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Using of Acoustic Models in Mechanical Diagnostics

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This paper presents an acoustical model for control and diagnostics of single stage gear wheels. The model is based on various methods and procedures that as a result provide information about the generator's condition, the gear in particular. The acoustical model is part of a complex system that units' different models to meet diagnostics of single stage gear wheels as precise as possible. Using the adaptive FIR filter, acoustical model enables the calculation of impulse response for different notch lengths between 0 and ac. The acoustical model consists of digital FIR filter, modified by LMS algorithm, used to calculate impulse responses in non-linear systems, the model for the calculation of any impulse response and the frequency analysis with the use of FFT for the simulation of frequency spectrums. Frequency spectrum of the simulated sound signal enables an analysis of the error that can be used for calculating the remaining service life and/or determining the control cycle of maintenance.

Keywords: mechanical fault diagnostics, acoustic models, impulse response, adaptive algorithm

0 INTRODUCTION

Undisturbed functioning of engineering systems is a prerequisite for effective operation of industrial processes. Therefore, a constant and efficient maintenance is crucial. This includes not only replacing the damaged and worn out parts and regular refits, but the use of various maintenance models that are based on surveillance and diagnostics of the current condition of technical systems. Such models of preventive maintenance use the method of condition maintenance. This means they help to discover damage in its early stage, i.e. in the initiation phase. Apart from considerable savings, this approach also ensures a continuous and reliable course of the engineering process without unnecessary interruptions. For control and diagnostics of the engineering system in question, sound and vibration are two key parameters.

Despite the generally accepted fact that vibrations are important and quality information carriers, the significance of sound has grown in recent years [1]. The main reason for that is a disadvantage of the vibration measurement method, since it requires an interference with into the measured system. The accelerometers need to be fastened (usually screwed) onto the specific spot, which damages the measured object and thus interferes with the measurement results, sometimes even shortening the object's service life. The sound measurement method avoids this inconvenience because there is no physical contact between the measured object and the microphone. For this reason, we initiated the development of a mathematical model that allows to use a sound signal simulation. The comparison of the measured and thus simulated sound signal enables to analyze tooth damage, determine its dimension and foretell its progress. It forms the basis for determining the remaining service life and/or the maintenance process.

1 ACOUSTIC DIAGNOSTIC SYSTEM

The whole complex diagnostic system consists of mathematical gear wheel simulator, an acoustic model and a model for calculating impulse response for any notch length between 0 and adop. It is an optimum system for detection of defects using acoustics, and it is the basis for estimation of the remaining lifespan or ascertainment of control cycles on the basis of acoustic responses. The structure of a system is shown in Figure 1. Mentioned modules are joined and simulated using the Simulink® simulation

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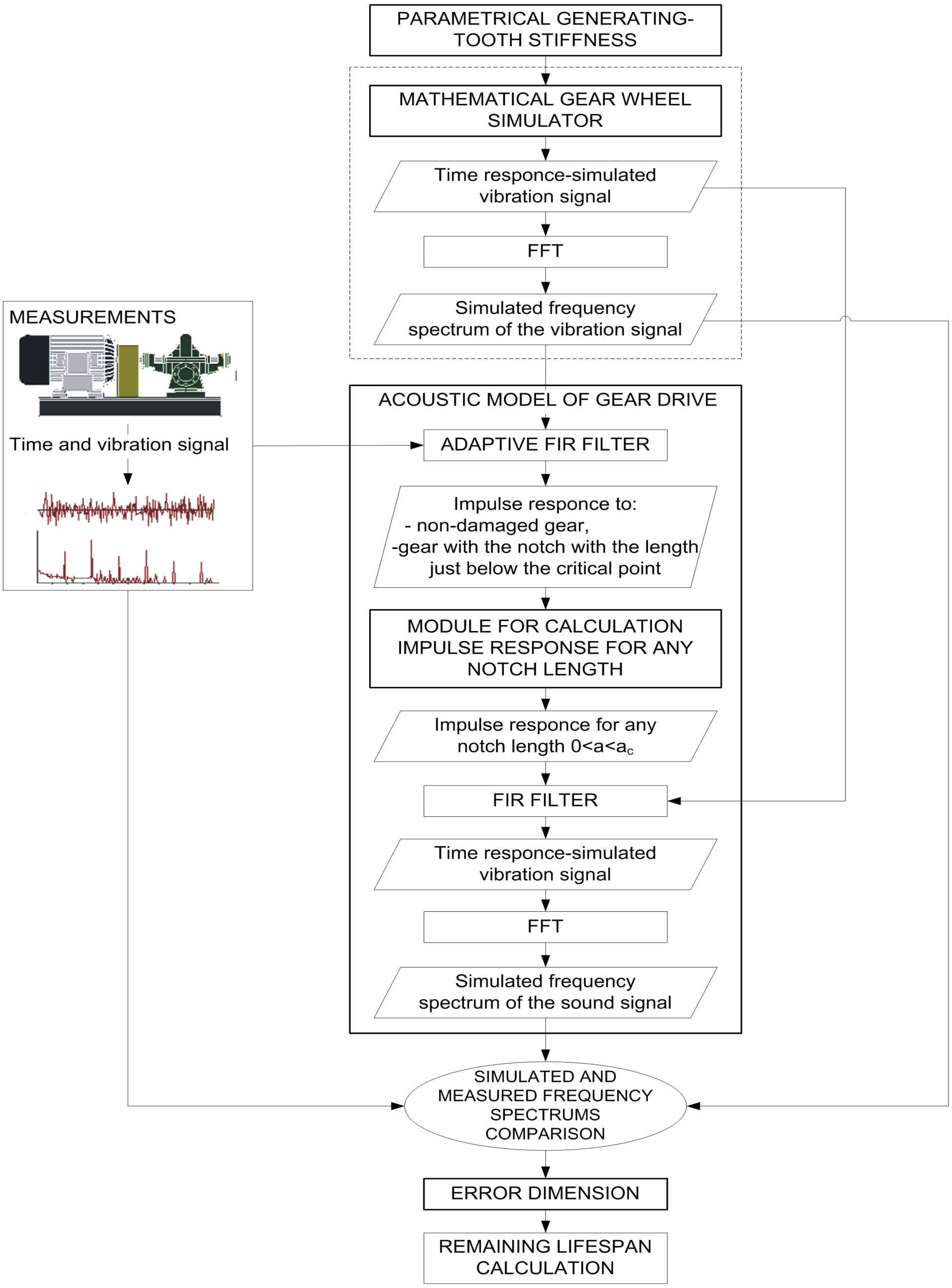

Fig. 1. *Structure of the acoustic diagnostic system*

Package. The gear wheel simulator represents modeling of the tooth grip with parametrical generating – tooth stiffness. The variable tooth stiffness is represented by perpendicular signal. The result of simulation is the temporal response which can be explained as moves or a simulated vibration signal. With the use of FFT, it is transformed into the frequency spectrum of the simulated vibration signal.

The acoustic model consists of a digital FIR-filter and an adaptive LMS-algorithm. The latter enables the weighted vector to be corrected in each iteration, which represents learning according to the principle of neural networks. From the measured sound signals and vibrations, it calculates the impulse response or the transfer function between them.

After joining gear wheel simulator and acoustic model, the thus calculated transfer function enables a simulation of the sound signal.

A model that enables calculation of the impulse response for any notch length on the interval from 0 to adop, based on the calculated impulse response for a gear wheel without a notch and one with a notch of critical length was also developed.

Since the measurement was done on a gear without a notch and on one with a notch of critical length, the impulse responses calculated with the use of the adaptive FIR-filter correspond to the gears without the notch of critical length and with it, respectively. The impulse response for any length of the notch between 0 and ac is calculated by using a module for calculating any impulse response. The impulse calculated in this way and the simulated vibration signal acquired from the mathematical module of the gear (both values for the same notch length, of course) is then inserted into the FIR-filter. The obtained result is a time signal that is called simulated sound signal. By the use of FFT it is transferred into the frequency spectrum of the simulated sound signal.

In the end, the frequency spectrums of simulated vibration signal and sound signal are compared to the frequency spectrums of measured vibration signal and sound signal. The result thus obtained is the dimension of the error that can be used in the module for calculating the remaining service life and/or determining the control cycle of maintenance [2].

2 GEAR WHEEL SIMULATOR

The mathematical gear wheel simulator represents modeling of the tooth grip with parametrical generating – tooth stiffness.

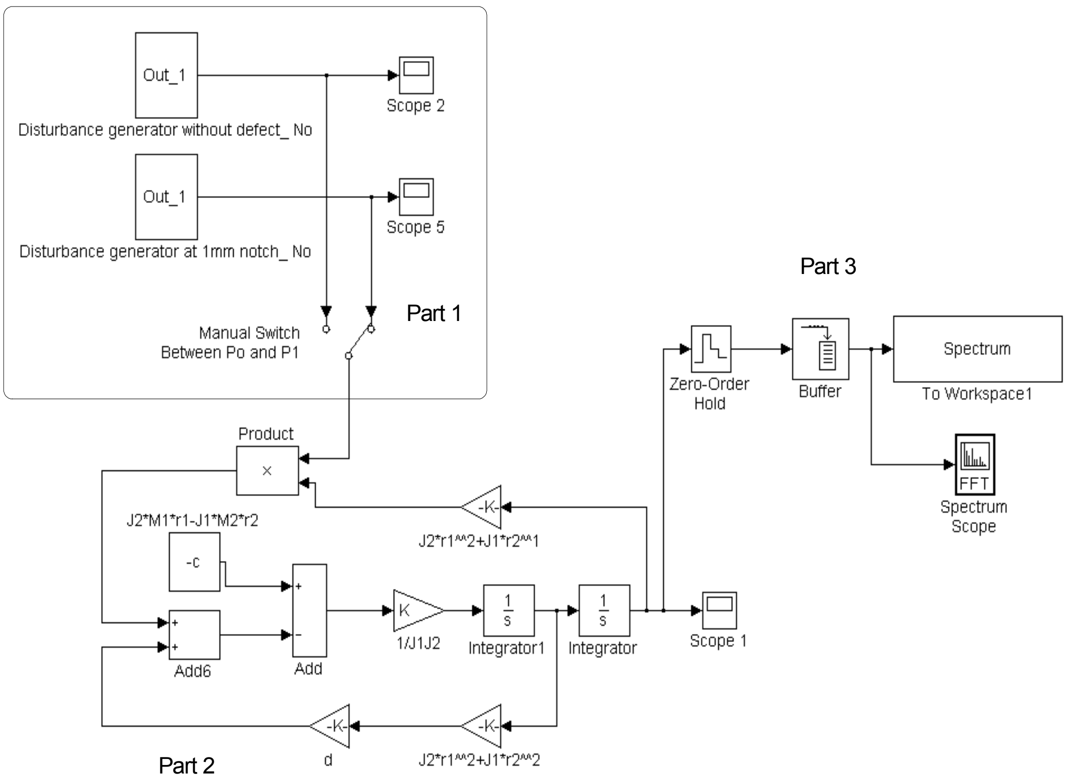

Fig. 2. *Block diagram of the gear wheel simulator*

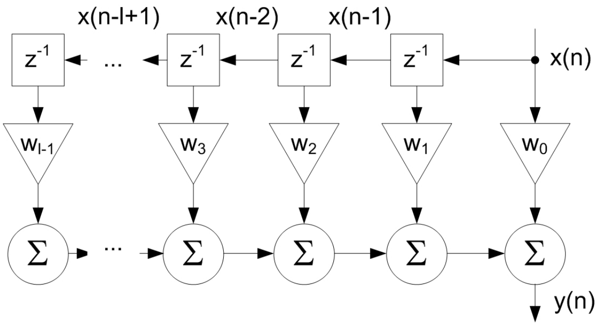

Fig. 3. *Digital FIR-filter*

The simulator is generated by a perpendicular signal that represents the variable tooth stiffness. The mechanical substitution model for deriving the gear simulator is described in detail by [3]. It consists of the gear wheel Z_1 with the mass m_1, basic circle radius r_1, mass inertia moment J_1 and is loaded with the torsion moment M_1, and the gear wheel Z_2, with the mass m_2, basic circle radius r_2, mass inertia moment J_2 and is loaded with the torsion moment M_2. The module has two degrees of freedom equivalent to generalized coordinates θ_1 and θ_2 representing the angles of rotation of both gear wheels. Tooth engagement is modeled with a spring damping element with the stiffness c and damping d. The module has no backlash, since no high dynamic forces that could cause tooth divergence to appear. By introducing the dynamic transmission the system of equation is translated into one degree of freedom [1].

The simulation module considers the length of the notch/crack up to the critical length a_c [4] and [5], as the crack expands exponentially.

Figure 2 shows the block diagram of the gear wheel simulator developed in Matlab® *Simulink* program package. It consists of four main parts. Part 1 contains blocks which represent parametric system excitation, part 2 combines blocks representing differential equation of the system and finally part 3 represents blocks which execute frequency analysis of calculated time response. Disturbance generator is marked with solid line frame.

3 ACOUSTIC MODEL OF GEAR DRIVE

This model consists of the adaptive FIR-filter used to calculate impulse responses in non-linear systems, the model for the calculation of any impulse response and the frequency analysis with the use of FFT for the simulation of frequency spectrums.

3.1 Adaptive FIR-Filter

The structure of the digital FIR-filter is shown in Figure 3. Digital filter is used in discrete regulation systems and plays a similar role as the transfer function in linear systems. FIR-filter and its coefficients describe the impulse response of the system. The digital FIR-filter represents filtration of the input signal $x(n)$ in steps or iterations (Figure 3), where its individual coefficients (l^{th} component of the weight vector $\vec{w}$) $w_l(n)$ represent or describe the impulse response of the system or transfer function.

The mathematical equation of the FIR algorithm is:

$$y(n) = \sum_{l=0}^{L} w_l(n)x(n-l) \qquad (1)$$

where L is the degree of the digital algorithm or the size of the FIR-filter.

The digital filter is only usable in linear systems. In our case, the notch in the tooth root is non-linear. For this reason, the FIR-filter has been joined with the adaptive algorithm, which in every iteration calculates the correction for improving the weight vector $\vec{w}$ from the previous step. Figure 4 shows a diagram representing the digital FIR-filter with the adaptive LMS-algorithm. $x(n)$ represents the input signal, $d(n)$ stands for the output of the system, $y(n)$ is output of the digital FIR-filter, whereas $e(n)$ is the error. $A(z)$ is the transfer function of the acoustic path.

It is clear from Figure 4 that the output of the digital filter $y(n)$ must equal the $d(n)$ output if the error is to be minimal. In order for the error to equal zero, the digital FIR-filter must describe the complete impulse response of the acoustic way $A(z)$ with vector **w**.

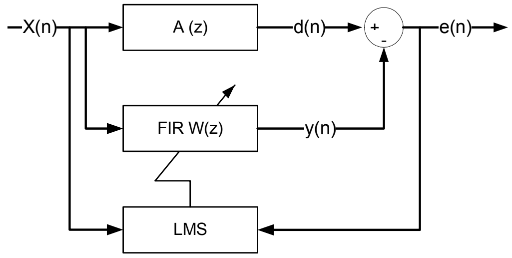

Fig. 4. *Adaptive digital FIR-filter*

The adaptive algorithm acquired in this way is therefore used with the purpose of simultaneous identification of the impulse response with the FIR-filter that is based on the LMS (Least Mean Square) of the [6].

Figure 5 shows the block scheme for calculating the impulse response using the Matlab® *Simulink* program package.

The two blocks on the left side of the scheme represent the input of the measured signals (vibration and sound) into the model; in the middle blocks, the impulse response is calculated and the data stored in the "Workspace"; the blocks on the right side of diagram carry out the frequency analysis of the input signals and store them in the "Workspace" too.

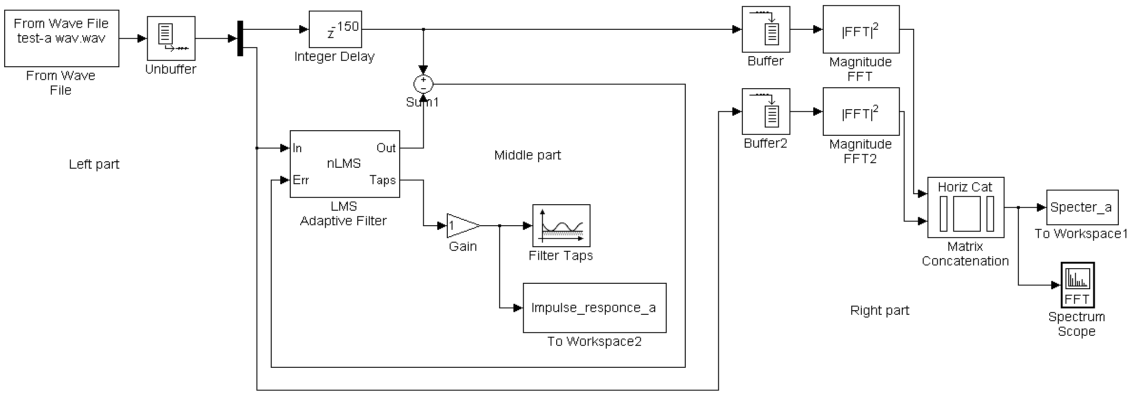

Fig. 5. *Block diagram for simulating the impulse response using the Matlab® Simulink*

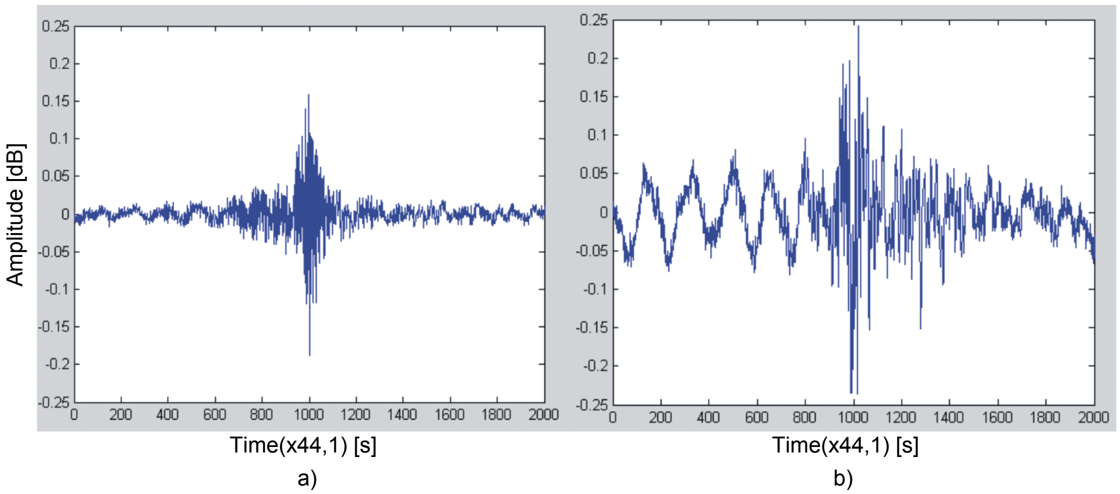

Fig. 6. *Impulse response of: a) undamaged gear drive; b) gear drive with 1mm notch in tooth root of gear wheel*

The block diagram is used for simulating the impulse response or the transfer function between the two measured signals from vibration into sound. Further on, the simulated impulse response is used in the combined mathematical model, which simulates a gear wheel and enables the calculation or simulation of the sound signal using the FIR-filter.

Figure 6 presents results of impulse response of undamaged gear drive and gear drive with 1mm notch in tooth root of gear wheel. It is obvious from the Figure 6 that in case of damage the impulse response amplitude is approximately 50 % higher. The impulse response in the first case is clear, while in the second case in scattered with many oscillations. Such result is expected since the tooth damage cause enlargement of acoustic and vibration signal.

3.2 Module for Calculation of any Impulse Response

These two calculated impulse responses (for the undamaged gear wheel and for the gear wheel with a critical damage) will be used as two referential responses for two extreme measured results. The crack appears in specific number of cycles N (point A) and expands exponentially until it reaches the critical length (a_c), which is followed by the tooth breakage. In practice, the crack is not allowed to expand to the size a_c, but only to the largest acceptable size a_{dop} ($a_{dop} \approx 0{,}5\, a_c$) [2] and [7]. For estimating the rest of the service life, the interval of the crack from 0 to a_{dop} is important. In the case of an undamaged gear, the impulse response represents the lower limit (point A); for a gear with a 1 mm notch in its root, however, the impulse response represents the upper limit (a_c). By selecting any point T, the impulse response for any notch length $a_t : 0 < a_t < a_c$ can be calculated. The impulse response values between points A and a_c can theoretically change with any function; however, let us presuppose they change with e^x as shown in [8] and [9].
The impulse response of an undamaged gear wheel can be represented by the vector:

$$\vec{r}\left(0\right)=\left(r_{1_0},r_{2_0},...,r_{n_0}\right) \quad (2)$$

and the impulse response of a gear with a critical notch length a_c by

$$\vec{r}\left(a_c\right)=\left(r_{1_{ac}},r_{2_{ac}},...,r_{n_{ac}}\right) \quad (3)$$

Let us presuppose that the values of impulse responses with the growing crack change with the ln function, which is also confirmed by [2] and [10].

The impulse response for any notch length a_t, $\vec{r}\left(a_t\right)=\left(r_{1_{at}},r_{2_{at}},...,r_{n_{at}}\right)$ is therefore calculated in parts:

$$r_{1_{at}}=r_{1_0}\cdot e^{\lambda_1\frac{t}{a_c}}=r_{1_0}\cdot\left(\frac{r_{1_{ac}}}{r_{1_0}}\right)^{\frac{t}{a_c}}, \text{ where } \lambda_1=\ln\frac{r_{1_{ac}}}{r_{1_0}}, \quad (4)$$

$$r_{2_{at}}=r_{2_0}\cdot e^{\lambda_1\frac{t}{a_c}}=r_{2_0}\cdot\left(\frac{r_{2_{ac}}}{r_{2_0}}\right)^{\frac{t}{a_c}}, \text{where } \lambda_2=\ln\frac{r_{2_{ac}}}{r_{2_0}}, \quad (5)$$

$$r_{n_{at}}=r_{n_0}\cdot e^{\lambda_1\frac{t}{a_c}}=r_{n_0}\cdot\left(\frac{r_{n_{ac}}}{r_{n_0}}\right)^{\frac{t}{a_c}}, \text{ where}$$

$$\lambda_n=\ln\frac{r_{n_{ac}}}{r_{n_0}} \text{ and } t\in\left[0,a_c\right]. \quad (6)$$

4 THE MODULE USED TO CALCULATE THE REST OF THE SERVICE ACOUSTIC MODEL OF GEAR DRIVE LIFE

Based on the models presented, we can analyze tooth damage level, which represents the basis for further calculation of the remaining service life [11] to [13]. We calculate the service life of the entire gear L_{GW} as a sum of the service lives of all gear wheels [14]:

$$L_{GW}=\left(\sum_{i=1}^{n}L_i^{-e}\right)^{-\frac{1}{e}}, \quad (7)$$

where: L_i is the service life of the gear wheel, e is Weilbull's exponent.

If the probability is taken into account too, Eq. (23) enables us to calculate the relevant service life of the entire gearing L_{GW} for the desired reliability different from 90 percent reliability.

$$\log\left(\frac{1}{S_{GW}}\right)= \log\left(\frac{1}{0.9}\right)\left[\left(\frac{L_{GW}}{L_1}\right)^{e_1}+\left(\frac{L_{GW}}{L_2}\right)^{e_2}+...\right] \quad (8)$$

Table 1. *Various parameters*

	Test 1	Test 2	Test 3	Test 4
Notch [mm]	0	0	1	1
Revolution speed [min^{-1}]	175,5	351	175,5	351

5 EXPERIMENTAL CONFIRMATION OF THE SYSTEM

The developed system was tested on the single stage gear wheel with helical gears having diagonal teeth. Sound and vibration measurements were carried out and frequency spectrums thus achieved were compared with the calculated ones. Measurements and simulations were carried out with various parameters (Table 1).

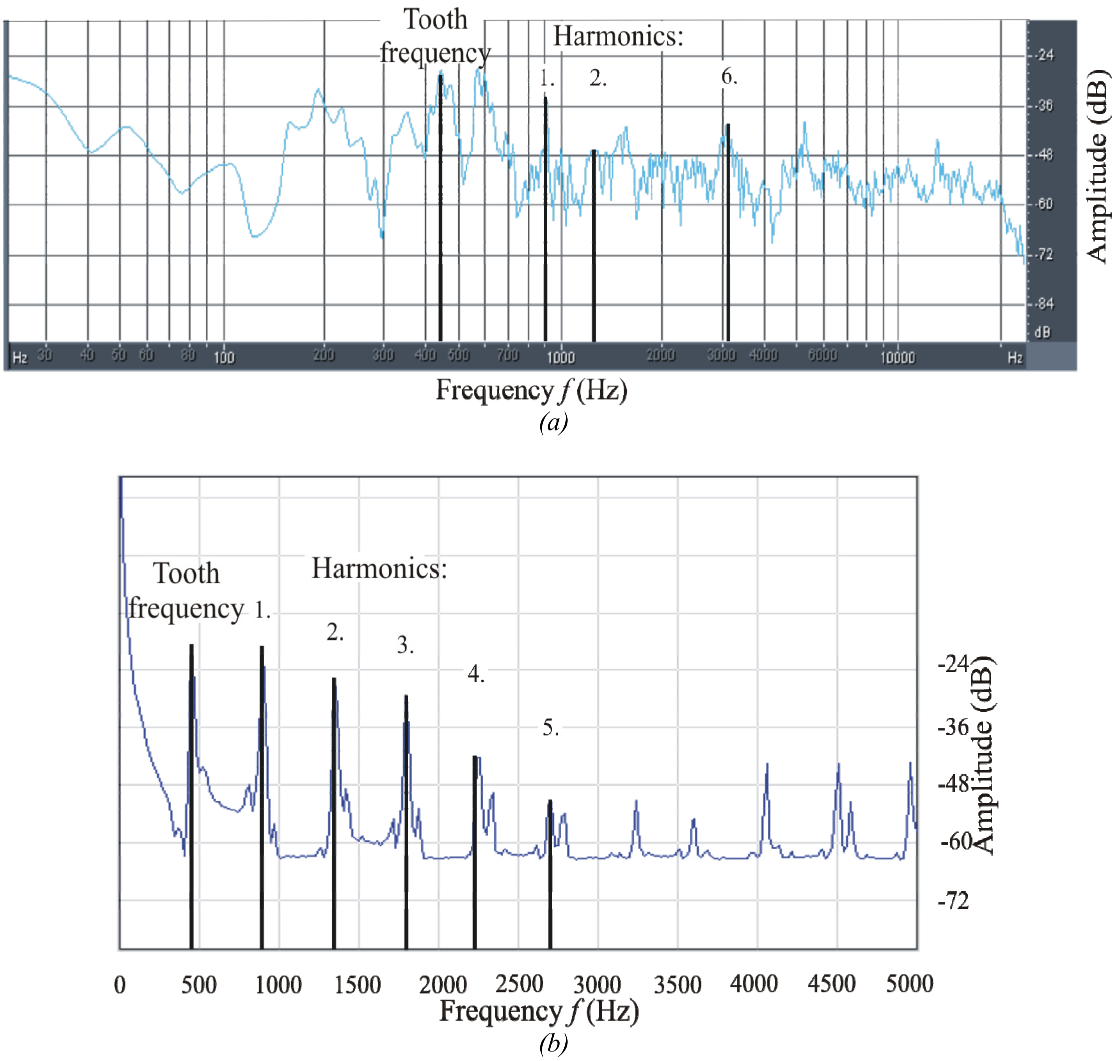

Fig. 7. *Frequency spectrum for an undamaged gear at the rotation speed 351 min^{-1} a) measured and b) simulated*

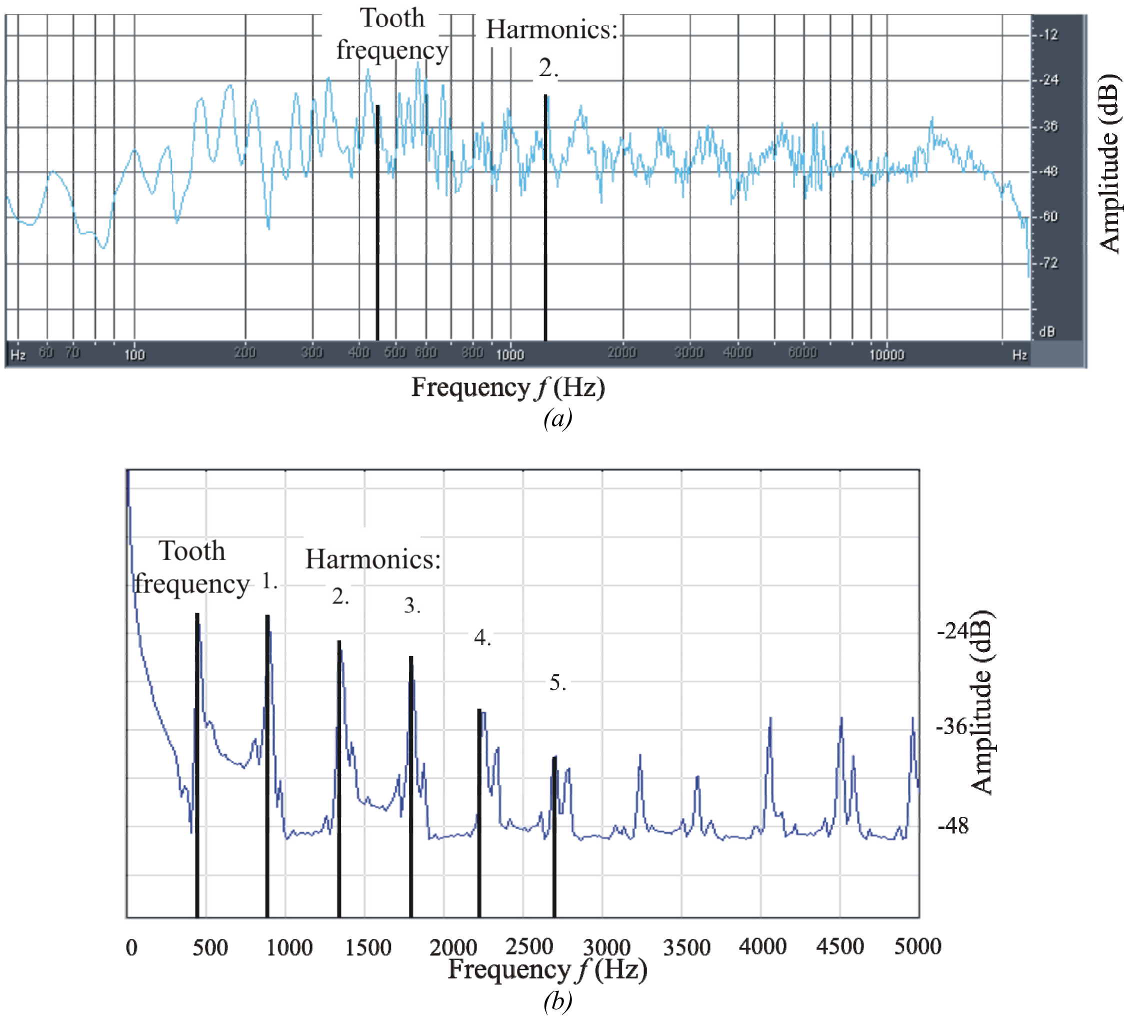

Fig. 8. *Frequency spectrum for damaged gear at the revolution speed 351 min^{-1} a) measured and b) simulated*

Figure 7 and 8 show the comparisons of measured frequency spectrums and simulated sound signals.

Simulated signal spectrums (Figures 7 b and 8 b) contain only the frequency components typical for tooth engage and their higher harmonics (marked in the figures), while the measured signal spectrums (Figures 7 a and 8 a) besides tooth frequencies contain the background noise and the frequency components of other machine parts.

This is understandable, since in the model we have considered only the parametrical generating with tooth stiffness and disregarded other sound sources.

It is noticed in the measured spectrums that some higher harmonics do not appear or stand out. This is caused by the electromotor working within the measurement area, which often blends the individual discrete frequency components.

Comparing Figures 7 and 8 that show frequency spectrums at the spinning speed of 351 min^{-1} without a notch and with a 1 mm one, we notice that the whole notch spectrum level is approximately 11 dB higher. Such difference enables to predict the error, which represents the basis for calculating the rest of the service life.

6 SUMMARY

The paper presents a complex diagnostic system consisting of four modules: the mathematical gear wheel simulator which translates the complete mechanical module into mathematical form, the adaptive FIR (Finite Impulse Response) filter that calculates impulse responses from the non-linear system, the model for calculating any impulse response, and the FFT (Fast Fourier Transform) frequency analysis used for simulating frequency spectrums. The comparison of simulated and measured sound frequency spectrums has shown that the simulated value is a good indicator of the shifts in the spectrum caused by the tooth root notch. The result of the diagnostic system is a frequency spectrum of sounds that allows tooth damage analysis and, furthermore, an estimate of the gear's remaining lifespan and/or the necessary maintenance. By introducing a model for calculating any impulse response, the system is additionally generalized for single stage gear wheels with the tooth size in the 1:5 proportion.

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