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Mathematical Modeling and Experimental Research of Characteristic Parameters Hydrodynamic Processes of a Piston Axial Pump

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An experimental research and mathematical modelling of non stationary high dynamic hydraulic processes in the pump cylinder, discharge space and intake and discharge pipe line in the function of the action angle of the shaft is the fundamental basis in developing the piston–axial pumps. Based on the experimental research findings and the results of the mathematical modeling, developing and application of the identification method of unknown parameters of a mathematical model, the computer AKSIP program, which enables sufficiently exact determination of some parameters of working processes of piston axial pumps, has been developed.

Keywords: axial piston pump, mathematical modeling, hydrodynamical processes

0 INTRODUCTION

Modern design of a piston axial pump, based on computer aided design (CAD), requires a description of all the processes and parameters in the pump. The complexity of hydrodynamic and dynamic processes in a piston-axial pump (pump cylinder, intake and discharge space, discharge valve and pipeline of high pressure) demands very studious physical and mathematical analysis of these processes. It is based on the experimental research findings and the results of the mathematical modeling developments and the application of the identification method of unknown parameters of the mathematical modeling of non stationary high dynamic processes and an optimization technique.

This way of solving the problem is only possible with a special computer program. Such program (AKSIP) has been developed and is presented [1] to [3].

1 MATHEMATICAL MODEL

For mathematical modeling of hydrodynamic and dynamic processes in a piston-axial pump (pump cylinder, intake and discharge space, discharge valve and a pipeline of high pressure) Fig. 1, the following general suppositions have been adapted [1]:

a) Changes of the fluid state are pseudo stationary, except in the discharge pipeline;
b) Kinetic energy of fluid in each control space, except in the discharge pipeline, is neglected;
c) Fluid flow through clearances (crevices between the piston and cylinder, the flow through a split panel and discharge valve) is pseudo stationary;
d) The processes in the control spaces are isothermic or isentropic.

Simultaneous integration of the previous non-linear differential equations of boundary conditions and partial differential equations of streaming in a discharge pipeline required the application of a computer and a corresponding computer program.

The program connecting and simultaneously solving all the listed differential equations, the equations of change of characteristic flow sections and changes of physical characteristics of fluid, required a corresponding structure and organization. The program was written in the programming language Digital Visual Fortran 5.0. and realized on the measuring and controlling system ADS 2000 [2]. The principles of structural and modular programming were used. The programming consists of the main program and a module.

The more important programs were written as complete modules mutually connected or with the main program, but they can be used individually as well.

On the basis of the previous equations, a program system named AKSIP was developed. It is used for mathematical modeling of

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streaming and hydro-dynamic processes for the complete time cycle of a piston-axial pump with combined distribution of working fluid. AKSIP program is modular outlined and consists of the main AKSIP program and its modules.

1.1 Mathematical Model of a Pump Process

Mathematical model is given for each element, considering the complexity of some processes and their mutual dependence, as well as the need for further mathematical modeling [8] to [14].

This makes programming module and their further improvement and monitoring much easier [1].

▪ Mass flow through the opening 1, on the entrance place into the intake space of the pump of fluid, Fig. 1:

$$\frac{dm_1}{dt} = \sigma_1 \mu_1 A_1 \sqrt{2\rho_s |p_u - p_s|}\,, \tag{1}$$

where are: $\sigma_1 = 1$ for $p_u \geq p_s$, $\sigma_1 = -1$ for $p_u < p_s$
A_1 - geometrical flow section of the intake pipe,
μ_1 - flow coefficient of the intake pipe.

▪ Mass flow of fluid through the split pump organ during filling one of the pump cylinders:

$$\frac{dm_u}{dt} = \sigma_u \mu_u A_u \sqrt{2\rho_s |p_s - p_c|}\,, \tag{2}$$

where are: $\sigma_u = 1$ for $p_s \gtrsim p_c$, $\sigma_u = -1$ for $p_s \lesssim p_c$
A_u - geometrical flow section of the intake split organ,
μ_u - flow coefficient.

▪ Mass balance of the intake space is:

$$\frac{dm_s}{dt} = \frac{dm_1}{dt} - \sum_{j=1}^{z_c} \frac{dm_{u,j}}{dt}\,, \tag{3}$$

where are: $j = 1, 2, ..., z_c$ order number of cylinder, z_c the numbers of cylinders.

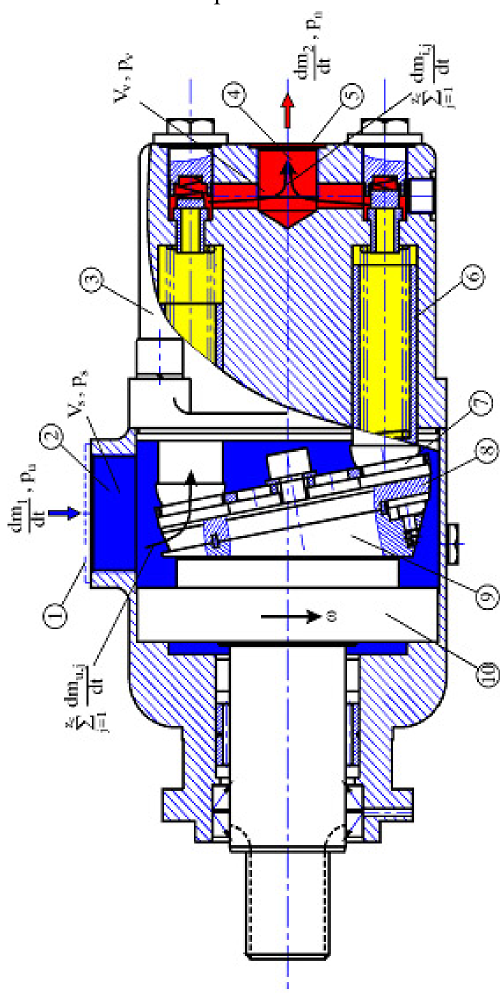
▪ Differential pressure equation in the intake pump space:

$$\frac{dp_s}{d\varphi} = \frac{E}{V_s \rho_s}\left(\frac{dm_1}{d\varphi} - \sum_{j=1}^{z_c} \frac{dm_{u,j}}{d\varphi}\right), \tag{4}$$

E - modulus of elasticity,
φ - the driving shaft angle.

Technical data:

- speed:1000 rot/min
- pressure: 210 bar
- specific volume: 75cm^3
- number of pistons 8.



Legend:

1. *intake pipe line connection*
2. *intake space of the pump*
3. *cylinder block*
4. *discharge space of the pump*
5. *discharge pipeline connection*
6. *piston*
7. *split panel*
8. *acute panel*
9. *the pump shaft*
10. *inbearing the pump shaft*

Fig. 1. *Piston axial pump*

- Differential pressure equation in the pump cylinder:

$$\frac{dp_c}{d\varphi}=\frac{E}{V_c}\left[\frac{A_c v_k}{\omega}+\frac{1}{\rho_c}\left(\frac{dm_u}{d\varphi}-\frac{dm_i}{d\varphi}\right)\right], \tag{5}$$

where are:
$V_c = V_{c\min} + V_{cx}$; $V_{cx} = A_c \cdot x_k$; immediate volume of the cylinder; the change of the volume of the pump cylinder caused by piston moving: $\frac{dV_c}{dt} = -A_c v_k$,

x_k - immediate displacement of the piston.

- Mass balance of the discharge space is:

$$\frac{dm_v}{dt}=\sum_{j=1}^{z_c}\frac{dm_{i,j}}{dt}-\frac{dm_2}{dt}, \tag{6}$$

where are: $j=1,2,...,$ z_c order number of cylinder, z_c the numbers of cylinders.

- Mass flow streaming out of the discharge space into the discharge pipe is:

$$\frac{dm_2}{dt}=\sigma_2\mu_2 A_2\sqrt{2\rho_t\left|p_v-p_n\right|}, \tag{7}$$

here are:

$\sigma_2=1$ for $p_v \gtrless p_n$, $\sigma_2 = -1$ for $p_v \lessgtr p_n$

A_2- geometrical flow section of the discharge pipe line.

- Differential pressure equation in the discharge pump space:

$$\frac{dp_v}{d\varphi}=\frac{E}{V_v\rho_v}\left(\sum_{j=1}^{z_c}\frac{dm_{i,j}}{d\varphi}-\frac{dm_2}{d\varphi}\right), \tag{8}$$

- Mass flow through a concentric clearance between the cylinder and the piston:

$$\frac{dm_z}{dt}=\frac{\pi\cdot D_c\cdot\Delta r^3}{12\cdot\eta\cdot x_k(\varphi)}\cdot\left(p_c-p_s\right)\cdot\rho_c, \tag{9}$$

where are: D_C - diameter of cylinder, Δr - radial clearance between the piston and the cylinder, η - dynamic viscosity, $x_k(\rho)$ - immediate displacement of the piston, p_c - the pressure in the cylinder, p_s - the pressure in the intake space, ρ_c - the density of the fluid in the cylinder.

1.2 Modeling the Streaming in the Intake and Discharge Pipe Line of the Pump

During mathematical modeling of a process in a pump, it is also necessary to include and consider a series of suppositions for a process modeling occurring in the intake and discharge pipe line of the pump.

For the most general model, the following suppositions for streaming of the operational fluid in the intake and discharge pipe line are taken and considered:

- The fluid streaming is one-dimensional. The pipes are of a constant cross section. Temperature and streaming fields per cross section of the pipe are homogeneous. Velocity vector laps the direction of the axis of the pipe at any moment and in any section.
- Viscosity friction between some layers of the fluid inside the pipe is neglected. The friction forces appear on the inside walls of the pipe.
- The processes in the pipes are isentropic. The change of entropy caused by friction, heat and mixing of fluid parts are neglected.
- Forces of the field (gravitational, magnetic, etc) are neglected.

In the scope of dynamic of one-dimensional streaming, such streaming is considered as “non stationary streaming in a streaming fiber”.

1.1.1 Continuity Equations

The equation of continuity of pressed fluid with functions p, w, ρ at the isentropic change of the state:

$$\frac{\partial p}{\partial t}+w\frac{\partial p}{\partial x}+a^2\rho\frac{\partial w}{\partial x}=0, \tag{10}$$

where: ρ and w – are values of density and velocity of fluid per cross section of the pipe,

$a=\sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$, the velocity of sound in the fluid,

where:
$p = p(t,x)$ and $\rho = \rho(t,x)$ are functions of time t and coordinate x.

1.1.2 Momentum Equations

$$w\frac{\partial\rho}{\partial t}+\rho\frac{\partial w}{\partial t}+w\frac{\partial}{\partial x}(\rho w)+\rho w\frac{\partial w}{\partial x}+\frac{\partial p}{\partial x}=-f_r\rho, \tag{11}$$

where is: f_Y - friction force per mass unit.

2 RESULTS OF MEASURING OF PARAMETERS OF WORKING PROCESSES OF THE PISTON AXIAL PUMP

In the scope of performed experimental testing, measuring of the pressure flow in the cylinder, discharge space and intake pipeline as well as vibration of the pump housing in dependence of the passed angle of the pump shaft was done. All pressures and vibrations were measured completely parallel on each cca 0.09° of the pump shaft (exactly 4.096 times per shaft rotation). As incremental giver of the angle an optical giver with 1024 pulses per rotation was used. Pulses of the giver of the angle were 4 times increased by the interface for the angle givers on the ADS 2000 system and so 4096 pulses per shaft rotation were obtained. In order to see the possible repetition of the consecutive cycles with the unchanged work regime, 10 consecutive cycles were measured. At the same time, a time interval from angle to angle was measured as well in order to determine an even angle speed of the shaft and work control of the incremental giver of the angle. All the analogue signals (pressure, vibrations) were paralelly converted into cipher form by means of four ultra speed converters working simultaneously (parallel). The total number of measured data was (4+1) x 4096 = 20480 per rotation (cycle), that is, 204800 for ten consecutive cycles. The number of samples of 4096 was not chosen by chance, but with the aim of the application of the fast Furie's transformation (FFT) of measured signals. Measures were done for seven working regimes. Fig. 2 (a to e) shows the measured pressure flow for individual, that is, ten consecutive cycles of the piston axial pump. The results relate to the experiment at the work regime p = 200 bar and n=875.6 rot/min.

There is a considerable similarity of measured pressures for the first of ten consecutive cycles (MERF) in relation to the middle of ten consecutive cycles (MERM).

Fig. 2a shows the measured pressure flow in the cylinder (p_c for one of ten consecutive cycles in the function of the angle of the shaft. The diagram shows the visual pressure gradients at the pressure stage and expansion as well as the appearance of piks during intake. Fig. 2a also shows the pressure flow in the discharge space p_v for one, that is the middle of the ten consecutive cycles in the function of the angle of the shaft. The pressure pulses in the discharge space depend on the number of the cylinders, which is obvious in this case because it deals with the pump with 8 cylinders. The appearance of piks at the intaking stage for one, that is, the middle of ten consecutive cycles at the angle interval of the shaft 120 to 270°, is shown in Figs. 2b and c.

Figs. 2d and e indicate the measured pressure flow in the cylinder (p_c) for one, that is the middle for ten consecutive cycles at the angle interval of the shaft of 278 to 307° with the aim to analyze the gradient growth of pressure at the stage of pressing in detail. The same diagram, at the same interval, shows the pressure pulses in the discharge space.

The figs. 2a to 2e stand for diagrams of measured pressure at the work regime n= 875.6 rot/min and p_n= 210 bar.

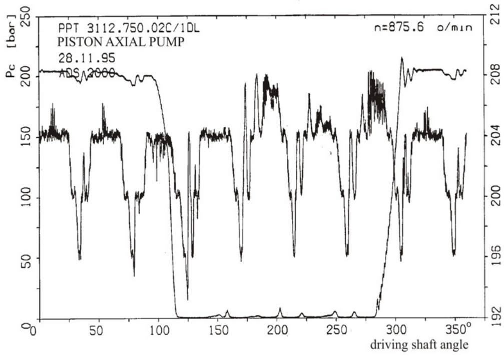


Fig. 2a. *The pressure history in the cylinder (p_c) and delivery chamber (p_v) for the average cycle*

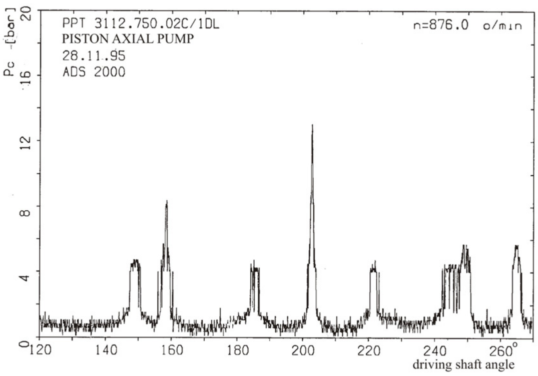


Fig. 2b. *The pressure history in the cylinder (p_c) in the 120 to 270^O interval for one cycle*

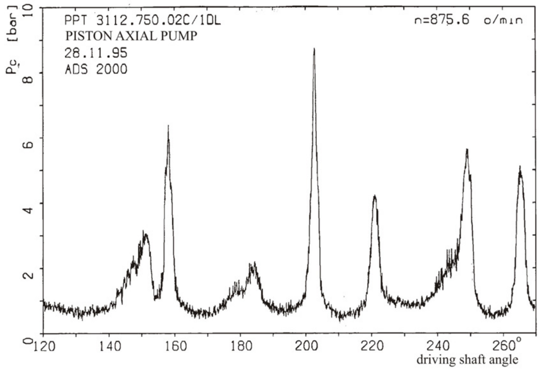


Fig. 2c. *The pressure history in the cylinder (p_c) in the 120 to 270^O interval for the average cycle*

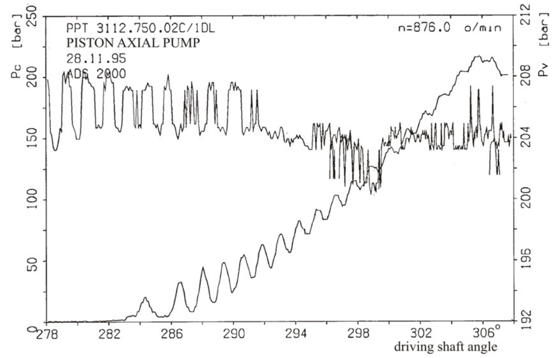


Fig.2d. *The pressure history in the cylinder (p_c) and delivery chamber (p_v) in 278 to 307° interval for one cycle*

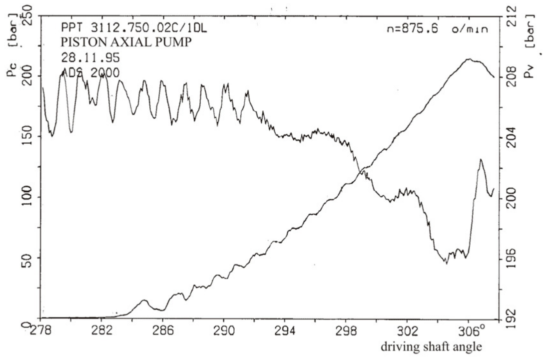


Fig.2e. *The pressure history in the cylinder (p_c) and delivery chamber (p_v) in 278 to 307° interval for the average cycle*

3 CONCLUSION

It is not possible to give a precise determination of parameters of hydrodynamic processes of a piston axial pump neither experimentally nor by a mere mathematical modeling. Sufficiently exact parameters can be obtained by combining the application of measuring the pressure flow in the cylinder, mathematical modeling of a real hydrodynamic process and the method of nonlinear optimisation which simultaneously enables the determination of systemic measuring errors and unknown parameters.

The computer AKSIP program gives possibilities to combine 56 influential pump parameters in order to achieve optimal solution, in regard to flow losses, flow inlet etc.

Further research is possible in the construction of the piston-axial pumps with a bent cylinder block and splitting of working fluid by means of a split panel. Mathematical model would be, in that case, expanded by a dynamic cylinder block and a hydrodynamic processes in clearances between the cylinder block and the split panel.

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