L$.

In the non-linear realization of $SU(2)_L \times SU(2)_R \to SU(2)_V$, and neglecting masses of $O(100\text{GeV})$ as appropriate to study the 1-3 TeV region, the corresponding next-to-leading order (NLO) Lagrangian density is

$$\mathcal{L} = \frac{1}{2}\left[1 + 2a\frac{h}{v} + b\left(\frac{h}{v}\right)^2\right]\partial_\mu\omega^i\partial^\mu\omega^j\left(\delta_{ij} + \frac{\omega^i\omega^j}{v^2}\right) + \frac{1}{2}\partial_\mu h\partial^\mu h$$
$$+ \frac{4a_4}{v^4}\partial_\mu\omega^i\partial_\nu\omega^i\partial^\mu\omega^j\partial^\nu\omega^j + \frac{4a_5}{v^4}\partial_\mu\omega^i\partial^\mu\omega^i\partial_\nu\omega^j\partial^\nu\omega^j + \frac{g}{v^4}(\partial_\mu h\partial^\mu h)^2$$
$$+ \frac{2d}{v^4}\partial_\mu h\partial^\mu h\partial_\nu\omega^i\partial^\nu\omega^i + \frac{2e}{v^4}\partial_\mu h\partial^\nu h\partial^\mu\omega^i\partial_\nu\omega^i \tag{13.1}$$

that was described in [3,4]. (Other researchers [5–7] are also pursuing this approach.) This Lagrangian has seven-parameters $(a, b, a_4, a_5, g, d, e)$, with the first two being LO and the other five NLO in the derivative expansion.

Two strategies can be followed. If the LHC run-II finds no new physics, it can conduct precision work to try to see a separation of the Standard Model $(1, 1, 0, 0, 0, 0, 0)$. Currently only $a \in (0.88, 1.3)$ is known at 2σ confidence level. Norma Mankoc triggered discussion on how Effective Theory is philosophically not too satisfactory as its predictivity is moderate and it cannot "solve" the various puzzles of the Standard Model. It remains a powerful descriptive tool to classify data. Full theories will manifest themselves as separations from the SM values of one of those parameters, and matching those UV completions to the Effective Theory allows to classify them and quickly discard or constrain families thereof. Example theories that can soon be tested include for example Left-Right models or Composite Higgs models that include spin-1 resonances within reach of the LHC [8,9].

If the LHC confirms resonant structures in $W_L W_L$ in the 2 TeV region, Effective Theory fails, since a derivative expansion cannot saturate unitarity. The second strategy then activates: the use of Unitarized Chiral Perturbation Theory based on the Lagrangian of Eq. (13.1) is appropriate to describe resonances [10–12].

13.2 The flavor problem and the SM parameters

The largest number of parameters in the Standard Model comes from the flavor sector. There is at present no compelling theory explaining them.

At the workshop, strong arguments were presented in favor of the existence of a fourth family. For example, the Ljubljana unified theory of spin and charge based on $SO(1, 13)$ predicts such a fourth fermion family. Also the concept of fermionization, by which SM fermions can be constructed from boson fields alone, was discussed by H. Nielsen and matching the number of degrees of freedom for the fundamental bosons and the generated fermions required that fourth family.

At present, strong phenomenological obstacles to this fourth family exist that require all its members to have high masses.

For a start, CKM unitarity closes very well with three families, so that the parameters of the unitarity triangle $\bar{\rho}$ and $\bar{\eta}$ are known to 20% and 3% respectively [13].

Second, direct searches at the LHC put excited quarks above the 3 TeV scale [14] so that they start being irrelevant for electroweak-scale physics.

Another hurdle for fourth-family extensions of the SM is that the number of relativistic degrees of freedom is tightly constrained from cosmology. For example, the Planck collaboration [15] reports an analysis of Baryon Acoustic Oscillations and the Cosmic Microwave Background that yields $N_{\rm eff} = 3.30 \pm 0.27$ for the effective number of relativistic degrees of freedom. This clearly excludes a fourth light neutrino, in agreement with LEP bounds at the Z-pole. The presumed fourth family therefore comes with an additional hierarchy problem in which $m_{\nu_4} \gg m_{\nu_{1,2,3}}$. (Planck finds that the sum of the three light neutrino masses is 0.23 eV.)

13.3 Other physics at very high scales: unification, condensates and gravity

13.3.1 Gauge symmetry groups

A topic widely discussed at the workshop is why nature chose the symmetry group $U(1) \times SU(2)_L \times SU(3)_c$ to charge the Standard Model fermions. Several possibilities were discussed. A widely accepted one is that the symmetry group at a very high energy scale is larger and we only perceive a remainder subgroup. Well-known are the $SU(5)$ and $SO(10)$ extensions [16] of the Standard Model, in strong tension with proton lifetime bounds. $SO(1,13)$ has also been presented as an important alternative because of the entailed unification of spins and charges [17] under a common framework.

The first type of groups under discussion do not involve space-time and thus make no statement about gravity. The unification happens at the level of internal degrees of freedom only on a fixed space-time background. The scale must then be smaller than the Planck scale and is usually taken around 10^{15} to 10^{16} GeV where the running couplings of the $U(1)$, $SU(2)$ and $SU(3)$ SM subgroups are all approximately equal (see fig. 13.1).

The second possibility entails unification of internal and space-time symmetries and is a more general concept.

Many questions remain open. One is why given a large group G, the symmetry breaking pattern brings us to the SM group, i.e. $G \to U(1) \times SU(2)_L \times SU(3)$. Currently we know of no good argument why fermion condensates perform exactly this breaking and not something else. (Arranging symmetry breaking by fundamental scalar fields is equally ad-hoc as the potentials must be tuned to produced the wanted results.) One recent alley of investigation [19] addresses the smallness of the SM group dimension. Should there be larger unbroken groups under which certain fermions would be charged, and all couplings being equal at the GUT scale, the finding is that these fermions would be very massive and beyond reach of current collider experiments. This comes about because the large antiscreening for 1-flavor of fermions charged under a large-dimension group (left plot of figure 13.2) forces chiral symmetry breaking at a much larger scale than QCD's $SU(3)$. The corresponding fermion mass is proportional to that scale, $M(0) \propto \Lambda_{\chi SB}$ (right plot of figure 13.2) and out of reach. Fermions charged under

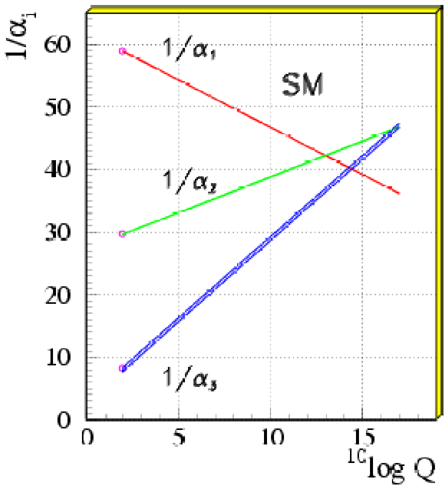

Fig. 13.1. Running coupling constants of $U(1)$, $SU(2)_L$ and $SU(3)_c$ in the absence of new physics through the GUT scale. Reprinted with permission of Particle Data Group [18]
.

$SU(4)$ would have masses of $O(10)$ TeV and be not too far in the energy scale, but larger groups yet would yield hopelessly heavy masses.

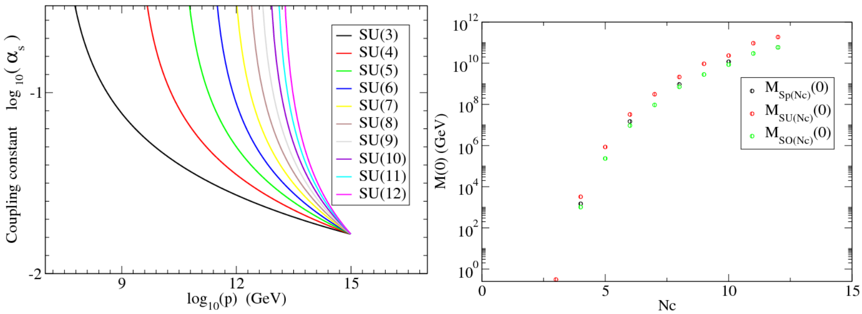

Fig. 13.2. Left: 1-loop running coupling constants for several groups (all equal at the GUT scale). Right: corresponding fermion masses due to chiral symmetry breaking. Were there fermions charged under $SU(4)$ or larger groups, their large mass would have impeded their production at colliders.

13.3.2 Condensates

Probably the biggest current embarrassment of physics is the smallness of the cosmological constant. The Planck collaboration [15] quotes $\Omega_\Lambda = 0.686(20)$ which is more than two thirds of the total energy density in the universe, but only slightly above 3 GeV per cubic meter in absolute value, or about 3×10^{-47} GeV^4. This is an absurdly small number by all SM measures. For example, the QCD condensate is typically found to be $(0.77(4)\Lambda_{\overline{\rm MS}})^3$ with the scale at 0.31(2) GeV [20], or about 0.023 GeV^3. The entailed energy density is 45 orders of magnitude off.

A solution for confining gauge theories such as QCD is to argue that this condensate is active only inside hadrons [21], that is, that the condensate itself is confined around quarks themselves. Dynamical studies of the corresponding domain walls between the condensed and uncondensed phases have to our knowledge not been carried out.

For the electroweak symmetry breaking sector the situation is worse since the corresponding vacuum energy density scale $v^4 = (246\text{GeV})^4$ is now off by 56 orders of magnitude. And it is not obvious that the fundamental scalar Higgs field reported so far will have anything to do with a confining gauge theory, so that the same mechanism can be invoked. In fact, technicolor theories were already discarded at the time of LEP. Likewise, condensates breaking higher symmetry groups will bring about energy densities disparate from the tiny number found by cosmologists.

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