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Siskov model toka nenewtonskih tekočin z metodo končnih prostornin

The Sisko Model For Non-Newtonian Fluid Flow Using The Finite-Volume Method

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Analizirali smo primernost metode končnih prostornin za izračun tokovnih razmer v nestisljivi viskozni nenewtonski tekočini ob uporabi Siskovega modela. Prav tako smo analizirali natančnost numeričnih rezultatov v odvisnosti od gostote mreže. Metodo smo testirali na primeru gnanega toka v kotanji in primeru toka v kanalu z nenadno zožitvijo. Numerično dobljene rezultate smo primerjali z vrednostmi iz literature. Za reološki model smo uporabili modelne parametre, dobljene na podlagi preskusa s cevnim viskozimetrom.

(Ključne besede: tekočine nenewtonske, modeli toka, metode končnih prostornin, modeli reološki)

We have analyzed the suitability of the finite-volume method for calculating incompressible, viscous, non-Newtonian fluid flow where the Sisko model was used. In addition, convergence criteria are presented and the convergence depending on the mesh size was analyzed. The method was tested for the driven-cavity case and flow in a channel with a sudden contraction. The numerical solution was compared with the results available in the open literature. For the rheological model, parameters obtained from an experiment with a capillary viscometer were used.

(Keywords: non-newtonian fluids, flow models, control-volume methods, rheological models)

0 UVOD

Nenewtonske tekočine so navzoče v vsakdanji inženirski praksi. Zaradi spremenljive viskoznosti je treba gibalne enačbe obravnavati brez običajnih poenostavitev. Viskoznost je lahko v splošnem odvisna od številnih parametrov, v tem prispevku pa obravnavamo posplošene newtonske tekočine, pri katerih je viskoznost odvisna od deformacijske hitrosti. Na področju posplošenih newtonskih tekočin obstaja več reoloških modelov z dvema, tremi ali štirimi parametri, npr. [5] do [7] in [9]. V pričujočem delu smo se osredotočili na Siskov triparametrični model.

Tokovne razmere v nestisljivi viskozni posplošeni newtonski tekočini smo obravnavali s sistemom parcialnih diferencialnih enačb osnovnih zakonov ohranitve, ki smo ga reševali z diskretno metodo končnih prostornin, opisano v [4] in povzeto po [1]. V skladu z opisanim načinom reševanja smo kontinuitetno in gibalno enačbo povezali z uporabo načela umetne stisljivosti. Konvektivne tokove smo računali po metodi karakteristik, difuzijske tokove in izvirne člene pa s končnimi razlikami (protivetrnimi in osrednjimi končnimi razlikami). Za napredovanje po

0 INTRODUCTION

Non-Newtonian fluids are very common in everyday engineering practice. Momentum equations can no longer be simplified with the usual simplifications due to variable viscosity. Viscosity is, in general, dependent on various parameters, whereas this paper deals with generalised Newtonian fluids, for which viscosity depends on the trace of the shear-rate tensor. There are several different rheological models, with two, three or four parameters among the generalised Newtonian fluids, see for example, [5] to [7] and [9]. In this study the main focus was on the Sisko three-parametric model.

The fluid conditions for an incompressible, generalised Newtonian fluid were treated as a system of partial differential equations of fundamental conservation laws, solved with a discrete, finite-volume method, as described in [4] and [1]. According to the described method the continuity and momentum equations were coupled with an artificial compressibility term. The convective fluxes were calculated using the method of characteristics, whereas the diffusive fluxes and source terms were computed using finite differences (upwind and central finite deferences). For time marching času smo uporabili izrecno metodo Runge-Kutta četrtega reda. Natančnost in konvergenco metode smo preskusili na primeru gnanega toka v kotanji in primeru toka v kanalu z nenadno zožitvijo. Za reološki model smo uporabili modelne parametre, dobljene na podlagi preskusa s cevnim viskozimetrom ([8] in [3]).

the Runge-Kutta fourth-order explicit method was used. The accuracy and convergence of the method was tested with the driven-cavity test case and flow in a channel with a sudden contraction. The model parameters of the fluid were obtained from an experiment with a capillary viscosimeter ([8] and [3]).

1 DEFINICIJA PROBLEMA

1 PROBLEM DEFINITION

Ravninski tok viskozne nestisljive tekočine lahko opišemo z gibalnima enačbama in kontinuitetno enačbo, ki jih ob zanemaritvi prostorninskih sil zapišemo v obliki:

The planar flow of a viscous, incompressible fluid can be described with two momentum equations and one continuity equation, which with neglected volume forces take the following forms:

$$\rho\frac{\partial v_i}{\partial t}+\rho\frac{\partial}{\partial x_j}(v_j v_i)=-\frac{\partial p}{\partial x_i}+\frac{\partial \tau_{ji}}{\partial x_j} \quad (1)$$

$$\frac{\partial v_i}{\partial x_i}=0 \quad (2).$$

Reševanje enačb ohranitve gibalne količine, zapisanih za osnovne fizikalne spremenljivke za tok nestisljive tekočine, je težavno, ker tlak ni termodinamična veličina. To pomanjkljivost odpravimo z vključitvijo dodatnega člena, časovnega odvoda, v kontinuitetno enačbo. Problem nihanja numerične rešitve odpravimo z navidezno stisljivostjo:

Solving the Navier-Stokes equations for incompressible fluid written with a primitive set of variables is difficult since the pressure is not a thermodynamic quantity. This drawback can be overcome with the introduction of an additional time derivative in the continuity equation. The problem of the oscillations of the numerical solution can be resolved with artificial compressibility:

$$\frac{1}{\beta}\frac{\partial p}{\partial t}+\frac{\partial(\rho v_i)}{\partial x_i}=0 \quad (3),$$

kjer je β umetna stisljivost.

Z vpeljavo brezrazsežnih spremenljivk, kjer sta L in v_∞, značilna dolžina in hitrost,

where the coefficient β is the artificial compressibility.

The introduction of nondimensional variables, where L and v_∞ are the characteristic length and the characteristic velocity, respectively:

$$x_i^*=\frac{1}{L}x_i \;,\quad v_i^*=\frac{1}{v_\infty}v_i \;,\quad p^*=\frac{1}{\rho_o v_\infty^2}p \;,\quad t^*=\frac{v_\infty}{L}t \;,\quad \beta^*=\frac{1}{v_\infty^2}\beta$$

prevedemo enačbi (1) in (2) v naslednjo obliko:

This results in the transformation of Equations (1) and (2) into:

$$\frac{1}{\beta^*}\frac{\partial p^*}{\partial t^*}+\frac{\partial v_i^*}{\partial x_i^*}=0,$$

$$\frac{\partial v_i^*}{\partial t^*}+\frac{\partial}{\partial x_j^*}(v_j^* v_i^*)=-\frac{\partial p^*}{\partial x_i^*}+\frac{\partial}{\partial x_j^*}\left((\mu_{NN}^*)(\frac{\partial v_i^*}{\partial x_j^*}+\frac{\partial v_j^*}{\partial x_i^*})\right)$$

V zgornjih enačbah se pojavljajo same brezrazsežne spremenljivke, zato bomo zaradi primernosti pisanje zvezdice (*) opustili.

Sistem enačb za ravninski tok nestisljive nenewtonske tekočine ob upoštevanju umetne stisljivosti lahko zapišemo v konzervativni obliki kot eno samo vektorsko enačbo ([1] in [10]):

These equations employ only nondimensional variables and we can, for convenience, skip the writing of (*).

The system of non-dimensional equations for the planar flow of an incompressible, non-Newtonian fluid with artificial compressibility can be written in conservative form as a single vector equation ([1] and [10]):

$$\frac{\partial Q}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial G}{\partial y}=\frac{\partial E_{vis}}{\partial x}+\frac{\partial G_{vis}}{\partial y} \quad (4),$$

kjer so:

where:

$$Q=\begin{Bmatrix} p/\beta \\ v_x \\ v_y \end{Bmatrix} \;,\quad E=\begin{Bmatrix} v_x \\ v_x^2+p \\ v_x v_y \end{Bmatrix} \;,\quad G=\begin{Bmatrix} v_y \\ v_x v_y \\ v_y^2+p \end{Bmatrix} \;,\quad E_{vis}=\begin{Bmatrix} 0 \\ \bar{\tau}_{xx} \\ \bar{\tau}_{xy} \end{Bmatrix} \;,\quad G_{vis}=\begin{Bmatrix} 0 \\ \bar{\tau}_{xy} \\ \bar{\tau}_{yy} \end{Bmatrix}$$

in / and:

$$\bar{\tau} = \mu_{NN} \begin{bmatrix} 2\frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \\ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} & 2\frac{\partial v_y}{\partial y} \end{bmatrix}$$

Z vpeljavo lokalnih koordinat (ξ, η), katerih povezavo s kartezijevim koordinatnim sistemom podamo z:

With the introduction of local coordinates (ξ, η) connected to the Cartesian coordinate system through the relation:

$$\frac{\partial}{\partial x} = \frac{\partial \xi}{\partial x} \cdot \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x} \cdot \frac{\partial}{\partial \eta} \quad , \quad \frac{\partial}{\partial y} = \frac{\partial \xi}{\partial y} \cdot \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial y} \cdot \frac{\partial}{\partial \eta}$$

prevedemo enačbo (4) v obliko

Equation (4) transforms into:

$$\frac{\partial \mathcal{Q}}{\partial t} + \frac{\partial \mathcal{E}}{\partial \xi} + \frac{\partial \mathcal{G}}{\partial \eta} = \frac{\partial \mathcal{E}_{vis}}{\partial \xi} + \frac{\partial \mathcal{G}_{vis}}{\partial \eta} \qquad (5),$$

kjer so:

where:

$$\mathcal{Q} = JQ \ , \ \mathcal{E} = J\left(\frac{\partial \xi}{\partial x}E + \frac{\partial \xi}{\partial y}G\right) \ , \ \mathcal{G} = J\left(\frac{\partial \eta}{\partial x}E + \frac{\partial \eta}{\partial y}G\right), \mathcal{E}_{vis} = J\left(\frac{\partial \xi}{\partial x}E_{vis} + \frac{\partial \xi}{\partial y}G_{vis}\right)$$

$$\mathcal{G}_{vis} = J\left(\frac{\partial \eta}{\partial x}E_{vis} + \frac{\partial \eta}{\partial y}G_{vis}\right) \quad \text{in} \quad J = \frac{\partial x}{\partial \xi} \cdot \frac{\partial y}{\partial \eta} - \frac{\partial x}{\partial \eta} \cdot \frac{\partial y}{\partial \xi}$$

Pri reoloških modelih posplošenih newtonskih tekočin je $\underline{\tau}$ podan z izrazom $\underline{\tau} = -\mu\left(\dot{\gamma}\right)\underline{\dot{\gamma}}$, kjer lahko tenzor deformacijske hitrosti in deformacijsko hitrost zapišemo kot:

Rheological models of generalised Newtonian fluids have $\underline{\tau}$ defined with the expression $\underline{\tau} = -\mu\left(\dot{\gamma}\right)\underline{\dot{\gamma}}$, where the shear-stress tensor and the shear rate are written as:

$$\underline{\dot{\gamma}} = \vec{\nabla}\vec{v} + \left(\vec{\nabla}\vec{v}\right)^T \quad , \quad \dot{\gamma} = \sqrt{\frac{1}{2}\left(\underline{\dot{\gamma}} : \underline{\dot{\gamma}}\right)}$$

kjer smo z operatorjem **:** označili sled tenzorja.

and the operator **:** means the trace of a tensor.

2 REOLOŠKI MODEL - SISKOV MODEL

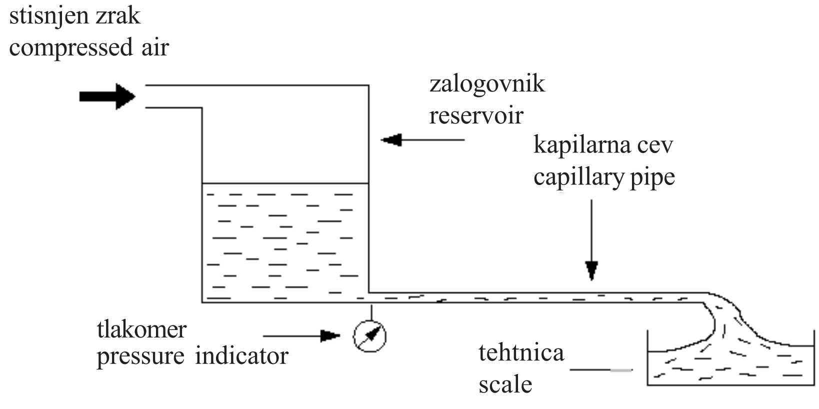
2 RHEOLOGICAL MODELS - SISKO MODEL

V okviru sodelave s Premogovnikom Velenje smo opravili meritve viskoznosti na cevnem viskozimetru, prikazanem na sliki 1 [7]. Bistvena sestavna dela viskozimetra sta zalogovnik in kapilarna cev, skozi katero se pri različnih tlakih pretaka zmes, katere viskoznost določamo.

Meritve smo opravljali na treh različnih mešanicah elektrofiltrskega pepela (ostanek iz termoelektrarne Šoštanj) in vode.

The setup of the viscosimeter is shown in Figure 1 [7], within the framework of a cooperation with Premogovnik Velenje. Measurements of viscosity using the capillary viscosimeter were performed. The essential parts of the viscosimeter are the container and the capillary pipe. The measured mixture flows through the pipe under various pressures.

The measurements were performed using three different mixtures of electro-filter ash (the product of the Šoštanj thermal powerplant) and water.



Sl. 1. *Vodoravni kapilarni viskozimeter*
Fig. 1. *Horizontal capilary viscosimeter*

Preglednica 1. *Masni deleži pepela in vode v zmeseh in gostota zmesi*
Table 1. *Mass fraction of ash and water in the mixtures and the density of the mixtures*

	$\dot{m}_{ash}/\dot{m}_{H_2O} \cdot 100$ [%]	ρ [kg/m^3]
A	50	1364
B	40	1469
C	37	1524

V preglednici 1 so podani masni deleži vode in pepela ter gostote izbranih zmesi.

Kakor smo že zapisali, smo izbrane zmesi modelirali s Siskovim zakonom tečenja kot reološkim modelom posplošenih newtonskih tekočin. Zapišemo ga v obliki:

Table 1 shows the mass fractions of the water and ash and the densities of the chosen mixtures.

The chosen mixtures were modelled using the Sisko model as a rheological model of generalized Newtonian fluids. It can be written in the following forms:

$$\underline{\tau} = -\mu(\dot{\gamma})\,\underline{\dot{\gamma}} = -\left(\mu_{\infty} + m \cdot \dot{\gamma}^{n-1}\right)\underline{\dot{\gamma}} \qquad (6)$$

oziroma / or

$$\mu\left(\dot{\gamma}\right) = \mu_{\infty} + m\dot{\gamma}^{n-1} \qquad (7).$$

Triparametrski model je sestavljen iz klasičnega potenčnega zakona tečenja z asimptotično vrednostjo dinamične viskoznosti μ_∞. μ_∞, m in n so vrednosti, podane za vsako tekočino posebej, in se določijo na temelju meritev viskoznosti. Če je $\mu_\infty = 0$, $n = 1$ in $m = \mu$, preide model v Newtonov zakon tečenja.

Na temelju meritev so ob uporabi metode najmanjšh kvadratov [8] izpeljani parametri Siskovega modela, in sicer:

The three-parameter model consists of the well-known power-law model with an added asymptotical coefficient of dynamic viscosity, μ_∞. The values μ_∞, m and n are specified for each fluid separately, and are obtained from experimental viscosity measurements. If one specifies $\mu_\infty = 0$, $n = 1$ and $m = \mu$ one gets an expression for a Newtonian fluid.

Based on the measurements performed, and with the use of the least-squares method [8], the following Sisko model parameters are obtained:

- za zmes A / for mixture A

$$\mu\left(\dot{\gamma}\right) = 0.001339 + 0.000059\dot{\gamma}^{1.6771} \qquad (8)$$

- za zmes B / for mixture B

$$\mu\left(\dot{\gamma}\right) = 0.000628 + 0.000124\dot{\gamma}^{1.6171} \qquad (9)$$

- za zmes C / for mixture C

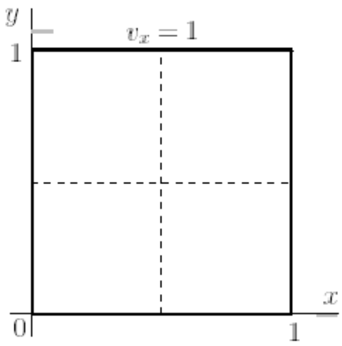
$$\mu\left(\dot{\gamma}\right) = 0.000008 + 0.006181\dot{\gamma}^{1.1719} \qquad (10).$$

3 REZULTATI

Model smo preskusili na primeru gnanega toka v kotanji. Gnana kotanja kot značilen dvorazsežni primer je pravokotnik s tremi mirujočimi stranicami in eno pomično stranico, ki se giblje z nespremenljivo hitrostjo. V našem primeru je to kvadrat z brezrazsežno dolžino stranice 1 in brezrazsežno hitrostjo pomika zgornje plošče 1. Geometrijska oblika kotanje je prikazana na sliki 2. Prekinjeni črti po sredini kotanje označujeta osrednji črti (navpično in vodoravno) po katerih smo primerjali hitrostne profile. Dodatna pričakovana težava tega primera je nezveznost v stičiščih mirujočih in gibajoče se stranice.

3 RESULTS

The model was tested with the driven-cavity flow. The driven cavity, as a typical two-dimensional example, is rectangular with three fixed walls and one moving wall that moves with constant velocity. In the selected case the cavity has a square form with a nondimensional side length of 1 and an nondimensional velocity of the top wall 1. The geometry is depicted in Figure 2. The dashed lines through the centre of the cavity show the locations (vertical and horizontal) where the velocity profiles were compared. An additional, but expected, difficulty with this case was the discontinuity in the top corners of the cavity, where the moving and fixed walls connect.



Sl. 2. *Geometrijska oblika in robni pogoji: gnana kotanja*
Fig. 2. *Geometry and boundary conditions: driven cavity*

V prvem delu rezultatov je prikazana konvergenca rezultatov v odvisnosti od gostote mreže. Rezultate smo primerjali z vrednostmi iz literature. Kot primerjalne vrednosti smo uporabili najnižje negativne vrednosti vodoravne in navpične komponente hitrosti na osrednjih črtah. Odstopanje od primerjalnih vrednosti smo definirali z izrazoma [4]:

In the first part of the results the convergence of the results due to mesh refinement is shown. The results were compared to the benchmark solution. The maximum negative values of the horizontal and vertical components of the velocity on the centre lines were used for comparison. Deviation from the benchmark solution was defined with the following expression [4]:

$$\Delta v_x = \frac{\left|v_{x_{ref}} - v_x\right|}{\left|v_{x_{ref}}\right|} \cdot 100 \quad [\%] \quad , \quad \Delta v_y = \frac{\left|v_{y_{ref}} - v_y\right|}{\left|v_{y_{ref}}\right|} \cdot 100 \quad [\%]$$

Konvergenca rezultatov za različne gostote mreže je prikazana v preglednici 1. Rezultati so primerjani z referenčnimi vrednostmi za newtonsko tekočino za *Re = 400*.

The convergence of the results for different mesh sizes is shown in Table 1. The results were compared to reference values for a Newtonian fluid with a Reynolds number value *Re=400*.

Preglednica 2. *Odstopanje rezultatov v odvisnosti od gostote mreže (Siskov model:* μ_∞ *= 0, m = 0,0025, n = 1)*
Table 2. *Deviation of the results for different mesh sizes (Sisko model:* μ_∞ *= 0, m = 0.0025, n = 1)*

	Δv_x %	Δv_y %
11×11	44.17	38.60
21×21	20.63	19.71
41×41	5.50	5.19
81×81	1.02	0.80
129×129	0.28	0.19

Pri redkejših enakomernih mrežah zaradi velikih gradientov ob trdnih stenah (predvsem zgornji) pride do večjih odstopanj od primerjalnih vrednosti. Z zgoščevanjem mreže se rezultati približujejo referenčnim vrednostim. Z najgostejšo mrežo (129×129) dobimo manj ko 0,3 % odstopanja od referenčnih vrednosti za izbrane parametre. Potek hitrostnih profilov v odvisnosti od gostote mreže je prikazan na sliki 3.

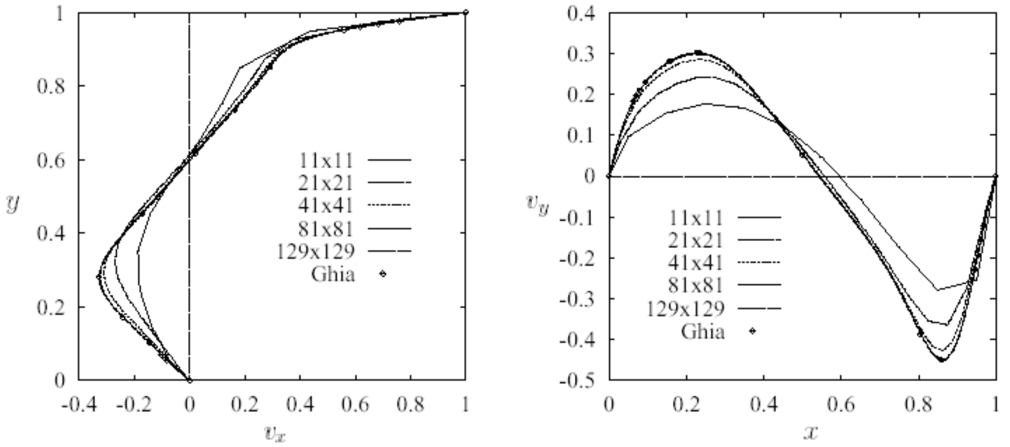
Na slikah 4, 5 in 6 so prikazani rezultati izračuna tokovnih razmer gnanega toka v kotanji z uporabo Siskovega modela.

Iz primerjave oblik profilov z rezultati iz literature za newtonske [2] tekočine je glede na gradiente hitrosti moč sklepati, da je zahtevnost izračuna za najredkejšo zmes (in s tem najmanj viskozno tekočino) primerljiva z izračunom newtonske tekočine za Reynoldsovo število *Re=1000*.

When using coarse equidistant meshes the differences from reference values are larger due to high velocity gradients near the walls (especially the top wall). With a refinement of the mesh the results approach reference values. With the finest mesh (129×129) a less-than-0.3% difference from the reference values for the chosen non-Newtonian model parameters was obtained. The velocity profiles for the different mesh densities are shown in Figure 3.

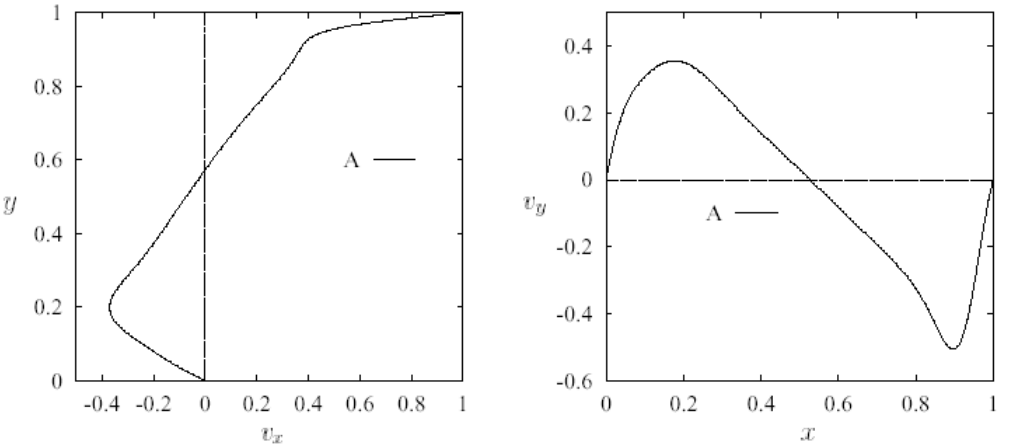
In Figures 4, 5 and 6 the results are shown for the calculation of the flow field in a driven cavity for model parameters of the Sisko model obtained from the experiment with a capilary viscosimeter.

A comparison of the calculated velocity profiles with the benchmark solution for Newtonian [2] fluids, taking into account velocity gradients, shows that the computation with the least dense mixture (meaning the least viscous fluid) corresponds to the computation of a Newtonian fluid with a Reynolds number value *Re=1000*.



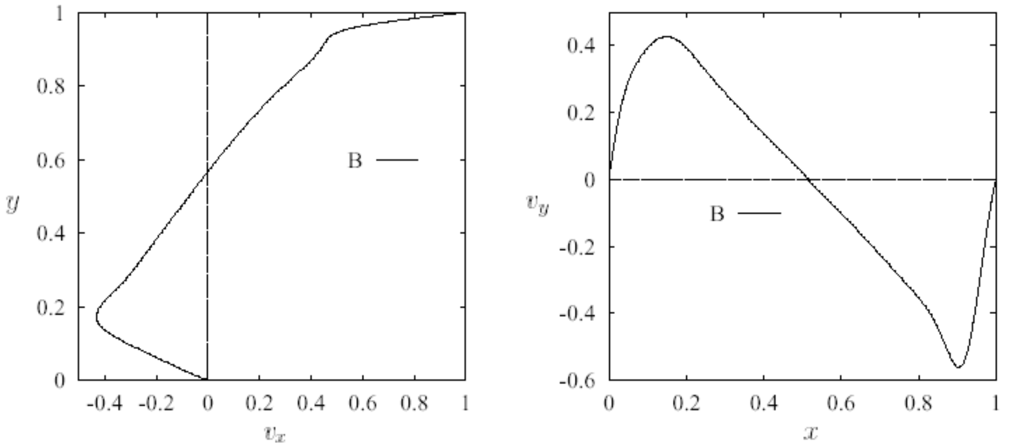
Sl. 3. *Profil vodoravne hitrosti v_x skozi navpično središčnico in navpične hitrosti v_y skozi vodoravno središčnico za različne gostote mreže (Sisko model: $\mu_\infty = 0$, m = 0,0025, n = 1)*

Fig. 3. *Horizontal velocity profile v_x through the vertical centreline and the vertical velocity profile v_y through the horizontal centreline for different mesh sizes (Sisko model: $\mu_\infty = 0$, m = 0.0025, n=1)*



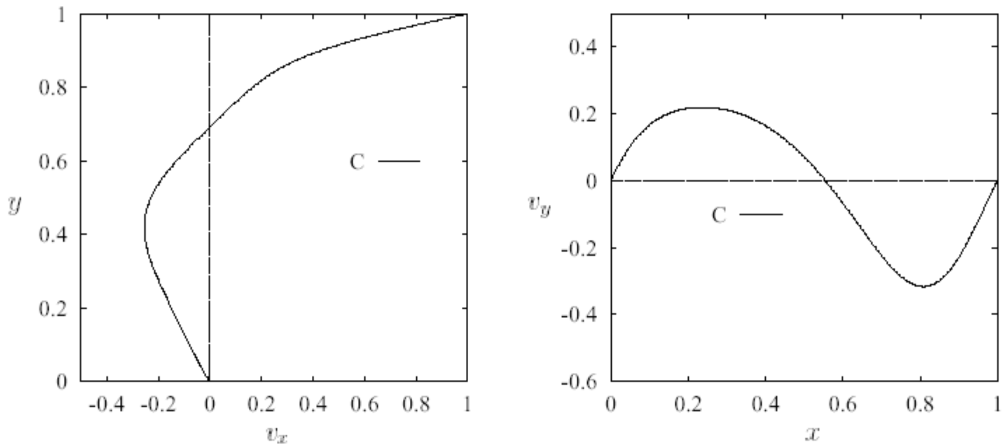
Sl. 4. *Profil vodoravne hitrosti v_x skozi navpično središčnico in navpične hitrosti v_y skozi vodoravno središčnico za zmes A (129×129 vozlišč)*

Fig. 4. *Horizontal velocity profile v_x through the vertical centreline and the vertical velocity profile v_y through the horizontal centreline for mixture A (129×129 nodes.)*



Sl. 5. *Profil vodoravne hitrosti v_x skozi navpično središčnico in navpične hitrosti v_y skozi vodoravno središčnico za zmes B (129×129 vozlišč)*

Fig. 5. *Horizontal velocity profile v_x through the vertical centreline and the vertical velocity profile v_y through the horizontal centreline for mixture B (129×129 nodes.)*



Sl. 6 *Profil vodoravne hitrosti* v_x *skozi navpično središčnico in navpične hitrosti* v_y *skozi vodoravno središčnico za zmes C (129×129 vozlišč)*

Fig. 6. *Horizontal velocity profile* v_x *through the vertical centreline and the vertical velocity profile* v_y *through the horizontal centreline for mixture C (129×129 nodes.)*

Model smo preskusili tudi na standardnem testnem primeru toka v kanalu z nenadno zožitvijo. Robni pogoji in geometrijska oblika so prikazani na sliki 7. Oba dela kanala merita 10 enot. Prvi del kanala je visok 4 enote, nato pa se kanal nenadno zoži na 1 enoto. Za izračun smo izdelali tri neenakomerne mreže, in sicer z 1500 (40×30 + 10×30), 6000 (80×60 + 20×60) in 13500 (120×90 + 30×90) končnimi prostorninami.

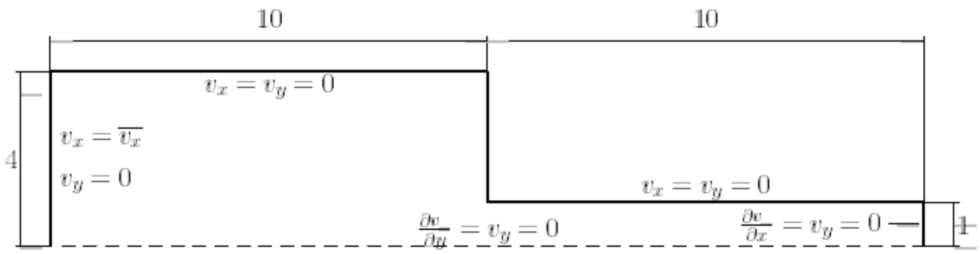
Za reološki model smo pri izračunih uporabili Siskov model za zmes A (enačba 8). Na stenah smo predpisali v_x, $v_y = 0$, na spodnji steni simetrijske robne pogoje ($\partial v_x/\partial y = v_y = 0$), na iztoku prosti iztok ($\partial v_x/\partial x = v_y = 0$) in na vtoku parabolični profil s povprečno hitrostjo v_{av} = 0,0393. (Ob uporabi newtonske dinamične viskoznosti μ = 0,01339 bi primer ustrezal *Re=400,* računano na izstopni prerez iz kanala).

Na sliki 8 je prikazana primerjava hitrostnih profilov na polovici dolžine kanala za vse tri gostote mreže. Po pričakovanju se profili hitrosti slabše ujemajo na mestu zožitve, kjer rezultati, dobljeni z najredkejšo mrežo, še dokaj odstopajo; rezultati, dobljeni z gostejšima mrežama, pa se že dokaj dobro ujemajo. Z zgoščevanjem mreže na področju razvijanja hitrostne mejne plasti rastejo gradienti hitrosti ob trdnih stenah in zaradi tega se hitrosti zmanjšujejo proti srednjici kanala.

The model was also tested on a standard test case of flow in a channel with a sudden contraction. The boundary conditions and geometry are shown in Figure 7. The inlet and outlet channels have a length of 10 units. The inlet channel has a height of 4 units and contracts to an outlet height of 1 unit. Three, non-equidistant meshes were generated: 1500 (40×30 + 10×30), 6000 (80×60 + 20×60) and 13500 (120×90 + 30×90) finite volumes.

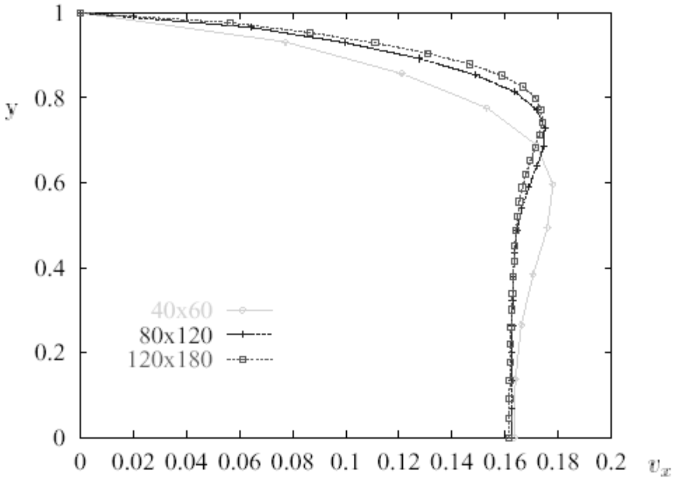
We used the Sisko rheological model with parameters for mixture A (Equation 8). We prescribed v_x, v_y = 0, on the walls, the symmetry boundary condition ($\partial v_x/\partial y = v_y = 0$) on the bottom edge, the developed profile ($\partial v_x/\partial x = v_y = 0$) at the outlet and the parabolic velocity profile with mean value v_{av} = 0.0393 at the inlet. (Using the Newtonian dynamic viscosity μ = 0.01339 and the outlet channel height, the equivalent Reynolds number would be *Re=400*).

In Figure 8 is the comparison of the velocity profiles on the contraction cross-section for three mesh densities. As expected, the profiles disagree in this area, especially for the coarsest mesh, whereas the two finest meshes show reasonably good agreement. With finer meshes in the area of the development of the boundary layer the velocity gradients near the wall increase and the velocity near the centreline of the channel decreases.

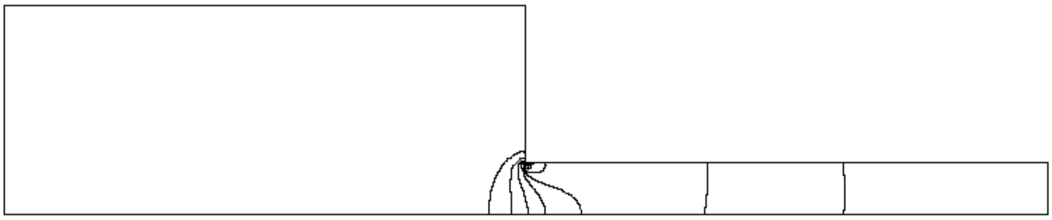


Sl. 7. *Geometrijska oblika in robni pogoji: nenadna zožitev (4:1)*

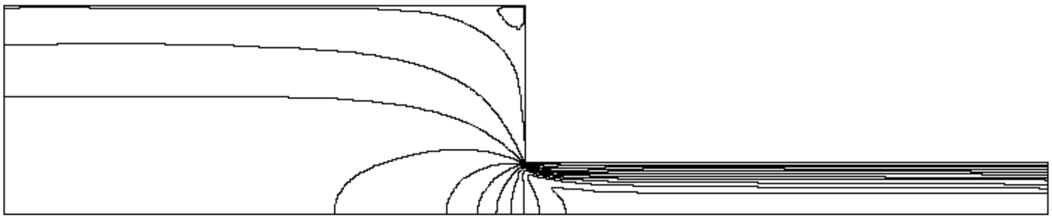
Fig. 7. *Geometry and boundary conditions: sudden contraction (4:1)*



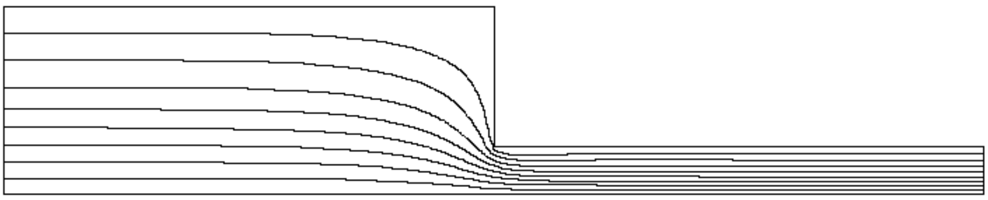
Sl. 8. *Primerjava profilov hitrosti na 1/2 dolžine kanala*
Fig. 8. *Comparison of velocity profiles on 1/2 the channel length*



Sl. 9. *Tlačno polje v kanalu*
Fig. 9. *Pressure field in the channel*



Sl. 10. *Izolinije hitrosti v smeri koordinatne osi x (v_x)*
Fig. 10. *Constant x component velocity contours (v_x)*



Sl. 11. *Tokovnice (zožitev 4:1)*
Fig. 11. *Streamlines (contraction 4:1)*

Na sliki 9 je prikazano tlačno polje v kanalu, izračunano z najgostejšo mrežo.

Na sliki 10 so prikazane izolinije hitrosti v smeri koordinatne osi *x* (najgostejša mreža).

Na sliki 11 so prikazane tokovnice v kanalu (najgostejša mreža).

Figure 9 shows the pressure field in the channel computed using the finest mesh.

Figure 10 shows the constant *x* component velocity contours (the finest mesh).

Figure 11 shows the streamlines in the channel (the finest mesh).

4 SKLEP

Metodo končnih prostornin smo uporabili za računanje tokovnih razmer v laminarni viskozni

4 CONCLUSIONS

The finite-volume method was used to compute the flow conditions for a laminar, viscous, nestisljivi nenewtonski tekočini, pri čemer smo kot reološki model uporabili Siskov model. Formulacija je testirana na primeru gnanega toka v kotanji. Prikazan je vpliv natančnosti rezultatov v odvisnosti od gostote mreže. Prikazani rezultati za primer gnanega toka v kotanji, dobljeni z najgostejšo mrežo (129×129 vozlišč) se zelo dobro ujemajo s primerjalnimi vrednostmi v literaturi. Modelni parametri Siskovega modela so določeni na temelju preskusa s cevnim viskozimetrom za tri različne gostote zmesi pepela in vode.

incompressible non-Newtonian fluid. The Sisko rheological model was used for capturing non-linear fluid properties. The proposed formulation was tested on driven-cavity flow. The influence of the mesh density on the accuracy of the numerical results was shown. The results obtained on the finest mesh (129×129 nodes) agree well with the values published in the literature. The computational examples conclude with three computations based on the model parameters of the Sisko model, obtained from an experiment with a capillary viscosimeter for three different mixtures of ash and water.

5 LITERATURA
5 REFERENCES

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