UDK621.3:(53+54+621+66), ISSN0352-9045 Informacije MIDEM 35(2005)3, Ljubljana

AFFINITY OF CONTACT MATERIALS TO FORM THE ELECTRIC DRAWN ARCS

[1]France Pavlovčič, [2]Janez Nastran

[1]Ministry for environment and spatial planning, Environmental Agency of the Republic of Slovenia

[2]University of Ljubljana, Faculty of electrical engineering, Ljubljana, Slovenia

Key words: drawn electric arc, arc formation, sliding contact, commutator, arc ignition voltage, contact material.

Abstract: Electro mechanic contacts are widely used in several appliances, because they are cheap, but also very robust, capable to withstand overvoltage and overcurrent surges, and when used in electric relays they enable galvanic separation of electric circuits. Especially they are necessary with commutators of universal electric motors for they are enabling commutation. These motors achieve high power per a unit of volume due to their high shaft speed, and they are produced at relatively low costs for a unit of power. So, they are economically very important for their numerous production, and they use sliding contacts for their operation. The sliding contacts are mechanical contacts, as switching contacts are, and so electric arcs ignite between their contact members during commutation. Therefore it is very important to determine contact materials with a small tendency to form the electric arcs. The arcs between electrodes of the sliding contacts are usually drawn arcs, so that affinity of the contact material to form the drawn arcs is very significant attribute of determining the right contact mate-rials. Conclusions made for the sliding contacts are also useful for the switching contacts.

Težnja kontaktnih materialov k tvorjenju električnih potegnjenih oblokov

Kjučne besede: potegnjeni električni oblok, nastanek obloka, drsni kontakt, komutator, nape-tost vžiga obloka, kontaktni material.

Izvleček: Elektromehanski kontakti so množično uporabljeni v številnih napravah, ker so poceni, vendar robustni, zmožni vzdržati prenapetostne in pretokovne preobremenitve in kadar so uporabljeni v električnih relejih omogočajo galvansko ločitev tokokrogov. Posebno so nepogrešljivi pri komutatorjih univerzalnih električnih motorjev, kajti omogočajo komutacijo. Ti motorji razvijejo veliko moč na enoto volumna zaradi velike hitrosti vrtenja in njihovi proizvodni stroški na enoto moči so relativno majhni. Zaradi njihove velikoserijske proizvodnje so ekonomsko zelo pomembni. Za svoje obratovanje pa uporabljajo drsne kontakte. Drsni kontakti so mehanski kontakti, kakor so tudi stikalni kontakti in zato se med njihovimi kontakti pri komutaciji vžgejo električni obloki. Zato je zelo pomembno določiti kontaktne materiale z majhno tendenco tvorjenja električnih oblokov. Obloki med drsnimi kontakti so ponavadi potegnjeni obloki, tako da je težnja kontaktnih materialov k tvorjenju potegnjenih oblokov zelo pomembna lastnost pri določanju pravega kontaktnega materiala. Zaključki, narejeni za drsne kontakte, so uporabni tudi za stikalne kontakte.

1. Introduction

Talking about affinity of the contact materials to form the electric drawn arcs, we must have in mind that it is not a physical quantity, but rather a conception, which is numerically estimated by one or more physical quantities. To begin with, we consider the voltage - current (*UI*) stationary electric arc characteristics, for instance of the copper contacts, shown in Fig. 1 as hyperbolae, in relation to the pure resistive load, shown in the same figure - the line marked with the label "break" /1/. This line illustrates the load characteristic of the electric arc at the breaking of an electric resistive circuit by the axial switching contacts. This drawn arc ignites at the voltage of 13.1 V, which is slightly over the voltage of the asymptote of the hyperbolae, and at the current of 1.95 A. The voltage of the asymptote is defined as the minimal arc voltage U_m by Holm /1/ and it is the property of the material. But we name it as the infimum arc voltage because it is the greatest lower limit voltage. The arc burns until its length is 0.8 mm and is extinguished at the voltage of 48.7 V and at the current of 0.73 A.

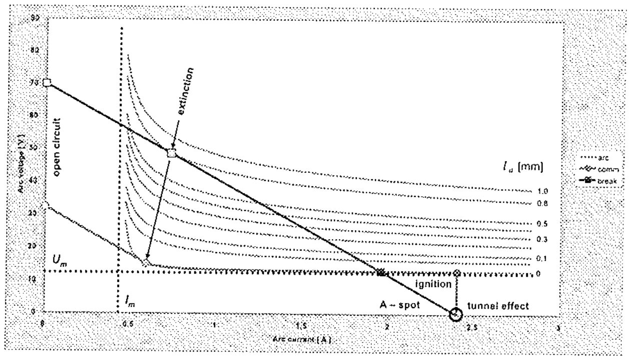

Fig. 1: The UI stationay and load arc characteristics.

Legend:

arc — *stationary arc characteristics,*

comm — *load arc characteristic at resistance commutation,*

break — *load arc characteristic at breaking resistive load.*

If another contact material is taken into consideration then the hyperbolic characteristics are the same curves, but voltage (U_m) and the current (I_m) the asymptotes have different values. The current asymptote is defined as the infimum arc current and is also the property of the material. If the value of the U_m - asymptote is higher than the one of copper the arc ignites at a lower current, and of course, at a higher voltage, and its length is shorter at the point of the extinction. The values of the arc voltage and the arc current at its extinction are also changed. Because the ignition and the length of the arc are depended on the U_m - asymptote, we decide that the infimum arc voltage is just the proper quantity to define affinity of the contact material to form the electric drawn arc.

It seems very simple to define affinity of the contact materials to form the drawn arcs between the axial switching contacts, but in the case of the sliding contacts, such as they are in the commutators, is a much more difficult task. Namely, the drawn arc ignites and burns at a macroscopic geometrical separation of the axial switching contacts, but researching the arc phenomenon with the sliding contacts it must be taken into account, that the drawn arc ignites and burns between the overlapping electrodes with no macroscopic geometrical separation of them /2/,/3/. If we consider only the *UI* stationary arc characteristics and the load characteristic of the arc during the resistance commutation, shown in Fig. 1 - the diagram marked with the label "comm", it is not selfevident that the infimum arc voltage is the measure of this kind of affinity. The arc ignites at the infimum value of the arc voltage and at the full value of the current through the contact. It burns at the voltage, which slightly rises from the infimum arc volt-age of 13 V up to 15.2 V for copper, and is extinguished at the latter value of the voltage and at the current of 0.59 A. Its length is in the range of some nanometres throughout its burning, so it is a short arc. It is exactly the same with another contact material: the ignition occurs at the infimum arc voltage and at the full current, the burning is at its length of some nanometres and at the corresponding voltage, and extinction is nearly at the current asymptote, which value is the infimum arc current. Therefore we have to establish the mathematical model of electric current conduction through the gap of some nanometres between the commutator bar and the brush during the commutation to make a clear definition of affinity of the contact materials to form the drawn arcs. When the commutator bar moves over the brush, the thickness of the gap between them varies, and so the way of the current conduction alters from tunnel effect to arc conducting mode - Fig. 2.

If the thickness of the gap is so large, that its corresponding voltage drop equals to or is greater than the infimum arc voltage U_m, the arc ignites. With the same thickness of the gap, some contact materials form the drawn arcs, but some not, depending on their infimum arc voltage. As this voltage is the property of the contact material, it fully determines affinity of the contact material to form the electric drawn arc.

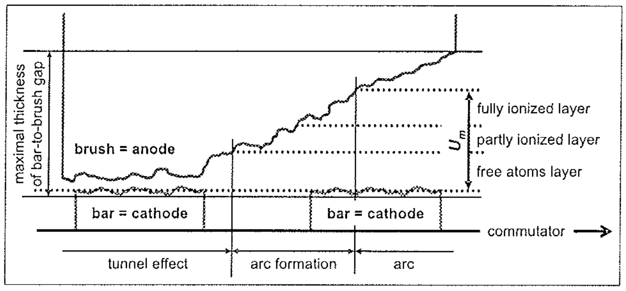

Fig. 2: The formation of the electric drawn arc during the commutation.

There are many physical constants and variables in this paper that are not explained in the text, so their definitions are present in the chapter of used symbols at the end of the paper. We also deal with many chemical elements, although they could not be used as pure materials in contact technique. The direct current case is considered by this model.

2. The mathematical model of the drawn arc formation

With the sliding contacts of the commutator, a very thin insulating film arises on the contact surface /1/,/4/. This film is the collector film. Beside it, there is also the gap between the contacts. The gap is filled with atoms and molecules of an external medium, and also with the atoms of the contact materials. The gap and the film thickness are up to a few nanometres, so that no ionization occurred.

The electric current flows due to tunnel effect. One contact member is a cathode, the other one is an anode. The charge carriers are free electrons in the electric field of the gap, so they are accelerated when moving along the path through the gap. They are emitted from the cathode by three ways:

- by the field emission, called also the cold emission;
- by the thermionic emission including also the Schottky effect, which is depended on the electric field, but its contribution is not included in the cold emission;
- by the emission due to the different work functions of the cathode and anode materials.

Considering the cathode is still cold, only the field emission is taken into account, and the current density is defined by the Fowler - Nordheim equation /5/,/6/ and /7/, which is equal to Eq. (1) for the electron emission from the smooth and unstained metal cathode in vacuum:

$$j_E = \frac{e^2}{8\cdot\pi\cdot h\cdot\frac{m_e^*}{m_e}}\cdot\frac{E^{*2}}{V_{\phi\kappa}^*}\cdot e^{-\frac{8\cdot\pi\cdot\sqrt{2\cdot m_e^*\cdot e}}{3\cdot h}\cdot\frac{V_{\phi\kappa}^{*\,3/2}}{E^*}+\frac{\sqrt{8\cdot m_e^*\cdot e^3}}{3\cdot h\cdot\varepsilon_0}\cdot V_{\phi\kappa}^{*\,-1/2}} \qquad (1).$$

Experiments show that the electric field /8/, the work function /8/,/9/, which is defined as the voltage V_{fk} , and the effective electron mass /10/,/11/ changed due to a contamination of the cathode, but the roughness of the cathode surface effects only the electric field intensity. So the following substitutions must take place to descride exploatation condition of the contacts:

$$E^* = \beta \cdot E \quad \Leftarrow \quad \beta \geq 1 \qquad (2),$$

$$V_{\phi\kappa}^* = \frac{V_{\phi\kappa}}{\upsilon} \quad \Leftarrow \quad \upsilon \geq 1 \qquad (3),$$

$$m_e^* = \frac{m_e}{\mu} \quad \Leftarrow \quad \mu \geq 1 \qquad (4).$$

The collector film and the moleculas of the air gases in the very thin gap are considered as the contanimation over the cathode surface. Therefore the equation, describing the current density due to the cold emission from the cathode, overlayed with the collector film and being in the air, is:

$$j_E = \frac{e^2 \cdot \mu}{8 \cdot \pi \cdot h} \cdot \frac{\beta^2 \cdot E^2 \cdot \upsilon}{V_{\phi\kappa}} \cdot e^{-\frac{8\pi \cdot \sqrt{2 \cdot m_e \cdot e}}{3 \cdot h \cdot \sqrt{\mu}} \cdot \frac{1}{\beta \cdot E} \cdot \sqrt{\frac{V_{\phi\kappa}^3}{\upsilon^3}} + \frac{\sqrt{8 \cdot m_e \cdot e^3}}{3 \cdot h \cdot \varepsilon_0 \cdot \sqrt{\mu}} \cdot \sqrt{\frac{\upsilon}{V_{\phi\kappa}}}} \qquad (5).$$

The current density is known in the most cases, so that the electric field intensity is calculated from Eq. (5) by some iterative method.

The commutator bar moves over the brush, so the thickness of the gap increases, as it is demonstrated in Fig. 2. Presuming the electric field is homogeneous the acceleration of the electrons in the gap is constant. The kinetic energy of the electron is /11/:

$$W_{ek} = \frac{m_e \cdot c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_e \cdot c^2 \qquad (6).$$

When the electron achieves such a velocity, that his kinetic energy is:

$$W_{ek} = \frac{W_{i1}}{N_A} \cdot \left(1 + \frac{m_e}{m_a}\right) \qquad (7),$$

the ionization of the atoms in the gap begins. The energy W_{i1} is the first ionization energy of one mole of the gaseous element in the gap.

Two values of the electron velocity are derived from Eqs (6) and (7). The first one is the average value of the linear motion of the electron /2/, and the second one is the rootmeansquare value needed for the ionization when collision between the electron and the atom is not centric. The average value of the electron velocity is:

$$v_{e\_avg} = c \cdot \sqrt{1 - \frac{m_e \cdot c^2}{\frac{W_{i1}}{N_A} \cdot \left(1 + \frac{m_e}{m_a}\right) + m_e \cdot c^2}} \qquad (8),$$

and the rootmeansquare value or, as it is also called, the effective value is:

$$v_{e\_rms} = c \cdot \sqrt{1 - \frac{m_e \cdot c^2}{2 \cdot \frac{W_{i1}}{N_A} \cdot \left(1 + \frac{m_e}{m_a}\right) + m_e \cdot c^2}} \qquad (9).$$

The motion of the electron is accelerated, so there is a constant force on the electron. This is the force of the electric field on the electron, and the vectors of the force and of the electric field are colinear:

$$F = e \cdot E \qquad (10).$$

This force is also the function of the electron mass and his acceleration /11/, and the vectors of the force and of the acceleration are colinear:

$$F = \frac{m_e}{\sqrt{\left(1 - \frac{v_{e\_rms}^2}{c^2}\right)^3}} \cdot \frac{dv}{dt} \qquad (11).$$

A path the electron must move over to get the sufficient velocity for the ionization of the atom is derived from Eqs (10) and (11). This path is:

$$s = \frac{m_e \cdot c^2}{e \cdot E} \left(\sqrt{1 + 2 \cdot \frac{W_{i1}}{N_A} \cdot \frac{1}{m_e \cdot c^2} \cdot \left(1 + \frac{m_e}{m_a}\right)} - 1 \right) \qquad (12).$$

Because the electric field is homogeneous, the voltage over this path is equal to:

$$U = \frac{m_e \cdot c^2}{e} \left(\sqrt{1 + 2 \cdot \frac{W_{i1}}{N_A} \cdot \frac{1}{m_e \cdot c^2} \cdot \left(1 + \frac{m_e}{m_a}\right)} - 1 \right) \qquad (13).$$

These two equations (12) and (13) are simplified according to the following rule of small numbers: $\sqrt{1 + 2 \cdot \delta} \approx 1 + \delta \quad \Leftarrow \quad \delta << 1$. So, the path and the corresponding voltage are:

$$s = \frac{W_{i1}}{e \cdot E \cdot N_A} \qquad (14),$$

$$U = \frac{W_{i1}}{e \cdot N_A} \qquad (15).$$

These are the basic equations that determine the path of the electron flow and the voltage over this path, which is sufficient for the ionization. If the gap is shorter than this path the current flows due to tunnel effect. The ionization means the end of tunnel effect, and it is also the beginning of the formation of the drawn arc. Comparing the drawn and the discharging arc, the ionization is present at the formation of them both, but for the latter one, the overvoltage between the contact members and the breakdown of the gap medium are the cause of the ignition. In the case of the drawn arc, the current conduction smoothly turns from tunnel effect to arc conducting mode, when the ionization begins.

While the tunnel conduction has only one kind of the charge carriers, which are the electrons emitted from the cathode, the drawn arc conduction has the following charge carriers:

- the electrons passed from the cathode by the field emission;
- the electrons passed from the cathode by the thermionic emission at the higher value of the cathode temperature as with tunnel effect;
- the cathode and the anode ions, which resulted from the ionization of the free atoms of the cathode and the anode materials in the conducting volume of the gap.

When the thickness of the gap sufficiently increases the ionization begins, but not the ionization of the cathode and the anode atoms at the same time. If the first ionization energy of the cathode material is lower as the first ionization energy of the anode material the ionization of the cathode atoms takes place, while the anode atoms are still not ionized. Further increase in the gap thickness - Fig. 2, causes that the electrons get the higher kinetic energy, sufficient to ionize the free anode atoms. So, there are three layers within the arc illustrated in Fig. 3:

- the first one, where the kinetic energy of the electrons is not sufficient to ionize any free atoms in this layer; the free cathode and anode atoms only are present there, therefore this one is named ***the free atoms layer***;
- the second one, where, in the discussed case, the cathode atoms are ionized, but the anode atoms are still neutral; this layer is ***the partly ionized layer***;
- the third one, where all atoms are ionized is called ***the fully ionized layer***.

The free atoms layer is in literature /1/,/4/ defined as ***a cathode layer***, but only the cathode atoms and ions are taken into account with its definition. To distinguish between our arc model, which considers the cathode and the anode atoms and ions, and the other arc model from the literature, we define the arc layers in this special way. The difference between the free atoms layer and the cathode layer, as defined in the literature, occurs when the first ionization energy of the cathode material is higher than that of the anode material.

When the fully ionized layer is established the drawn arc ignites. The question arises about the length of the path through the fully ionized layer. It must be so large that the ions, while moving along this path, get the sufficient kinetic energy to heat the cathode to such amount that the cathode atoms pass from the cathode to the gap by vaporization or by sublimation, and that the thermionic emission of

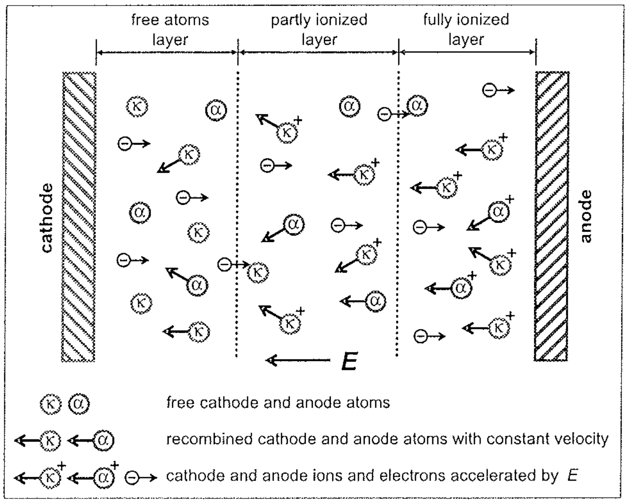

Fig. 3: The arc layers.

the electrons is established. Further on, travelling toward the cathode, they are recombined, and their collisions against the cathode are nonperpendicular. There must also be equilibrium of the charge carriers, so that the arc is neutral to the external medium.

The mathematical model of the drawn arc formation has the following presumptions:

- there is only the field emission with tunnel effect;
- the electric field intensity is constant while the current conduction passes from tunnel effect to arc conducting mode; the electric field is also homogeneous;
- the contact current is constant while the current conduction passes from tunnel effect to arc conducting mode;
- the motion of the electrons in the gap between the cathode and the anode is linear;
- the collisions of the particles are not centric, nor they are perpendicular;
- the ions are recombined before they collide against the cathode;
- the arc is externally neutral;
- the arc temperature is equal to the cathode temperature at the instant of the arc formation;
- the ratio between the number of the free cathode atoms and ions and the number of all free atoms and ions in the conducting volume is calculated as the cathode and the anode temperatures are equal;
- the external medium is air under the normal pressure, or is optionally vacuum;
- the total pressure of gases in the conducting volume between the cathode and the anode is equal to the pressure of the external medium, unless it is determined by the thermionic emission to be higher.

There are the cathode and the anode atoms in the gap, which are ionized by the collisions with the electrons. The first ionization energy of the cathode atoms is not equal to the one of the anode atoms. Therefore the electrons that ionize the cathode atoms have the different average and effective values of the velocity than the electrons that ionize the anode atoms. The ratio between the number of the free cathode atoms and ions and the number of all free atoms and ions in the conducting volume must be determined to calculate the overall average velocity of the electrons. This ratio is according to the ideal gas law:

$$\eta_\kappa = \left.\frac{p_{v\kappa}(T_\kappa)}{p_{v\kappa}(T_\kappa)+p_{v\alpha}(T_\alpha)}\right|_{T_\alpha=T_\kappa} = \frac{p_{v\kappa}(T_\kappa)}{p_{v\kappa}(T_\kappa)+p_{v\alpha}(T_\kappa)} \quad (16).$$

In Eq. (16), there are the saturated vapour pressures of the cathode and the anode atoms and ions. They are determined according to the Clapeyron - Clausius equations /11/:

$$\frac{d\,p_{v\kappa}(T_\kappa)}{d\,T_\kappa} - \frac{L_{v\kappa}(T_\kappa)\cdot p_{v\kappa}(T_\kappa)}{R\cdot T_\kappa^2\cdot\left(1-\frac{V_{\mu l\kappa}}{V_{\mu v\kappa}}\right)} = 0 \quad (17),$$

$$\frac{d\,p_{v\alpha}(T_\kappa)}{d\,T_\kappa} - \frac{L_{v\alpha}(T_\kappa)\cdot p_{v\alpha}(T_\kappa)}{R\cdot T_\kappa^2\cdot\left(1-\frac{V_{\mu l\alpha}}{V_{\mu v\alpha}}\right)} = 0 \quad (18).$$

The $L_{v\kappa}$ and $L_{v\alpha}$ are the values of the latent heat of vaporization of the cathode and the anode materials depended on the cathode temperature. Their functions are presented as the linear interpolated equations between the boiling and the critical temperatures:

$$L_{v\kappa}(T_\kappa) = L_{vb\kappa}\cdot\frac{T_{c\kappa}-T_\kappa}{T_{c\kappa}-T_{b\kappa}} \quad (19),$$

$$L_{v\alpha}(T_\kappa) = L_{vb\alpha}\cdot\frac{T_{c\alpha}-T_\kappa}{T_{c\alpha}-T_{b\alpha}} \quad (20).$$

The ratios between the volume of one mole of the material in liquid state and the one in gaseous state are neglected in Eqs (17) and (18) as $\frac{V_{\mu l\kappa}}{V_{\mu v\kappa}} \approx 0 \wedge \frac{V_{\mu l\alpha}}{V_{\mu v\alpha}} \approx 0$. So, the saturated vapour pressures for the cathode and the anode materials are:

$$p_{v\kappa}(T_\kappa) = p_{vb}\cdot e^{-\frac{L_{vb\kappa}}{R\cdot(T_{c\kappa}-T_{b\kappa})}\left(\frac{T_{c\kappa}}{T_\kappa}-\frac{T_{c\kappa}}{T_{b\kappa}}-\ln\frac{T_\kappa}{T_{b\kappa}}\right)} \quad (21),$$

$$p_{v\alpha}(T_\kappa) = p_{vb}\cdot e^{-\frac{L_{vb\alpha}}{R\cdot(T_{c\alpha}-T_{b\alpha})}\left(\frac{T_{c\alpha}}{T_\kappa}-\frac{T_{c\alpha}}{T_{b\alpha}}-\ln\frac{T_\kappa}{T_{b\alpha}}\right)} \quad (22),$$

where $p_{vb} = 101.325\ [\mathrm{kPa}]$ is the saturated vapour pressure of any material at the boiling temperature by the definition of the boiling point.

Presuming the current density is known, the density of the electrons is calculated:

$$n_e = \frac{j}{e\cdot\left(\eta_\kappa\cdot v_{e\kappa\_avg} + (1-\eta_\kappa)\cdot v_{e\alpha\_avg}\right)} \quad (23),$$

where the v_{ek_avg} and v_{ea_avg} are the average values of the velocities of the electrons that are to ionize the cathode and the anode atoms respectively. According to the ideal gas law we get the following equation:

$$p_{v\kappa}(T_\kappa)+p_{v\alpha}(T_\kappa) = \max\left(\frac{n_e}{\alpha\cdot N_A}\cdot R\cdot T_\kappa, p_{ext}\right) \quad (24).$$

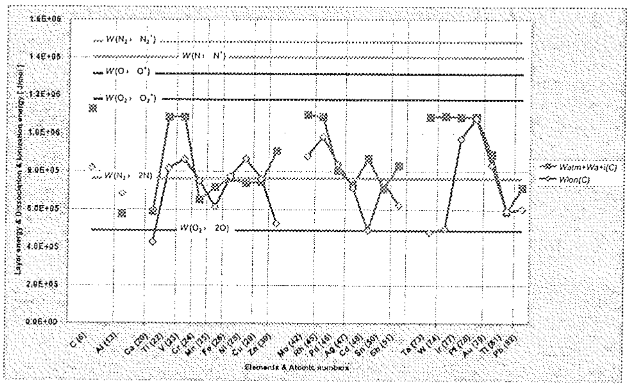

Fig. 4: The layer energy in comparison to the ionization and dissociation energy of nitrogen and oxygen.

Legend:

Watm+Wa+i(C) *layer energy of free atoms layer and partly ionized layer together - anode is carbon,*

Wion(C) *layer energy of fully ionized layer - anode is carbon,*

$W(N_2 \rightarrow N_2^+)$ *the first ionization energy of nitrogen molecules,*

$W(N \rightarrow N^+)$ *the first ionization energy of nitrogen atoms,*

$W(N_2 \rightarrow 2N)$ *dissociation energy of nitrogen molecules,*

$W(O_2 \rightarrow O_2^+)$ *the first ionization energy of oxygen molecules,*

$W(O \rightarrow O^+)$ *the first ionization energy of oxygen atoms,*

$W(O_2 \rightarrow 2O)$ *dissociation energy of oxygen molecules.*

The ionization coefficient α is less than 1 with the whole conducting volume of the gap. The cathode temperature is calculated from this Eq. (24) by some iterative method.

The pressure of the external medium is $p_{ext} = 101.325\ [\mathrm{kPa}]$ when it is air, but zero with vacuum.

The question is, why the external medium is so important. The external medium is air, so it contains the nitrogen molecules (78%) and the oxygen molecules (21%). If the saturated vapour pressure of the catode and the anode atoms and ions together is less than the pressure of external medium the molecules of the external gas are also present in the gap, but their partial pressure is less than the external pressure. There are also the collisions between the electrons and the nitrogen and the oxygen molecules. This collisions are elastic, unless the kinetic energy of the electons is sufficient to ionize the nitrogen and the oxygen moleculas, or to dissociate them and further on, to ionize the gaseous atoms. Though the collisions are elastic, the velocity vector of the electrons changes by impacts, and cosequently the final kinetic energy of the elecrons changes. The sufficient kinetic energy of the electrons for full ionization is gained after they have moved along the path through some layer, so we define this energy to be the layer energy. According to Eq. (14) the layer energy is not depended on the particles, which are either the electrons or the ions. The layer energy of the free atoms layer and the partly ionized layer together and the layer energy of the fully ionized layer for several cathode materials, while the anode is carbon, are compared with the first ionization energy of the molecules, the dissociation energy and the first ionization energy of the atoms of nitrogen and of oxygen. The results are shown in Fig. 4. The dissociation energy of the nitrogen and the oxygen molecules is achieved with some cathode materials in the first two layers of the arc. But no ionization energy of the nitrogen and the oxygen particles is ever achieved. The elastic collisions and the dissociation of the nitrogen and the oxygen molecules waste the kinetic energy of the electrons, so that the total ionization of the cathode and the anode atoms is not attained in the fully ionized layer. To establish the arc, the cathode temperature is increasing, so that the total saturated vapour pressure of the cathode and the anode atoms and ions forces the nitrogen and the oxygen particles out of the conducting volume of the gap. Then the conditions for the arc formation exist.

The current density due to the thermionic emission is defined by the Richardson - Dushman equation /5/:

$$j_T = \frac{4\cdot\pi\cdot m_e\cdot e\cdot k^2}{h^3}\cdot T_K^{\,2}\cdot e^{\frac{e}{k\cdot T_K}\left(-\frac{V_{\phi K}}{\upsilon}+\sqrt{\frac{e\cdot\beta\cdot E}{4\cdot\pi\cdot\varepsilon_0}}\right)} \qquad (25).$$

The presumptions have the following effects on the total current density and on the crosssection of the conducting path between the cathode and the anode, that is the crosssection of the arc:

$$E = const \;\Rightarrow\; j = j_E + j_T \qquad (26),$$

$$I = const \;\Rightarrow\; A_{arc} = A_{scl}\cdot\frac{j_E}{j_E + j_T} \qquad (27).$$

The current density increases, but the cross-section constricts. A narrow ionized conducting channel, something similar as a pilot streamer at discharges /12/, arises. It is a hot flow of the electrons, the cathode and the anode ions and atoms. The absence of the ions of the air gases of the external medium is the main difference between the drawn and the discharge arcs. There is a loop in this procedure: Eq. (26) effects Eq. (23), but the algorithm converges.

The results of the temperature calculations for the electrodes, both of the same metal, are presented in Fig. 5. The comparison is carried out between the air medium and the vaccum, and further on, the cathode temperatures toward the melting and the boiling temperatures. The cathode temperatures are somehow between the melting and the boiling points, but there are some exceptions.

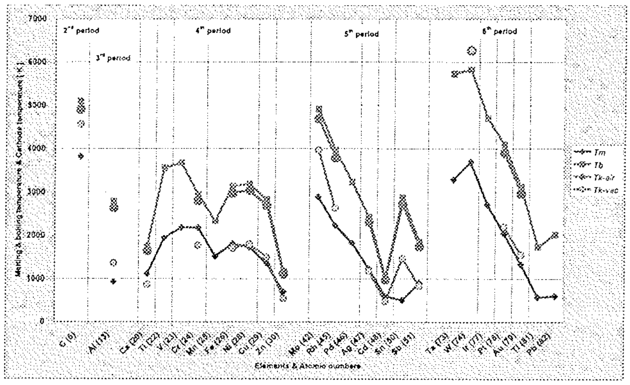

Fig. 5: The cathode temperatures, melting and boiling temperatures.

Legend:

Tm	*melting temperature,*
Tb	*boiling temperature,*
Tk-air	*cathode temperature in air,*
Tk-vac	*cathode temperature in vacuum.*

So far we presume that both, the cathode and the anode atoms and ions are present in the conducting volume of the gap. The number of the cathode atoms and ions N_{a+ik} and the number of the anode atoms and ions N_{a+ia} are natural numbers unequal to zero. But it could happen, especially with refractory materials, that either one of them is zero. It means, that it is possible with the great mathematical confidence, that the atoms and ions of one material are absent in the tiny conducting volume of the gap. The Dirac and Heaviside functions are introduced to include such cases into the mathematical model. The used Dirac function is symmetrical, but the Heaviside function is asymmetrical:

$$\begin{aligned} \upsilon_+(x \le 0) &= 0 \\ \upsilon_+(x > 0) &= 1 \end{aligned} \qquad (28).$$

While the electron moves along the sufficient path, it achieves the energy to ionize the cathode atom, which has the corresponding voltage drop, derived from Eq. (15), and equals to:

$$U_K = \frac{W_{i1K}}{e\cdot N_A}\cdot\upsilon_+\left(N_{a+iK}\right) \qquad (29).$$

The corresponding voltage for the ionization of the anode atoms is:

$$U_{\alpha} = \frac{W_{i1\alpha}}{e \cdot N_A} \cdot \upsilon_{+}\left(N_{a+i\alpha}\right) \quad (30).$$

The voltage drop of the free atoms layer is:

$$U_{atm} = \min_{U \neq 0}(U_{\kappa}, U_{\alpha}) = \\ = \frac{\min(W_{i1\kappa} \cdot (\delta(N_{a+i\kappa})+1), W_{i1\alpha} \cdot (\delta(N_{a+i\alpha})+1))}{e \cdot N_A} \quad (31).$$

The voltage of the partly ionized layer is:

$$U_{a+i} = \max(U_{\kappa}, U_{\alpha}) - \min_{U \neq 0}(U_{\kappa}, U_{\alpha}) = \\ = \frac{\max(W_{i1\kappa} \cdot \upsilon_{+}(N_{a+i\kappa}), W_{i1\alpha} \cdot \upsilon_{+}(N_{a+i\alpha}))}{e \cdot N_A} - U_{atm} \quad (32).$$

All atoms are ionized at the far end of the partly ionized layer, looking from the cathode. Then the full ionized layer begins. In this layer, the ions are accelerated toward the cathode by the electric field. But, when they leave this layer, they are recombined, because they are not immune to the collisions from the electrons. From this point on, the recombined atoms move toward the cathode uniformly with the constant velocity, hence there is no force on them due to the electric field, for they are neutral. The velocity of the ions is gained in the fully ionized layer, and the corresponding kinetic energy must be sufficient to heat the cathode to cause the vaporization or the sublimation of the cathode material, and to cause also the thermionic emission of the electrons. The average kinetic energy of the ion, either it is of the cathode or of anode material, is:

$$W_{ionk} = \frac{\eta_{\kappa} \cdot L_{v\kappa}}{N_A} + \frac{e \cdot V_{\phi\kappa}}{\upsilon} \quad (33),$$

Because the energy of the work function is:

$$\phi_{\kappa} = e \cdot V_{\phi\kappa} \quad (34)$$

the voltage drop of the full ionized layer, derived from Eq. (15), is:

$$U_{ion} = \frac{\eta_{\kappa} \cdot L_{v\kappa}}{e \cdot N_A} + \frac{\phi_{\kappa}}{e \cdot \upsilon} \quad (35).$$

The infimum arc voltage is the sum of the layers voltages - Eqs (31), (32) and (35), and it is:

$$U_m = U_{atm} + U_{a+i} + U_{ion} = \\ = \frac{\max(W_{i1\kappa} \cdot \upsilon_{+}(N_{a+i\kappa}), W_{i1\alpha} \cdot \upsilon_{+}(N_{a+i\alpha}))}{e \cdot N_A} + \\ + \frac{\eta_{\kappa} \cdot L_{v\kappa}}{e \cdot N_A} + \frac{\phi_{\kappa}}{e \cdot \upsilon} \quad (36).$$

There is one parameter in the calculation of the cathode temperature - Eq. (24), which has not been fully defined. It is the ionization coefficient of the whole conducting volume. The volume occupied only by the ions is the volume of the fully ionized layer. There are some ions also in the partly ionized layer, but we neglect them when estimating the ionization coefficient. The density of the ions and atoms together is constant throughout the whole conducting volume, so the coefficient is the ratio of the volume of the fully ionized layer and the whole conducting volume. Presuming the crosssection area is uniform the coefficient is the ratio of the thickness of the fully ionized layer and the arc path. Because the electric field is homogeneous the ionization coefficient becomes:

$$\alpha = \frac{U_{ion}}{U_m} \quad (37).$$

So, this equation leads to another loop in the mathematical model - Eq. (24), but the convergence still exists.

The results of this mathematical model are shown in Fig. 6. The infimum arc voltage directly depends on the energy of the first ionization of the cathode and the anode material, on the latent heat of vaporization of the cathode material, and on the energy of the work function of the cathode. These quantities are the properties of the contact materials. But, this voltage also depends on the cathode temperature indirectly through the ratio between the number of the free cathode atoms and ions and the number of all free atoms and ions. The whole model is recalculated at the contact current I = 1 A, at the crosssection of the conducting path A_{scl} = $5\times10^{\,6}$ m^2, and the coefficients b, u and μ being unit.

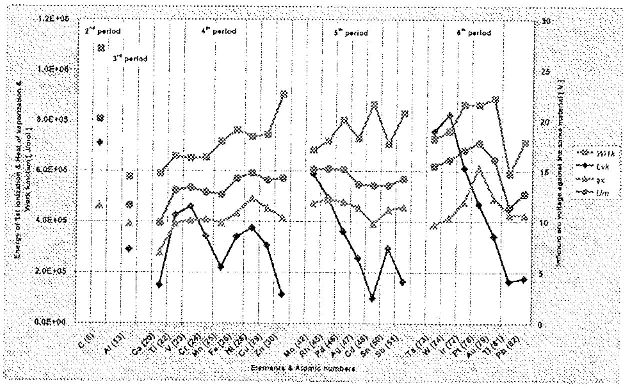

Fig. 6: The infimum arc voltage with its influential quantities depended on the cathode material against the anode of the same material.

Legend:

Wi1k *energy of 1st ionization of cathode material,*
Lvk *latent heat of vaporization of cathode material,*
ϕk *energy of work function,*
Um *infimum arc voltage.*

It has to be emphasized that the cathode temperature depends mainly on the latent heat of vaporization of the cathode and the anode material, their boiling and critical temperatures, and the external pressure.

3. The comparison of the results

The infimum arc voltage is determined not only by the cathode material, but also by the anode material. The diagrams in Fig. 7 show the infimum arc voltage depended on the cathode material when facing the anode made by the same material, and the one of tungsten and of carbon. The next diagrams in Fig. 8 present the comparison of the infimum arc voltage achieved by our model and the values of the minimal arc voltages according to Holm, Fink and Gaulrapp /1/, which are considered as experimental values. But, as stated in the literature /1/ and presented in Fig. 8, several researchers got very different results of their experimental determination of the minimal arc voltage, although they are determined by the contact electrodes of the same material. These authors considered that minimal arc voltage is essentially determined only by the cathode material, nevertheless the contact electrodes are made of the different material. That is the significant difference from our model.

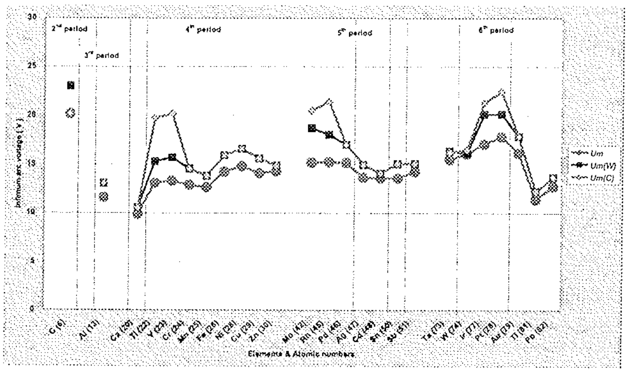

Fig. 7: The comparison of the infimum arc voltage of several combinations of materials.

Legend:
Um *infimum arc voltage with cathode and anode of the same material,*
Um(W) *infimum arc voltage with anode of tungsten,*
Um(C) *infimum arc voltage with anode of carbon.*

Some researcher of the commutation phenomenon /2/,/3/ discovered by their experiments, that there was practically no difference in the infimum arc voltage whether the cathode was made of carbon and the anode of copper, or just the opposite, the cathode was of copper and the anode of carbon. And their observations are confirmed by the model: the cathode against the anode C-Cu 12.5 V, Cu-C 15.6 V by the model, C-Cu or Cu-C from 14 V to 20 V by their measurements /2/, and C-Cu 20 V and Cu-C 13 V by Holm /1/, and further on, the cathode temperature 2835 K by the model and up to 2273 K by their estimation of the experiment /2/.

The comparison between the model results and the experimental results by several researchers /1/ is illustrated in Fig. 8. Having in mind uncertainties of the experimental results, and the fact that the results of the model are presented without any contamination of the electrodes and with no roughness of the contact surfaces, the results are estimated as very good and useful.

4. Conclusions

The evaluation of affinity of the contact materials to form the drawn electric arc carried out by defining the infimum arc voltage for several combination of the contact materials is only one aspect of determining the proper contact materials. The second parameter to be taken into account is the cathode temperature, which is also obtainable by this model, especially, when reducing it under the calculated value by heat transfer to the external medium, and so avoiding the arc.

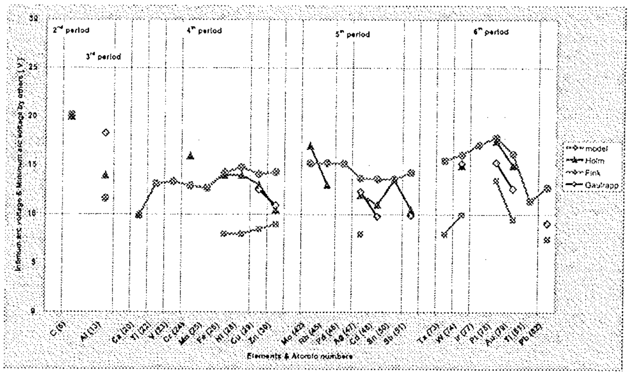

Fig. 8: The comparison of the infimum arc voltage and minimal arc voltage with the electrodes of the same material according to several researchers /1/.

Legend:
model *infimum arc voltage by our model,*
Holm *minimal arc voltage by Holm /1/,*
Fink *minimal arc voltage by Fink /1/,*
Gaulrapp *minimal arc voltage by Gaulrapp /1/.*

There are many mechanical properties, for instance elasticity, plasticity and hardness and the frictional wear of the materials and the electric properties as it is the contact and the bulk resistance, which must be considered when determining the contact materials for an application. Nevertheless, this model is a good mathematical tool to determine the contact material from the viewpoint of the drawn arc formation. The model is applicable to the switching contacts, and these conclusions are also valid for them.

5. Used symbols

α coefficient of ionization
β enhancement factor of electric field intensity
δ Dirac function
υ decreasing factor of work function
η_k ratio between number of free cathode atoms and ions and number of all free atoms and ions
ϕ_k work function energy in vacuum
ε_0 dielectric constant in vacuum
μ factor between rest and effective electron mass
A_{arc} cross-section of arc
A_{scl} cross-section of path of current conducting by t unnel effect
c light velocity in vacuum
e elementary charge
E electric field intensity
E^* enhanced electric field intensity
e^x exponential function
F force
h Planck constant
I contact current
j contact current density
j_E current density due to cold emission
j_T current density due to thermionic emission
k Boltzmann constant
L_{vk} molar latent heat of vaporization of cathode material
$L_{v\alpha}$ molar latent heat of vaporization of anode material
L_{vbk} molar latent heat of vaporization of cathode material at boiling temperature
$L_{vb\alpha}$ molar latent heat of vaporization of anode material at boiling temperature
m_a rest atom mass
m_e rest electron mass
m_e^* effektive electron mass
N_A Avogadro number
N_{a+ik} number of free cathode atoms and ions
$N_{a+i\alpha}$ number of free anode atoms and ions
n_e electron density
p_{vk} saturated vapour pressure of cathode atoms and ions
$p_{v\alpha}$ saturated vapour pressure of anode atoms and ions
p_{vb} saturated vapour pressure of boiling point
R general gaseous constant
s path through layer
T_k cathode temperatrure
T_{bk} cathode boiling temperature
$T_{b\alpha}$ anode boiling temperature
T_{ck} cathode critical temperature
$T_{c\alpha}$ anode critical temperature
U layer voltage
U_{a+i} partly ionized layer voltage
U_{atm} free atoms layer voltage
U_{ion} fully ionized layer voltage
U_m infimum arc voltage
v_+ asymmetrical Heaviside unit function
v particle velocity
v_{e_avg} average electron velocity
v_{ek_avg} average electron velocity up to ionization of cathode atom
$v_{e\alpha\_avg}$ average electron velocity up to ionization of anode atom
v_{e_rms} effective (root-mean-squere) electron velocity
$V_{\phi k}$ work function voltage
$V_{\phi k}^*$ decreaced work function voltage
$V_{\mu l\alpha}$ mole volume of anode material in liquid state
$V_{\mu lk}$ molar volume of cathode material in liquid state
$V_{\mu v\alpha}$ molar volume of anode material in vapour state
$V_{\mu vk}$ molar volume of cathode material in vapour state
W_{ek} electron kinetic energy
W_{i1} molar energy of 1st ionization
W_{i1k} molar energy of 1st ionization of cathode atoms
W_{i1a} molar energy of 1st ionization of anode atoms
W_{ionk} average ion kinetic energy

References

/1/ Holm, R.: *Electric contacts, theory and application*, fourth completely rewritten edition, Springer - Verlag, Berlin - Heidelberg - New York, 1979.

/2/ Rejec, J.: *Vpliv komutacije na življensko dobo univerzalnih motorjev* (An influence of commutation on the operating ability duration of universal motors), doktorska disertacija, Univerza v Ljubljani, Fakulteta za elektrotehniko, Ljubljana, 2004.

/3/ Benedičič B.: *Model komutacije univerzalnega motorja* (A model of commutation of the universal motor), doktorska disertacija, Univerza v Ljubljani, Fakulteta za elektrotehniko, Ljubljana, 2004.

/4/ Slade, P.G.: *Electrical Contacts, Principles and Applications*, Marcel Dekker, Inc., New York/Basel, 1999.

/5/ Sproull, R.L.: *Modern Physics*, second edition, John Wiley & Sons, Inc., New York - London, 1966.

/6/ Bender, J.P.; Wager, J.F.: *ACTFEL Device Modeling via SPICE*, Department of Electrical and Computer Engineering, Center for Advanced Materials Research, Oregon State University, Corvallis, Oregon 97731-3211.
http://ece.oregonstate.edu/matdev/pub/ACTFEL-SPICE.ppt, 26.03.2003/01.07.2005.

/7/ Habermann, T.: *Rastermikroskopische Untersuchung der Feldemission von Metall- und Diamantkathoden*, Dissertation zur Erlangung des Doktorgrades des Fachbereichs Physik der Bergischen Universität Wuppertal, Fachbereichs Physik der Bergischen Universität Gesamthochschule Wuppertal, Wuppertal 1999.
http://www.bib.uni wuppertal.de/elpub/fb08/diss1999/habermann, 26.03.2003/01.07.2005

/8/ F.R. Newman Laboratory for Elementary-Particle Physics (LEPP), a research laboratory of the Cornell University Department of Physics: *Basics of SRF Cavities 3.1: Fowler-Nordheim Theory, Superconducting RF Cavities: A Primer*.

http://www.lns.cornell.edu/public/CESR/SRF/BasicSRF/SRFBas31.html, 26.03.2003/01.07.2005.

/9/ Sauer, D.; Stuve E.M.: *Changes in Work Function*, INTEL, MS/AL3-66. 5200 N.E. Elam Young Park-way, Hillsborough, OR 97124; University of Washington, Department of Chemical Engineering, BF-10, Seat-tle, WA 98195; November 1994.

http://www.mcallister.com/papers/stuve.html, 28.03.2003.

/10/ Zeghbroeck, Bart Van: *Principles of Semicon-ductor Devices*, Copyright B. Van Zeghbroeck, 2002 (Chap-ter 3.9: Currents through insulators).

http://ece.colorado.edu/~bart/book//chapter3/pdf/ch3_9.pdf/, 26.03.2003/01.07.2005.

/11/ Yavorsky, B.; Detlaf, A.: *Handbook of Physics*, English translation, Mir publishers, Moscow, 1975.

/12/ Perez, R.: *Handbook of Electromagnetic Compatibility*, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, Academic Press, San Diego - New York - Boston - London - Sydney - Tokyo - Toronto, 1995.

Asst Prof. Dr France Pavlovčič, univ.dipl.ing.
Ministry for environment and spatial planning,
Environmental Agency of the Republic of Slovenia,
Vojkova 1b, SI - 1000 Ljubljana, Slovenia
tel.: +386 (0)1 4784 098,
fax: +386 (0)1 4784 054,
E-mail: france.pavlovcic@gov.si

Prof. Dr Janez Nastran, univ.dipl.ing.
University of Ljubljana, Faculty of electrical engineering, Tržaška 25, SI - 1000 Ljubljana, Slovenia
tel.: +386 (0)1 4768 282,
fax: +386 (0)1 4264 647,
E-mail: janez.nastran@fe.uni-lj.si

Prispelo (Arrived): 19.09.2005 Sprejeto (Accepted): 30.09.2005