Strojniški vestnik - Journal of Mechanical Engineering 54(2008)1, 11-24
UDC 621.83.052
Paper received: 10.7.2006
Paper accepted: 19.12.2007

Vibration Analysis to Determine the Condition of Gear Units

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The use of the most up-to-date production technologies and a high level of production stability without any unscheduled outages are of utmost importance; they are affected primarily by monitoring the condition and by adequate maintenance of mechanical systems.

Life cycle design of machines and devices is nowadays gaining ground rather quickly; users want that machines and devices operate with a high level of accuracy and reliability and with as few outages as possible. Thus, by monitoring the condition, not only the presence of changes but also predictions related to the type and size of damage or error jeopardising the high quality of operation during the remaining life cycle of a machine is established.

Keywords: gear units, fault detection, vibration analysis, probability analysis, operation reliability

0 INTRODUCTION

Life cycle design represents a modern approach in relation to designing machine parts and structures that undergo dynamic loads. The traditional design is based on comparing the working stress with the permissible stress, whereas life cycle design is based on defining the number of loading cycles N that a mechanical part will endure at a specified loading σ.

Damages caused to machine parts and structures that are under dynamic load are referred to as fatigue failures. If compared to machine parts under static load, damages are caused to machine parts under substantially lower dynamic loads, however, only after a certain number of loading cycles N. Generally, the stages of fatigue process caused to materials of machine parts [1] are as follows:

- microcrack initiation,
- short crack propagation,
- long crack propagation,
- damage formation.

In relation to engineering analysis, the first two stages are usually dealt with as crack initiation, whereas the second two stages are dealt with as crack propagation. The life cycle of a machine part (the number of loading cycle N before the final damage is formed):

$$N = N_i + N_p \tag{1}$$

N_i is the number of loading cycles prior to crack initiation, and N_p stands for the number of loading cycles between the initial and the critical crack length, i.e. till final damage is formed.

During fatigue process, it is usually difficult to define precisely the borderline between fatigue crack initiation and propagation. The crack initiation stage represents the major part of a life cycle (usually above 90%) in relation to small loads (as a rule, substantially below proof stress) (Fig. 1). By increasing the load, the crack initiation stage decreases, whereas crack propagation stage increases.

Crack initiation is one of the most significant stages in the fatigue process of machine parts and structures. Position, size and the way of fatigue crack initiation depend primarily on material microstructure, loading type and geometry of the machine part. As a rule, cracks are initiated on the machine parts surface, where intrusions and extrusions appear due to pushing out slip planes of dislocations, which can be dealt with as microcracks (Fig. 1). In case of dynamic load, they propagate through individual crystal grains into the interior of the machine part, along the slip planes of the shearing stress (stage I). After the initial microcrack propagates through a certain number of crystal grains, crack propagation (stage II) appears. In this stage, the crack propagates along slip planes perpendicularly to the working load. At a certain critical crack length, when the remaining transverse section cannot endure the working load any longer, fracture occurs.

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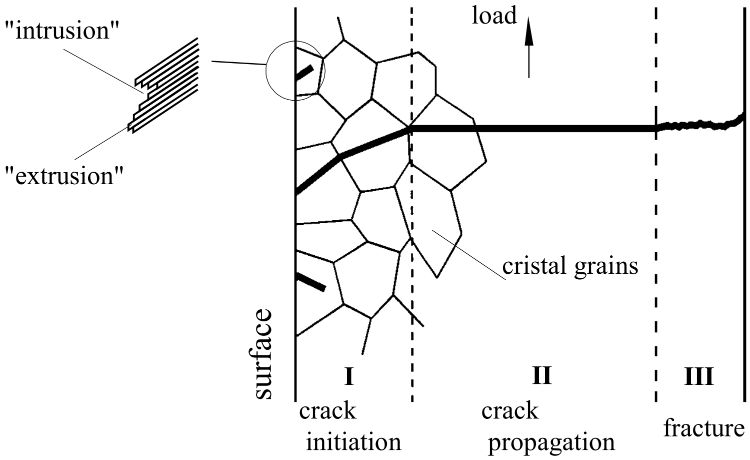

Fig. 1. *Schematic presentation of damage formation in relation to a machine part [3]*

The appearance of fracture surface (e.g. fracture of the axis in Fig. 2) is typical of a fatigue damage. Initial crack propagates from the peak of transverse section into the interior, which is clear also from a relatively smooth fracture surface (*permanent fracture*). After a certain number of loading cycles *N* when the crack length becomes critical, the remaining transverse section breaks instantaneously (*instant fracture*).

1 ESTABLISHING THE CONDITION BY MEANS OF TIME-FREQUENCY ANALYSIS OF VIBRATIONS

The objectives of maintenance are to detect, control and foresee the condition as well as to avoid and repair any damages with the purpose to maintain the characteristics of a technical system at the most favourable or still acceptable level of operation.

Maintenance is associated with assessing the condition of a technical system on the basis of collecting, analysing, comparing and processing data, acquired by means of different methods. In relation to this it is possible to reduce maintenance costs, improve operation reliability and reduce the frequency and complexity of damages. Without the capability of collecting precise data and without adequate data processing it is impossible to control mechanical systems.

Gear units are the most frequent machine parts or couplings. They are of different types and sizes, and consist of a housing, toothed wheels, bearings and a lubricating system. The majority of durable damages in gear units result from geometrical deviations or unbalanced component parts, material fatigue, caused by the engagement of a gear pair, or by damages of roller bearings.

Modern monitoring of the condition of a mechanical system is primarily associated with the

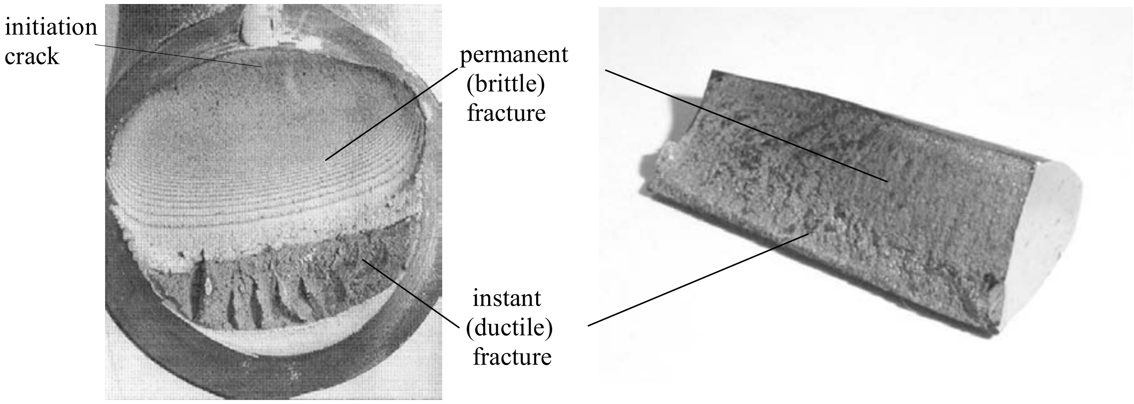

Fig. 2. *Appearance of fatigue fracture surface of an axis [4] and a gear tooth*

methods of measuring vibrations which is the most suitable way to acquire data on a gear unit. Values obtained by measurements are then additionally analysed, using adequate tools; at the same time features indicating the presence of damages and faults are defined.

A gear unit is a set of elements enabling the transmission of rotating movement; it represents a complex dynamic model. In spite of its complexity, its movement is usually periodic, and faults and damages represent a disturbing quantity or impulse. The disturbance is indicated by local and time changes in vibration signals, therefore, time-frequency changes can be expected. This idea is based on kinematics and operating characteristics.

The factor that affects reliability of operation and achievement of adequate quality of operation of gear units most negatively is the presence of cracks in gear units, this is followed by wear and tear of teeth flanks and eccentricity caused by backlash in bearings, and errors when assembling and manufacturing gear units. Monitoring the condition on the basis of measured vibrations is the most frequent method for determining the condition of a gear unit. It is usually attempted to determine deviations from reference values on the basis of a frequency spectrum. A gear unit is a complex mechanical system with changeable dynamic reactions, therefore it is impossible to establish modifications of a frequency component in time, consequently, the approach based on time-frequency methods is more appropriate.

Frequency analysis is a tool very commonly used in diagnostics, however, it may well be stated that good results are obtained only in relation to periodical processes without any local changes. As a consequence of the presence of a damage or fault, dynamic parameters of a mechanical system change, which is reflected in the frequency spectrum. Monitoring frequency reaction is one of the most common spectral methods used with the purpose to establish the condition of a gear unit. In relation to classical frequency analysis, time description of vibration is transformed into frequency description, changes within a signal are averaged within the entire time period observed. Therefore, local changes are actually lost in the average of the entire function of vibrations. As a consequence, identification of local changes is very difficult or impossible.

Time-frequency analysis represents a more thorough approach as it eliminates deficiencies mentioned above – local changes that deviate from the global periodical oscillation are namely expressed with the appearance or disappearance of individual frequency components in a spectrogram. Thus, a signal is presented simultaneously in time and frequency.

In signals related to technical diagnostics, individual frequency components often appear only occasionally. Classical frequency analysis of such signals does not indicate when certain frequencies appear in the spectrum. The purpose of time-frequency analysis is to describe in what way frequency components of transient signals change with time and to determine their intensity levels.

Fourier, adaptive and wavelet transforms, and Gabor expansion are representatives of various time-frequency algorithms. The basic idea of all linear transforms, including Fourier transform, is to carry out comparison with elementary function determined in advance. By means of various elementary functions, different signal presentations are obtained.

On the basis of previous research results [5], the applicable value of a windowed Fourier transform for evaluating the condition of gear units has been established, along with its shortcomings, which have required the use of a more advanced signal analysis method, such as adaptive transform.

2 ADAPTIVE METHOD RELATED TO VIBRATION ANALYSIS

Adaptive transform of a signal was, to a large extent, enhanced and concluded by Qian [6] although many authors had been developing algorithms without interference parts that reduce usability of individual transforms as opposed to Cohen’s class. The entire presentation of the following adaptive method is based on [6].

Adaptive transform of a signal *x(t)* is expressed as follows:

$$x(t)=\sum_{p} B_p \cdot h_p(t) \qquad (2),$$

where analysis coefficients are determined by means of the following equations:

$$B_p = \langle x, h_p \rangle \qquad (3),$$

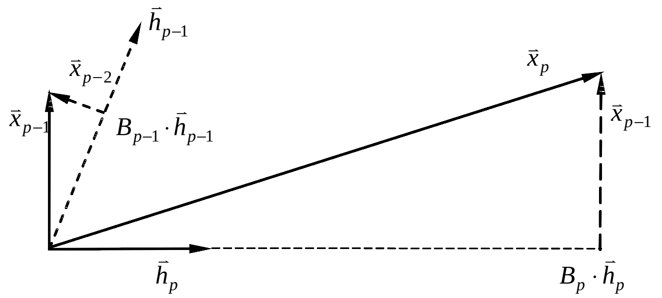

Fig. 3. *Process of adaptive signal decomposition*

expressing similarity between the measured signal $x(t)$ and elementary functions $h_p(t)$ of transform. In Figure 3, the process of adaptive signal decomposition is presented.

The original signal represents the starting point with parameter values $p=0$ and $x_0(t)=x(t)$. In the set of desired elementary functions, $h_0(t)$ is searched for that is most similar to $x_0(t)$ in the following sense:

$$\left|B_p\right|^2 = \max_{h_p} \left|\left\langle x_p(t), h_p(t)\right\rangle\right|^2 \quad (4),$$

for $p = 0$. The next step is associated with the calculation of the remaining $x_1(t)$

$$x_{p+1}(t) = x_p(t) - B_p \cdot h_p(t) \quad (5).$$

Without giving up the generalisation idea, $h_p(t)$ is to have a unit of energy representation of a signal. Therefore:

$$\left\|h_p(t)\right\|^2 = 1 \quad (6).$$

The energy contained in the remaining signal:

$$\left\|x_{p+1}(t)\right\|^2 = \left\|x_p(t)\right\|^2 - \left|B_p\right|^2 \quad (7).$$

The Equation (5) is repeated to find $h_1(t)$ that would suit best $x_1(t)$, etc. In each step one elementary function $h_p(t)$ that suits best $x_p(t)$ is found. The main objective of adaptive signal representation is to find a set of elementary functions $\{h_p(t)\}$ that are most similar to time-frequency structure of a signal, and at the same time satisfy Equations (2) and (3).

If Equation (6) is expressed as:

$$\left\|x_p(t)\right\|^2 = \left\|x_{p+1}(t)\right\|^2 + \left|B_p\right|^2 \quad (8)$$

it indicates that the remaining energy of a signal at pth level can be determined on the basis of $p + 1$ level and the rest B_p. If this process is continued, the result is as follows:

$$\left\|x(t)\right\|^2 = \sum_{p=0}^{\infty}\left|B_p\right|^2 \quad (9).$$

This is an equation related to energy conservation and it is similar to Parseval's relation in the Fourier transform.

The result of using Wigner-Ville distribution for both sides of the Equation (2), and organising equations into two groups is as follows:

$$P_{WV}x(t,\omega) = \sum_p B_p^2 \cdot P_{WV}h_p(t,\omega) + \sum_{p\neq q} B_p \cdot B_q * P_{WV}\left(h_p, h_q\right)(t,\omega) \quad (10).$$

The first group represents elementary signal components, whereas the second one represents cross interference terms.

Due to the relation described in (9) and the given value of energy conservation, it is evident that:

$$\frac{1}{2\cdot\pi} \cdot \iint \sum_{p\neq q} B_p \cdot B_q * P_{WV}\left(h_p, h_q\right)(t,\omega) = 0 \quad (11).$$

This is the reason that a new time-dependent adaptive spectrum can be defined as:

$$P_{ADT}(t,\omega) = \sum_p \left|B_p\right|^2 \cdot P_{WV}h_p(t,\omega) \quad (12).$$

As this is an adaptive spectrum based on representations it is referred to as adaptive spectrogram. It includes no interferences and no cross terms – in this respect, it differs from Wigner-Ville distribution, and it also satisfies the condition related to energy conservation.

$$\left\|x(t)\right\|^2 = \frac{1}{2\cdot\pi} \cdot \iint P_{ADT}(t,\omega) \cdot dt \cdot d\omega \quad (13).$$

The basic issue related to linear presentations is the selection of elementary functions. In case of Gabor expansion, a set of elementary functions comprises time-shifted and frequency modulated prototype window function *w(t)*. In relation to wavelets, elementary functions are obtained on the basis of scaling and shifting of a mother wavelet $\psi(\tau)$. In these two examples, structures of elementary functions are determined in advance. Elementary functions related to adaptive representation are rather demanding.

Generally speaking, adaptive transform is independent from the choice of elementary functions $h_p(t)$ as it permits arbitrary elementary functions.

As a rule, elementary functions, used for adaptive representation of a signal with equation (2) are very general. In practice, however, this is not always the case. In order to stress time dependence of a signal, it is desirable that elementary functions are localised in regard to time and frequency. They also have to be able to use the presented algorithm in a relatively simple way. A Gauss type signal has very favourable features and it is considered a basic choice in relation to adaptive representation. It is expressed as follows:

$$h_p(t) = \left(\frac{\alpha_p}{\pi}\right)^{0,25} \cdot e^{-\frac{\alpha_p}{\pi}\cdot(t-T_p)^2} \cdot e^{-j\cdot\Omega_p\cdot t} \quad (14),$$

where (T_p, Ω_p) is a time-frequency centre of an elementary function, whereas α_p^{-1} stands for a variance of Gauss function at (T_p, Ω_p).

The variance, acquired by means of the ordinary Gabor transform, is stable, in (13) it is adaptive. Gauss functions, normally used in the ordinary Gabor transform, are located at fixed points $(mT, n\Omega)$ of time and frequency grid, whereas the centres of elementary functions in (13) are not fixed and can be located anywhere. The adaptation of the variance value can make elementary functions longer or shorter; the adaptation of parameters (T_p, Ω_p) changes time and frequency centres of elementary functions. Adapting both variance and time-frequency centres results in increased suitability of local time-frequency features of the signal *x(t)*.

Wigner-Ville distribution of time-frequency density of adaptive Gauss functions is expressed as:

$$P_{WV} h_p(t,\omega) = 2 \cdot e^{-\left(\alpha_p\cdot(t-T_p)^2 + \frac{(\omega-\Omega_p)^2}{\alpha_p}\right)} \quad (15).$$

The function of time-frequency density of the adaptive Gauss function forms an ellipse with the centre at (T_p, Ω_p). The energy concentration of Gauss functions is an optimum one. By means of Gauss functions of various variances and of a different time-frequency centre, the local behaviour of each analysed signal *x(t)* is characterised by the Gauss function. If the signal features rapid time change, the Gauss function with a small variance value suitable for the sudden change has to be applied. If stable frequency over a longer period of time is typical of the signal, the Gauss function with greater variance value is to be used.

By entering (14) into (2), the following equation is acquired:

$$x(t) = \sum_p B_p \cdot h_p(t) = \sum_p B_p \cdot \left(\frac{\alpha_p}{\pi}\right)^{0,25} \cdot e^{-\frac{\alpha_p}{\pi}\cdot(t-T_p)^2} \cdot e^{-j\cdot\Omega_p\cdot t} \quad (16).$$

This is adaptive Gabor transform, which resembles an ordinary Gabor transform.

The adaptive transform has the same functions of analysis and synthesis, which makes it different from the Gabor transform. When the optimum function of synthesis $h_p(t)$ is achieved, it is possible to calculate adaptable coefficients B_p, on the basis of ordinary operation of internal product:

$$B_p = \int x_p(t) \cdot h_p^*(t) \cdot dt = \left(\frac{\alpha_p}{\pi}\right)^{0,25} \cdot \int x_p(t) \cdot e^{-\frac{\alpha_p}{\pi}\cdot(t-T_p)^2} \cdot e^{-j\cdot\Omega_p\cdot t} \cdot dt \quad (17),$$

which guarantees that local signal behaviour is really expressed by B_p.

By entering (15) into (12), a Gauss adaptive spectrogram, which is based on the function, the following equation is produced:

$$P_{ADT}(t,\omega) = 2 \cdot \sum_p \left|B_p\right|^2 \cdot e^{-\left(\alpha_p\cdot(t-T_p)^2 + \frac{(\omega-\Omega_p)^2}{\alpha_p}\right)} \quad (18).$$

As time-frequency resolutions of Gauss function are determined by one parameter α_p, it is relatively easy to calculate optimum $h_p(t)$. Thus, it is relatively easy to calculate adaptive Gabor expansion when elementary functions $\{h_p(t)\}$ are determined. There is, however, a question regarding the choice of elementary functions $\{h_p(t)\}$. As already mentioned before, $h_p(t)$ should suit $x_p(t)$ best in the sense of Equation (4), where Gabor function determined in Equations (14) and (4), represents the optimum solution in regard to $(\alpha_p, T_p, \Omega_p)$. Generally speaking, there is no analytical solution in relation to Equation (4) as, similarly to the approximation process to the final result, it is possible to obtain a numerical approximation with a corresponding deviation.

The process of calculating adaptive spectrogram begins in a wide time range of a measured signal. Then the range has to be reduced, depending on what is wanted to be achieved. As Fourier integral is included in the elementary operations of searching for a suitable elementary function, the described calculation process is very effective [7]. The accuracy of approximation depends primarily on the size of time-frequency interval. The narrower the intervals are, the better is the accuracy of representation, along with increased time of calculation. Therefore a compromise between the accuracy of approximation and the efficiency has to be found.

3 SIGNAL RESAMPLING PROCEDURE FOR LIMITED SPEED FLUCTUATION

Vibration measuring can represent measuring of a dynamic quasi periodical signal as the gear consists of rotating elements (shafts with gears and bearings). This represents complex repeated rotating movements. The information obtained by measuring the rotation speed, making it possible to follow the stability of rotation speed during the measuring, is important for the frequency analysis [8]. During measuring, rotation speed can oscilate, even more so when gears are used in machines, which are under load. This results in an unreliable frequency spectrum, which is even more expressed in case of frequency analysis of higher harmonics; additional sidebands namely appear. In relation to time-frequency analysis it is also more adequate to assess the results on the basis of stable rotation frequency [9]. By following the rotation speed signal (TTL signal), the beginning and the end of rotations are located and their length and time of duration are measured.

There are two techniques for producing synchronously sampled data; the traditional approach which uses specialised hardware to dynamically adapt the sample rate and a technique where the vibration signals and a tachometer signal are asynchronously sampled, that is, they are sampled conventionally at equal time increments (Fig. 4). From the asynchronously sampled tachometer signal re-sample times required to produce synchronous sampled data are calculated. In this paper, analysis of rotation frequency stability and, if required, suitable resampling are carried out prior to each analysis.

Recently, there has been interest in calculating synchronous sampled signals from asynchronously sampled data. Both the vibration signals and tachometer are sampled at constant time intervals, that is, asynchronously. From these signals, synchronously sampled vibration data is synthesised in a computer code.

A computed re-sampling system is generally based on two stage algorithm: the tachometer pulse trigger, with angle evaluation procedure and the signal interpolation. Tachometer determines when the edge of the pulse arrives. From these timings the angular motion of the shaft is resulting in resample times. The asynchronously sampled vibration signal is interpolated and re-sampled at these times thus producing synchronously sampled data.

To calculate re-sample times at constant angular increments, the angular motion of the shaft has to be estimated from the tachometer pulses. In the first instance constant angular speed may be assumed, which results in linear interpolation between the shaft angle at consecutive tachometer pulses.

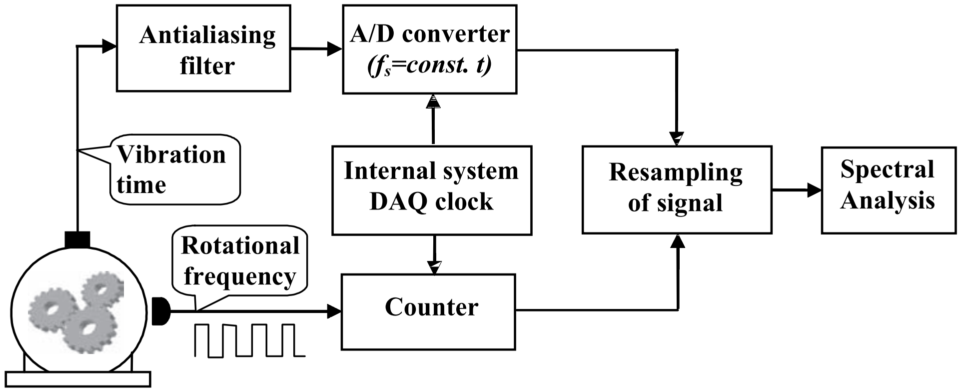

Fig. 4. *Data acquisition with fixed sampling frequency*

To produce the desired synchronously sampled signal, the value of the vibration signal must be estimated at the re-sample times.

Linear evaluation is the simplest signal interpolation method. This approach is adequate for signals with slow oscillation rotation speed.

B-spline evaluation is the most useful signal interpolation method which produces a pth-order piecewise polynomial fit to the data. The method is especially interesting because up to the (p - l)th derivative of the resulting signal is continuous. Using the weighted sum of B-spline basis functions, the signal is approximated. In the analysis cubic B-spline evaluation is used, giving good interpolation flexibility and appropriate basis functions.

A piecewise polynomial fit to the data samples is obtained through Lagrange interpolation (a spline interpolation method) [10]. A pth-order polynomial is usually used to fit (p + 1) data points; this is, however, limited with the condition that the polynomial passes through every sample. Piecewise polynomial signals are created by interpolation; the derivatives across the boundaries (at the sample points) are not constrained to be continuous, which is different from B-spline interpolation. Using the latter one (i.e. B-spline interpolation), a more original signal is actually created. The presented method is successfully applied in relation to the change in rotation frequency by 2%.

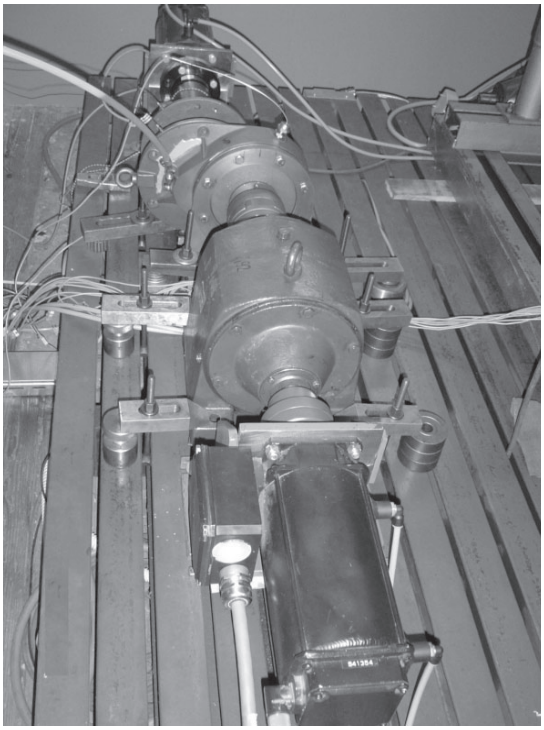

Fig. 5. *Test plant and a part of measuring equipment*

The trend of rotation speed oscillation and its stability being observed contribute to a more precise and clearer spectrogram, even to a larger extent with high harmonics and their side frequencies.

4 PRACTICAL EXAMPLE

All the measures have been carried out in the test plant of the Computer Aided Design Laboratory at the Faculty of Mechanical Engineering, University of Maribor. The test plant is presented in Figure 5, with a one-stage helical gear unit at the spot where vibration measurements have been carried out.

Some changes have been caused to a standard gear unit with the purpose to (artificially) produce faults and damages in a gear couple and to adapt some design-related features to the test plant. Simultaneously, appropriate measurements of vibration accelerations have been carried out – they serve as a tool for determining the presence of individual changes in a gear unit. As the method of determining the condition of a gear unit is based on comparing the measured signal of a faultless gear unit with the signal of a faulty gear unit, a faultless and faulty gear units were tested.

A single stage gear unit EZ5.B3.132 produced by “Strojna tovarna” in Maribor was tested. Into the gear unit, a helical gear unit with straight teeth was integrated. A more detailed description and technical documentation of the gear unit are in [5].

Measures of a gear unit with a fatigue crack in the tooth root of a pinion were carried out, under operating conditions that would be normally associated with this type of a gear unit. A ground gear pair with a crack is a standard gear pair, with the teeth quality 6, but with the presence of a crack in the tooth root of a pinion; it is presented in Figure 6.

The length of the crack in Figure 6, the presence of which can be observed on one of the teeth, is 4.8 mm. The whole measurement process and preparations for analysis are described in [5].

Own computer program for signal analysis for a family of time-frequency transformations has been developed in the LabView 7.1 program

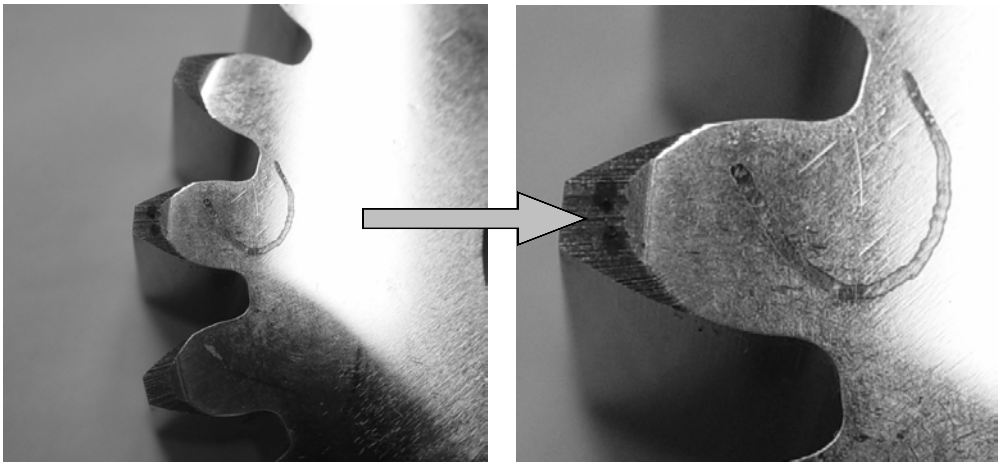

Fig. 6. *Pinion with a crack in the tooth root*

package, and tested and evaluated on the basis of a certain number of basic mathematically generated signals [11].

Adaptive transform was used to determine the presence of a crack in the tooth root. In relation to adaptive spectrogram, adaptive representation for signal decomposition, prior to Wigner-Ville distribution, was used. Additionally, only the sum of Wigner-Ville distribution of basic signal components was taken into account, whereas cross-interference terms were completely ignored.

Adaptive spectrogram has a fine adaptive time-frequency resolution because the features of elementary functions are limited. Time-frequency resolution of the transform is, therefore, adapted to signal characteristics. As an elementary function, Gauss function (impulse) and linear chirp with Gauss window can be used. If a signal is composed of linear chirps that are the consequence of a linear change in the rotation frequency of a gear unit, an adaptive spectrogram can be applied to establish in what ways a possible frequency modulation is reflected in the time-frequency domain. A possible presence of non-linear frequency modulation may represent a problem as a spectrogram may include a certain level of distortion because adaptive representation is approaching non-linear modulation in the form of a linear combination of chirps with linear frequency

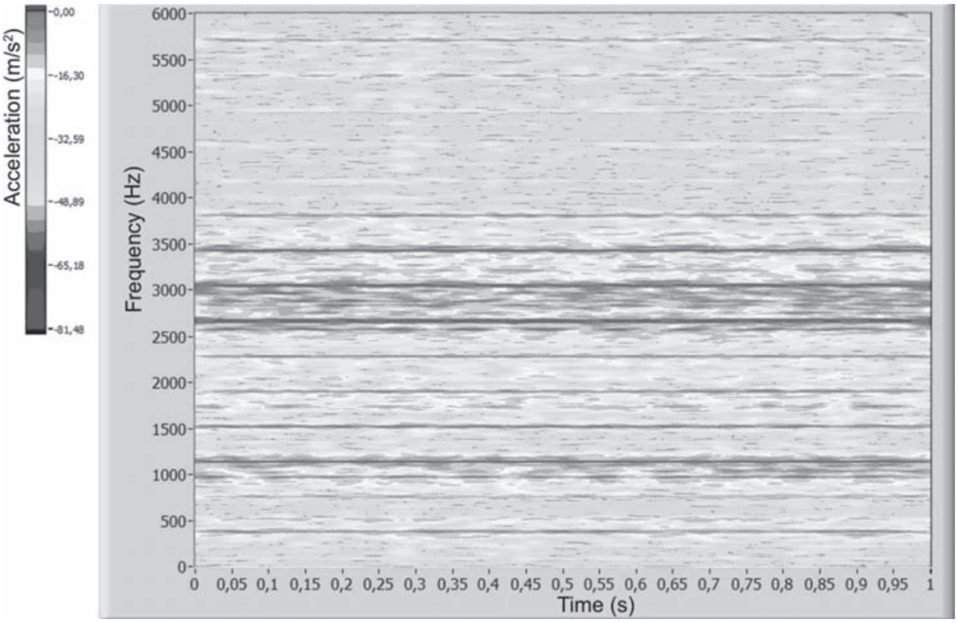

Fig. 7. *Gabor's spectrogram of a faultless gear unit*

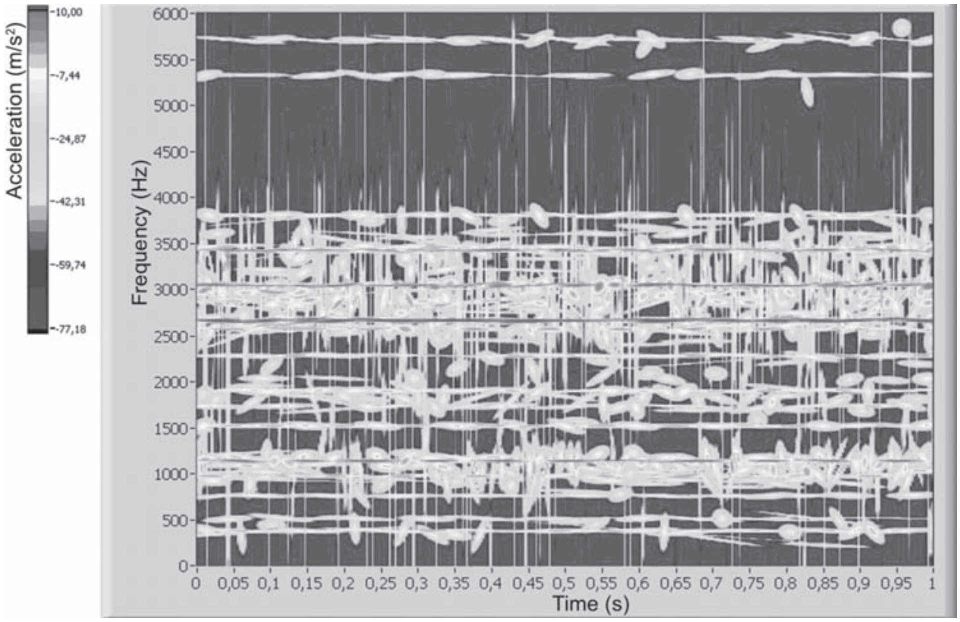

Fig. 8. *Adaptive spectrogram of a faultless gear unit*

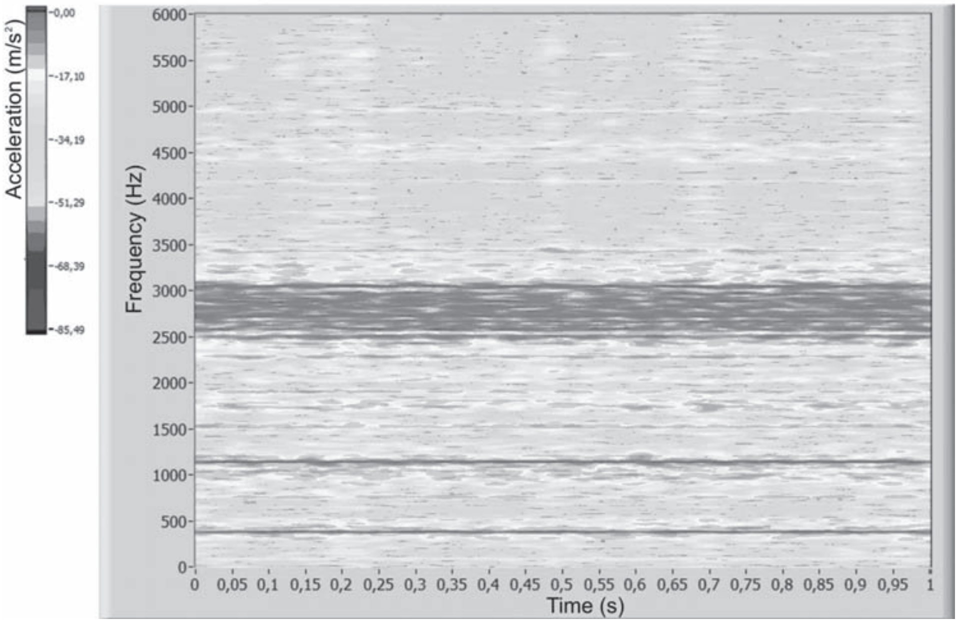

Fig. 9. *Gabor's spectrogram of a gear unit with a cracked pinion*

modulation. The time required for transform calculation increases, along with the increased amount of data and the number of cycles necessary to search for an adequate elementary function.

The signal of measured values was 1 s long and composed of, on an average, 12500 measuring points. Rotation frequency was 20 Hz at the time of measurement. The number of teeth of the pinion was 19, and of the gear unit 34. For comparison, spectrograms related to Gabor and window Fourier transforms are presented, with the length of the window being 700 points, which is 10% more that the length of the period of one rotation of a gear couple.

Calculation time required in relation to adaptive spectrogram is at least 15 times longer than the time required for the calculation associated with the Fourier transform. The resolution of the adaptive transform, however, is on an average, 1.7 times better.

In Figure 7, Gabor spectrogram is presented; no rhythmic pulsation of harmonics can be observed, with the exception of typical frequencies, determined on the basis of power spectrum. In relation to adaptive spectrogram (Fig. 8), featuring a higher level of energy accumulation in the origins, some pulsation sources are indicated but they are not very expressed. It is particularly interesting to monitor the increase or decrease (complete disappearance) in appropriate frequency components with rotation frequency of 20 Hz. This phenomenon is typical of the 3rd harmonic, 1530 Hz is expressed only in relation to the presence of a crack. The phenomenon is much more expressed in the adaptive spectrogram (Fig. 10) than in the Gabor spectrogram (Fig 9). In Figure 10, pulsation (the area, marked with a continuous line) is expressed reflecting a single engagement of a gear couple with a crack within

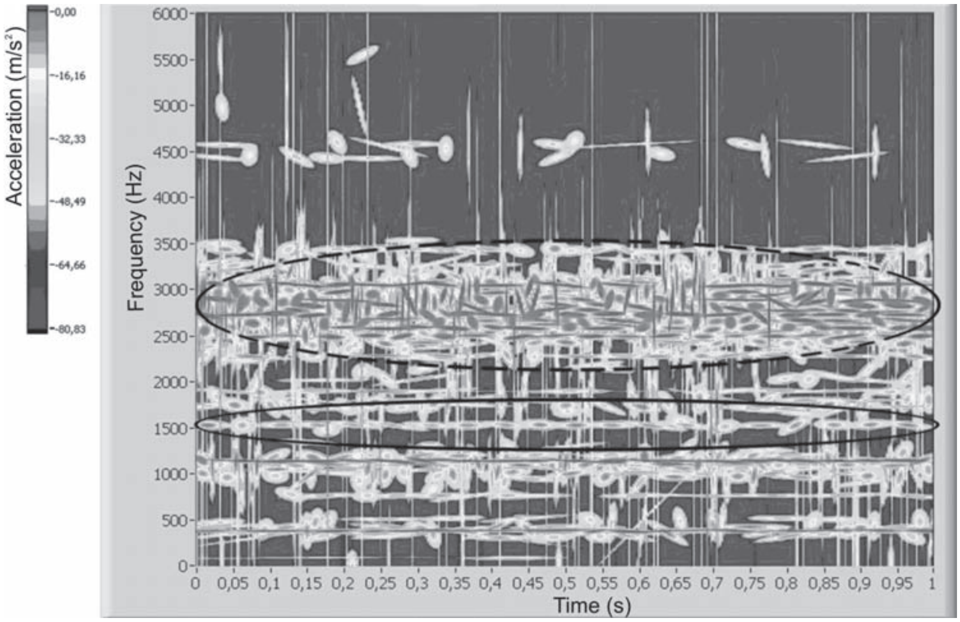

Fig. 10. *Adaptive spectrogram of a gear unit with a cracked pinion*

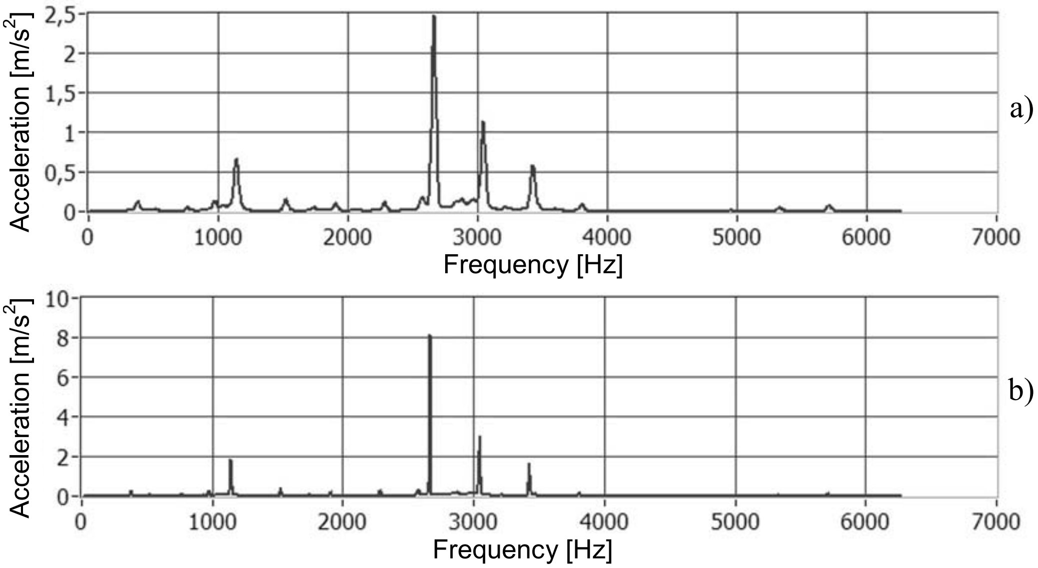

Fig. 11. *Average frequency specter of the spectrogram of vibrations produced by the faultless couple of gears with ground surface teeth: a) Gabor and b) adaptive*

one rotation of a shaft. Similarly, between the 6th and the 9th harmonics (area marked with a dashed line) rich with sources indicating pulsating portions of individual components with the frequency of 20 Hz.

The spectrogram evaluation can be based on an average spectrogram representing an amplitude spectrum of a Fourier or adaptive transform of a measured signal and by observing pulsating frequencies of individual frequency components.

In Figure 11, average frequency spectres of spectrograms of vibrations produced by the faultless gear unit are presented. It is evident that the greatest average values are achieved at the 6th, 7th, 2nd and 8th harmonics. The Figure indicates a larger width of a frequency component round individual harmonics, which is more expressed in relation to the Gabor transform, whereas in relation to the adaptive transform the concentration of frequency components is very distinguished.

Table 1. *Amplitudes and frequencies of the pulsation of harmonics in relation to the average Gabor spectrogam of vibrations of the faultless couple of gears with ground surface teeth*

h	f [Hz]	A_{pos} [m/s^2]	f_{ut1} [Hz]	f_{ut2} [Hz]	f_{ut3} [Hz]
0	379.175	0.116	1.998	4.994	10.987
1	758.350	0.044	8.989	10.987	1.998
2	1137.524	0.651	10.987	8.989	2.996
3	1516.699	0.151	10.987	0.999	7.990
4	1895.874	0.093	1.998	10.987	0.999
5	2275.049	0.103	0.999	5.993	6.992
6	2654.224	2.080	1.998	12.984	3.995
7	3033.398	0.917	1.998	0.999	12.984
8	3412.573	0.436	2.996	0.999	10.987
9	3791.748	0.068	10.987	0.999	9.988
10	4170.923	0.006	10.987	5.993	1.998
11	4550.098	0.005	2.996	8.989	6.992
12	4929.272	0.006	3.995	2.996	1.998
13	5308.447	0.027	10.987	9.988	12.984
14	5687.622	0.036	10.987	9.988	0.999
15	6066.797	0.002	6.992	3.995	4.994

Table 2. *Amplitudes and frequencies of the pulsation of harmonics in relation to the average adaptive spectrogram of vibrations of the faultless couple of gears with ground surface teeth*

h	f [Hz]	A_{pos} [m/s^2]	f_{ut1} [Hz]	f_{ut2} [Hz]	f_{ut3} [Hz]
0	379.175	0.271	11.011	14.015	1.001
1	758.350	0.088	11.011	9.009	20.021
2	1137.524	1.363	11.011	7.007	10.010
3	1516.699	0.216	11.011	8.008	20.021
4	1895.874	0.086	11.011	12.012	1.001
5	2275.049	0.082	9.009	11.011	10.010
6	2654.224	0.670	5.005	11.011	3.003
7	3033.398	0.551	11.011	2.002	9.009
8	3412.573	0.129	9.009	11.011	1.001
9	3791.748	0.053	20.021	11.011	9.009
10	4170.923	0.001	152.158	54.056	103.107
11	4550.098	0.001	131.136	128.133	132.137
12	4929.272	0.001	7.007	20.021	6.006
13	5308.447	0.007	11.011	20.021	2.002
14	5687.622	0.009	11.011	20.021	22.023
15	6066.797	0.001	25.026	154.160	129.134

In Tables 1 and 2, the amplitudes of individual harmonics along with corresponding frequencies are shown. In addition, dominating frequencies of the pulsation of individual harmonics are indicated, which makes the evaluation of changes in frequency components over a period of time easier.

In Figure 12, the vibrations of the 3rd harmonic over a period of time are shown; it has been established that it is very suitable for observing the pulsation in relation to all types of irregularities; it is evident that expressed pulsations (rhytmic changes) are not present. The frequency, which is also present, is 11 Hz and it represents rotation speed of the gear unit, whereas other parts of frequency components are much lower.

In Figure 13, average spectrograms of the vibrations of a gear unit are presented, on the basis of which it is clear that the greatest average values are associated with the 6th and 7th harmonics as well as with the 2nd harmonic and with the meshing frequency. Very expressed sideband frequencies are associated only with the 6th and 7th harmonics,

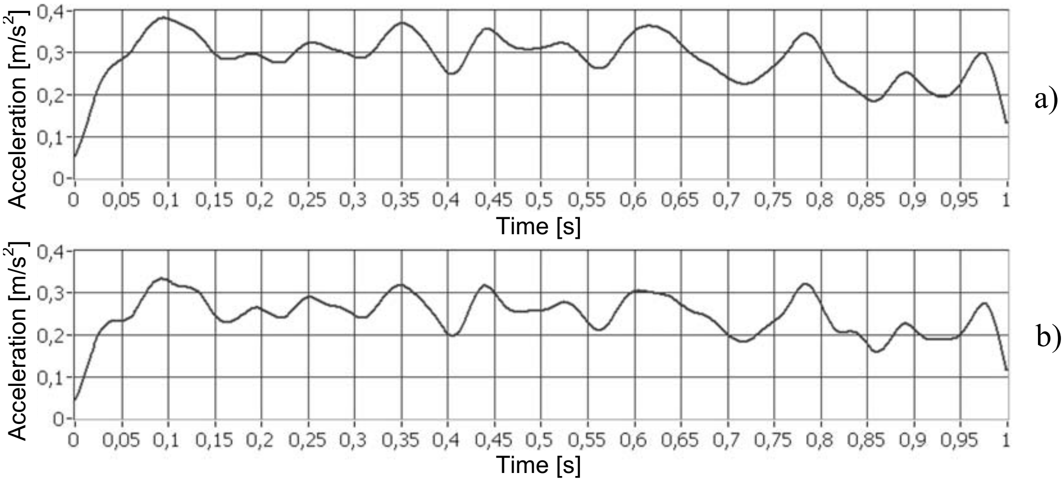

Fig. 12. *Time presentation of the 3rd harmonic (1516.7 Hz) of the spectrogram of vibrations of the faultless couple of gears with ground surface: a) Gabor and b) adaptive*

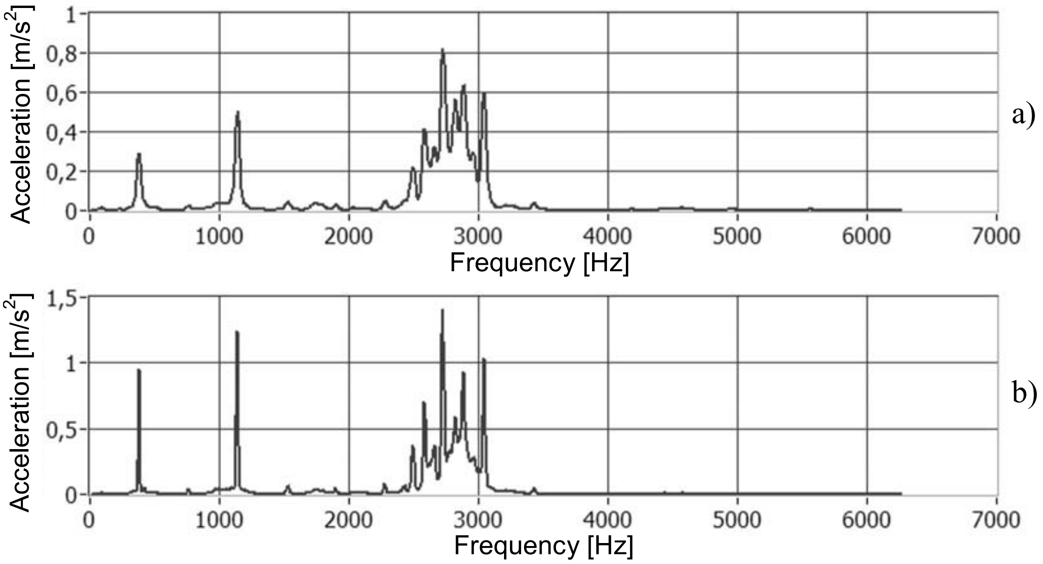

Fig. 13. *Average frequency spectrogram of vibrations of a couple of gears with a crack: a) Gabor and b) adaptive*

which is typical when a crack is present. In addition to that, the figure shows a larger width of frequency components round individual harmonics, which is particularly expressed in relation to the window Gabor transform, whereas in relation to the adaptive transform, the concentration is much larger round harmonics.

Tables 3 and 4 show amplitudes of individual harmonics with corresponding frequencies, along with dominating frequencies of the pulsation of individual harmonics. Thus, pulsating frequencies are changed and the pulsation of rotation frequencies of the shaft of the pinion (19.96 Hz) and of the gear (10.98 Hz) are dominant. Partial frequencies and higher harmonics of the rotation frequencies of both shafts appear as well. The pulsation of the 3rd harmonic (Fig. 14) is particularly interesting; it is very clear and periodic (20 Hz) and it appears only in relation to the couple of gears with a crack in the tooth root.

Table 3. *Amplitudes and frequencies of the pulsation of harmonics of an average Gabor spectrogram of the vibrations produced by a couple of gears with a crack*

h	f [Hz]	A_{pos} [m/s^2]	f_{ut1} [Hz]	f_{ut2} [Hz]	f_{ut3} [Hz]
0	378.751	0.290	10.975	5.986	19.954
1	757.502	0.023	10.975	19.954	11.972
2	1136.253	0.486	10.975	19.954	12.970
3	1515.004	0.035	19.954	1.995	18.956
4	1893.755	0.028	10.975	1.995	21.949
5	2272.506	0.041	10.975	19.954	33.921
6	2651.257	0.292	19.954	39.907	1.995
7	3030.008	0.463	10.975	19.954	2.993
8	3408.759	0.021	19.954	10.975	11.972
9	3787.510	0.002	3.991	19.954	12.970
10	4166.261	0.004	13.968	4.988	10.975
11	4545.012	0.007	10.975	19.954	7.981
12	4923.763	0.005	19.954	2.993	3.991
13	5302.514	0.002	10.975	22.947	2.993
14	5681.265	0.001	10.975	3.991	13.968
15	6060.016	0.001	3.991	1.995	10.975

Table 4. *Amplitudes and frequencies of the pulsation of harmonics of an average adaptive spectrogram of the vibrations produced by a couple of gears with a crack*

h	f [Hz]	A_{pos} [m/s^2]	f_{ut1} [Hz]	f_{ut2} [Hz]	f_{ut3} [Hz]
0	378.751	0.950	11.027	6.015	5.012
1	757.502	0.039	11.027	2.005	10.025
2	1136.253	1.238	11.027	2.005	13.032
3	1515.004	0.032	20.050	40.099	2.005
4	1893.755	0.028	11.027	5.012	2.005
5	2272.506	0.055	11.027	20.050	2.005
6	2651.257	0.289	20.050	8.020	5.012
7	3030.008	0.289	12.030	6.015	11.027
8	3408.759	0.010	20.050	11.027	2.005
9	3787.510	0.001	78.194	68.169	156.387
10	4166.261	0.001	78.194	156.387	68.169
11	4545.012	0.002	11.027	4.010	7.017
12	4923.763	0.001	9.022	10.025	23.057
13	5302.514	0.001	91.226	19.047	74.184
14	5681.265	0.001	91.226	78.194	169.420
15	6060.016	0.001	78.194	91.226	169.420

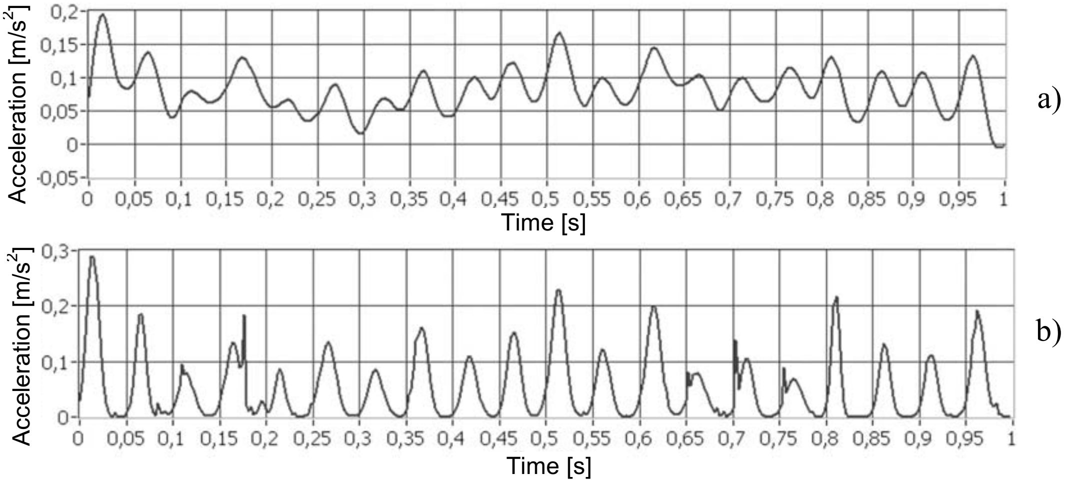

Fig. 14. *Time presentation of the 3rd harmonic (1515 Hz) of the spectrogram of vibrations of a couple of gears with a crack in the pinion: a) Gabor and b) adaptive spectrograms*

5 CONCLUSION

Fault detection, presented in this paper, in relation to industrial gear units is based on vibration analysis; it increases the safety of operation and, consequently, of monitoring operational capabilities.

Appropriate spectrogram samples and a clear presentation of the pulsation of individual frequency components that, in addition to the average spectrum, represent a criterion for assessing the condition of a gear unit make it possible to monitor life cycle of a gear unit more reliably. Adaptive time-frequency representation represents, above all, a reliable prediction as the representation is clearer, without increased dissemination of signal energy into the surroundings.

In relation to the life cycle design of machine parts and structures, it is therefore possible, by using an adequate method or criterion, to monitor the actual condition of a device and of its vital components that can affect its operational capability considerably. By detecting faults or damages, the reliability of operation is controlled to such an extent that the probability and reliability of detecting faults make a positive contribution related to the prediction of the remaining life cycle of a gear unit.

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