INFLUENCE OF K₀ on the creep properties of marl

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Abstract

The influence of the stress state on the deformation response of a rock mass has been experimentally examined on uniaxial, bi-axial and three-axial specimens of marl at short loading and in creep tests for periods of 3 to 180 days. The lateral (horizontal) pressure significantly changes the deformation behaviour of both the initial deformations induced by the stress change and creep deformations. The influence of lateral pressure on the vertical deformation can be well approximated by a linear function.

кeywords

creep, soft rock, marl, rheological model, lateral stress, laboratory testing

1 INTRODUCTION

In order to formulate the rheological model for soft rocks with time-dependent deformations presented in this paper, laboratory creep tests were carried out on marl. While formulating the rheological model, specific rheological material coefficients and parameters need to be defined. The task therefore becomes more complex, because, on the one hand, a mathematical model is required to encompass as many characteristics of the materials as possible; while, on the other hand, the number of parameters and the constants of the materials quantifying those characteristics is limited by the real possibility to define parameters, i.e., material constants by measurements "in situ" or on specimens in the laboratory.

The rheological modelling of rock mass represents an extremely complex problem and thus while formulating the mathematical model, no matter how complex it is, it is necessary to make certain approximations. The level of the approximations, on one hand, in the model should provide the necessary accuracy of the results of the considered stress-deformation phenomenon of the rock mass, and on the other hand, it should enable a simple determination of the material constants and parameters used in the mathematical model.

Most of the research referring to rock behaviour in the conditions of long-term loading at room temperature published so far has been carried out on samples of rock salt [3-11]. There are also fewer published experimental research articles on the room-temperatures behaviour of marl or similar rocks, characterized by significant creep deformations [2-8], representing a real work environment under which many underground structures are constructed.

Various authors published a number of mathematical models that define rock creeping, which are mainly based on the results of tests on rock salt with significant deformation over time. A far smaller number of developed constitutive relations, which include rock creeping, can apply a correct description of different phenomena in rock mechanics due to their general character, in terms of the type of rock material and the direction of the stress and stress states. Due to the previously mentioned, it is necessary to examine the behaviour of different types of rocks, and thus in the first place to develop the most common constitutive models that can adequately describe their time-dependent stress-strain behaviour.

Experimental research on marl specimens, part of which is presented in this paper, under conditions of short-term loading, long-term load and unloading was conducted for the purpose of the formulation of a rheological model of the behaviour of soft rock under different stress conditions. It is also important and necessary to provide the appropriate material parameters of the rock matrix, which was the objective of this experimental research, through the testing of marl as a selected representative rock from the group of soft rocks.

Uniaxial tests are relatively easy to perform, which makes it possible to obtain the material parameters that correspond with the uniaxial stress state. Having in mind the previously mentioned, creep tests were carried out on uniaxial marl specimens and based on the obtained data, the rheological model was formulated and the material constants and parameters were obtained. The stress state during the analysis of the stress-deformation phenomenon in rock mechanics, for example, the stress state of tunnel tubes, significantly departs from the uniaxial state. Due to that, along with uniaxial tests, tests on bi-axially loaded plate specimens were performed under conditions of the plane stress state and the application of different forces in the vertical and horizontal directions as a simulation of the different primary stress states. A number of short creep tests in a conventional three-axial cell with a rotation symmetrical stress state were conducted.

Based on the creep tests on uniaxial specimens, material constants and the substantial parameters of the materials were obtained, and the rheological model was formulated for uniaxial stress, which describes the stress-deformation behaviour of the materials under the conditions of short- and long-term load [10]. Furthermore, a comparative analysis resulted in the establishment of a correlation between all the deformations at the uniaxial, bi-axial and three-axial rotation symmetrical stress states. This enables a good evaluation of the complete time deformation with complex stress states based on simple tests on uniaxially loaded specimens. This paper presents the results of a three-year laboratory testing on marl under short- and long-term loading and unloading, which refers to the impact of K_0 on the creep properties.

2 SAMPLING AND MATERIAL Properties of the tested Marl

The block was taken out from a section above coal deposits from the "Potrlica" coal mine in the town of Pljevlja (northern Montenegro), from a bench that

mainly consists of a thick layered marl. The thickness of the marl layer varies from several to over 100 meters (marl layers mostly lie directly on the coal deposits). A complex layer of marl, 80-100 cm thick, estimated as suitable, was chosen to be taken out as a block (suitable in terms of the dimensions of a specimen for the following laboratory testing), which is shown with arrows in Fig. 1. The layer is at a 15-20 m depth from the actual natural surface of the terrain, but it took more time to do the excavations at 1-3 metres from the surface of the bench.

The marl block that was taken out was a relatively regular form, 380x90x80 cm. A boring mining hole was made in the block and it clearly indicated the natural position of the block, that is, it defined the vertical direction in its natural position. The prismatic and plate specimens for testing were obtained by cutting, and the cylindrical samples were obtained by taking out the core using the rotation drill.

The tested marl, according in its chemical contents, contains about 48% CaCO₃, while the content of the insoluble remainder (clayey+quartzite) is nearly 52%. Regarding the mineralogical contents, calcite (46-48%) and quartz (12-13%) represent the dominant mineral phases, while in the clayey phase there are illite and smektite, montmorillonite, kaolinite, glauconite, transformed feldspar and mica. The sample moisture was 8-11%, which is why all the stress-deformation analyses were given for the total stresses. The uniaxial strength of the material was about $\sigma_c = 8.8$ *MPa* and the average volume weight was 18.8 kN/m³.



Figure 1. Layer of grey marl where specimens was taken.

With rocks, some material parameters, which describe the mechanical behaviour in general, and thus the creep itself, besides stress and temperature, depend on the moisture of the rock material, the moisture change and the humidity of the air in the surroundings, as well. In the experimental research carried out on the marl samples, the influence of these effects was minimized (by coating the sample with paraffin, the change of the humidity was slowed and limited, and direct contact with the air was prevented as well) in order to perform an unbiased research on the stress effects and the phenomena on deformations in the case of long-term load. During the creep test, relevant measurements on the specimens' moisture, their shrinkage, the temperature and the air humidity were conducted.

3 CREEP-TESTS APPARATUS

3.1 UNIAXIAL CREEP TESTS ON MARL

The uniaxial marl creep test in the experimental research presented here was performed on two groups with three prismatic specimens, 15x15x40 cm each. (This major group of specimen tests was preceded by the "zero" test serial of six specimens on which the creep test was carried out within a period of three months, which was used to verify the measurement techniques and to obtain preliminary values of the measured ones. Apart from this, 12 uniaxial tests were conducted under short loading on the specimens for the purpose of defining the



Figure 2. Device for uniaxial loading at creep test.

material parameters). A device with "dead" weight and a system of levers was used for loading and the "preservation" of the force over time, as presented in Fig. 2. Each of the six used devices was equipped with ring load cells (constructed by the author of this paper), which enabled variations of the force on the specimen during the test to be less than 0.3%.





Figure 3. Prismatic sample: a) scheme of measurement points; b) samples during the creep test (coated with paraffin).

The test was performed in three phases, loading, unloading and reloading to a higher level of stress, while maintaining its constant value after the stress change. The total duration of the test was 360 days (I phase: 180 days, II phase: 30 days and III phase: 150 days). After loading, the creep deformations were measured in the direction of the vertical (longitudinal) and horizontal specimen axes (on four unloaded sides of the specimen), as shown in Fig. 3. a. The measurements were conducted with a mechanical deformation meter (type "Pfender", accuracy 1/1000 mm), 1, 3, 6, 12 and 24 hours after loading, then after 3, 7 and 15 days, and in the remaining period every 30 days. The measurement intervals were determined so as to have the differences between the previous and the current deformation nearly the same.

3.2 TEST ON PLATE SPECIMENS

The testing of the plate specimens was conducted with the devices shown in Fig. 4. The vertical loading in the device was imposed through the system of a lever and a dead load, while the horizontal force was applied through two inter-connected presses. The vertical force was applied through the press, which was set between the load cell and trapeze conveying element, while the horizontal force transfers to the horizontal press, set between the secondary frame and the trapeze element, following the law of connecting tanks. Different ratios of the vertical and horizontal pressures can be obtained by the application of presses whose pistons have different cross-sections. By the application of this device the specimen can be "preserved" for months under the same vertical and horizontal pressures.

The plate specimens (Fig. 5 and 6) were loaded uniaxially and bi-axially, in their plane, in increments of 0.5 MPa in the period of an hour to a vertical stress of 2.0 MPa, which makes about 25% of the peak uniaxial strength of the examined marl. Three specimens were loaded uniaxially, and in the following three groups the ratio of the horizontal and vertical pressures was varied $P_h/P_v = 0.3, 0.5$ and 1.0. The initiated stress state (with the ratio $P_h/P_v = \text{const.}$) was preserved in the following 45 days with a measurement of the deformation fields on both sides of the plate. This phase of the test was projected in such a way as to establish the influence of the lateral pressure on the creeping in the vertical direction, and to establish the variation of the measured deformations (short and long), depending on the form of the specimens, i.e., prismatic or plate.

The measurement of creep deformations was performed on the network of measuring spots (shown in Fig. 6) 1, 6 and 24 hours after loading, then after 3 and 7 days and in the following period every 15 days. The measurement intervals were determined so as to have the differences between the previous and current measurement deformations nearly equal.

The network of measuring spots consists of interconnected triangles with 92 sides in total. Since the measurements contain certain errors, it was necessary to make a correction of the measurements so that the



Figure 4. Device for bi-axial loading - characteristic view.



Figure 5. Plate specimen loading scheme.



Figure 6. Plate specimen: a) scheme of measuring spots; b) specimen equiped with measuring bases.

lengths of the sides of all the measuring bases-sides of the triangles in the network give the same coordinates of the nodes. For the purpose of finding coordinates of the nodes of the network, the software "SIGMA" was used, which is intended for the "settlement" of the geodetic network based on the over-squares method. For the purpose of calculating the coordinates of the triangles' apexes of the referential network, 62 lengths of the sides correctly positioned on the network were sufficient, while the software used supernumerary measurements to improve the accuracy.

3.3 CONVENTIONAL TESTS OF TRIAX-IAL COMPRESSION

The measurement of the specimens' deformations by conventional three-axial tests in the standard Hook's cell included the use of a strain gauge and rosettes for the measurement of deformation in the two normal directions (accuracy 10⁻⁵ mm, producer: TML-Japan). Glue P2 (TML-Japan), with a high deformation power, was used for the gluing of the rosettes, which enabled the measurement of the high strain expected in the examined material.

Each specimen was equipped with three rosettes or a strain gauge set in the middle of the specimen height under the central angle of 120°, seen through the cross-section of the specimen, as shown in Fig. 7. A stripe cable, about 0.3mm thick, was used to connect the strain gauge with the "data logger". After the cable was set and

the rosettes were connected, a protective membrane was put over the specimens. A total of 24 specimens were prepared and examined in the three-axial device along with measurements of the deformations and about ten more specimens were led to failure without any measurement of the deformations (with a measurement of the axial stress of the failure and the lateral pressure).

Conventional tests of the triaxial compression give the possibility of analysing the influence of the lateral stress on the deformations. The lateral pressure $\sigma_2 = \sigma_3$ (which varies in the set of tests) does not usually exceed a value of 50% of the peak strength obtained in a uniaxial test with free lateral deformations. Generally speaking, this type of test can be conducted with controlled forces, with a load against the specimen or with a control of the velocity of the axial deformation. A creep test carried out on marl in a shorter period of time (seven days), represents a specific type of three-axial test with controlled force that is stress state, after which the application remains constant during the test.

A creep test on marl in a conventional triaxial device was carried out on 10 specimens with a constant vertical load of 2.0 MPa in the period of 3 days and with 4.0 MPa of load within the following 3 days. Three specimens were tested without lateral pressure, the fourth one had a lateral pressure of 1.0 MPa, and three specimens had a 2.0 MPa lateral pressure. This short creep test offered some qualitative indicators of behaviour of the examined rock over time with the influence of the rotation symmetrical lateral pressure.



Figure 7. Cylindrical specimen for a conventional three-axial test; a) disposition of measurement rosettes; b) rosettes attached to the specimen.

4 RESULTS OF THE CREEP TESTS ON MARL

4.1 RESULTS OF THE UNIAXIAL CREEP TESTS ON MARL

The results of the measurement of creep deformations on prismatic specimens, at an axial stress from 2.0 MPa to 4.0 MPa, are shown in Fig. 8. The diagram clearly shows the zone of intensive creep of materials in the axial direction in the first 20 days after the loading. The deformation increase in this period of time is non-linear in relation to the time. However, after this period the creep deformations increase linearly. It can also be noticed that the gradient of deformation increase during a period of time is bigger with specimens under higher pressure stress.



Figure 8. Comparative diagram of the creep in the axial direction under different stresses.



Figure 9. Comparative creep diagram after the total or partial unloading of prismatic samples in the axial direction.

Fig. 9 presents a comparative creep diagram after unloading in the axial direction in these two groups of prismatic samples. This diagram clearly shows that during the process of unloading, besides the instantaneous elastic deformation, there is also a significant time-dependent deformation - reverse creep (for details see [10]).

4.2 RESULTS OF THE CREEP TEST ON THE PLATE SPECIMENS

4.2.1 Results of the creep test on the uniaxially loaded plate specimens

The material's low level of tensile strength made conducting the uniaxial test on the plate specimens difficult. Namely, small bumps on the specimens on the contact points with the steel trapeze elements that transfer the load resulted in a local concentration of the tension stress (like the "Brazilian" test by cleavage). Under a vertical load from 1.0 to 2.0 MPa, the tension stresses, induced in the horizontal direction, acquire the strength in materials, referring to tightening, the consequence of which is the appearance of vertical cracks.

At the contact point between the steel construction, i.e., conveying the trapeze elements for the steady loading of the plate specimens, a double Teflon foil was placed in order to eliminate any contact friction. This measure reduced the friction to minimum, which was verified by a measurement of the deformations on the plate specimens. Measuring the moisture of the plate specimens was conducted directly after breaking the specimens by taking three pieces of marl. The average value of the moisture of the plate specimens was established to be 13.21%.

Due to the above-mentioned difficulties in performing the creep tests on uniaxially loaded plates, the test was performed on two plates with some cracks. The plate marked PL-1 is divided into two plates by a 60x30 cm crack, while the plate marked PL-4 was reduced by a vertical crack to a size of 60x50 cm.

After several unsuccessful attempts to load plate specimens uniaxially to a 2.0 MPa stress without cracks, the problem was, in the subsequent work, resolved by the replacement of two layers of Teflon foil by 5mm rubber beds placed on the contact points with steel trapeze conveying elements. The plate marked PL-7 (which was used in the first phase as a compensation for the shrinkage and temperature) was loaded through the rubber layers up to a stress of 2.0 MPa without the appearance of cracks. This plate was used for the analysis of short deformations of the plate specimens.

The results of the first phase of the creep test on the plate specimens are presented in the diagram in Fig. 10. Some bigger deformations, measured on plate marked PL-1, were probably the consequence of the smaller dimensions of the horizontal plate (60x30 cm), i.e., a smaller number of prevented side deformations. The approximate creep deformations of the prismatic specimens are shown as comparative values.



Figure 10. Diagram of the creep of uniaxially loaded plate specimens with 2.0 MPa stress.

The measured deformations on the plate specimens, with the same vertical stress, are bigger than the measured deformations on prismatic specimens. The ratio of the initial vertical deformations of the plate and prismatic specimens is 1.19. The same ratio was established between the deformations on the cylindrical and prismatic specimens. The ratio of the creep deformation of the plate and the prismatic specimens is bigger and amounts 1.85 (30 days after loading).

The diagram in Fig. 10 shows that an increase of the deformation of the creep significantly reduces in the horizontal direction after 10-15 days. This phenomenon can be explained by the behaviour of the materials in the vertical and horizontal directions, i.e., the exceptional anisotropy of the material ($E_h/E_v=2$, E-elasticity module), which is the consequence of the establishment of the microstructure of the marl during sedimentation. The creep of the vertically loaded samples in the horizontal direction is evidently predominantly occurring immediately after loading, i.e., in the zone of the primary creep component - delayed elasticity. When full, elastic deformations are developed, including the delayed deformations, the increase of the creep deformations is drastically reduced. A similar effect is noticed for the bi-axial load specimens – Fig.11.

4.2.2 Results of the creep test on the bi-axially loaded plate specimens

The lateral load of the bi-axially loaded plate specimens significantly changes the deformation picture, both the initial deformation induced by the stress change and the creep deformation. Fig. 11 shows the results of the measurement of the deformation on the three groups of specimens loaded under different relations of horizontal and vertical stresses. All the diagrams show the average total deformation of the uniaxially loaded prismatic specimens with 2.0MPa vertical stress as a comparative value.

In the relation $K_o = \sigma_h / \sigma_v = 1.0$ with plate specimens, the initial and time vertical deformation differs very slightly from the deformation of the uniaxially loaded prismatic specimens, as shown in the diagram in Fig. 11.a. The increase in the time deformations was slightly bigger with the plate specimens compared to the prismatic specimens 10-15 days after loading. A larger increase of time deformations was accordingly the consequence of larger secondary creep with the plate specimens (secondary creep becomes more important as the time passes, while primary creep after a period of 10 days had practically no further influence on the deformation increase).

With a reduction of the relation $K_o = \sigma_h / \sigma_v$ from the value 1.0 towards the uniaxial stress state, an increase of the initial and time-dependent vertical deformations increases as shown in the diagrams in Fig. 11 a, b and c, which is the consequence of the release of the specimen's longitudinal extension. The horizontal deformation, with bi-axially loaded plate specimens, showed fluctuations in the first 5-7 days after loading. In the further period, creep in the horizontal direction could be registered only for specimens loaded with the same vertical and horizontal pressure. With all the other specimens with ratios $K_o = \sigma_h / \sigma_v = 0.5$ the horizontal deformation and in ratio $K_o = \sigma_h / \sigma_v = 0.3$ the horizontal deformations were



Figure 11. Diagram of the creep of the bi-axially loaded plate specimens.

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almost the same as the horizontal deformations of the prismatic specimens. In the ratios $K_o = \sigma_h/\sigma_v = 1.0$ and 0.5 the horizontal deformation is the shrinkage deformation and in the ratios $K_o = \sigma_h/\sigma_v = 0.3$ the horizontal deformation is the extension deformation.

4.3 RESULTS OF THE CREEP TESTS IN THE CONVENTIONAL TRIAXIAL DEVICE

The diagram in Fig. 12 shows simultaneously the results of the creep tests (strain-time) of the uniaxially loaded cylindrical specimens (d/h=5.4x10.8cm) and the average creep values on the prismatic specimens (15x15x40cm)

under uniaxial stresses of 2.0 and 4.0 MPa. The test results indicate that there is no significant difference in the measured values of the time-dependent deformations on the prismatic and cylindrical specimens.

The diagram in Fig. 13 shows the creep-test results of the triaxially loaded cylindrical specimens with different lateral pressures. The full lines indicate the axial (vertical) strain and the dash lines indicate the tangential (horizontal) strain.

The radial deformation with all the specimens after one day of loading preserves the obtained level, regardless of the intensity of the radial stress in the considered domain, while the axial deformation indicates the pres-



Figure 12. Comparative results of the uniaxial creep tests on the prismatic and cylindrical specimens with uniaxial stresses of 2.0 and 4.0. MPa.



Figure 13. Results of the triaxial creep tests on the cylindrical specimens and of the uniaxial test on the prismatic specimens with an axial stress of 2.0 Mpa. (Legend: PR - prismatic sample; T_v/T_t - vertical/horizontal strain on the cylindrical sample; p – the lateral pressure).

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ence of the influence of the lateral stress on the flow of the time deformations.

4.4 RHEOLOGICAL MODEL

Due to the common characteristics of rock features, which can be encompassed, and the formulation for the three-axial stress state, as a suitable basis for a description of the stress-deformation behaviour of different types of rock material, the constitutive Wallner Law was used /1983/ (developed on the basis of tests on rock salt). In a comparative analysis of the time-dependent deformation component of the marl after loading, it was determined that Wallner's rheological model, based on the results of the rock-salt testing, approximates well the creep of the marl after loading. In a comparative analysis of time deformations after total or partial unloading, it was found that the model does not include a reverse creep deformation, which is the consequence of the assumption that the primary creep depends on the current stress state.

As proposed by the author of this paper, and based on the results of uniaxial creep tests on marl specimens, a modified rheological model that describes the behaviour of soft rock after loading and after total or partial unloading was formulated. In this model, the creep was primarily formulated as a function that is dependent on the stress difference that preceded the creep compared to the original model, where this component of creep depends on the current stress state. The abovementioned correction enabled a correct description of the behaviour after total or partial unloading, which has significant implications in the event of modelling the state of stress around the tunnel opening. The mechanical model of the constitutive equations of the stressdeformation behaviour of the rock according to Wallner and the author of this paper is presented in Fig. 14.

For a practical application of the presented constitutive model a mathematical description of the rheological model of the rock mass is needed. The mathematical formulation of the rheological model of marl, when stresses are under the plasticity limit, is summarized in the following text (for details, readers can refer to the bibliography [3], [4] and [10). The equations define the components of deformations and generally refer to three-dimensional cases. Having in mind the fact that deformations are time-dependent equations and are given as derivatives of time.



Figure 14. Mechanical model of the constitutive equations of the stress-deformational behaviour of the rock according to M. Wallner and the author of this paper.

According to Wallner's model, all the components of deformations are irreversible, except for the elastic component, while according to the corrected model, the component of secondary creep is reversible as well. Volumetric deformation appears in the elastic domain during tertiary creep and in failure due to the strengthening. The above-mentioned rule applies when there is no volumetric change in the primary and secondary creep.

The total deformation with stresses under the conditions of plasticity is obtained as a sum of three components:

$$\varepsilon = \varepsilon_e + \varepsilon_p + \varepsilon_s$$
; (1)

where:

- $\varepsilon_{\rm e}$ the elastic strain (induced by stress change),
- ε_p the strain of primary creep (delayed elasticity),
- ε_s the strain of secondary creep.

4.4.1 elastic component of deformations

The relation between the stress and the strain in the domain of elasticity in a common form is formulated as a differential equation:

$$\left\{\frac{\partial \varepsilon^{e}}{\partial t}\right\} = [C] \left\{\frac{\partial \sigma}{\partial t}\right\}; \quad (2)$$

where:

 ε - the vector of the strain components,

 σ - the vector of component stresses,

[*C*] - the matrix of the coefficients of flexibility.

When the stress-strain relation is independent of the time, equation (2) is reduced to the following equation

$$\{\varepsilon\} = [D]^{-1}\{\sigma\} = [C]\{\sigma\}$$
 . (3)

In the case of an arbitrary spatial stress state and strain, the equations of the generalized Hook's Law apply.

4.4.2 primary creeping

The common form of the relation between the stress and the strain in primary creep in a three-dimensional case according to the corrected model is described in the differential equation:

$$\frac{\partial \varepsilon^{p}}{\partial t} = \frac{1}{\eta_{p}} G \left\{ \frac{\partial (\Delta \sigma_{eff})}{\partial \{\sigma\}} \right\} ; \qquad (4)$$

where:

$$G = E_p \left[\left(\frac{\Delta \sigma_{eff}}{E_p} \right)^m - \Delta \varepsilon_{eff}^p \right]; \quad (4.1)$$

$$\Delta\sigma_{eff} = \sqrt{\frac{3}{2}} \left(\Delta S_x^2 + \Delta S_y^2 + \Delta S_z^2 + 2\Delta \tau_{xy}^2 + 2\Delta \tau_{yx}^2 + 2\Delta \tau_{zx}^2 \right); (4.2)$$
$$\Delta \varepsilon_{eff}^p = \sqrt{\frac{2}{3}} \left(\Delta \varepsilon_x^2 + \Delta \varepsilon_y^2 + \Delta \varepsilon_z^2 + \frac{1}{2} \Delta \gamma_{xy}^2 + \frac{1}{2} \Delta \gamma_{yx}^2 + \frac{1}{2} \Delta \gamma_{zx}^2 \right); (4.3)$$

 E_p - is the strain-hardening modulus

 $\eta_p^{'}$ - is the viscosity (for primary creep)

m - is the stress exponent (for primary creep)

The primary creep is described by three specific material parameters. E_p , η_p and m.

For the plastic potential σ_{eff} one invariant of the deviant part of the stress tensor was adopted ($\sigma_{eff} = \frac{3}{\sqrt{2}} \tau_{okt}$, where τ_{okt} is the octahedron of the shearing stress) and describes the departure from the hydrostatic stress state ($\sigma_1 = \sigma_2 = \sigma_3$), while ε_{eff}^p is an adequate invariant of the deviatory part of the deformations tensor. The plastic potential σ_{eff} and ε_{eff}^p are defined by the components of the deviatory part of the stress tensor S_x , S_y , S_z , σ_{xy} , σ_{yz} , σ_{zx} , i.e., the components of the deviatory part of the deformations tensor ε_x , ε_y , ε_z , ε_{xy} , ε_{yz} , ε_{zx} .

The change of the plastic potential with the stress change from σ_i to σ_j is expressed as $\Delta \sigma_{e\!f\!f}$, i.e., $\Delta \sigma = \sigma_j - \sigma_i$. An adequate invariant of the deviatory part of the deformations tensor is $\varepsilon^{p}_{e\!f\!f}$.

4.4.3 secondary creep

The common form of the relation between the stress and the strain of the secondary creep is described by the differential equation:

$$\left\{\frac{\partial\varepsilon^{s}}{\partial t}\right\} = \frac{1}{\eta_{s}}H\left\{\frac{\partial\sigma_{eff}}{\partial\{\sigma\}}\right\} ; \qquad (5)$$

where:

$$H = P_o \left(\frac{\sigma_{eff}}{P_o}\right)^n ; \qquad (5.1)$$

$$\eta_s = \frac{P_o}{a} ; P_o = 1.0 MPa ; \qquad (5.2)$$

a - is the creep parameter

n - is the normalized stress exponent (for secondary creep)

Secondary creep is described by two parameter constants of material *a* and *n*.

In a regression analysis based on the results of the creep tests on uniaxial specimens, the material parameters of the marl were defined, which describe the creep after total or partial unloading. The components of the rheological model that appear as soon as the conditions of release are reached have not been the subject matter of this research, and they are not presented in detail (for details readers can refer to the papers of Wallner [11] and Doring and Kiehl [8]).

4.4.4 equation for uniaxial stress state

Considering that the conducted laboratory creep tests were carried out under a uniaxial stress state, a transfer to the creep equation for this specific stress case from general equations is of the most significant interest for the following analysis. After the inclusion of the uniaxial stress state and the appropriate processing, an equation describing the creep under a uniaxial stress state is obtained:

$$\varepsilon = \frac{1}{E} \Delta \sigma_1 + \frac{3}{2} \left(\frac{\Delta \sigma_1}{E_p} \right)^m \left(1 - e^{-\frac{2E_p}{3\eta_p}t} \right) + a \left(\Delta \sigma_1 \right)^n t \ ; (6)$$

where the first element describes the elastic, the second element describes the primary creep and the third element describes the secondary creep. The material parameters of the marl were defined and they describe the components of the primary and secondary creep (Table 1) for the uniaxial stress state (for details, readers can refer to the papers of Tomanovic [10]).

Table 1. Values of the material parameters of the marl forprimary and secondary creep after loading.

	Primary creep			Secondary creep	
Parameters	E_p	$\mu_{\rm p}$	т	а	п
	[MPa]	[MPa.d]		[1/d]	
Marl	225	425	0.06	$2.71^{*}10^{-4}$	2.5
Rock salt	180	900	2	3,40*10-10	5

5 INFLUENCE OF K, ON Vertical deformation

5.1 INFLUENCE OF HORIZONTAL STRESS ON THE VERTICAL DEFORMATION WITH PLANE STRESS

A comparative diagram of the aggregate vertical deformation (stress dependent + creep deformation) of plate specimens shows that the applied lateral (horizontal) pressure reduces the vertical deformation. The diagram in Fig. 15 shows the values of the approximate measured vertical strain 45 days after the loading for different ratios of the horizontal and vertical stresses applied to the specimen.

The influence of the lateral pressure on the vertical deformation can be sufficiently well approximated with a linear function. The regression analysis with the application of the over-squares method resulted in an empirical dependence:

$$\varepsilon_1 = \varepsilon_1 (\sigma_h = 0) - 0.88 K_o ; \qquad (7)$$

where:

$$\begin{split} \varepsilon_1 & \text{- the vertical strain for } K_0 = 0 \text{ to } 1.0; \\ \varepsilon_1(\sigma_h = 0) & \text{- the vertical strain of the plate specimen} \\ & \text{when the lateral pressure equals zero;} \\ K_0 & \text{- the ratio of the horizontal and vertical stress} \\ & K_o = \sigma_h / \sigma_v \; . \end{split}$$

According to the above-mentioned vertical strain with plane stress under the lateral pressure the influence can be reduced by up to 30% (where $K_0 = 1.0$) compared to the vertical strain developed without the influence of lateral pressure.



Figure 15. Total vertical strain of plate specimens 45 days after creep, depending on the ratios of the applied horizontal and vertical stresses.

5.2 INFLUENCE OF LATERAL STRESS $\sigma_2 = \sigma_3$ on the axial deformation with a rotationally symmetrical stress state

The comparative diagram of the total vertical-axial deformations (stress dependent + creep deformations) of the cylindrical specimens of marl with a conventional three-axial test shows that the applied lateral-radial pressure influences the reduction of the axial deformations. The diagram in Fig. 16 shows the values of the approximate measured deformation 3 days after the moment of loading for different ratios of the applied lateral and axial stresses on the specimen. The total deformation of the examined marl is two times smaller, with ratios $\sigma_h/\sigma_v = 1.0$, in relation to the uniaxially loaded specimens with $\sigma_h = 0$.

The influence of the lateral pressure on the vertical deformation can be well approximated in a linear function. The regression analysis with the application of the method of the over-squares method led to an empirical dependence, as follows:

$$\varepsilon_1 = \varepsilon_1 (\sigma_k = 0) - 0.994 K_o$$
; (8)

where:

 ε_1 - the vertical stain for $K_0 = 0$ to 1.0; $\varepsilon_1(\sigma_h = 0)$ - the vertical-axial strain of the cylindrical specimen where the lateral stress equals zero K_0 - the ratio of the horizontal (radial) and axial (vertical) stresses $K_o = \sigma_h / \sigma_v$;

According to the previously illustrated, the total axial deformation (stress dependent + time dependant

component) in the conditions of rotationally symmetrical pressure can be smaller by up to 100% (where $K_0 = 1.0$) than the axial deformation developed without the impact of lateral pressure. With the plate specimens loaded in their plane at the plane stress state, the influence of the horizontal stress leads to a reduction of the vertical deformation up to a maximum of 30%, which indicates that borderline conditions have a large influence on the current deformations induced by the stress change and on the time-dependent deformations.

6 CONCLUSION

Creep tests conducted on uniaxial loaded prismatic specimens indicate that marl with a constant uniaxial pressure shows significant time deformations, i.e., creep, and thus the creep deformation that develops within 6 months almost reaches the value of the current deformation induced by the initial change of stress. In the first 20 days after loading, the increase in the deformation was non-linear in relation to the time. This is the zone of intensive creep of the material in the axial direction, i.e., primary creep. After this period the deformations of the creep are smaller and the increase in the deformations is almost linear, i.e., secondary creep. Additionally, the gradient of the deformation increase over time is larger with specimens loaded with a higher stress pressure.

The lateral (horizontal) pressure with bi-axially loaded plane specimens significantly changes the deformation picture of both initial deformations induced by the stress change and the creep deformations as a consequence of the different boundary conditions of different test methods. With ratios $K_o = \sigma_h / \sigma_v = 1.0$ for the plate



Figure 16. Total deformation of cylindrical specimens 3 days after creep, depending on the ratio of the applied horizontal and vertical stresses.

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specimens, the initial and time-dependent vertical deformation very slightly differ from the deformation of the uniaxially loaded prismatic specimens. With a reduction of the ratios $K_o = \sigma_h / \sigma_v$ from value 1.0 towards the uniaxial stress state, the increase in the initial and time-dependent vertical deformations grows, which is the consequence of the release of the lateral expanding of the specimen.

The comparative diagram of the overall vertical deformation (stress dependent + creep deformation) of the plate specimens shows that the applied lateral (horizontal) pressure reduces the vertical deformation and this influence can be sufficiently well approximated in an empirical function (7). The vertical deformation at the plane stress state due to the impact of the lateral pressure can be reduced by 30% (where $K_0 = 1.0$) in comparison to the vertical deformation, developed without the influence of lateral pressure, i.e., under a uniaxial stress state.

The effect of the influence of lateral pressure, the stress on the axial deformation during a conventional test of triaxial compression is significantly larger compared to the influence of the lateral pressure during a bi-axial load. The axial deformation in the conditions of rotationally symmetrical pressure can be reduced by as much as 100% (where $K_0 = 1.0$) compared to the axial deformation, developed without the impact of the lateral pressure. The influence of the lateral pressure can, with a rotationally symmetrical stress state, be sufficient to approximate an empirical function (8).

For practical applications with an acceptable approximation, the materials constants can be used as well as the parameters of the materials obtained in uniaxial tests, where the obtained values of all the deformations (short + creep deformations) should be corrected in line with the developed empirical formulas for plane and rotationally symmetrical stress states based on the dominant stress state of the matter considered.

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