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Dvodimenzijski matematični model transporta lebdečih plavin

A Two-Dimensional Mathematical Model of Suspended-Sediment Transport

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Po kratki predstavitvi teoretičnih osnov hidrodinamičnih značilnosti tokov, ki jih obravnavamo kot dvorazsežne v vodoravni ravnini, z globinsko povprečnimi hitrostmi, je v prispevku predstavljena enačba transporta lebdečih plavin, ki je uporabljena v matematičnem modelu. To je advekcijsko-difuzijska enačba z dodatnim izvornim členom, ki opisuje spremembo koncentracije plavin, povzročeno z usedanjem ali dvigovanjem materiala z dna struge. Globinsko povprečno vrednost koncentracije lebdečih plavin smo dobili na podlagi analize transportne enačbe v navpični ravnini. Izračun izvornega člena sloni na transportni enačbi, definirani v navpični ravnini, ki daje značilen razpored koncentracije lebdečih plavin z najmanjšo koncentracijo na površini in največjo na dnu struge. Rezultati izračuna so odvisni od razlike med vtočno (izračunano) globinsko povprečno koncentracijo plavin in povprečno vrednostjo koncentracije plavin v ravnovesnem stanju, za določene hidrodinamične pogoje znotraj kontrolnega volumna. Kot primer uporabe matematičnega modela je obdelana problematika Ptujskega jezera. To je izpostavljeno vplivu usedanja lebdečih plavin, ki ga nosi reka Drava. Prikazani so rezultati opravljenih meritev, postopek umerjanja hidrodinamičnega dela matematičnega modela ter rezultati modela za račun prenosa lebdečih plavin.
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(Ključne besede: modeli matematični, transport materiala, plavine lebdeče, Ptujsko jezero)

After a brief review of the theoretical basis of the hydrodynamic characteristics of two-dimensional depth-averaged flow in a horizontal plane, in this paper we present an equation for suspended sediment transport. It is an advective-diffusion equation with an added source term that describes the concentration of a suspended sediment caused by sedimentation or erosion. The depth-averaged concentration of the suspended load is a result of an analysis of the transport equation in the vertical plane. The source-term definition is based on the transport equation in the vertical plane, which gives a characteristic concentration distribution of the suspended load with a minimum concentration at the surface and a maximum at the bottom of the bed. The calculation results depend on the difference between the inflow (calculated), depth-averaged concentration of the suspension and the averaged equilibrium suspension concentration in a numeric cell under certain hydrodynamic conditions. As an example of the application of the mathematical model, the problem of Ptuj lake (Slovenia) is presented. It is very exposed to the sedimentation of suspended sediment that is brought by the river Drava. The results of the measurements, the procedure of the hydrodynamic part of the mathematical model calibration and the results of the suspended-load module are presented.
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(Keywords: mathematical model, materials transport, suspended sediment load, Ptuj lake)

0 UVOD

Eden izmed zelo pomembnih problemov, s katerim se srečujemo pri gospodarjenju z umetnimi akumulacijami, je problem gibanja plavin. To je pojav, ki je odvisen od hidrodinamičnih sil ter drugih pogojev, ki so definirani z značilnostmi plavin. Gibanje plavin povzroča dotok materiala iz zaledja porečja in njegovo usedanje, spremembo ravni podtalnice in veča stopnjo ogroženosti zaradi morebitnega iztekanja vode iz struge in poplavljanja obrečnega območja.

0 INTRODUCTION

The transport of suspended sediment is a very real problem in the management of artificial reservoirs. It depends on the kinds of hydrodynamic forces present and on other conditions that are defined by the characteristic of the sediment. Suspended-sediment transport takes into account the processes of erosion, the transport of materials and their settling. The major consequences of settling material are: the filling up of riverbeds, the lifting of the river bottom and, in some

Pojav usedanja plavin je posebej pomemben pri akumulacijah, zgrajenih na rekah z večjo transportno zmogljivostjo, ker povzroča zmanjšanje prostornine akumulacije in kakovosti vode, tako da postaja vprašljiva večnamenska uporabnost akumulacije.

Z nalogami, ki so s časom postajale vse bolj zapletene, se je razvijala tudi metodologija načina prenosa plavin. Od prvih, bolj grobih napovedi, zasnovanih na empiričnih in polempiričnih enačbah, je dandanes večji poudarek na matematičnih modelih, ki naj bi bili zmožni napovedati koncentracijo lebdečih plavin in bilanco proda v prostoru in času z uporabo diferencialnih enačb, ki opisujejo ta fizikalni pojav. Zaradi spremembe koncentracije plavin, povzročene z njihovim usedanjem ali erozijo, je treba prenos plavin obravnavati kot prenos nekonservativne snovi.

Za določanje bilance prenosa plavin obstajajo različne merske in računske metode. Ne glede na razpoložljivo mersko in računalniško opremo je še vedno težko napovedati bilanco plavin in spremembo konfiguracije dna. Za potrebe kalibracije bolj točnih in tudi bolj zahtevnih matematičnih modelov, je treba razpolagati z večjim številom podatkov, ki jih je mogoče dobiti le s celovitimi in dolgotrajnimi meritvami (v določenih primerih tudi do 10 let). Zato je treba, glede na kakovost in število razpoložljivih podatkov, racionalno izbrati kompleksnost matematičnega modela. Na ta način se lahko izognemo uporabi parametrov, za katere nimamo znanih vrednosti.

Problem prenosa plavin je v splošnem trodimenzijski problem. Glede na geometrijske značilnosti toka, je mogoče pri reševanju praktičnih problemov, uvesti ustrezne poenostavitve. Najbolj pogoste so:

- dvodimenzijski pristop (2D), kadar problematiko obravnavamo v navpični ali vodoravni ravnini (kot model enotne širine ali povprečen po globini) in
- enodimenzijski (1D) pristop.

Za analizo tokov na krajših odsekih v bližini izpustov tekočine, kjer je snov pod vplivom turbulence v navpični ravnini že enakomerno premešana ali pa v plitvih jezerih s prevladujočimi izmerami v vodoravni ravnini, se po navadi uporabljajo dvodimenzijski, po globini povprečni matematični modeli. Takšen model je uporabljen tudi pri analizi tokov in prenosa lebdečih plavin v Ptujskem jezeru.

1 HIDRODINAMIČNI MATEMATIČNI MODEL

1.1 Osnovne enačbe

Tok vode je temeljni parameter prenosa snovi v obliki raztopine ali delcev. Zato je natančno poznavanje hidrodinamičnih parametrov, kot sta

cases, an increased danger of floods, etc. This process has a special importance for retention basins that are located on gravel-bed rivers, because it causes a reduction in the basin's volume and water quality, so its multipurpose usage might become questionable.

As the practical problems have become more complex, the methodology of the approach to sediment-transport problems has developed. The first, more approximate, predictions were based on empirical and half-empirical equations; today, the main stress is on mathematical models, which should be able to predict the concentration of suspended load and the balance of the load in space and time by using differential equations for describing certain physical events. Sediment transport should be treated as the transport of non-conservative matter, as its concentration changes with settling and erosion.

Different measurement and calculation methods are present for the defining the balance of sediment transport. Regardless of the available measuring and computer equipment, it is still difficult to predict the balance of sediment transport and changes to the bottom configuration. If the aim is to calibrate a more accurate and more complex mathematical model, it is necessary to have more data available, which could be provided by integral and long-term measurements (sometimes for 10 years). So, depending on the quantity and quality of the available data, an appropriate mathematical model should be chosen. In this way it is possible to avoid the need for parameters for which there are no available data.

Matter-concentration transport is, in general, a three-dimensional problem. However, by taking into account the geometrical characteristics of water flow in practical work, the solving of hydrodynamic and concentration transport equations can be simplified. The most common simplifications are:

- a two-dimensional treatment of the problems - in a vertical or a horizontal plane (as a depth-averaged model),
- a one-dimensional treatment.

A two-dimensional, depth-averaged treatment is usually used for situations where problems appear in the region near the outflow into the recipient, where the matter is already well mixed in the vertical profile by the influence of turbulence and bottom friction, or in shallow lakes with the largest dimensions in the horizontal plain. Such a two-dimensional hydrodynamic mathematical model was applied to our sediment-transport analysis of the Ptuj lake.

1 HYDRODYNAMIC MATHEMATICAL MODEL

1.1 Basic equations

The basic parameter of the solution or parts transport is water flow. So, hydrodynamic parameters, such as velocity and flow depth, should be accurately hitrost in globina toka, zelo pomembno. Izračun prenosa plavin je mogoče opraviti šele potem, ko imamo na voljo preverjeno hidrodinamično sliko toka.

Hidrodinamične značilnosti opišemo z dvodimenzijsimi, po globini povprečnimi enačbami. Uporabljene kontinuitetna in dinamični enačbi v konservativni obliki, ki opisujejo stalni tok v kartezičnem pravokotnem koordinatnem sistemu, so naslednje:

known. The sediment-transport calculation can only be performed when the available hydrodynamic conditions are checked.

The hydrodynamic characteristics can be described by two-dimensional, depth-averaged equations. The continuity and dynamic equations for the cartesian orthogonal coordinate system are used in the following form:

$$\frac{\partial(hu)}{\partial x}+\frac{\partial(hv)}{\partial y}=0 \quad (1)$$

$$\frac{\partial(hu^2)}{\partial x}+\frac{\partial(huv)}{\partial y}=-gh\frac{\partial h}{\partial x}-gh\frac{\partial z_b}{\partial x}-ghn^2\frac{u\sqrt{u^2+v^2}}{h^{4/3}}+\frac{\partial}{\partial x}(h\upsilon_{ef}\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(h\upsilon_{ef}\frac{\partial u}{\partial y}) \quad (2)$$

$$\frac{\partial(huv)}{\partial x}+\frac{\partial(hv^2)}{\partial y}=-gh\frac{\partial h}{\partial y}-gh\frac{\partial z_b}{\partial y}-ghn^2\frac{v\sqrt{u^2+v^2}}{h^{4/3}}+\frac{\partial}{\partial x}(h\upsilon_{ef}\frac{\partial v}{\partial x})+\frac{\partial}{\partial y}(h\upsilon_{ef}\frac{\partial v}{\partial y}) \quad (3).$$

Ker se lahko ponekod pojavljajo območja z bolj izraženim vrtinčnim tokom ter v skladu z dosedanjimi izkušnjami pri matematičnem modeliranju tokov v rekah in jezerih z uporabo dvodimenzijskih modelov in z namenom, da bi dololočili vrednosti dodatnih turbulentnih napetosti, je uporabljena k - ε verzija hidrodinamičnega modela [2]. Dodatne transportne enačbe za turbulentno kinetično energijo k in njeno disipacijo ε so uporabljene v naslednji obliki:

As the appearance of regions with vortices in the flow is possible, and based on our experience of the mathematical modeling of river and lake flows using two-dimensional models, and in order to define additional turbulent stresses, the k - ε turbulent model seemed to be justified [2]. Additional transport equations for the turbulent kinetic energy k and its dissipation ε are used in the following form:

$$\frac{\partial(huk)}{\partial x}+\frac{\partial(hvk)}{\partial y}=\frac{\partial}{\partial x}(h\frac{\upsilon_{ef}}{\sigma_k}\frac{\partial k}{\partial x})+\frac{\partial}{\partial y}(h\frac{\upsilon_{ef}}{\sigma_k}\frac{\partial k}{\partial y})+hG-c_D h\varepsilon+hP_{kv} \quad (4)$$

$$\frac{\partial(hu\varepsilon)}{\partial x}+\frac{\partial(hv\varepsilon)}{\partial y}=\frac{\partial}{\partial x}(h\frac{\upsilon_{ef}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x})+\frac{\partial}{\partial y}(h\frac{\upsilon_{ef}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y})+c_1\frac{\varepsilon}{k}hG-c_2\frac{\varepsilon^2}{k}h+hP_{\varepsilon v} \quad (5),$$

kjer so: h – globina vode; u in v - komponente hitrosti v smereh x in y; z_b - kota dna; n - Manningov koeficient hrapavosti; υ_{ef} - kinematični koeficient efektivne viskoznosti; g – gravitacijski pospešek; k - turbulentna kinetična energija na enoto mase in ε - stopnja njene disipacije. Izrazi za G (produkcija k zaradi vodoravnih gradientov hitrosti) ter P_{kv} in P_{sv} (izvorna člena zaradi trenja ob dno) so skupaj s stalnicami, ki se pojavljajo v modelu turbulence (c_D, c_μ, c_1, c_2, $c_{\varepsilon\Gamma}$, σ_k in σ_ε), bolj natančno razloženi v literaturi [2].

where is: h – water depth; u and v – velocity components in the x and y directions; z_b – bottom level; n – Manning's roughness coefficient; υ_{ef} – kinematic coefficient of effective viscosity; g – acceleration due to gravity; k – turbulent kinetic energy per mass unit; and ε – its level of dissipation. The term G (the production of k caused by horizontal velocity gradients), P_{kv} and P_{sv} (the source terms caused by bottom friction), together with the constants that appear in the turbulence model (c_D, c_μ, c_1, c_2, $c_{\varepsilon\Gamma}$, σ_k and σ_ε), are explained in the literature [2].

1.2 Metoda reševanja in računalniški program

Sistem nelinearnih parcialnih diferencialnih enačb (1) do (5) rešujemo numerično z metodo kontrolnih volumnov, ki je natančno prikazana v literaturi [2] in [6]. Njene glavne značilnosti so premaknjena numerična mreža, hibridna shema (kombinacija centralnodiferenčne in sheme vzvodnih razlik) ter iteracijsko reševanje na podlagi popravkov globin. Izbrana shema je kompromis med točnostjo in ekonomičnostjo ter daje fizikalno realne rezultate pri vseh Pecletovih številih (razmerje med konvekcijskim in difuzijskim prenosom), razen v bližini vrednosti ±2.

1.2 Solution method and computer program

The control-volume method, described in [2] and [6], was used for solving the partial differential equations (1) to (5). Its main characteristics are a staggered numeric grid, a hybrid scheme (a combination of the central-differences and upwind schemes) and an iterative solution based on water-depth corrections. The chosen scheme is a compromise between accuracy and economy. It gives physically “real” results across the whole range of Peclet numbers (the ratio between advective and diffusion transport) accept near the value of ±2.

Osnova hidrodinamičnega matematičnega modela je zelo razširjen računalniški program TEACH [4]. Program je dopolnjen z možnostjo računanja tokov s povprečno globino z uporabo k - ε modela turbulence ob upoštevanju spremenljive geometrijske oblike ([2] do [4]). Tako je nastala nova različica programa, ki smo jo poimenovali PCFLOW2D.

Enačbe od (1) do (5) so nelinearne parcialne diferencialne enačbe drugega reda, hiperboličnega tipa. Zato potrebujemo začetne in robne pogoje za hitrosti u in v ter globino h na vseh štirih robovih. Tudi za preračun kinetične energije in njene disipacije, k in ε, potrebujemo robne pogoje na vseh štirih robovih računskega področja. Podrobneje so ti pogoji navedeni v literaturi [3] in [4].

The basis of the hydrodynamic model is the well-known computer program called TEACH [4]. The computer program is also able to calculate the depth-averaged flows using the k - ε turbulence model with respect to variable geometry ([2] to[4]). This resulted in a new version of the computer program called PCFLOW2D.

Equations (1) to (5) are non-linear, partial differential equations of the second-order, hyperbolic type. The initial and boundary conditions for the velocities u and v and the water depth h should be defined on all four boundaries. Also, for the production and dissipation of the kinetic energy, k and ε, the boundary conditions should be known for all four boundaries of the calculation area. These conditions are described in detail in [3] and [4].

2 MATEMATIČNI MODEL PRENOSA LEBDEČIH PLAVIN

2 MATHEMATICAL MODEL FOR THE TRANSPORT OF SUSPENDED SEDIMENT

2.1 Osnovne enačbe

2.1 Basic equation

Advekcijsko-difuzijska enačba, ki opisuje prenos snovi v dvodimenzijskem prostoru vodoravne ravnine, se lahko napiše v naslednji obliki:

The advection-diffusion equation for matter transport in a two-dimensional space in a horizontal plane can be written in the following form:

$$\frac{\partial}{\partial x}\left(h\bar{c}\,\bar{u}\right)+\frac{\partial}{\partial y}\left(h\bar{c}\,\bar{v}\right)=\frac{\partial}{\partial x}\left(h\frac{\upsilon_{ef}}{\sigma_{sm}}\frac{\partial \bar{c}}{\partial x}\right)+\frac{\partial}{\partial y}\left(h\frac{\upsilon_{ef}}{\sigma_{sm}}\frac{\partial \bar{c}}{\partial y}\right)\pm S_C \quad (6).$$

Oznake v enačbi (6) pomenijo: h - globino vode, $\bar{u}$ in $\bar{v}$ - globinsko povprečni komponenti hitrosti v smereh x in y, $\bar{c}$ - globinsko-povprečno koncentracijo lebdečih plavin, υ_{ef} - kinematični koeficient efektivne viskoznosti, σ_{sm} - Schmidtovo število in S_C - izvorni člen, s katerim opišemo spremembo koncentracije plavin v računskem polju, odvisno od dejanske koncentracije in hidrodinamičnih pogojev.

Izvorni člen S_C smo definirali na podlagi dvodimenzijske analize toka in razporeda koncentracije lebdečih plavin v navpični ravnini kontrolnega volumna [1]. Dvodimenzijska oblika diferencialne enačbe, ki opisuje prenos lebdečih plavin v navpični ravnini, se glasi:

using the following terms: h – water depth, $\bar{u}$ and $\bar{v}$ – depth-averaged components of the velocities in the x and y directions, $\bar{c}$ – depth-averaged concentration of suspended sediments, υ_{ef} – kinematic coefficient of effective viscosity, σ_{sm} – Schmidt number, and S_C – source term, describing the change of sediment concentration in a calculation cell, depending on the real concentration and the hydrodynamic conditions.

The source term S_C is determined on the basis of a two-dimensional analysis of flow and the distribution of the suspended load concentration in a vertical plane in a certain control volume [1]. The suspended sediment transport in a vertical plane is described by the following differential equation in a two-dimensional form:

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}-w_s\frac{\partial c}{\partial z}=\frac{\partial}{\partial z}\varepsilon_{s,z}\frac{\partial c}{\partial z} \quad (7).$$

Pomen členov v enačbi je:

- u hitrost toka v vzdolžni smeri (x) (prevladujoča smer toka);
- w_s hitrost usedanja zrna plavine v navpični smeri (os z), pozitivna smer hitrosti je usmerjena v negativni smeri osi z;
- t čas;
- c koncentracija lebdečih plavin in
- $\varepsilon_{s,z}$ koeficient difuzije plavin.

Enačba (7) se po navadi rešuje tako, da se ob definiranih začetnih in robnih pogojih kot rešitev dobi razpored koncentracije lebdečih plavin c po globini

Where:

- u is the velocity in the longitudinal (x) direction (dominant flow direction);
- w_s is the settling velocity in the vertical direction (z axis), the positive direction is opposite to the positive direction of the z axis;
- t is the time;
- c is the suspended load concentration,
- $\varepsilon_{s,z}$ is the suspension diffusive coefficient.

Equation (7) is usually solved so that the solution is a suspension concentration distribution c along the water depth under defined initial and toka. Pri tem se vpliv neenakomerne razporeditve velikosti zrna po globini po navadi ne upošteva. Obravnavano enačbo lahko rešujemo tudi neodvisno za vsako posamezno frakcijo plavin. Pomanjkljivost takega načina je v tem, da ni upoštevan medsebojni vpliv posameznih frakcij. Pri reševanju enačbe (7) smo uporabili prvi način, ob upoštevanju naslednjih robnih pogojev ([7] in [8]):

- na gladini, kjer je $z = z_d + h = z_g$:

boundary conditions. The influence of a non-uniform distribution of the grain diameter along the water depth was not taken into account. It is possible to solve the treated equation for each grain fraction independently. The problem with this approach is that the mutual influence of each grain size is not considered. The first approach was used in the solving of Equation (7), and the following conditions were considered ([7] and [8]):

- on the free surface, where $z = z_d + h = z_g$:

$$\varepsilon_{s,z} \frac{\partial c}{\partial z} + w_s c = 0 \qquad (8),$$

- na dnu robni pogoj izraža pretok mase plavin kot vsoto usedlega in erodiranega materiala:

- the bottom condition is explained as sediment discharge, as the sum of the settled and eroded material:

$$-\left(\varepsilon_{s,z} \frac{\partial c}{\partial z} + w_s c \right)_{z_d} = -\alpha w_s (c - c_*)_{z_d} \qquad (9).$$

V enačbi (9) je pomen oznak naslednji:

- c_* – ravnotežna koncentracija, odvisna od hidrodinamičnih pogojev in značilnosti plavin,
- α – bredimenzijski koeficient, ki izraža medsebojni vpliv dna in plavin. Zmnožek αw_s je znan kot dejanska hitrost usedanja. Vrednost $\alpha = 0$ pomeni idealen odziv dna, dokler velike vrednosti tega koeficienta ustrezajo adsorbirajoči površini (idealen ponor).

Prvi člen na desni strani enačbe (9) ($-\alpha w_s c$) pomeni pretok plavin v navpični smeri, ki se useda. Drugi člen desne strani, ($+\alpha w_s c_*$) pomeni pretok snovi, ki se erodira z dna in vključuje v tok. Ustrezno enačbi (9) je pretok plavin (usedanje ali erozija zrn) sorazmeren razliki med dejansko koncentracijo plavin in transportne zmogljivosti toka.

Z integriranjem pretoka plavin po globini, upoštevaje enačbo (7), dobimo vrednost globinsko povprečene koncentracije plavin $\bar{c}$ na razdalji $x = |\vec{V}|\, t$ od vtoka v kontrolni volumen, kjer je vtočna koncentracija $\bar{c}_0$. Pri tem je $|\vec{V}|$ velikost rezultante vektorja hitrosti v kontrolnem volumnu in t čas zadrževanja delcev znotraj kontrolnega volumna [5]:

The meaning of the terms in equation (9) is as follows:

- c_* is the equilibrium concentration, and it depends on the hydrodynamic conditions and the sediment characteristics,
- α is a non-dimensional coefficient that explains the mutual influence of the bottom and the sediments. The term αw_s is known as the actual settled velocity. The value of $\alpha = 0$ corresponds to an ideal bottom response, a larger value corresponds to an adsorption plain (ideal sink).

The first term on the right-hand side of equation (9), $-\alpha w_s c$, represents the sediment discharge in the vertical direction as settling. The second term, $+\alpha w_s c_*$, is the sediment discharge of the eroded material that entrainments in a flow. According to Equation (9) the sediment discharge (the settling or erosion of grains) is proportional to the difference between a certain sediment concentration and the flow transport capacity.

The depth-averaged sediment concentration $\bar{c}$ is calculated by integrating the sediment discharge over the depth, by considering Equation (7), at a distance $x = |\vec{V}|\, t$ from the inflow boundary of the control volume, at the point of the inflow concentration $\bar{c}_0$. There is a $|\vec{V}|$ velocity vector magnitude in a control volume and a t dwell time of the particles inside the control volume [5]:

$$\bar{c} = (\bar{c}_0 - \bar{c}_*) \exp\left(-\frac{\alpha w_s}{h} t \right) + \bar{c}_* \qquad (10),$$

kjer so: $\bar{c}_*$ - ravnovesna globinsko-povprečna koncentracija lebdečih plavin, α - brezdimenzijski koeficient, ki izraža medsebojni vpliv med dnom in plavinami in w_s - hitrost usedanja delcev plavin.

Izvorni člen iz konvekcijsko-difuzijske enačbe transporta lebdečih plavin (6) pa je definiran kot razlika med vteklo koncentracijo in dejansko koncentracijo v kontrolnem volumnu:

where the following terms are used: $\bar{c}_*$ – equilibrium, depth-averaged concentration of the suspended load, α – non-dimensional coefficient expressing the interacting influence between the bottom and the sediments, and w_s – the settling velocity of the particles.

The source term from the advective-diffusive equation of the suspended sediment (6) is defined as the difference between the inflow concentration and the actual concentration in the control volume:

$$S_C = \bar{c} - \bar{c}_0 \qquad (11).$$

2.2 Robni, začetni pogoji in računalniški program

Pri reševanju advekcijsko-difuzijske enačbe je bila podana znana koncentracija lebdečih plavin na vtočnem robu matematičnega modela in na iztočnem robu je bil robni pogoj definiran tako, da je gradient koncentracije v prevledujoči smeri toka enak nič. Na zaprtih (neprepustnih) robovih je postavljen pogoj neprepustne stene tako, da skozi takšen rob ni pretoka plavin.

Na začetku iterativnega postopka je treba podati vrednosti koncentracije lebdečih plavin v vseh kontrolnih volumnih. Pri razvoju računalniškega programa smo ugotovili, da se iteracijski postopek najhitreje konča, če je v vseh kontrolnih volumnih (razen v tistih z znano koncentracijo) podana vrednost enaka nič. Da bi se izognili težavam, ki bi lahko nastale z deljenjem z ničlo, podamo za začetno vrednost koncentracije kakšno majhno vrednost.

Za reševanje enačbe (6) je podobno, kakor že pri reševanju hidrodinamičnih enačb, uporabljena metoda kontrolnih volumnov s hibridno shemo. Sistem algebrajskih enačb se v iteracijskem postopku rešuje z istim podprogramom, prevzetim iz hidrodinamičnega modela.

3 PRIMER UPORABE MATEMATIČNEGA MODELA NA PTUJSKEM JEZERU

3.1 O Ptujskem jezeru

Ptujsko jezero je umetna akumulacija, zgrajena za potrebe proizvodnje električne energije na reki Dravi. To je zadnja akumulacija v slovenskem delu verige dravskih jezer. Od pregrade v Markovcih do strojnice HE Formin je zgrajen dovodni kanal.

2.2 Boundary, initial conditions and computer program

During the solving of the advective-diffusive equation, the known concentration of the suspended load was prescribed at the inflow boundary of the mathematical model, and the outflow boundary was defined such that the gradient of the concentration in the dominant direction of the flow is equal to zero; at the closed, non-permeable boundaries it was defined as the condition of a non-permeable wall, which means there is no sediment flow through these boundaries.

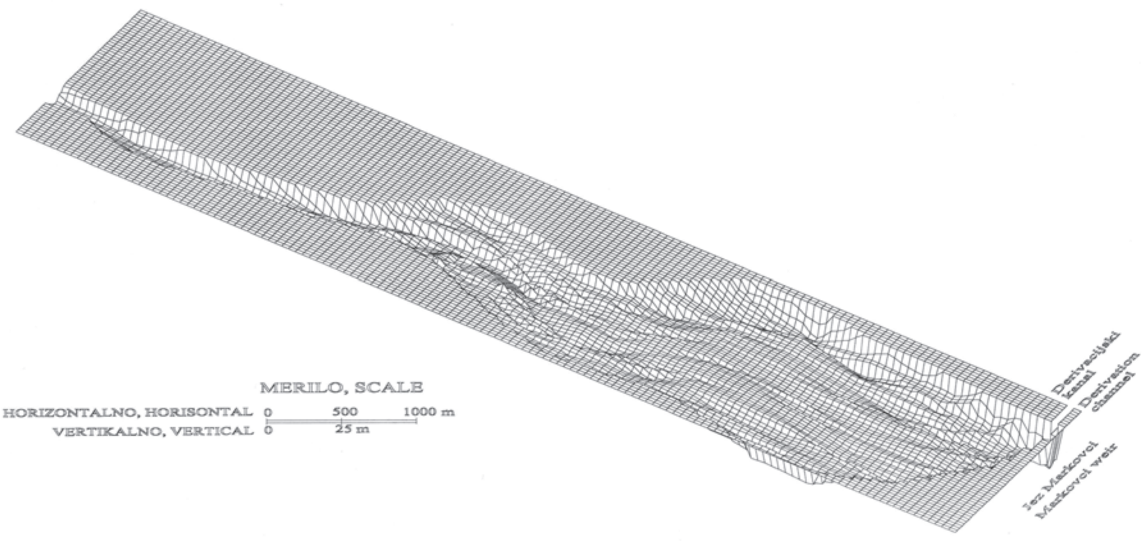
The concentration values should be defined on all the control volumes at the start of the iterative process. During the development of the computer program it was found that the iterative process takes the shortest time if all the control volumes (except those with a known concentration) had defined values of zero. To avoid the difficulties caused by dividing by zero, the starting concentration was chosen to be a small value.

Like during the solving of hydrodynamic equations, for the solving of Equation (6) the control volume method with a hybrid scheme was used. A system of algebraic equations was solved in the subprogram adopted from the hydrodynamic model.

3 AN EXAMPLE OF USING THE MATHEMATICAL MODEL FOR PTUJ LAKE

3.1 About Ptuj lake

Ptuj lake is a reservoir on the Drava river; its main purpose is the production of electric power. It is the last reservoir in a chain of several hydro-electric power plants on the Drava river. From the weir in Markovci to HEPP Formin, there is a derivation



Sl. 1. *Aksonometrična slika razporeda dna Ptujskega jezera*
Fig. 1. *An axonometric view of the bottom configuration of Ptuj lake*

Prostornina akumulacije je 22 milijonov m^3. Jezero je dolgo okoli 5,5 km, njegova povprečna širina je 1 km in povprečna globina 5 m. Največja globina je v bližini pregrade in znaša 12 m. Aksonometrični prikaz dna akumulacije je podan na sliki 1.

Ob reševanju elektroenergetskih problemov so bila z izgradnjo akumulacije zaščitena obrežna zemljišča reke Drave pred poplavljanjem. Izgradnja akumulacije je povzročila spremembo okolja in je imela tudi negativne posledice, predvsem na živalske vrste, značilne za to območje. Poleg tega se je zaradi pojavov evtrofikacije v usedlinah jezera poslabšala kakovost vode. Do te težave je prišlo zaradi porušitve ravnovesja med dotokom in prenosom plavin na obravnavanem odseku. Povečano usedanje materiala je povzročilo zmanjšanje globine (na nekaterih mestih se je znižala na 0,5 m) in prostornine zbiralnika (za 11% do leta 1993).

Da bi rešili omenjeno problematiko, so bile leta 1993 na Fakulteti za gradbeništvo in geodezijo, Univerze v Ljubljani, organizirane terenske meritve in opravljeni hidrodinamični računi toka v Ptujskem jezeru.

Te dejavnosti so obsegale:

1. Terenske meritve hitrosti toka na Ptujskem jezeru.
2. Zbiranje vzorcev vode in usedlega materiala z dna ter njihovo laboratorijsko analizo.
3. Pripravo topografskih, hidravličnih in hidroloških podatkov ter robnih pogojev za 2D hidrodinamični račun tokov.
4. Dvodimenzijske račune tokov za značilne pretoke in verifikacijo dobljenih rezultatov s pomočjo merjenih vrednosti.
5. Dvodimenzijske račune prenosa lebdečih plavin na podlagi izmerjenih značilnosti plavin in ustrezno izbranih koeficientov.

Opozoriti je treba, da so meritve temperature in drugih parametrov kakovosti vode po globini jezera pokazale, da stratifikacija ni opazna, kar upravičuje uporabo dvodimezijskega matematičnega modela.

3.2 Terenske meritve

Glede na dejstvo, da se večina materiala prenaša v času večjih pretokov, je bilo treba terenske meritve časovno uskladiti s pojavom ustreznih pogojev. Opravljene so bile oktobra leta 1993 pri srednji vrednosti pretoka 964 m^3/s, od tega je približno 460 m^3/s teklo skozi turbine, preostalih 497 m^3/s pa preko prelivnih polj. Merili smo naslednje parametre:

- hitrosti toka z uporabo hidrometričnih kril na območju dela zbiralnika z večjimi hitrostmi ter elektromagnetnih sond in plovcev na območju z manjšimi hitrostmi toka (pod 0,2 m/s). Gibanje plovcev smo časovno spremljali z geodetskim opazovanjem. Obdelani rezultati vseh meritev so podani na sliki 2;

channel. The volume of the reservoir is 22 million m^3. The length of the lake is about 5.5 km, with a main width of 1 km and a main depth of 5 m. The maximum depth is near the weir, and it is 12 m. An axonometric view of the lake bottom is presented in Figure 1.

The building of the reservoir solved some electro-power problems and also protects the surrounding lands of the river Drava from flooding. At the same time, it has caused environmental changes and resulted in some negative consequences, especially to fauna that is characteristic of the area. The quality of the water was decreased by the eutrofication process. This was caused by upsetting the balance in the inflow and the available sediment transport along the reservoir. A decreasing in the depth of the water (at certain parts of the lake it is less than 0.5 m) and the volume (11% since 1993) was caused by the settling of sediments.

In an attempt to solve these problems, in 1993 at the Faculty of Civil and Geodetic Engineering of the University of Ljubljana, field measurements were arranged and hydrodynamic calculations of the flow in Ptuj lake were performed.

These activities were:

1. Field measurements of the velocities in Ptuj lake.
2. Water and bottom-sediment sampling and laboratory analyses.
3. Preparing topographic, hydraulic and hydrological data and boundary conditions for the 2D hydrodynamic flow calculation.
4. 2D calculations for the characteristic discharges and result verifications with the help of measured values.
5. 2D calculation of the suspended-sediment transport based on measured sediment characteristics and suitably chosen coefficients.

It should be noted that the measurement of temperature and other parameters of the water quality along the depth of reservoir showed that the lake was not stratified and that the use of the 2D model was justified.

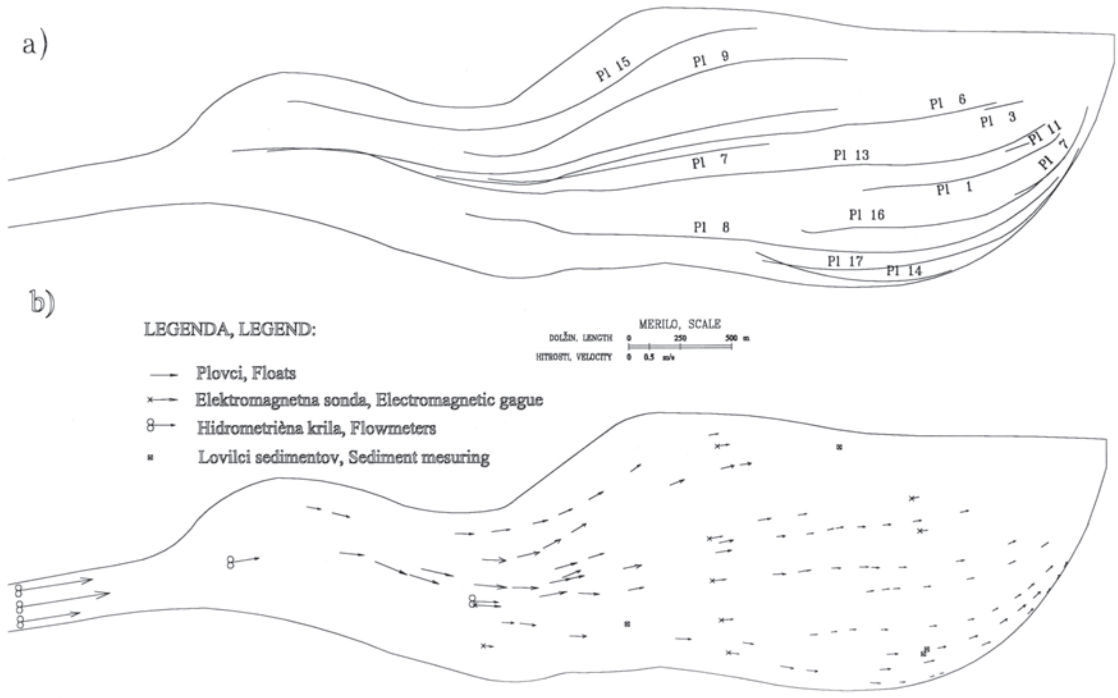
3.2 Field measurements

Because most of the material is transported during times of high discharges, measurements were adjusted appropriately with suitable conditions. The measurements were performed in October 1993 under the following conditions: discharge of 964 m^3/s, 460 m^3/s flew through turbines. The remaining discharge was overtopped on the weir. The following parameters were measured:

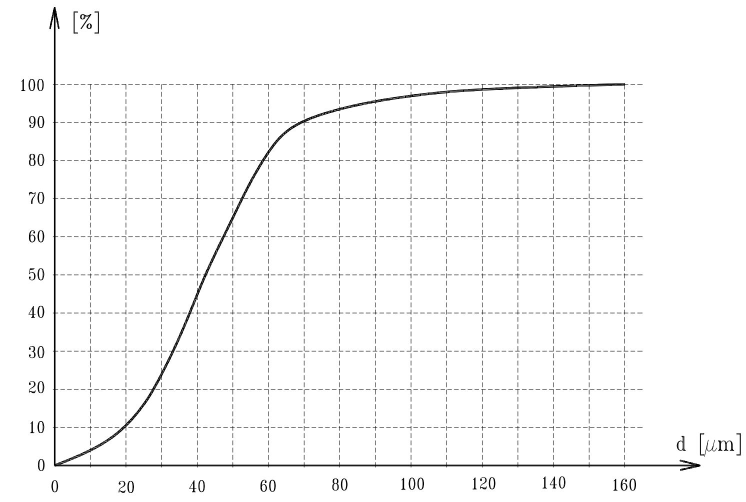
- velocities were measured with flowmeters in the region of the reservoir with higher velocities, and with electro-magnetic gauges and floats in the region with smaller flow velocities (smaller than 0.2 m/s). The movement of the floats was followed using geodetic observation. The results of all the measurements are presented in Fig. 2,

- hitrosti vetra, ki so bile izvedene na treh mestih na in ob jezeru z namenom, da bi ugotovili morebitni vpliv vetra na smer in hitrosti toka vode. Ugotovljeno je bilo, da je bila hitrost vetra v času meritev majhna in da je njegov vpliv na tok vode zanemarljiv;
- vzorčevanje vode smo opravili zato, da bi pridobili podatke o značilnostih lebdečih plavin, njihovi koncentraciji, gostoti in zrnavostni sestavi. Vzorčili smo na več globinah v jezeru, tako da smo lahko dobili ustrezno sliko razporeda koncentracij lebdečih plavin. Primer rezultatov analize zrnavosti je podan na sliki 3. Po globini povprečena koncentracija lebdečih plavin v korenu akumulacije je bila

- the wind velocity was measured at three locations on the lake surface and on the banks to ascertain if there was any wind influence on the water stream. We found that the wind velocity was so small that its influence on the water flow could be neglected,
- the water sampling was performed to obtain data on the suspended sediment's characteristics, its concentration, specific weight and grain size. The sampling was performed on several verticals in the lake, so the distribution of the suspended sediment's concentration was determined. The result of the grain size analysis is presented in Figure 3. The depth-averaged concentration in the



Sl. 2. *Rezultati opravljenih meritev na Ptujskem jezeru (26.10.1993), a) tokovnice, b) merjene hitrosti*
Fig. 2. *Results of hydrodynamic measurement on Ptuj lake (26.10.1993), a) streamlines, b) velocities*



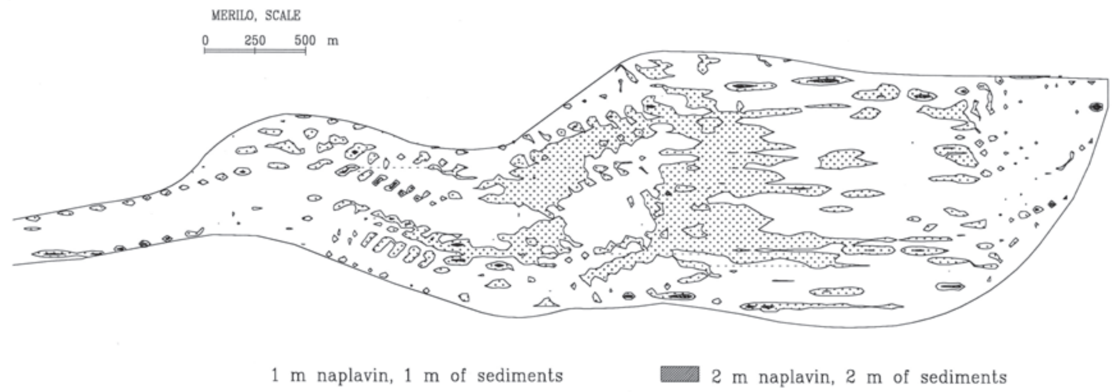
Sl. 3. *Zrnavostna krivulja lebdečih plavin na vtoku v akumulacijo (26.10.1993)*
Fig. 3. *Grain size curve of the suspended load in the upper part of the lake (26.10.1993)*

$\bar{c}_0 = 525$ g/m^3, medtem ko je bila zaradi prisotnosti organskih snovi gostota $\rho_s = 2{,}2\times10^3$ kg/m^3.

Na podlagi občasnih meritev prečnih profilov, ki se opravljajo vsakih pet let, je izvedena analiza višine usedlih plavin. Rezultati dveh snemanj, ki sta bili opravljenji v letih 1981 in 1991, sta matematično obdelani s programom QuickSurf (delujočega pod programom AutoCAD). Rezultat obdelave je prikazan na sliki 4, kjer so podane izočrte višine usedlega materiala. Opozoriti je treba, da je usedanje materiala opazno na vsem območju akumulacije, čeprav daje slika (zaradi precejšnjega razmika izočrt) nasproten vtis, saj je višina usedlega materiala na večjem delu zbiralnika manjša od 1 m. Rezultat, prikazan na sliki 4, je bil podlaga za umerjanje matematičnega modela za račun prenosa lebdečih plavin.

upper part of the lake was $\bar{c}_0 = 525$ g/m^3, with specific weight $\rho_s = 2.2\times10^3$ kg/m^3.

Based on periodic measurements of the cross-sections, which were made every five years, the height of the settled material was analysed. Two measurements, made in 1981 and 1991, were analyzed with the QuickSurf program (acting in AutoCAD). The result is presented in Figure 4 with isolines of the settled material depth. It should be pointed out that the sedimentation was present over the whole of the lake, even though Figure 4 gives (as the equidistance is relatively high) the opposite impression. The height of the sediments for the major region of the lake was less than 1 m. Figure 4 was the basis for the calibration of the mathematical model used to calculate the transport of the suspended sediment.



Sl. 4. *Višina usedlih lebdečih plavin v Ptujskem jezeru v obdobju od 1981 do 1991*
Fig. 4. *Sediment height in Ptuj lake during the period from 1981 to 1991*

3.3 Rezultati matematičnega modela

Pred simulacijo prenosa lebdečih plavin v Ptujskem jezeru je bilo treba opraviti umerjanje hidrodinamičnega dela modela. To vključuje določanje ustreznih vrednosti in razporeda koeficienta hrapavosti dna. Rezultati opravljenih računov so pokazali največjo podobnost z izmerjenimi vrednostmi hitrosti ob upoštevanju enotnega koeficienta hrapavosti po vsem dnu jezera, čigar vrednost znaša po Manningu $n = 0{,}026$ sm$^{-1/3}$. Na podlagi dosedanjih izkušenj ter opravljenih analiz zasipavanja akumulacije smo ugotovili, da večino materiala prinašajo pretoki enoletne povratne dobe ali večji. Ker je bil v prvi fazi pripravljen relativno preprost način računanja z uporabo modela za račun stalnega toka, je bil kot reprezentativen izbran pretok, ki ustreza petletni povratni dobi $Q_5 = 1239$ m^3/s. Izračunano polje hitrosti in tokovnice za ta pretok so podane na sliki 5.

Na podlagi podatkov o značilnostih lebdečih plavin, višini usedlega materiala in hidrodinamičnih rezultatov, je bilo mogoče nadaljevati z modeliranjem prenosa lebdečih plavin. Določiti je

3.3 Results of the mathematical model

Before the transport of the suspended sediments in Ptuj lake was calculated the hydrodynamic model needed to be calibrated. It took into account the determination of a suitable value and the distribution of the roughness coefficient of the bottom of the lake. Several calculations showed that the best agreement with the measured velocities was reached with uniform distribution over the whole region of the lake, and with a value of Manning's coefficient $n = 0.026$ sm$^{-1/3}$. Based on experience and the analysis of filling up the reservoir, we found that the largest amount of material brought discharges of a one year return period or higher. As for the first phase of the analysis, it was prepared with the relatively simple approach of a steady-state calculation. The representative discharge was from five-year return period, $Q_5 = 1239$ m^3/s. The calculated velocity field with streamlines for that discharge is presented in Figure 5.

Based on the data of the suspended sediment's characteristics – the height of the sediments and the hydrodynamics results – it was possible to continue with the modeling of the suspended-sediment transport. The Schmidt number σ_{sm} and the nonbilo treba vrednosti Schmidtovega števila σ_{sm} in bredimenzijskega koeficienta α iz enačbe (9).

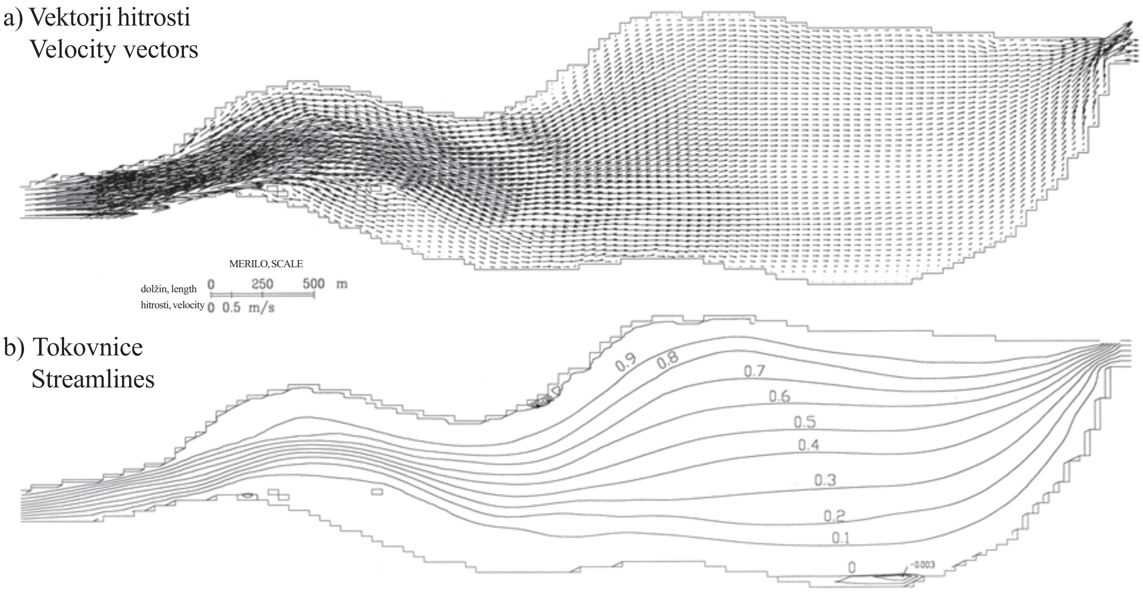
Po podatkih iz literature se vrednost Schmidtovega števila giblje med 0,5 in 1,0. Ker so analize pokazale precejšnjo neobčutljivost rezultatov matematičnega modela za račun prenosa lebdečih plavin na vrednost Schmidtovega števila v priporočenih mejah, tega problema v nadaljevanju nismo več obravnavali. V modelu smo upoštevali zgornjo priporočeno vrednost, σ_{sm} =1.

Določiti smo morali še vrednost koeficienta α. S testnimi računi smo ugotovili, da se doseže najboljše ujemanje z merjenimi vrednostmi z upoštevanjem α= 0,1. Rezultati računov za obdobje od leta 1981 do leta 1991 so podani na sliki 6. Zaradi nepopolnih podatkov o značilnostih materiala v dnu akumualcije (poroznosti in podobno), prikaz rezultatov je podan samo primerjalno. Z znanimi manjkajočimi podatki, bi bilo

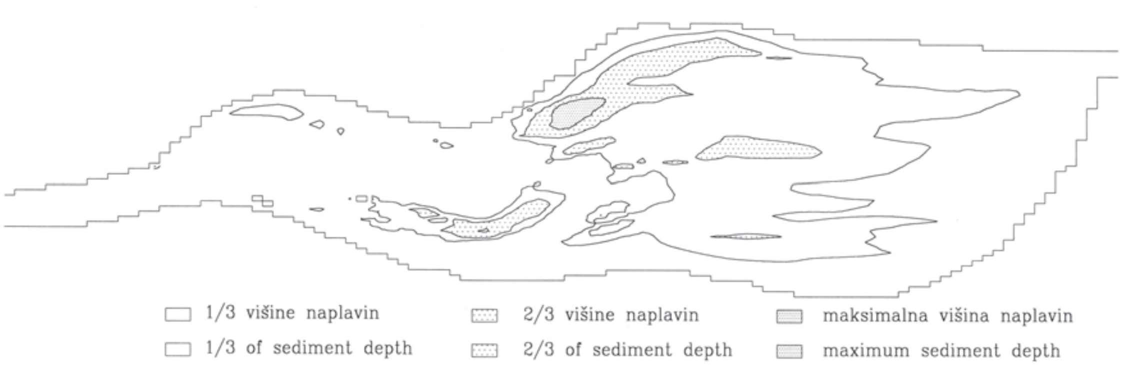
dimensional coefficient α from the equation (9) needed to be determined.

Using data from the literature, the value of the Schmidt number was between 0.5 and 1.0. Since the analysis showed a rather low sensitivity of the mathematical model's results for the calculation of the suspended sediment transport of the chosen value of the Schmidt number in the recommended boundaries this problem was not treated further. The upper recommended value of σ_{sm} = 1 was used in the model.

The value of coefficient α should also be determined. Using test calculations we found that the best agreement with the measured results was obtained for α = 0.1. The results for the period 1981 to 1991 are presented in Figure 6. As the data for the characteristics of the sediment were not full (porosity etc.), the result is presented only for a comparison. By including the missing data it would be possible to



Sl. 5. *Hidrodinamični rezultati računov v Ptujskem jezeru,Q=1239 m^3/s, n=0,026 $sm^{-1/3}$, a) hitrostno polje, b) tokovnice*
Fig. 5. *Results of the hydrodynamic calculations in Ptuj lake Q=1239 m^3/s, n=0.026 $sm^{-1/3}$, a) velocity field, b) streamlines*



Sl. 6. *Rezultati izračuna višine usedlih plavin v Ptujskem jezeru v obdobju od 1981 do 1991*
Fig. 6. *Calculated sediment height in Ptuj lake for the period from 1981 to 1991*

mogoče določiti dejansko višino usedlih plavin na posameznih delih dna zbiralnika.

4 SKLEPI

Matematični model je omogočil naslednje:
- s hidrodinamičnim modelom je bila določena dvodimenzijska razporeditev hitrosti toka v jezeru pri različnih pretokih. Poznavanje hidrodinamične slike je pomembno zaradi več razlogov, predvsem pa za boljše razumevanje biokemičnih procesov in pravilno napovedovanje dogajanj, povezanih s prenosom plavin;
- model prenosa lebdečih plavin je omogočil kakovostno simulacijo usedanja lebdečih plavin v Ptujskem jezeru na podlagi razpoložljivih merskih podatkov. Ugotovimo lahko relativno dobro ujemanje rezultatov matematičnega modela in izmerjenimi vrednostmi (sl. 4, 6). Predvidevamo, da bi z ustrezno krivočrtno numerično mrežo dobili boljše ujemanje izračunane in izmerjene hidrodinamične slike. To bi prispevalo k še boljšim rezultatom računa prenosa lebdečih plavin ter s tem tudi k bolj natančni sliki višine usedlega materiala. Upoštevati je treba tudi to, da uporabljeni model zdaj obravnava problematiko prenosa plavin kot stalni pojav pri nekem referenčnem pretoku. To pomeni, da vpliva usedlega materiala in posledično s tem tudi spremembe konfiguracije dna na tokove ni bilo mogoče upoštevati. V naslednjem modelu unestalnega toka bo treba z modelom zajeti časovno spremembo konfiguracije dna in njen vpliv na hidrodinamično sliko ter s tem tudi na usedanje materiala.

determine the real sediment height and the distribution over the reservoir.

4 CONCLUSIONS

The mathematical model made possible the following:
- by using the hydrodynamic model, a two-dimensional distribution of the velocities in the lake was determined in several discharges. Knowledge of the hydrodynamic characteristics of the lake is important for regularly predicting the sediment transport;
- a qualitative simulation of the settling suspended sediments in Ptuj lake, based on the available measured data. The agreement between the results of the mathematical model and the measured values is relatively good (Figures 4 and 6); however, a calculation with a curvilinear numeric mesh should offer better agreement between the calculated and measured results. This will contribute to better calculation results of the suspended sediments and a higher accuracy for the sediment height. It should be considered that the mathematical model used treats the sediment transport as a semi-steady-state process with a certain reference discharge. This means that the influence of settled material and consequently the changes of the bottom configuration on the water flow were not taken into account. The next mathematical model of the unsteady-state flow should take into account changes of the bottom configuration and its influence on the hydrodynamic picture as well as on the sedimentation of the load.

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