BLED WORKSHOPS
IN PHYSICS
VOL. 14, NO. 2

Proceedings to the 16th Workshop
What Comes Beyond . . . *(p. 113)*
Bled, Slovenia, July 14-21, 2013

10 *Spin-Charge-Family* Theory is Explaining Appearance of Families of Quarks and Leptons, of Higgs and Yukawa Couplings

N.S. Mankoč Borštnik

Department of Physics, FMF, University of Ljubljana,
Jadranska 19, SI-1000 Ljubljana, Slovenia

Abstract. The so far observed three families of quarks and leptons, the vector gauge fields of the fermions charges and the scalar Higgs responsible for masses of fermions and weak bosons, all these confirming the *standard model*, make most of physicists to declare that the Higgs was the last missing particle to be confirmed. But can this at all be true? Is it not self evident that there must be additional scalar fields which manifest effectively the appearance of the Yukawa couplings and that the Yukawa couplings can only be understood if we understand the origin of families? The *spin-charge-family* theory [1–4] is offering a possible explanation for the origin of families and also for several scalar fields, which are responsible for masses of fermions and weak vector boson fields. The theory is offering the explanation also for other assumptions of the *standard model*. The theory predicts at the observable regime two decoupled groups of four families. The fourth family, coupled to the measured three, will be observed at the LHC. The fifth family is the candidate for the dark matter. Masses of each group of the four families and of each of the two corresponding vector bosons are triggered by a different group of condensates. The theory explains why the scalar fields are doublets with respect to the weak charge, while they are triplets with respect to the family groups. The accuracy with which the fourth family masses can be predicted in this theory depends strongly on the accuracy with which the two mixing matrices will be measured. Correspondingly might the properties of the scalar fields (the low energy effective representation of which is the observed Higgs) be estimated also from the mass matrices of quarks and leptons. The main progress this year in the *spin-charge-family* theory is that I can "pedagogically" explain: i. Why the scalar fields are doublets with respect to the weak charge, carrying in addition the appropriate hyper charge. ii. Why the two groups of four families have so different masses although both groups of the scalar fields contributing to masses of the upper and lower four families, contribute also to masses of the weak bosons, while the second (not yet observed) $SU(2)$ gauge vector field have much higher masses. iii. The numerical calculations have improved so that we shall hopefully soon be able to say more about the intervals of masses of the fourth to the so far observed three families.

Povzetek. Doslej smo izmerili tri družine kvarkov in leptonov, tri vrste vektorskih polj, s katerimi so kvarki in leptoni sklopljeni ter skalarni Higgsov delec, ki je odgovoren za mase fermionov in šibkih bozonov. Vsa ta fermionska in bozonska polja so v skladu s *standardnim modelom*. Večina fizikov meni, da je Higgs zadnji delec, ki ga je bilo treba potrditi. Ali je to sploh lahko res? Ali ni očitno, da je skalarnih polj več, ki se efektivno kažejo kot Yukawine sklopitve in da lahko Yukawine sklopitve razumemo le, če razumemo izvor družin? Teorija

spinov-nabojev-družin [1–4] ponuja razlago za izvor družin in napoveduje, da določajo mase fermionov in šibkih vektorskih bozonov dva tripleta skalarnih polj, ki nosijo družinska kvantna števila in trije singleti, ki se sklapljajo z vsakim družinskim članom drugače. Teorija pojasni, zakaj so vsa skalarna polja šibki dubleti in zakaj nosijo tudi hyper naboj. Teorija razloži tudi ostale predpostavke *standardnega modela*. Teorija napove dve skupini štirih družin, ki nista sklopljeni in se razlikujeta po masah, ker sodelujejo pri nastanku mas vsake od skupin drugačna skalarna polja in pri eni od obeh tudi kondenzat desnoročnih nevtrinov. Četrto družino, sklopljeno s prvimi tremi že izmerjenimi, bodo opazili na LHC. Peta družina pojasni temno snov.

Natančnost, s katero lahko v tej teoriji izračunamo masne matrike in napovemo mase četrte družine, je odvisna od natačnosti meritev mas in matričnih elementov mešalnih matrik za tri poznane družine. Iz masnih matrik pa lahko sklepamo tudi na nekatere lastnosti skalarnih polj, ki smo jih doslej opazili kot Higgsovo skalarno polje in Yukavine sklopitve. Od lanskega zbornika je napredek teorije *spinov-nabojev-družin* predvsem v tem: i. Da lahko "pedagoško" razložim: i. Zakaj so skalarna polja dubleti glede na šibki naboj, in nosijo hipernaboj, da „obleęjo" desnoročne družinske člane v prava kvantna števila? ii. Zakaj imata dve skupini štirih družin tako različne mase, in zakaj sta tako zelo različnih mas tudi obe umeritveni polji, vsaka s svojo grupo $SU(2)$ (šibke bozone poznamo, druge vrste pa še ne), čeprav obe skupini skalarnih polj, ki sicer prispevata vsaka k masam svoje skupine štirih družin, prispevata k masi šibkih bozonov? II. Numerični izračuni so napredovali, tako da bo kmalu lahko podrobneje določiti intervale za mase četrte družine in njihove sklopitve s poznanimi tremi.

10.1 Introduction

The (extremely) efficient *standard model* is built on several assumptions, chosen to be in agreement with the data: i. There exist before the electroweak break massless coloured quarks and colourless leptons, left handed weak charged and right handed weak chargeless. ii. There exist families of fermions. iii. There exist the gauge fields to the observed charges of the family members. iv. There exists the boson – the scalar field and the anti-scalar field with the non zero vacuum expectation values after the elecroweak break and the properties to successfully "dress" the right handed fermions, giving them properties of the left handed ones and manifesting as doublets when interacting with the weak bosons. v. There exist the Yukawa couplings, distinguishing among the family members, to ensure right properties of families of fermions.

The questions are: a. Where do the families originate from and how many of them might be observable at the low energy regime? b. Where do the scalar fields and the Yukawa couplings originate from? c. Why is the Higgs a scalar boson manifesting as a doublet in the weak charge, while all the other bosons are in the vector representations with respect to all the charges, if they are not singlets [4]? d. Do we understand the appearance of the charges?

There are many other open questions, but the most urgent ones are to my understanding the first two, if we want to make a step towards understanding the *standard model* assumptions.

We should be able to predict what will the extremely expensive experiments measure in the near future.

There are several inventive proposals in the literature [6–14] extending the *standard model*. No one explains, to my knowledge, the origin of families. There are several proposals in the literature trying to explain the mass spectrum and mixing matrices of quarks and leptons [15] and properties of the scalar fields [16–19]. All of them just assuming on one or another way the number of families.

I am proposing the *spin-charge-family* theory [1–3,20–23], which does offer the explanation for the assumptions of the *standard model*:

- For the origin of massless families, explaining also the appearance of the family members with their charges.
- For the origin of the vector gauge fields.
- For the origin of several scalar fields which manifest effectively in the low energy regime as the Higgs and Yukawa couplings, explaining, why do the scalar fields and consequently the Higgs manifest as doublets with respect to the weak charge and carry the appropriate hyper charge and why do the family members manifest so different properties.

The theory is consequently able to make the *prediction* for the *number of families* and their *properties* and for the *number of scalar fields* and their *properties*, measurable in the today experiments. It is explaining also the *appearance of the dark matter*.

My starting assumption is a simple action in $d > (3+1)$ which leads to:

1. The Weyl equation for massless fermions couple to vielbeins and the spin connections of two kinds: The ones which are the gauge fields of $S^{ab} = \frac{i}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a)$, where γ^a are the Dirac γ^a's defined in any d, and the ones which are the gauge fields of $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$, where $\tilde{\gamma}^a$ are the second kind of the Clifford algebra objects, anti-commuting with the Dirac ones, again defined in any d.
2. The first kind of the Clifford algebra objects, γ^a, describes the spin in any d and after the break of the starting symmetry the spin in $d = (3+1)$ and all the so far observed charges, conserved and non-conserved.
3. The second kind of the Clifford algebra objects, since defining the equivalent representations with respect to the Dirac one, while there are only two kinds of the Clifford algebra objects (connected with the left - the Dirac one - and the right - my $\tilde{\gamma}^a$ - multiplication of any Clifford algebra object, which is a polynomial of powers of γ^a), the second kind must be used to describe families, which form the equivalent representations with respect to spin and charges.
4. The equations for boson fields, the vielbeins and spin connections of both kinds, are linear in the curvature.
5. d is chosen to be $(13+1)$ since one massless Weyl representation in $d = (13+1)$ contains, if analysed with respect to the *standard model* spin and charge groups, all the members of one family and their antiparticles: The left handed weak charged and the right handed weak chargeless coloured quarks of by the *standard model* required hyper charges and the left handed weak charged and the right handed weak chargeless colourless leptons - neutrinos and electrons - with by the *standard model* required hyper charges and their antiparticles according to the requirements of the ref. [5]. There are $2^{\frac{d}{2}-1}/2$

of massless particle plus antiparticle states if we pay attention to states of particular handedness and helicity only once.

6. The break of the starting $SO(13+1)$ symmetry first to **i.** $SO(7,1) \times U(1)_{II} \times SU(3)$, when (still massless) left handed weak charged and right handed weakless fermions and left handed weakless and right handed weak charged antifermions, differ further in the baryon quantum number ($U(1)_{II}$ ($\pm\frac{1}{6}$, for quarks $(+)$ and for antiquarks $(-)$ and $\mp\frac{1}{2}$, for leptons $(-)$ and antileptons $(+)$) while quarks and leptons differ further in the colour (quarks are triplets, antiquarks antitriplets, leptons are colourless singlets and antileptons anticolourless singlets), leaves these family members in $2^{\frac{7}{2}-1} = 8$ massless families, which stay massless also in the further breaks to **ii.** $SO(3,1)\times SU(2)_{I} \times SU(2)_{II}\times U(1)_{II} \times SU(3)$.
7. At the further two breaks, to $SO(3,1)\times SU(2)_{I} \times U(1)_{I} \times SU(3)$, when a weakless and hyper chargeless condensate of the right handed neutrinos carrying the quantum numbers of the upper four families brings masses to the $SU(2)_{II}$ gauge vector bosons, and to the electroweak break to $SO(3,1)\times U(1) \times SU(3)$, fermions, coupled to particular gauge scalar fields, which are vielbeins and spin connections with the scalar index with respect to $(3+1)$ and gain nonzero vacuum expectation values, become massive.
8. At the breaks some of the gauge fields stay massless (the colour vector bosons) and the final ($U(1)$) vector gauge field - the electromagnetic field - while the two $SU(2)$ vector gauge bosons become massive when the corresponding symmetry is broken.
9. The *standard model* can be interpreted as a low energy manifestation of the *spin-charge-family* theory.

In this talk I briefly present the *spin-charge-family* theory (already presented in several talks and papers): The fermions and gauge bosons starting action and the action after breaks, sect. 10.2, the fermion representations, sect. 10.2.1, and the scalar and vector representations, sect. 10.2.2. I answer the question why do scalar gauge bosons, carrying the family quantum numbers, manifest as weak (fermion) doublets, while they behave as triplets with respect to the family groups 10.2.2. I discuss a possible answer to the question: Why do the two gauge fields appearing in this theory, the gauge fields of the two kinds of the charges, $SU(2)_{II}$ and $SU(2)_{I}$, distinguish so much in their masses (the $SU(2)_{II}$ gauge vector boson has not yet been observed), although the two groups of the scalar fields, one responsible for the masses of the upper four families and another for the masses of the lower four families, are all weak ($SU(2)_{I}$) doublets and the hyper charge singlets 10.2.3.

I discuss predictions of the *spin-charge-family* theory: The properties of the fourth family coupled to the observed three [21,25], of the stable fifth family, of the scalar fields and of the accuracy of measurements needed that predictions will be more accurate, sect. 10.3, 10.2.2. To predict the fourth family properties (masses of the family members and the mixing matrix elements coupling the fourth family members to the observed three ones) accurately enough the two 3×3 mixing (sub)matrices should be measured pretty much more accurately. Properties of several scalar fields, leading effectively in the low energy regime to the scalar

Higgs and the Yukawa couplings, manifest in the mass matrices and can therefore some of their properties be evaluated by analysing properties of mass matrices.

The *spin-charge-family* theory opens several questions like: How many dimensions does the space have? Are there non-observable dimensions curled into compact or non-compact spaces? And many others.

10.2 Brief presentation of the *spin-charge-family* theory

In this section the *spin-charge-family* theory is briefly presented, first the simple starting action for massless fermions and massless gauge fields, with which I start and which includes families of fermions. I follow in this part to high extent the ref. [4]. In subsect. 10.2.1 the fermion representations are discussed, leading to mass matrices of family members.

The explanation is presented for why does the starting action manifest effectively, after several breaks up to the electroweak one, two decoupled groups of massive four families of quarks and leptons, three of the lower four already observed, and to the known gauge fields, the scalar Higgs and the Yukawa couplings. Each group of four families are coupled to their own kind of the scalar fields, the gauge fields with the scalar index with respect to $d = (3+1)$ of the two kinds of the Clifford algebra objects. Both groups of scalar fields gain nonzero vacuum expectation values. There is also the condensate, ref. 10.2.3, of the right handed neutrinos with the family quantum numbers of the upper four families, with the $SU(2)_{II}$ charge equal to 1, weakless and with the hyper charge equal to zero, bringing mass to the $SU(2)_{II}$ vector gauge fields. The scalars interacting with the lower four families determine, in loop corrections in all orders together with other fields, mass matrices of quarks and leptons, the three of which are the known ones. Mass matrices of all the family members, quarks and leptons, belonging to the lower four families, manifest the same symmetry 10.2.2. All these scalars are doublets with respect to the weak charge, while they carry appropriate hyper charge Y, 10.2.2, and manifest effectively at low energies as the Higgs and the Yukawa couplings.

The theory assumes that the spinor carries in $d(=(13+1))$-dimensional space two kinds of the spin, no charges [1,2,4,3]: **i.** The Dirac spin, described by $\gamma^{\mathbf{a}}$'s, defines the spinor representations in $d = (13+1)$ $(SO(13,1))$, and correspondingly in the low energy regime, after several breaks of symmetries and before the electroweak break, the spin $(SO(3,1))$ and all the charges (the colour $SU(3)$, the weak $SU(2)$, the hyper charge Y and the non conserved hyper charge Y') of quarks and leptons. There are the left handed weak charged and the right handed weak chargeless quarks and leptons. Handedness is determined by the spin properties in $d = (3+1)$, in agreement with the *standard model*. **ii.** The second kind of the spin [27,28,26], described by $\tilde{\gamma}^{\mathbf{a}}$'s ($\{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}$) and anticommuting with the Dirac γ^a ($\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$), defines the families of spinors, which at the symmetries of $SO(3,1) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ manifests two groups of four massless families, each belonging to different $SU(2) \times SU(2)$ symmetry, namely: $(\tilde{SU}(2)_R \times \tilde{SU}(2)_{II}) \times (\tilde{SU}(2)_L \times \tilde{SU}(2)_I)$, the first one determines the symmetries of one of the four families and the second one of the second one of four families.

One can understand the appearance of the (only) two kinds of the Clifford algebra objects as follows: If the Dirac one corresponds to the multiplication of any spinor object B (any product of the Dirac γ^a's, which represents a spinor state when being applied on a spinor vacuum state $|\psi_0>$) from the left hand side, can the second kind of the Clifford objects be understood (up to a factor, determining the Clifford evenness ($n_B = 2k$) or oddness ($n_B = 2k+1$) of the object B) as the multiplication of the object from the right hand side

$$\tilde{\gamma}^a B\,|\psi_0>:= i(-)^{n_B} B\gamma^a\,|\psi_0>_{fam}, \qquad (10.1)$$

with $|\psi_0>_{fam}$ determining the vacuum state on which B applies. Accordingly we have

$$\begin{aligned}
&\{\gamma^a,\gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a,\tilde{\gamma}^b\}_+, \quad \{\gamma^a,\tilde{\gamma}^b\}_+ = 0,\\
&S^{ab} := (i/4)(\gamma^a\gamma^b-\gamma^b\gamma^a), \quad \tilde{S}^{ab} := (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b-\tilde{\gamma}^b\tilde{\gamma}^a), \quad \{S^{ab},\tilde{S}^{cd}\}_- = 0.
\end{aligned} \qquad (10.2)$$

More detailed explanation can be found, for example in appendix of the ref. [4] and in the refs [3,28,27].

The *spin-charge-family* theory proposes in $d = (13+1)$ a simple action for a Weyl spinor and for the corresponding gauge fields

$$\begin{aligned}
S\; =& \int d^dx\; E\;\mathcal{L}_f + \\
& \int d^dx\; E\;(\alpha\, R + \tilde{\alpha}\,\tilde{R}),
\end{aligned} \qquad (10.3)$$

$$\begin{aligned}
\mathcal{L}_f &= \frac{1}{2}\,(\bar{\psi}\,\gamma^a p_{0a}\psi) + h.c.,\\
p_{0a} &= f^\alpha{}_a p_{0\alpha} + \frac{1}{2E}\,\{p_\alpha, Ef^\alpha{}_a\}_-,\\
p_{0\alpha} &= p_\alpha - \frac{1}{2}S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha},\\
R &= \frac{1}{2}\,\{f^{\alpha[a}f^{\beta b]}\;(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\,\omega^c{}_{b\beta})\} + h.c.\,,\\
\tilde{R} &= \frac{1}{2}\,f^{\alpha[a}f^{\beta b]}\;(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\tilde{\omega}^c{}_{b\beta}) + h.c.\,.
\end{aligned} \qquad (10.4)$$

Here [1] $f^{\alpha[a}f^{\beta b]} = f^{\alpha a}f^{\beta b} - f^{\alpha b}f^{\beta a}$. To see that the action (Eq.(10.3)) manifests after the breaks of symmetries [2,4,3] all the known gauge fields and the scalar

[1] $f^\alpha{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^\alpha{}_b = \delta^a{}_b,\; e^a{}_\alpha f^\beta{}_a = \delta^\beta_\alpha$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

fields and the mass matrices of the observed families, let us rewrite formally the action for a Weyl spinor of (Eq.(10.3)) as follows

$$\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\gamma^n (p_n - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_n)\psi + \\
&\{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\; \psi\} + \text{the rest}\,, \\
p_{0s} &= p_s - \frac{1}{2} S^{tt'}\omega_{tt's} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}, \qquad (10.5)
\end{aligned}$$

where $n = 0, 1, 2, 3$ with

$$\begin{aligned}
\tau^{Ai} &= \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab}, \\
\{\tau^{Ai}, \tau^{Bj}\}_- &= i\delta^{AB} f^{Aijk}\tau^{Ak}. \qquad (10.6)
\end{aligned}$$

All the charges (τ^{Ai}, Eqs. (10.6), (10.8), (10.9)) and the spin (Eq. (10.7)) operators are expressible with S^{ab}, which determine all the internal degrees of freedom of one family: the spin and the charges.

$$\vec{N}_{\pm}(= \vec{N}_{(L,R)}) := \frac{1}{2}(S^{23} \pm i S^{01}, S^{31} \pm i S^{02}, S^{12} \pm i S^{03})\,, \qquad (10.7)$$

determine representations of the two $SU(2)$ subgroups of $SO(3,1)$, while

$$\begin{aligned}
\vec{\tau}^1 &:= \frac{1}{2}(S^{58} - S^{67},\, S^{57} + S^{68},\, S^{56} - S^{78})\,, \\
\vec{\tau}^2 &:= \frac{1}{2}(S^{58} + S^{67},\, S^{57} - S^{68},\, S^{56} + S^{78})\,, \qquad (10.8)
\end{aligned}$$

determine representations of $SU(2)_I\times SU(2)_{II}$ of $SO(4)$, which is the subgroup of $SO(7,1)$ and

$$\begin{aligned}
\vec{\tau}^3 &:= \frac{1}{2}\{S^{9\,12} - S^{10\,11}\,, S^{9\,11} + S^{10\,12},\, S^{9\,10} - S^{11\,12}, \\
&\quad S^{9\,14} - S^{10\,13},\, S^{9\,13} + S^{10\,14}\,, S^{11\,14} - S^{12\,13}\,, \\
&\quad S^{11\,13} + S^{12\,14},\, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}, \\
\tau^4 &:= -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})\,, \qquad (10.9)
\end{aligned}$$

determine representations of $SU(3) \times U(1)$, originating in $SO(6)$.

Family quantum numbers, expressible with $\tilde{S}^{ab}$,

$$\vec{\tilde{N}}_{\pm}(= \vec{\tilde{N}}_{(L,R)}) := \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03})\,, \qquad (10.10)$$

determine representations of the two $SU(2)$ subgroups of $SO(3,1)$ in the $\tilde{S}^{ab}$ sector, while

$$\begin{aligned}
\vec{\tilde{\tau}}^1 &:= \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67},\, \tilde{S}^{57} + \tilde{S}^{68},\, \tilde{S}^{56} - \tilde{S}^{78})\,, \\
\vec{\tilde{\tau}}^2 &:= \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67},\, \tilde{S}^{57} - \tilde{S}^{68},\, \tilde{S}^{56} + \tilde{S}^{78}), \qquad (10.11)
\end{aligned}$$

determine representations of $SU(2)_I \times SU(2)_{II}$ of $SO(4)$, which is the subgroup of $SO(7,1)$ again in the $\tilde{S}^{ab}$ sector.

Families gain masses through the interaction with the scalar fields $\frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}$, the gauge fields of the family charges ($\tilde{SU}(2)_R \times \tilde{SU}(2)_{II}$ the upper four families and $\tilde{SU}(2)_L \times \tilde{SU}(2)_I$ the lower four families), where we assume that after the breaks we end up with $(a,b) \in \{n, s\}$, $n = (0,1,2,3)$ and $s = (7,8)$. The upper four families and the vector gauge fields of the group $SU(2)_{II}$ gain masses also through the interaction with the right handed neutrinos condensate (sect. 10.2.3, Table 10.6), which is weakless, hyper chargeless and the electromagnetic chargeless, belonging to the $SU(2)_{II}$ triplet, carrying τ^{23} equal 1 and $\tau^4 = -1$.

At the electroweak break the scalar fields which are the gauge fields of $\tilde{SU}(2)_L \times \tilde{SU}(2)_I$ contribute to masses of the lower four families, while the scalars, the gauge fields of Q, Q' and Y' contribute to masses of all the eight families, distinguishing among the family members (sect. 10.2.2, 10.2.2). All these scalar gauge fields, since they are doublets with respect to the weak charge, carrying also the hyper charge, contribute to the masses of the weak bosons.

Correspondingly index A in Eq. (10.6) enumerates all possible spinor charges and g^A is the coupling constant to a particular gauge vector field A^{Ai}_n. τ^{3i} describe the colour charge ($SU(3)$), τ^{1i} the weak charge ($SU(2)_I$), τ^{2i} the second $SU(2)_{II}$ charge, τ^4 determines the $U(1)_{II}$ charge and $\tau^{23} = Y$ describes also the hyper charge, $Q = Y + \tau^{13} = S^{56} + \tau^4$ is the electromagnetic charge, $Q' = \tau^{13} - Y\tan^2\theta$ and $\tau^{\pm} = \tau^{11} \pm i\tau^{12}$.

The theory starts with one (massless, left handed) Weyl representation of $SO(13,1)$ spinors in $2^{d/2-1}$ families. In the breaks of the starting symmetry to the symmetry of $SO(7,1) \times SU(3) \times U(1)_{II}$ only eight ($2^{(7+1)/2-1}$) of them stay massless [2]. Families stay massless also after breaks to $SO(3,1) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$.

In the further two breaks, the first to $SO(3,1) \times SU(2)_I \times U(1)_I \times SU(3)$, triggered by the right handed neutrino condensate, carrying the family quantum numbers of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, and belonging to the $SU(2)_{II}$ triplet with $\tau^{23} = 1$ and $\tau^4 = -1$, and correspondingly with zero electromagnetic, weak and hyper charges, and the electroweak break caused by the scalar fields which gain nonzero vacuum expectation values, all the fermions become massive. All the scalar fields, which contribute in the breaks, are doublets with respect to the weak charge carrying also the hyper charge Y(sect. 10.2.2).

In Eq. (10.12) the effective action for fermions at the electroweak is presented. The second line manifests the covariant momentum for fermions as seen by the *standard model* in agreement with the so far observed fermion and vector boson fields. The third line presents the contribution to the covariant momentum of the massive $SU(2)_{II}$ gauge fields, coupled through Y' and $\tau^{2\pm}$ to fermions. To masses of these vector gauge bosons mostly the condensate of the right handed neutrinos contributes. The fourth line determines the mass term for both groups of four

[2] We proved that it is possible to have massless fermions after a break if one starts with massless fermions and assume particular boundary conditions or particular vielbeins and spin connections causing the breaks [23,24] and taking care of massless and mass protected families after the break.

families on the tree level. It is assumed that the symmetries in the $\tilde{S}^{ab}\,\tilde{\omega}_{abc}$ and $S^{ab}\omega_{abc}$ part break in a correlated way. The generators $\tilde{S}^{ab}$ (Eqs. (10.10), (10.11)) transform each member of one family into the same family member of another family, due to the fact that $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$. The generators S^{ab} transform the family member into another one, keeping family quantum number unchanged.

$$
\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\,(\gamma^m\, p_{0m} - M)\psi\,,\\
p_{0m} &= p_m - \{e\,Q\,A_m + g^{Q'}\,Q'\,Z^{Q'}_m + \frac{g^1}{\sqrt{2}}\,(\tau^{1+}\,W^{1+}_m + \tau^{1-}\,W^{1-}_m) + \\
&+ g^{Y'}\,Y'\,A^{Y'}_m + \frac{g^2}{\sqrt{2}}\,(\tau^{2+}\,A^{2+}_m + \tau^{2-}\,A^{2-}_m)\,,\\
\bar{\psi}\,M\,\psi &= \bar{\psi}\,\gamma^s\,p_{0s}\psi\\
p_{0s} &= p_s - \{\tilde{g}^{\tilde{N}_R}\,\vec{\tilde{N}}_R\,\vec{\tilde{A}}^{\tilde{N}_R}_s + \tilde{g}^{\tilde{Y}'}\,\tilde{Y}'\,\tilde{A}^{\tilde{Y}'}_s + \frac{\tilde{g}^2}{\sqrt{2}}\,(\tilde{\tau}^{2+}\,\tilde{A}^{2+}_s + \tilde{\tau}^{2-}\,\tilde{A}^{2-}_s)\\
&+ \tilde{g}^{\tilde{N}_L}\,\vec{\tilde{N}}_L\,\vec{\tilde{A}}^{\tilde{N}_L}_s + \tilde{g}^{\tilde{Q}'}\,\tilde{Q}'\,\tilde{A}^{\tilde{Q}'}_s + \frac{\tilde{g}^1}{\sqrt{2}}\,(\tilde{\tau}^{1+}\,\tilde{A}^{1+}_s + \tilde{\tau}^{1-}\,\tilde{A}^{1-}_s)\\
&+ e\,Q\,A^{Q}_s + g^{Q'}\,Q'\,Z^{Q'}_s + g^{Y'}\,Y'\,A^{Y'}_s\,\},\;\; s \in \{7, 8\}\,.
\end{aligned}
\tag{10.12}
$$

The term $\bar{\psi}\,M\,\psi$ determines the tree level mass matrices of quarks and leptons. The two groups of four families are decoupled due to different family quantum numbers: One group carries the quantum numbers of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, the other of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Since the condensate contributes in loop corrections only to one of the two groups, the first one, the mass matrices are expected to appear at two different energy scales. Also the scalar fields couple to either the upper or to the lower four families.

Since all the scalar fields, which gain nonzero vacuum expectation values - those with the quantum numbers of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, with $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, and those with Q, Q', Y' - are doublets with respect to the weak charge carrying also the hyper charge (10.2.2), all contribute to masses of the vector bosons W and Z. It is, namely, $-2iS^{0s}$, $s = 7, 8$, which transform the right handed weak chargeless quarks and leptons into the corresponding left handed weak charged partners, transforming at the same time the hyper charge Y. The gauge scalar fields have correspondingly the weak and hyper charges.

10.2.1 Fermions through breaks

I discuss properties of quarks, u and d, and leptons, ν and e, all left and right handed, for two decoupled groups of four families, before and after they gain masses, triggered by the vacuum expectation values of the scalar fields with which each of the two groups couples.

At the stage of the symmetry

$$
\begin{aligned}
&SO(3,1)_\gamma \times SO(3,1)_{\tilde{\gamma}} \times SU(2)_{I\,\gamma} \times SU(2)_{I\,\tilde{\gamma}}\\
&\times SU(2)_{II\,\gamma} \times SU(2)_{II\,\tilde{\gamma}} \times U(1)_{II\,\gamma} \times U(1)_{II\,\tilde{\gamma}}\\
&\times SU(3)_\gamma,
\end{aligned}
\tag{10.13}
$$

the eight families are assumed to be massless. The two indices γ and $\tilde{\gamma}$ are to point out that there are two kinds of subgroups of $SO(7,1)$, those defined by S^{ab} responsible for properties (spin and charges) of family members and those defined by $\tilde{S}^{ab}$ responsible for the appearance of families.

To manifest how do the operators presented in Eqs. (10.7, 10.8, 10.9) transform one family member into another one of the same family, in Table 10.1 quarks of a particular colour charge are presented in the spinor technique [28]. A brief introduction into the technique can be found also in Appendix of this talk. Spinor states are defined as products of nilpotents ($[\stackrel{ab}{(k)}]^2 = 0$) and projectors ($[\stackrel{ab}{[k]}]^2 = \stackrel{ab}{[k]}$) (Eq. (10.37) in Appendix 10.4)

$$\begin{aligned}
\stackrel{ab}{(\pm i)}: &= \frac{1}{2}(\gamma^a \mp \gamma^b), \; \stackrel{ab}{[\pm i]} := \frac{1}{2}(1 \pm \gamma^a \gamma^b), \quad \text{for } \eta^{aa}\eta^{bb} = -1, \\
\stackrel{ab}{(\pm)}: &= \frac{1}{2}(\gamma^a \pm i\gamma^b), \; \stackrel{ab}{[\pm]} := \frac{1}{2}(1 \pm i\gamma^a \gamma^b), \quad \text{for } \eta^{aa}\eta^{bb} = 1, \qquad (10.14)
\end{aligned}$$

chosen to be eigen states of S^{ab}. They are at the same time also the eigenstates of $\tilde{S}^{ab}$ (Eq. (10.38) in Appendix 10.4).

$$S^{ab} \stackrel{ab}{(k)} = \frac{k}{2} \stackrel{ab}{(k)}, \quad S^{ab} \stackrel{ab}{[k]} = \frac{k}{2} \stackrel{ab}{[k]}, \quad \tilde{S}^{ab} \stackrel{ab}{(k)} = \frac{k}{2} \stackrel{ab}{(k)}, \quad \tilde{S}^{ab} \stackrel{ab}{[k]} = -\frac{k}{2} \stackrel{ab}{[k]} \, . \qquad (10.15)$$

The choice of the Cartan subalgebra of the commuting operators is made as follows:

$$\begin{aligned}
&S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14}, \\
&\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\,10}, \tilde{S}^{11\,12}, \tilde{S}^{13\,14} \, . \qquad (10.16)
\end{aligned}$$

Let the reader note that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$ (Eq. (10.39) in Appendix 10.4)

$$\gamma^a \stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]}, \; \gamma^b \stackrel{ab}{(k)} = -ik \stackrel{ab}{[-k]}, \; \gamma^a \stackrel{ab}{[k]} = \stackrel{ab}{(-k)}, \; \gamma^b \stackrel{ab}{[k]} = -ik\eta^{aa} \stackrel{ab}{(-k)} \, , \qquad (10.17)$$

$$\tilde{\gamma}^a \stackrel{ab}{(k)} = -i\eta^{aa} \stackrel{ab}{[k]}, \; \tilde{\gamma}^b \stackrel{ab}{(k)} = -k \stackrel{ab}{[k]}, \; \tilde{\gamma}^a \stackrel{ab}{[k]} = \; i \stackrel{ab}{(k)}, \; \tilde{\gamma}^b \stackrel{ab}{[k]} = -k\eta^{aa} \stackrel{ab}{(k)} \qquad (10.18)$$

The nilpotents and projectors of Table 10.1 operate on a vacuum state, not presented in the table. The states solve the Weyl equation Eq.(10.19)

$$\begin{aligned}
\gamma^0 \gamma^a p_a \psi = 0 &= \gamma^0 \, (\gamma^m p_m + \sum_{s=7,8} \bar{\psi}\gamma^s \, p_s \,)\psi \\
&= \gamma^0 \, (\stackrel{78}{(-)} \; p_- + \stackrel{78}{(+)} \; p_+)\psi \, , \\
\stackrel{78}{(\pm)} &= \frac{1}{2} \, (\gamma^7 \; \pm i \, \gamma^8) \, , \\
p_\pm &= (p_7 \mp i \, p_8) \, , \qquad (10.19)
\end{aligned}$$

for free massless spinors in the coordinate system where $p^a = (p^0, 0, 0, p^3, \vec{0})$, $\vec{0}$ stays for all the components in $d > 4$.

There are $2^{\frac{d}{2}-1} = 64$ basic spinor states of one family representation in $d = (13+1)$, defining the spinors (colour triplets quarks and antitriplets antiquarks and colourless leptons and anticolourless antileptons). Family members of a particular colour or the colourless ones form $2^{(7+1)/2=8}$ states and so do anticoloured and colourless spinors. One easily sees that the operator $\gamma^0 \stackrel{78}{(\pm)} I_{\vec{x}_3}$, $I_{\vec{x}_3}$ reflecting (x^1, x^2, x^3) into $(-x^1, -x^2, -x^3)$, transforms the state u_{1R} from the first line into the state u_{1L} from the seventh line, while $\vec{\tau}^3$ transforms any of the quark states of the starting colour charge into otherwise the same states but in general of of another colour charges.

$S^{9\,10}$, for example, transforms the u_{1R} quark from the first line into the ν_{1R} lepton from the first line in Table 10.2. Such transformations are after the breaks not allowed. Following the proposal from the ref. [5] for the definition of the discrete symmetries in cases of the Kaluza-Klein kind for d even

$$
\begin{aligned}
\mathcal{C}_{\mathcal{N}}\, \psi(x^0, \vec{x}) &= \Gamma^{(3+1)}\, \gamma^2\, K\, \psi(x^0, x^1, x^2, x^3, x^5, -x^6, x^7, -x^8, \ldots, x^{d-1}, -x^d) \\
&= \Gamma^{(3+1)}\, \gamma^2\, K\, I_{6,8,\cdots,d}\, \psi(x^0, \vec{x})\,, \\
\mathcal{T}_{\mathcal{N}}\, \psi(x^0, \vec{x}) &= \Gamma^{(3+1)}\, \gamma^1\, \gamma^3\, K\, \psi(-x^0, x^1, x^2, x^3, -x^5, x^6, -x^7, \ldots, -x^{d-1}, x^d) \\
&= \Gamma^{(3+1)}\, \gamma^1\, \gamma^3\, K\, I_{x^0}\, I_{5,7,\cdots,d-1}\, \psi(x^0, \vec{x})\,, \\
\mathcal{P}^{d-1}_{\mathcal{N}}\, \psi(x^0, \vec{x}) &= \gamma^0\, \Gamma^{(3+1)}\, \Gamma^{(d)}\, \psi(x^0, -x^1, -x^2, -x^3, x^5, x^6, \ldots, x^{d-1}, x^d) \\
&= \gamma^0\, \Gamma^{(3+1)}\, \Gamma^{(d)}\, I_{\vec{x}_3}\, \psi(x^0, \vec{x})\,,
\end{aligned}
\tag{10.20}
$$

where $I_{\vec{x}_3}$ reflects (x^1, x^2, x^3), $I_{6,8,\cdots,d}$ reflects $(x^6, x^8, \cdots, x^d)$, I_{x^0} reflects the time component x^0 and $I_{5,7,\cdots,d-1}$ reflects $(x^5, x^7, \cdots, x^{d-1})$, it is $\mathcal{C}_{\mathcal{N}} \cdot \mathcal{P}^{d-1}_{\mathcal{N}}$, which transforms the positive energy states into the corresponding negative energy states, staying within the same Weyl, while either $\mathcal{C}_{\mathcal{N}}$ or $\mathcal{P}^{d-1}_{\mathcal{N}}$ jumps out of the starting Weyl representation.

Emptying the negative energy state obtained by the application of the $\mathcal{C}_{\mathcal{N}} \cdot \mathcal{P}^{d-1}_{\mathcal{N}}$ on the single particle state put on the top of the Dirac sea, one creates the corresponding antiparticle state with the positive energy and put on the top of the Dirac sea, carrying all the properties of the starting particle, except the S^{03} value and the charges [5].

The above requirements can be expressed as follows.

Statement: *The antiparticle state put on the top of the corresponding Dirac sea follows from the particle state put on the top of this Dirac sea by applying on the particle state the operator $\mathbb{O}_{\mathcal{N}}$*

$$
\begin{aligned}
\{\mathbb{O}_{\mathcal{N}} &= \mathrm{emptying} \times \mathcal{C}_{\mathcal{N}}\, \mathcal{P}_{\mathcal{N}} \\
&= \gamma^0 \prod_{\gamma^a \in \Im, a \neq 2} \gamma^a\, \Gamma^{(3+1)}\, I_{\vec{x}_3} I_{6,8,\ldots,d} \Gamma^{(d)}\} \text{ particle state.}
\end{aligned}
\tag{10.21}
$$

The corresponding antiparticle state on the top of the Dirac sea also solves the Weyl equation (10.19).

Using Eq. (10.21) it is easy to find the antiparticle state of positive energy (which are put on the top of the Dirac sea) to the particle states (which are put on the top of the Dirac sea), presented in Tables (10.1, 10.2). The corresponding two tables are presented in Tables (10.3, 10.4).

ψ_i^{pos}	positive energy state	$\frac{p^0}{\vert p^0\vert}$	$\frac{p^3}{\vert p^3\vert}$	$(-2iS^{03})$	$\Gamma^{(3+1)}$	τ^{13}	τ^{23}	τ^4	Y	Q
$u_{1\,R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	$+1$	0	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
$u_{2\,R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	$+1$	0	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
$d_{1\,R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	$+1$	0	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
$d_{2\,R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	$+1$	0	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
$d_{1\,L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
$d_{2\,L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
$u_{1\,L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
$u_{2\,L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$

Table 10.1. One $SO(7,1)$ sub representation [5] of the representation of $SO(13,1)$, the one representing quarks, which carry the colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$). All members have $\Gamma^{(13+1)} = -1$. The states representing particles are put on the top of the Dirac sea. All states are the eigenstates of the Cartan subalgebra ($S^{03}, S^{12}, S^{56}, S^{7,8}, S^{9\,10}, S^{11\,12}, S^{13\,14}$) and solve the Weyl equation (10.19) for the choice of the coordinate system $p^a = (p^0, 0, 0, p^3, 0, \ldots, 0)$ for free massless fermions. The infinitesimal generators of the weak charge $SU(2)_I$ group ($\vec{\tau}^1$) and of another $SU(2)_{II}$ group ($\vec{\tau}^2$) are defined in Eq. (10.8) and of the τ^4 charge and the colour charge group ($\vec{\tau}^3$) in Eq. (10.9). $Y = \tau^{23} + \tau^4$, $Q = \tau^{13} + Y$. Nilpotents $\stackrel{ab}{(k)}$ and projectors $\stackrel{ab}{[k]}$ operate on the vacuum state $|vac>_{fam}$ not written in the table.

ψ_i^{pos}	positive energy state	$\frac{p^0}{\lvert p^0\rvert}$	$\frac{p^3}{\lvert p^3\rvert}$	$(-2iS^{03})$	$\Gamma^{(3+1)}$	τ^{13}	τ^{23}	τ^4	Y	Q
ν_{1R}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[+]}\;\;\stackrel{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	+1	+1	+1	+1	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
ν_{2R}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[+]}\;\;\stackrel{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	+1	−1	−1	+1	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
e_{1R}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[+]}\;\;\stackrel{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	+1	+1	+1	+1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	−1	−1
e_{2R}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[+]}\;\;\stackrel{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	+1	−1	−1	+1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	−1	−1
e_{1L}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[+]}\;\;\stackrel{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	+1	−1	−1	−1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	−1
e_{2L}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[+]}\;\;\stackrel{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	+1	+1	+1	−1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	−1
ν_{1L}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[+]}\;\;\stackrel{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	+1	−1	−1	−1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
ν_{2L}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{[+]}\;\;\stackrel{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	+1	+1	+1	−1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0

Table 10.2. One $SO(7,1)$ sub-representation [5] of the representation of $SO(13,1)$, the one representing the colourless leptons, when the state is put on the top of the Dirac sea. The rest of definitions is the same as in Table 10.1.

$\bar{\psi}^{pos}_i$	positive energy state	$\frac{p^0}{\|p^0\|}$	$\frac{p^3}{\|p^3\|}$	$(-2iS^{03})$	$\Gamma^{(3+1)}$	τ^{13}	τ^{23}	τ^4	Y	Q
$\bar{u}_{1\,L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\,\|\,\stackrel{56}{[-]}\stackrel{78}{[-]}\,\|\|\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\;e^{-i\|p^0\|x^0-i\|p^3\|x^3}$	+1	−1	−1	−1	0	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
$\bar{u}_{2\,L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\,\|\,\stackrel{56}{[-]}\stackrel{78}{[-]}\,\|\|\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\;e^{-i\|p^0\|x^0+i\|p^3\|x^3}$	+1	+1	+1	−1	0	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
$\bar{d}_{1\,L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\,\|\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\|\|\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\;e^{-i\|p^0\|x^0-i\|p^3\|x^3}$	+1	−1	−1	−1	0	$+\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
$\bar{d}_{2\,L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\,\|\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\|\|\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\;e^{-i\|p^0\|x^0+i\|p^3\|x^3}$	+1	+1	+1	−1	0	$+\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
$\bar{d}_{1\,R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\|\,\stackrel{56}{(+)}\stackrel{78}{[-]}\,\|\|\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\;e^{-i\|p^0\|x^0+i\|p^3\|x^3}$	+1	+1	+1	+1	$+\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
$\bar{d}_{2\,R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\|\,\stackrel{56}{(+)}\stackrel{78}{[-]}\,\|\|\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\;e^{-i\|p^0\|x^0-i\|p^3\|x^3}$	+1	−1	−1	+1	$+\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
$\bar{u}_{1\,R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\|\,\stackrel{56}{[-]}\stackrel{78}{(+)}\,\|\|\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\;e^{-i\|p^0\|x^0+i\|p^3\|x^3}$	+1	+1	+1	+1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
$\bar{u}_{2\,R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\|\,\stackrel{56}{[-]}\stackrel{78}{(+)}\,\|\|\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\;e^{-i\|p^0\|x^0-i\|p^3\|x^3}$	+1	−1	−1	+1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$

Table 10.3. One $SO(7,1)$ sub representation [5] of the representation of $SO(13,1)$, the one representing antiquarks, which carry the anticolour charge ($\tau^{33} = -1/2$, $\tau^{38} = -1/(2\sqrt{3})$) and it is put on the top of the Dirac sea. All the other comments are the same as in Table 10.1.

Let us find now, according to Eq. (10.21), the antilepton states (to be put on the top of the Dirac sea) to the states, presented in Table 10.2. One finds the states (to be put on the top of the Dirac sea), presented in Table 10.4

One can easily check that $\gamma^0 \stackrel{78}{(+)} I_{\vec{x}_3}$ transforms the weakless antiparticle state put on the top of the Dirac sea $\bar{u}_L$ with the hyper charge $Y = -\frac{2}{3}$ from the first line in Table 10.3 into the weak charged antiparticle state $\bar{u}_R$, put on the top of the Dirac sea from the seventh line in the same table. $\bar{u}_R$ has $Y = -\frac{1}{6.}$. Similarly does $\gamma^0 \stackrel{78}{(+)} I_{\vec{x}_3}$ transform the weakless $\bar{e}_{1L}$ from the third line in Table 10.4 with $Y = 1$ into the weak charged $\bar{e}_R$ from the fifth line in the same table, with $Y = \frac{1}{2}$, both antiparticle states put on the top of the Dirac sea.

One sees that the term $\gamma^0 \sum_{s=7,8} \gamma^s p_{0s}$ determines the mass term as soon as a superposition of the fields $\tilde{\omega}_{abs}$ or of the fields ω_{abs}, or both superposition, gain nonzero vacuum expectation values. I shall demonstrate this in the next subsection.

Families of fermions Here I again follow a lot the ref.[4]. The generators $\tilde{N}^{\pm}_{R,L}$ and $\tilde{\tau}^{(2,1)\pm}$ (Appendix 10.4, Eq. (10.50)), which are superposition of $\tilde{S}^{ab}$, transform each member of one family into the same member of another family, due to the fact that $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$ (Eq.(10.2)).

The eight families of the first member of the eight-plet of quarks from Table 10.1, for example, that is of the right handed u_{1R} quark, are presented in the left column of Table 10.5 [4]. In the right column of the same table the equivalent eight-plet of the right handed neutrinos ν_{1R} are presented. All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N^{\pm}_{R,L}$ and $\tau^{(2,1)\pm}$ on this particular member.

The eight-plets separate into two group of four families: One group contains doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, these families are singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Another group of families contains doublets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, these families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$.

The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ couple only to the four families which are doublets with respect to these two groups. The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ couple only to the four families which are doublets with respect to these last two groups.

Masses of fermions We saw in subsect. 10.2.1 that the term $\psi^\dagger \gamma^0 M \psi$ in Eq. (10.12) causes the appearance of masses of fermions as soon as the corresponding scalar fields, presented in the covariant momentum in the fifth, sixth and seventh line of the same equation gain nonzero expectation values.

$\bar{\psi}^{pos}_i$	positive energy state	$\frac{p^0}{\lvert p^0\rvert}$	$\frac{p^3}{\lvert p^3\rvert}$	$(-2iS^{03})$	$\Gamma^{(3+1)}$	τ^{13}	τ^{23}	τ^4	Y	Q
$\bar{\nu}_{1\,L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	+1	−1	−1	−1	0	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0
$\bar{\nu}_{2\,L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	+1	+1	+1	−1	0	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0
$\bar{e}_{1\,L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	+1	−1	−1	−1	0	$+\frac{1}{2}$	$+\frac{1}{2}$	+1	+1
$\bar{e}_{2\,L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	+1	+1	+1	−1	0	$+\frac{1}{2}$	$+\frac{1}{2}$	+1	+1
$\bar{e}_{1\,R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	+1	+1	+1	+1	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	+1
$\bar{e}_{2\,R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	+1	−1	1	+1	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	+1
$\bar{\nu}_{1\,R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	+1	+1	+1	+1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0
$\bar{\nu}_{2\,L}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	+1	−1	−1	+1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0

Table 10.4. One $SO(7,1)$ sub representation [5] of the representation of $SO(13,1)$, the one representing the colourless antileptons, when the state is put on the top of the Dirac sea. The rest of definitions is the same as in Table 10.1.

I	$u_{I\,1R}$	$\overset{03}{(+i)}\,\overset{12}{[+]}\,\vert\,\overset{56}{[+]}\,\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{I\,1R}$	$\overset{03}{[+i]}\,\overset{12}{(+)}\,\vert\,\overset{56}{[+]}\,\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$
I	$u_{II\,1R}$	$\overset{03}{[+i]}\,\overset{12}{(+)}\,\vert\,\overset{56}{[+]}\,\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{II\,1R}$	$\overset{03}{(+i)}\,\overset{12}{[+]}\,\vert\,\overset{56}{(+)}\,\overset{78}{[+]}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$
I	$u_{III\,1R}$	$\overset{03}{(+i)}\,\overset{12}{[+]}\,\vert\,\overset{56}{(+)}\,\overset{78}{[+]}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{III\,1R}$	$\overset{03}{(+i)}\,\overset{12}{[+]}\,\vert\,\overset{56}{[+]}\,\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$
I	$u_{IV\,1R}$	$\overset{03}{[+i]}\,\overset{12}{(+)}\,\vert\,\overset{56}{(+)}\,\overset{78}{[+]}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{IV\,1R}$	$\overset{03}{[+i]}\,\overset{12}{(+)}\,\vert\,\overset{56}{(+)}\,\overset{78}{[+]}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$
II	$u_{I\,1R}$	$\overset{03}{[+i]}\,\overset{12}{[+]}\,\vert\,\overset{56}{[+]}\,\overset{78}{[+]}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{I\,1R}$	$\overset{03}{[+i]}\,\overset{12}{[+]}\,\vert\,\overset{56}{[+]}\,\overset{78}{[+]}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$
II	$u_{II\,1R}$	$\overset{03}{(+i)}\,\overset{12}{(+)}\,\vert\,\overset{56}{[+]}\,\overset{78}{[+]}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{II\,1R}$	$\overset{03}{(+i)}\,\overset{12}{(+)}\,\vert\,\overset{56}{[+]}\,\overset{78}{[+]}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$
II	$u_{III\,1R}$	$\overset{03}{[+i]}\,\overset{12}{[+]}\,\vert\,\overset{56}{(+)}\,\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{III\,1R}$	$\overset{03}{[+i]}\,\overset{12}{[+]}\,\vert\,\overset{56}{(+)}\,\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$
II	$u_{IV\,1R}$	$\overset{03}{(+i)}\,\overset{12}{(+)}\,\vert\,\overset{56}{(+)}\,\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{IV\,1R}$	$\overset{03}{(+i)}\,\overset{12}{(+)}\,\vert\,\overset{56}{(+)}\,\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$

Table 10.5. Eight families of the right handed u_{1R} (10.1) quark with spin $\frac{1}{2}$, the colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$, and of the colourless right handed neutrino ν_{1R} of spin $\frac{1}{2}$ (10.2) are presented in the left and in the right column, respectively. They belong to two groups of four families, one (I) is a doublet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$) and a singlet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$), the other (II) is a singlet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$) and a doublet with with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$). All the families follow from the starting one by the application of the operators ($\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$), Eq. (10.50). The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) (Eq. (10.50)) transform u_{1R} to all the members of one family of the same colour. The same generators transform equivalently the right handed neutrino ν_{1R} to all the colourless members of the same family.

If the operators γ^7 and γ^8 in Eq. (10.12) are expressed in terms of the nilpotents $\overset{78}{(\pm)}$, the mass term can be rewritten as follows

$$
\begin{aligned}
\bar{\psi} M \psi &= \sum_{s=7,8} \bar{\psi}\gamma^s\, p_{0s}\, \psi = \psi^{\dagger}\,\gamma^0\,(\overset{78}{(-)}\; p_{0-} + \overset{78}{(+)}\; p_{0+})\psi\,, \\
\overset{78}{(\pm)} &= \frac{1}{2}\,(\gamma^7 \;\pm i\gamma^8)\,, \\
p_{0\pm} &= (p_{07} \mp i\, p_{08}) = (p_7 \mp i\, p_8) - (\Phi^{Ai}_7 \mp i\,\Phi^{Ai}_8)\,, \\
\Phi^{Ai}_{\mp} &= \{\vec{\tilde{A}}^{\tilde{N}_R}_{\mp}, \vec{\tilde{A}}^{2}_{\mp}, \vec{\tilde{A}}^{\tilde{N}_L}_{\mp}, \vec{\tilde{A}}^{1}_{\mp}, A^{Q}_{\mp}, Z^{Q'}_{\mp}, A^{Y'}_{\mp}\}\,.
\end{aligned}
\tag{10.22}
$$

We clearly see that all the scalars $\Phi^{Ai}_{\mp}$ are doublets with respect to the weak charge, carrying also the hyper charge, $(\tau^{13}, Y)\,\Phi^{Ai}_{-} = (\frac{1}{2}, -\frac{1}{2})\;\Phi^{Ai}_{-}$, $(\tau^{13}, Y)\,\Phi^{Ai}_{+} = (-\frac{1}{2}, \frac{1}{2})\;\Phi^{Ai}_{+}$, since they obviously bring the right quantum numbers to the right handed partners, to (u_R, ν_R) the scalars Φ^{Ai}_{-}, and to (d_R, e_R) the scalars Φ^{Ai}_{+}, as we have checked in Tables 10.1 and 10.2, manifesting that $\gamma^0\;\overset{78}{(-)}$ transforms (u_R, ν_R) into (u_L, ν_L), and equivalently for other quarks and leptons. We shall discuss properties of scalar fields also in subsect. 10.2.2, 10.2.2.

To masses of one of the two groups of four families only the scalar fields, which are the gauge fields of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ contribute, to masses of the other group of four families only the gauge fields of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ contribute.

The scalars A^{Q}_{s}, $Z^{Q'}_{s}$ and $A^{Y'}_{s}$ from the last line in Eq. (10.12) contribute to all eight families, distinguishing among the family members and not among the families.

In loop corrections also all the gauge fields which couple to fermions contribute. To the upper four families contributes in addition the (assumed to be) condensate of the right handed neutrinos (10.2.3), carrying the spin equal zero, $Q = 0 = Y$, $\tau^{13} = 0$, $\tau^{23} = 1$ and $\tau^{4} = -1$. It also carries the $\tilde{\tau}^{23} = 1$ and $\tilde{N}^{3}_{R} = 1$ charges.

The mass matrix of any family member belonging to any of the two groups of four families manifests, due to the $\widetilde{SU}(2)_{(R,L)} \times \widetilde{SU}(2)_{(II,I)}$ (either (R, II) or (L, I)) structure of the quantum numbers of the scalar fields which are the gauge fields of the $\vec{\tilde{N}}_{R,L}$ and $\vec{\tilde{\tau}}^{2,1}$, the symmetry presented in Eq. (10.23)

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^{\alpha} , \tag{10.23}$$

the same for all the family members $\alpha \in \{u, d, \nu, e\}$. The properties of the mass matrices and the procedure how to extract from the observed properties of the lower three families of the lower group of four families the masses and mixing matrix elements is discussed in the contribution to this proceedings [25] and in the refs. [3,4]. All the parameters of the mass matrix are determined by the tree level contributions and the loop corrections in all orders of all the fields, which couple to particular family member of one of the two groups of four families.

If assuming that the mass matrix elements are real then there are 6 free parameters for each family member. The mixing matrix for quarks has then 6 free parameters and so has the corresponding one for leptons. Since any $(n-1)\times(n-1)$ sub-matrix of the $n \times n$ unitary matrix determines for $n \geq 4$ the unitary matrix uniquely, we would be able to calculate from two times three masses and the mixing matrix elements of the 3×3 sub-matrix the fourth family members masses for the accurately enough experimental data.

We have not yet started to study the CP violation.

Let us learn [4,3] how do fermions interact with the scalar fields. Let $\psi^{\alpha}_{(L,R)}$ denote massless and $\Psi^{\alpha}_{(L,R)}$ massive four vectors for each family member $\alpha = (u_{L,R}, d_{L,R}, \nu_{L,R}, e_{L,R})$ after taking into account loop corrections in all orders [3,22], for any of the two groups of four families. $\psi^{\alpha}_{(L,R)} = V^{\alpha}_{(L,R)}\, \Psi^{\alpha}_{(L,R)}$,

$$\begin{aligned} \psi^{\alpha}_{(L,R)} &= V^{\alpha}\, \Psi^{\alpha}_{(L,R)} , \\ V^{\alpha} &= V^{\alpha}_{(o)}\, V^{\alpha}_{(1)} \cdots V^{\alpha}_{(k)} \cdots . \end{aligned} \tag{10.24}$$

It then follows

$$\begin{aligned} < \psi^{\alpha}_{L}|\gamma^{0}\, M^{\alpha}\, |\psi^{\alpha}_{R} > = < \Psi^{\alpha}_{L}|\gamma^{0}\, (V^{\alpha})^{\dagger}\, M^{\alpha}\, V^{\alpha}\, |\Psi^{\alpha}_{R} >= \\ < \Psi^{\alpha}_{L}|\gamma^{0}\, diag(m^{\alpha}_{1}, \cdots, m^{\alpha}_{4})|\Psi^{\alpha}_{R} > . \end{aligned} \tag{10.25}$$

It follows then that $V^{\alpha\dagger}\, \mathcal{M}^{\alpha}\, V^{\alpha} = \Phi^{\alpha}_{\Psi}$ determines the superposition of the scalar dynamical fields which couple with the coupling constants m^{α}_{k} (in some units) to the family member belonging to the k^{th} family

$$(\Phi^{\alpha}_{\Psi})_{k\,k'}\,\Psi^{\alpha\,k'} = \delta_{k\,k'}\,m^{\alpha}_{k}\,\Psi^{\alpha\,k}\,. \qquad (10.26)$$

Let us repeat that to loop corrections two kinds of scalar dynamical fields contribute, those originating in $\tilde{\omega}_{abs}$ ($\tilde{g}^{\tilde{N}_R}\,\vec{\tilde{N}}_R\vec{\tilde{A}}^{\tilde{N}_R}_s$, $\tilde{g}^2\,\vec{\tilde{\tau}}^2\,\vec{\tilde{A}}^2_s$ to the upper four families and $\tilde{g}^{\tilde{N}_L}\vec{\tilde{N}}_L\,\vec{\tilde{A}}^{\tilde{N}_L}_s$ to the lower four families) those originating in ω_{abs} ($e\,Q\,A^Q_s$, $g^1\,Q'\,Z^{Q'}_s$ and $g^{Y'}Y'\,A^{Y'}_s$ to all eight families), the vector gauge fields from Eq.(10.12), the fermion fields and to the upper four families also the condensate.

Even if we are able to reproduce the mass matrices, as we are trying in the ref. [25], it is not easy to extract some properties of the scalar fields from the known mass matrices.

10.2.2 Scalars and gauge fields through breaks

In the *spin-charge-family* theory there are the vielbeins $e^s{}_\sigma$

$$e^a{}_\alpha = \begin{pmatrix} \delta^m{}_\mu & 0 \\ 0 & e^s{}_\sigma \end{pmatrix}$$

in a strong correlation with the spin connection fields of both kinds, $\tilde{\omega}_{ab\sigma}$ $((a,b) \in \{0,\dots,3,5,\dots,8\}$, $\sigma \in \{7,8\}$) and with $\omega_{st\sigma}$ $((s,t) \in \{5,6,7,8\}, \sigma \in \{5,6,7,8\})$, which manifest in $d = (3+1)$-dimensional space as scalar fields after particular breaks of the starting symmetry. Phase transitions are (assumed to be) triggered by nonzero vacuum expectation values of the fields $f^{\alpha}{}_s\,\tilde{\omega}_{ab\alpha}$ and $f^{\alpha}{}_s\,\omega_{ab\alpha}$ [4] and the fermion (the right handed neutrinos from the upper four families) condensate.

The gauge fields then correspondingly appear as

$$e^a{}_\alpha = \begin{pmatrix} \delta^m{}_\mu & 0 \\ e^s{}_\mu = e^s{}_\sigma E^\sigma{}_{Ai} A^{Ai}_\mu & e^s{}_\sigma \end{pmatrix},$$

with $E^{\sigma Ai} = \tau^{Ai}\,x^\sigma$, where A^{Ai}_μ are the gauge fields, corresponding to (all possible) Kaluza-Klein charges τ^{Ai}, manifesting in $d = (3+1)$. Since the space symmetries include only S^{ab} ($M^{ab} = L^{ab} + S^{ab}$) and not $\tilde{S}^{ab}$, there are no vector gauge fields of the type $e^s{}_\sigma \tilde{E}^\sigma{}_{Ai}\tilde{A}^{Ai}_\mu$, with $\tilde{E}^\sigma{}_{Ai} = \tilde{\tau}_{Ai}\,x^\sigma$. The gauge fields of $\tilde{S}_{ab}$ manifest in $d = (3+1)$ only as scalar fields.

There occurs two successive breaks from $SO(3,1)\times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SO(3,1) \times U(1) \times SU(3)$.

I assume that the first break, that is to $SO(3,1) \times SU(2)_I \times U(1)_I \times SU(3)$, is triggered by the right handed neutrinos belonging of the upper four families forming a condensate, 10.2.3, with the quantum numbers (spin equal zero, $Q = 0 = Y$, $\tau^{13} = 0$, $\tau^{23} = 1$, $\tau^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{N}^3_R = 1$, or any other $\tilde{\tau}^{23}$ and $\tilde{N}^3_R$ values). It couples correspondingly to the gauge fields $\vec{A}^2_m$, bringing them masses (leaving the weak bosons massless). The condensate, carrying the family quantum numbers $\tilde{\tau}^{23} = 1$, $\tilde{N}^3_R = 1$ of the upper four families, couples also to the upper four families.

At the electroweak break, when all the scalar fields gain nonzero vacuum expectation values, all the family members of both groups of four families become massive. Since all the scalar fields are doublets with respect to the weak charge and carry also the hyper charge, their nonzero vacuum expectation values contribute on the tree level to the masses of Z_m and $W^{\pm}_m$ according to

$$(\frac{1}{2})^2\,(g^1)^2\,v_I^2(\frac{1}{(\cos\theta_1)^2}\,Z^{Q'}_m Z^{Q'\,m} + 2\,W^+_m W^{-\,m})\,, \tag{10.27}$$

where v_I are the contribution to the vacuum expectation value of all the scalar fields $\Phi^{I\,Ai}_{\mp}$. Eq. (10.27) is in agreement with the *standard model*.

To know the properties of the scalar fields one should study in details breaks, in which the condensate of the right handed neutrinos, and the scalar fields carrying the weak and hyper charges and the family quantum numbers participate, which is not an easy job.

However, from the mass matrices and the interactions of the scalar fields with fermions we can still learn something about properties of the scalar fields.

I demonstrate in subsect. 10.2.2 that all the scalar fields are doublets with respect to the weak charge and that they carry a hyper charge. I comment in subsect. 10.2.2 that the symmetry of mass matrices are the same for all the family members and that loop corrections keep this symmetry. I demonstrate the properties of the condensate in subsect. 10.2.3 and comment on why do the two groups of four families differ in masses, and why do the two gauge vector fields, carrying the $SU2_{II}$ and $SU(2)_I$ quantum numbers, respectively, differ in masses.

Scalar fields - doublets with respect to weak charge and carrying hyper charge

We saw in sect. 10.2.1, Eqs. (10.12, 10.22) that the operators $\stackrel{78}{(-)}\,\Phi^{Ai}_{-}$ and $\stackrel{78}{(+)}\,\Phi^{Ai}_{+}$ transform the right handed u_R-quarks and ν_R-leptons and the right handed d_R-quarks and e_R-leptons, respectively, into the corresponding left handed partners for all the scalar fields, independent of the family quantum numbers. Scalar fields $\Phi^{Ai}_{\mp}$ ($\Phi^{Ai}_{\mp}$ stay for $\{\vec{\tilde{A}}^{\tilde{N}_R}_{\mp}, \vec{\tilde{A}}^{2}_{\mp}, \vec{\tilde{A}}^{\tilde{N}_L}_{\mp}, \vec{\tilde{A}}^{1}_{\mp}, A^{Q}_{\mp}, Z^{Q'}_{\mp}, A^{Y'}_{\mp}\}$ (Eq. (10.22)) with nonzero vacuum expectation values must accordingly carry the appropriate quantum numbers. All these scalar fields appear in Eqs. (10.12, 10.22) as follows

$$\psi^{\dagger}\gamma^0\,\sum_{Ai}(\stackrel{78}{(-)}\;\Phi_{-} + \stackrel{78}{(+)}\;\Phi_{+})\psi\,,$$
$$\Phi_{\mp} = \Phi_7 \pm i\Phi_7\,. \tag{10.28}$$

Let us analyse their properties. Eqs. (10.8, 10.9) and Table 10.1 require [4] that

$$\vec{\tau}^1 = \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{6,8}, S^{56} - S^{7,8})\,,$$
$$\vec{\tau}^2 = \frac{1}{2}(S^{58} + S^{6,7}, S^{57} - S^{6,8}, S^{56} + S^{7,8})\,,$$
$$Y = \tau^{23} + \tau^4\,,\quad \tau^4 = -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})\,. \tag{10.29}$$

Any vector A^d has the transformation property

$$(S^{ab})^c{}_d A^d = i(\eta^{ac}\,\delta^b_d - \eta^{bc}\,\delta^a_d)\,A^d\,. \tag{10.30}$$

Correspondingly one finds the following properties of the fields

$$\begin{aligned}
\tau^4\,(\Phi^7 \pm i\Phi^8) &= 0\,, \quad Y\,(\Phi^7 \pm i\Phi^8) = \mp\frac{1}{2}(\Phi^7 \pm i\Phi^8)\,,\\
\tau^{13}\,(\Phi^7 \pm i\Phi^8) &= \pm\frac{1}{2}(\Phi^7 \pm i\Phi^8)\,,\\
\tau^{1+}\,(\Phi^7 + i\Phi^8) &= -(\Phi^5 + i\Phi^6)\,, \quad \tau^{1-}\,(\Phi^7 + i\Phi^8) = 0\,,\\
\tau^{1-}\,(\Phi^7 - i\Phi^8) &= (\Phi^5 - i\Phi^6)\,, \quad \tau^{1+}\,(\Phi^7 - i\Phi^8) = 0\,,\\
\tau^{1+}\,(\Phi^5 + i\Phi^6) &= 0\,, \quad \tau^{1-}\,(\Phi^5 + i\Phi^6) = -(\Phi^7 + i\Phi^8)\,,\\
\tau^{1+}\,(\Phi^5 - i\Phi^6) &= (\Phi^7 - i\Phi^8)\,, \quad \tau^{1-}\,(\Phi^5 - i\Phi^6) = 0\,,\\
\tau^4\,(\Phi^5 \pm i\Phi^6) &= 0\,, \quad \tau^{13}\,(\Phi^5 \pm i\Phi^6) = \mp\frac{1}{2}(\Phi^5 \pm i\Phi^6)\,,\\
Y(\Phi^5 \pm i\Phi^6) &= \mp\frac{1}{2}(\Phi^7 \pm i\Phi^8)\,.
\end{aligned} \tag{10.31}$$

In Eq. (10.31) the fields $(\Phi^7 \pm i\Phi^8) = \Phi_{\mp}$ stay for all $\Phi^{Ai}_{\mp}$.

It is, therefore, just proved that the scalar fields $\Phi^{Ai}_{\mp}$ with nonzero vacuum expectation values contribute on the tree level to the mass term of fermions with which they interact, "dressing" at the same time the right handed u_R-quarks and ν_R-leptons with the weak charge $\tau^{13} = \frac{1}{2}$ and the hyper charge $Y = -\frac{1}{2}$, while they "dress" the right handed d_R-quarks and e_R-leptons with the weak charge $\tau^{13} = -\frac{1}{2}$ and the hyper charge $Y = \frac{1}{2}$.

Why are symmetries of mass matrices kept in all orders of loop corrections? I have checked, together with the coauthor [31], that the symmetry of the mass matrix, Eq. 10.23, suggested by the *spin-charge-family* theory, stays unchanged in all orders of loop corrections, for several types of loop contributions. The evaluations were done in the massless basis. The final proof is under investigations and looks promising.

10.2.3 Do we understand why do two groups of four families distinguish in masses and why do two vector boson $SU(2)$ fields distinguish in masses?

All the scalar fields, which gain nonzero vacuum expectation values, are doublets with respect to the weak charge carrying also the hyper charge, as we have seen in the above discussions. This is true independently of what family quantum numbers the scalar fields carry. Correspondingly all the scalar fields contribute to the masses of Z_m and $W^{\pm}_m$ vector bosons. Each of the two groups of four families carry different family charges, coupling correspondingly only to those scalars, which are the gauge fields of their family groups.

How can then the two groups of families have so different masses? And why are the masses of the vector gauge fields of the group $SU(2)_{II}$ so much larger than those of the vector bosons Z_m and $W^{\pm}_m$?

The right handed neutrinos with the family quantum numbers of the upper group of four families are solving this problem, provided that they form a condensate with quantum numbers $Q = 0 = Y$, $\tau^{13} = 0$, $\tau^{23} = 1$, $\tau^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{N}^3_R = 1$, different values of $\tilde{\tau}^{23}$, $\tilde{N}^3_R$ are also acceptable. Such a condensate couples to the gauge fields $\vec{A}^2_m$ and, in loop corrections, to the upper four families. It does not couple to the lower four families and also not to the vector bosons Z_m and $W^{\pm}_m$. The condensate causes a non conservation of the fermion quantum number, keeping (3× quark minus lepton) quantum number unbroken, as long as Y is a conserved quantity.

In Table 10.6 a triplet of the group $SU(2)_{II}$ with the generators τ^{2i} is presented: The condensate of the right handed neutrinos and the two partners, all carrying τ^4 equal to -1. The family quantum numbers $\tilde{\tau}^{23} = 1$ and $\tilde{N}^3_R$ are chosen. Any of the rest possibilities for these two family quantum numbers values, or all of them are acceptable as well.

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^4	Y	Q	$\tilde{\tau}^{23}$	$\tilde{N}^3_R$
$(\lvert\nu_{1R}>_1 \lvert\nu_{2R}>_2)_{\mathcal{A}}$	0	0	0	1	-1	0	0	1	1
$(\lvert\nu_{1R}>_1 \lvert e_{2R}>_2)_{\mathcal{A}}$	0	0	0	0	-1	-1	-1	1	1
$(\lvert e_{1R}>_1 \lvert e_{2R}>_2)_{\mathcal{A}}$	0	0	0	-1	-1	-2	-2	1	1

Table 10.6. The condensate of two right handed neutrinos ν_R, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, together with its two partners, is presented. The condensate has $Q = 0 = Y$. The triplet carries $\tau^4 = -1$, $\tilde{\tau}^{23} = 1$ and $\tilde{N}^3_R = 1$ (All belong to the family IVR of the group II from Table 10.5). The family quantum numbers IVR are not noted on the states. Index $_{\mathcal{A}}$ stays for anti symmetrization.

There could be condensates also from the anti-neutrinos, right handed and belonging to the upper four families with the same family quantum numbers, or with other possible family quantum numbers of the same group. The corresponding condensate of two anti-neutrinos to the neutrinos presented in Table 10.6 would carry $\tau^{23} = -1$ and $\tau^4 = -1$.

It stays an open question, what does make the right handed neutrinos (or antineutrinos), belonging to the upper four families, to form such a condensate.

10.3 Conclusions and predictions of *spin-charge-family* theory

I demonstrate in this talk that the *spin-charge-family* theory is offering the explanation for the appearance of families, explaining as well the appearance of several scalar fields and of so far observed charges of fermions and the corresponding gauge fields. I demonstrate why are these scalar fields doublets with respect to the weak charge and singlets with respect to the hyper charge. I also offers predictions of the theory.

The theory predicts that there are two decoupled massless four families at some low energy scale, which stay massless also after they become massive, since each of the two groups carries different family quantum numbers.

There are two kinds of triggers responsible for the appearance of fermion masses: **i.** The condensate of the right handed neutrinos, carrying the family quantum numbers of the upper four families. Carrying the quantum numbers of the $SU(2)_{II}$ gauge vector field, the condensate makes this gauge field massive. Carrying the family quantum numbers of only one of the two groups, the condensate contribute to masses of the upper four families. **ii.** The scalar fields after they gain nonzero vacuum expectation values. The scalar fields belong to three groups: *ii.a.* The two scalar triplets with respect to the family quantum numbers of the upper four families bring masses to the upper four families. *ii.b.* The two scalar triplets with respect to the family quantum numbers of the lower four families bring masses to the lower four families. These two kinds of scalar fields do not distinguish among family members. *ii.c.* The third kind of the scalars are singlets which carry the quantum numbers (Q, Q', Y') of the family members, distinguishing correspondingly among the family members and not among families. They contribute to masses of all the eight families.

I demonstrate that all the scalar fields are doublets with respect to the weak charge carrying also the hyper charge, just as the so far observed Higgs is. They "dress" correspondingly the right handed u_R-quark and ν_R- lepton with the weak $\tau^{13} = \frac{1}{2}$ and the hyper charge $Y = -\frac{1}{2}$, while they "dress" the right handed d_R-quark and e_R- lepton with the weak $\tau^{13} = -\frac{1}{2}$ and the hyper charge $Y = \frac{1}{2}$. Correspondingly all the scalar fields contribute to masses of Z_m and $W^{\pm}_m$.

I demonstrate properties of one representation of the $SO(13,1)$, which includes all the family members, left and right handed, coloured and colourless, as well as their antiparticles, and the properties of families of all these quarks and leptons and the antiquarks and antileptons, using our spinor technique.

The appearance of several scalar fields manifest at the low energy regime as the Higgs, explaining the Yukawa couplings.

I offer the answer to the question: Why are the two $SU(2)$ gauge fields, $SU(2)_{II}$, which is not yet observed, and the weak $SU(2)_{I}$ so different in masses and why are also the two groups of four families so different in masses. The condensate of the right handed neutrinos with the family quantum numbers of the upper four families resolves this problem, since it couple only to the $SU(2)_{II}$ gauge bosons and to the upper four families.

The theory predicts that there are two times decoupled four families at the low energy.

The lowest of the upper four families is stable and is the candidate to form the dark matter [21]. The fourth of the lower four families will be observed at the LHC. Accurately enough measured mixing 3×3 sub matrices of quarks and leptons will enable to determine the masses of the fourth family members accurately. The ref. [25] is reporting on this calculations.

The *spin-charge-family* theory is treating all the family members, quarks and leptons, equivalently. I report on the trial to prove that the symmetry of mass matrices predicted by the theory, the same one for all the family members, is kept in all loop corrections. Loop corrections in all orders are needed to understand why are mass matrices so different in values for different family members, while they all demonstrate the same symmetry.

10.4 APPENDIX: Short presentation of technique [27,28]

I make in this appendix a short review of the technique [28], initiated and developed by me when proposing the *spin-charge-family* theory [1–4,20,21] assuming that all the internal degrees of freedom of spinors, with family quantum number included, are describable in the space of d-anti-commuting (Grassmann) coordinates [27], if the dimension of ordinary space is also d. There are two kinds of operators in the Grassmann space, fulfilling the Clifford algebra which anti-commute with one another. The technique was further developed in the present shape together with H.B. Nielsen [28] by identifying one kind of the Clifford objects with γ^s's and another kind with $\tilde{\gamma}^a$'s. In this last stage we constructed a spinor basis as products of nilpotents and projections formed as odd and even objects of γ^a's, respectively, and chosen to be eigenstates of a Cartan subalgebra of the Lorentz groups defined by γ^a's and $\tilde{\gamma}^a$'s. The technique can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

The objects γ^a and $\tilde{\gamma}^a$ have properties

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}\,, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}\,, \quad , \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,, \tag{10.32}$$

for any d, even or odd. I is the unit element in the Clifford algebra.

The Clifford algebra objects S^{ab} and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$\begin{aligned}
S^{ab} &:= (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)\,,\\
\tilde{S}^{ab} &:= (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,,\\
\{S^{ab}, \tilde{S}^{cd}\}_- &= 0\,,\\
\{S^{ab}, S^{cd}\}_- &= i(\eta^{ad}S^{bc} + \eta^{bc}S^{ad} - \eta^{ac}S^{bd} - \eta^{bd}S^{ac})\,,\\
\{\tilde{S}^{ab}, \tilde{S}^{cd}\}_- &= i(\eta^{ad}\tilde{S}^{bc} + \eta^{bc}\tilde{S}^{ad} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac})\,,
\end{aligned} \tag{10.33}$$

We assume the "Hermiticity" property for γ^a's and $\tilde{\gamma}^a$'s

$$\gamma^{a\dagger} = \eta^{aa}\gamma^a\,, \qquad \tilde{\gamma}^{a\dagger} = \eta^{aa}\tilde{\gamma}^a\,, \tag{10.34}$$

in order that γ^a and $\tilde{\gamma}^a$ are compatible with (10.32) and formally unitary, i.e. $\gamma^{a\,\dagger}\gamma^a = I$ and $\tilde{\gamma}^{a\,\dagger}\tilde{\gamma}^a = I$.

One finds from Eq.(10.34) that $(S^{ab})^\dagger = \eta^{aa}\eta^{bb}S^{ab}$.

Recognizing from Eq.(10.33) that two Clifford algebra objects S^{ab}, S^{cd} with all indices different commute, and equivalently for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

$$\begin{aligned}
&S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\;d}, \quad \text{if} \quad d = 2n \geq 4,\\
&S^{03}, S^{12}, \cdots, S^{d-2\;d-1}, \quad \text{if} \quad d = (2n+1) > 4\,,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\;d}, \quad \text{if} \quad d = 2n \geq 4\,,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \cdots, \tilde{S}^{d-2\;d-1}, \quad \text{if} \quad d = (2n+1) > 4\,.
\end{aligned} \tag{10.35}$$

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) in any d

$$\Gamma^{(d)} := (i)^{d/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n,$$
$$\Gamma^{(d)} := (i)^{(d-1)/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n+1\,. \tag{10.36}$$

One can proceed equivalently for $\tilde{\gamma}^a$'s. We understand the product of γ^a's in the ascending order with respect to the index a: $\gamma^0\gamma^1\cdots\gamma^d$. It follows from Eq.(10.34) for any choice of the signature η^{aa} that $\Gamma^\dagger = \Gamma,\ \Gamma^2 = I$. We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$) , while for d odd it commutes with γ^a ($\{\gamma^a, \Gamma\}_- = 0$).

To make the technique simple we introduce the graphic presentation as follows (Eq. (10.14))

$$\stackrel{ab}{(k)}: = \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \qquad \stackrel{ab}{[k]}:= \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,,$$
$$\stackrel{+}{\circ}: = \frac{1}{2}(1+\Gamma)\,, \qquad \stackrel{-}{\bullet}:= \frac{1}{2}(1-\Gamma), \tag{10.37}$$

where $k^2 = \eta^{aa}\eta^{bb}$. One can easily check by taking into account the Clifford algebra relation (Eq.10.32) and the definition of S^{ab} and $\tilde{S}^{ab}$ (Eq.10.33) that if one multiplies from the left hand side by S^{ab} or $\tilde{S}^{ab}$ the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, it follows that

$$S^{ab}\stackrel{ab}{(k)}= \frac{1}{2}k\stackrel{ab}{(k)}\,, \qquad S^{ab}\stackrel{ab}{[k]}= \frac{1}{2}k\stackrel{ab}{[k]}\,,$$
$$\tilde{S}^{ab}\stackrel{ab}{(k)}= \frac{1}{2}k\stackrel{ab}{(k)}\,, \qquad \tilde{S}^{ab}\stackrel{ab}{[k]}= -\frac{1}{2}k\stackrel{ab}{[k]}\,, \tag{10.38}$$

which means that we get the same objects back multiplied by the constant $\frac{1}{2}k$ in the case of S^{ab}, while $\tilde{S}^{ab}$ multiply $\stackrel{ab}{(k)}$ by k and $\stackrel{ab}{[k]}$ by $(-k)$ rather than (k). This also means that when $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors of S^{ab}. We further recognize (Eq. 10.17,10.18) that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, never to $\stackrel{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$, never to $\stackrel{ab}{[-k]}$

$$\gamma^a\stackrel{ab}{(k)}= \eta^{aa}\stackrel{ab}{[-k]},\ \gamma^b\stackrel{ab}{(k)}= -ik\stackrel{ab}{[-k]},\ \gamma^a\stackrel{ab}{[k]}=\stackrel{ab}{(-k)},\ \gamma^b\stackrel{ab}{[k]}= -ik\eta^{aa}\stackrel{ab}{(-k)}\,,$$
$$\tilde{\gamma^a}\stackrel{ab}{(k)}= -i\eta^{aa}\stackrel{ab}{[k]},\ \tilde{\gamma^b}\stackrel{ab}{(k)}= -k\stackrel{ab}{[k]},\ \tilde{\gamma^a}\stackrel{ab}{[k]}= \ i\stackrel{ab}{(k)},\ \tilde{\gamma^b}\stackrel{ab}{[k]}= -k\eta^{aa}\stackrel{ab}{(k)} \tag{10.39}$$

From Eq.(10.39) it follows

$$
\begin{aligned}
&S^{ac} \stackrel{ab}{(k)}\stackrel{cd}{(k)} = -\frac{i}{2}\eta^{aa}\eta^{cc} \stackrel{ab}{[-k]}\stackrel{cd}{[-k]}\,, \qquad \tilde{S}^{ac} \stackrel{ab}{(k)}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{aa}\eta^{cc} \stackrel{ab}{[k]}\stackrel{cd}{[k]}\,,\\
&S^{ac} \stackrel{ab}{[k]}\stackrel{cd}{[k]} = \frac{i}{2} \stackrel{ab}{(-k)}\stackrel{cd}{(-k)}\,, \qquad \tilde{S}^{ac} \stackrel{ab}{[k]}\stackrel{cd}{[k]} = -\frac{i}{2} \stackrel{ab}{(k)}\stackrel{cd}{(k)}\,,\\
&S^{ac} \stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa} \stackrel{ab}{[-k]}\stackrel{cd}{(-k)}\,, \qquad \tilde{S}^{ac} \stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa} \stackrel{ab}{[k]}\stackrel{cd}{(k)}\,,\\
&S^{ac} \stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc} \stackrel{ab}{(-k)}\stackrel{cd}{[-k]}\,, \qquad \tilde{S}^{ac} \stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc} \stackrel{ab}{(k)}\stackrel{cd}{[k]}\,.
\end{aligned}
\tag{10.40}
$$

From Eqs. (10.40) we conclude that $\tilde{S}^{ab}$ generate the equivalent representations with respect to S^{ab} and opposite.

Let us deduce some useful relations

$$
\begin{aligned}
&\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0\,, \quad \stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa} \stackrel{ab}{[k]}\,, \quad \stackrel{ab}{(-k)}\stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]}\,, \quad \stackrel{ab}{(-k)}\stackrel{ab}{(-k)} = 0\,,\\
&\stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}\,, \quad \stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0\,, \quad \stackrel{ab}{[-k]}\stackrel{ab}{[k]} = 0\,, \quad \stackrel{ab}{[-k]}\stackrel{ab}{[-k]} = \stackrel{ab}{[-k]}\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0\,, \quad \stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}\,, \quad \stackrel{ab}{(-k)}\stackrel{ab}{[k]} = \stackrel{ab}{(-k)}\,, \quad \stackrel{ab}{(-k)}\stackrel{ab}{[-k]} = 0\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)}\,, \quad \stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0, \quad \stackrel{ab}{[-k]}\stackrel{ab}{(k)} = 0\,, \quad \stackrel{ab}{[-k]}\stackrel{ab}{(-k)} = \stackrel{ab}{(-k)}\,.
\end{aligned}
\tag{10.41}
$$

We recognize in the first equation of the first line and the first and the second equation of the second line the demonstration of the nilpotent and the projector character of the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, respectively. Defining

$$
\stackrel{ab}{(\tilde{\pm i})} = \frac{1}{2}\,(\tilde{\gamma}^a \mp \tilde{\gamma}^b)\,, \quad \stackrel{ab}{(\tilde{\pm 1})} = \frac{1}{2}\,(\tilde{\gamma}^a \pm i\tilde{\gamma}^b)\,,
\tag{10.42}
$$

one recognizes that

$$
\stackrel{ab}{(\tilde{k})}\stackrel{ab}{(k)} = 0\,, \quad \stackrel{ab}{(\tilde{-k})}\stackrel{ab}{(k)} = -i\eta^{aa} \stackrel{ab}{[k]}\,, \quad \stackrel{ab}{(\tilde{k})}\stackrel{ab}{[k]} = i \stackrel{ab}{(k)}\,, \quad \stackrel{ab}{(\tilde{k})}\stackrel{ab}{[-k]} = 0\,.
\tag{10.43}
$$

Recognizing that

$$
\stackrel{ab}{(k)}^{\dagger} = \eta^{aa} \stackrel{ab}{(-k)}\,, \quad \stackrel{ab}{[k]}^{\dagger} = \stackrel{ab}{[k]}\,,
\tag{10.44}
$$

we define a vacuum state $|\psi_0>$ so that one finds

$$
\begin{aligned}
&< \stackrel{ab}{(k)}^{\dagger} \stackrel{ab}{(k)} >= 1\,,\\
&< \stackrel{ab}{[k]}^{\dagger} \stackrel{ab}{[k]} >= 1\,.
\end{aligned}
\tag{10.45}
$$

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for d-dimensional space, with d even or odd.

For d even we simply make a starting state as a product of $d/2$, let us say, only nilpotents $\stackrel{ab}{(k)}$, one for each S^{ab} of the Cartan subalgebra elements (Eq.(10.35)), applying it on an (unimportant) vacuum state. For d odd the basic states are products of $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$. Then the generators S^{ab}, which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

$$
\begin{array}{c}
\stackrel{0d}{(k_{0d})}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})} \cdots \stackrel{d-1\,d-2}{(k_{d-1\,d-2})}\, \psi_0 \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{[-k_{12}]}\stackrel{35}{(k_{35})} \cdots \stackrel{d-1\,d-2}{(k_{d-1\,d-2})}\, \psi_0 \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{[-k_{35}]} \cdots \stackrel{d-1\,d-2}{(k_{d-1\,d-2})}\, \psi_0 \\
\vdots \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})} \cdots \stackrel{d-1\,d-2}{[-k_{d-1\,d-2}]}\, \psi_0 \\
\stackrel{od}{(k_{0d})}\stackrel{12}{[-k_{12}]}\stackrel{35}{[-k_{35}]} \cdots \stackrel{d-1\,d-2}{(k_{d-1\,d-2})}\, \psi_0 \\
\vdots
\end{array}
\qquad (10.46)
$$

All the states have the handedness Γ, since $\{\Gamma, S^{ab}\} = 0$. States, belonging to one multiplet with respect to the group $SO(q, d-q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrate that for d even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object γ^a being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

For d odd a Weyl spinor has besides a product of $(d-1)/2$ nilpotents or projectors also either the factor $\stackrel{+}{\circ} := \frac{1}{2}(1+\Gamma)$ or the factor $\stackrel{-}{\bullet} := \frac{1}{2}(1-\Gamma)$. As in the case of d even, all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of $(1+\Gamma)$ and $(d-1)/2$ nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. But γ^a's, being applied from the left hand side, do not change the handedness of the Weyl spinor, since $\{\Gamma, \gamma^a\}_- = 0$ for d odd. A Dirac and a Weyl spinor are for d odd identical and a "family" has accordingly $2^{(d-1)/2}$ members of basic states of a definite handedness.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$ for either d even or odd.

While S^{ab} which do not belong to the Cartan subalgebra (Eq. (10.35)) generate all the states of one representation, generate $\tilde{S}^{ab}$ which do not belong to the Cartan subalgebra(Eq. (10.35)) the states of $2^{d/2-1}$ equivalent representations.

Making a choice of the Cartan subalgebra set of the algebra S^{ab} and $\tilde{S}^{ab}$

$$S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14},\\ \tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\,10}, \tilde{S}^{11\,12}, \tilde{S}^{13\,14}, \tag{10.47}$$

a left handed ($\Gamma^{(13,1)} = -1$) eigen state of all the members of the Cartan subalgebra, representing a weak chargeless u_R-quark with spin up, hyper charge (2/3) and colour $(1/2\,,1/(2\sqrt{3}))$, for example, can be written as

$$\stackrel{03}{(+i)}\stackrel{12}{(+)}|\stackrel{56}{(+)}\stackrel{78}{(+)}||\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;|\psi\rangle = \\ \frac{1}{2^7}(\gamma^0-\gamma^3)(\gamma^1+i\gamma^2)|(\gamma^5+i\gamma^6)(\gamma^7+i\gamma^8)|| \\ (\gamma^9+i\gamma^{10})(\gamma^{11}-i\gamma^{12})(\gamma^{13}-i\gamma^{14})|\psi\rangle\,. \tag{10.48}$$

This state is an eigen state of all S^{ab} and $\tilde{S}^{ab}$ which are members of the Cartan subalgebra (Eq. (10.16)).

The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (10.16)), generate families from the starting u_R quark, transforming u_R quark from Eq. (10.48) to the u_R of another family, keeping all the properties with respect to S^{ab} unchanged. In particular $\tilde{S}^{01}$ applied on a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,,1/(2\sqrt{3}))$ from Eq. (10.48) generates a state which is again a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,,1/(2\sqrt{3}))$

$$\tilde{S}^{01}\;\stackrel{03}{(+i)}\stackrel{12}{(+)}|\stackrel{56}{(+)}\stackrel{78}{(+)}||\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)} = -\frac{i}{2}\;\stackrel{03}{[+i]}\stackrel{12}{[+]}|\stackrel{56}{(+)}\stackrel{78}{(+)}||\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}\;. \tag{10.49}$$

Below some useful relations [2] are presented

$$N^{\pm}_{+} = N^1_{+} \pm i\,N^2_{+} = -\stackrel{03}{(\mp i)}\stackrel{12}{(\pm)}, \quad N^{\pm}_{-} = N^1_{-} \pm i\,N^2_{-} = \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)}, \\ \tilde{N}^{\pm}_{+} = -\stackrel{03}{\tilde{(\mp i)}}\stackrel{12}{\tilde{(\pm)}}, \quad \tilde{N}^{\pm}_{-} = \stackrel{03}{\tilde{(\pm i)}}\stackrel{12}{\tilde{(\pm)}}, \\ \tau^{1\pm} = (\mp)\;\stackrel{56}{(\pm)}\stackrel{78}{(\mp)}, \quad \tau^{2\mp} = (\mp)\;\stackrel{56}{(\mp)}\stackrel{78}{(\mp)}, \\ \tilde{\tau}^{1\pm} = (\mp)\;\stackrel{56}{\tilde{(\pm)}}\stackrel{78}{\tilde{(\mp)}}, \quad \tilde{\tau}^{2\mp} = (\mp)\;\stackrel{56}{\tilde{(\mp)}}\stackrel{78}{\tilde{(\mp)}}\;. \tag{10.50}$$

Acknowledgements

The research was financed by the Slovenian Research Agency, project no. P1-0188.

References

1. N.S. Mankoč Borštnik, Phys. Lett. **B 292** (1992) 25, J. Math. Phys. **34** (1993) 3731 Int. J. Theor. Phys. **40** 315 (2001), Modern Phys. Lett. **A 10** (1995) 587, Proceedings of the 13^{th} Lomonosov conference on Elementary Particle Physics in the EVE of LHC, World Scientific, (2009) p. 371-378, hep-ph/0711.4681 p.94, arXiv:0912.4532 p.119.
2. A. Borštnik, N.S. Mankoč Borštnik, hep-ph/0401043, hep-ph/0401055, hep-ph/0301029, Phys. Rev. **D 74** (2006) 073013, hep-ph/0512062.
3. N.S. Mankoč Borštnik, http://arxiv.org/abs/1011.5765, http://arXiv:1012.0224, p. 105-130.
4. N.S. Mankoč Borštnik, J. of Modern Physics **4** No.6, (2013) 823-847; doi:10.4236/jmp.2013.46113; http://arxiv.org/abs/1207.6233; http://arxiv.org/abs/1212.3184. New J. of Phys. **10** (2008) 093002, hep-ph/0606159, hep/ph-07082846, hep-ph/0612250, p.25-50.
5. N.S. Mankoč Borštnik, H.B. Nielsen, "Discrete symmetries in the Kaluza-Klein-like theories", http://arxiv.org/abs/1212.2362. T. Troha, D. Lukman, N.S. Mankoč Borštnik, in this proceedings p. 212.
6. R. Jackiw, K. Johnson, Phys. Rev. **D 8** (1973) 2386.
7. S. Weinberg, Phys. Rev. **D 19** (1979) 1277, S. Dimopoulos, L. Susskind, Nucl. Phys. **B 155** (1979) 237.
8. T. Appelquist, J. Terning, Phys. Rev. **D 50** (1994) 2116.
9. P.Q. Hung, Phys. Rev. Lett. **80** (1998) 3000, P.H. Frampton, P.Q. Hung, M. Sher, Phys. Rept. **330** (2000) 263.
10. S.J. Huber, C. A. Lee, Q. Shafi, Phys. Lett. **B 531**(2002) 112.
11. A.J. Buras, B. Duling, T. Feldmann, arxiv:1002.2126v3.
12. Yu.A. Simonov, arxiv:1004.2672v1.
13. T.A. Ryttov, R. Shrock, arxiv: 1004.2075.
14. Z. Kakushadze, S.H.H. Tye, arxiv:9605221.
15. H. Fritzsch, Phys. Lett. **B 73**(1978)317.
16. R.S. Chivukula and H. Georgi, Phys. Lett. **B 188** (1987) 99.
17. G. D'Ambrosio, G. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. **B 645** (2002) 155–187, arXiv:hep-ph/0207036.
18. R. Alonso, M.B. Gavela, L. Merlo, S. Rigolin, arXiv: 1103.2915v1[hep-ph].
19. C.D. Froggatt, H.B. Nielsen, Nucl. Phys. **B 147** (1979) 277.
20. G. Bregar, M. Breskvar, D. Lukman, N.S. Mankoč Borštnik, New J. of Phys. **10** (2008) 093002, hep-ph/0606159, hep/ph-07082846, hep-ph/0711.4681, http://dark09.klapdor-k.de/mankoc.pdf, http://viavca.in2p3.fr/ .
21. G. Bregar, N.S. Mankoč Borštnik, Phys. Rev. **D 80** (2009) 083534.
22. A. Hernandez-Galeana, N.S. Mankoč Borštnik, "Masses and Mixing matrices of families of quarks and leptons within the Spin-Charge-Family theory, Predictions beyond the tree level", arXiv:1112.4368 p. 105-130, arXiv:1012.0224 p. 166-176.
23. N. S. Mankoč Borštnik, H. B. Nielsen, Phys. Lett. **B 633** (2006) 771-775, hep-th/0311037, hep-th/0509101, Phys. Lett. **B 644** (2007)198-202, hep-th/0608006, Phys. Lett. **B** 10.1016 (2008).
24. D. Lukman, N. S. Mankoč Borštnik, H. B. Nielsen, New J. of Phys. **13** (2011) 103027, hep-ph/0412208.
25. G. Bregar, N.S. Mankoč Borštnik, "Can we predict the fourth family masses for quarks and leptons?", in this proceedings p. 31; arxiv:1212.3184.
26. D. Lukman and N.S. Mankoč Borštnik, "Families of spinors in $d = (1 + 5)$ with a zweibein and two kinds of spin connection fields on an almost S^2", http://arxiv.org/abs/1212.2370; M. Štimulak, A. Založnik, N.S.Mankoč Borštnik, http://arxiv.org/abs/1212.2374.

27. N.S. Mankoč Borštnik, J. of Math. Phys. **34** (1993) 3731.
28. N.S. Mankoč Borštnik, H.B. Nielsen, J. of Math. Phys. **43** (2002) 5782, hep-th/0111257, J. of Math. Phys. **44** (2003) 4817, hep-th/0303224.
29. G. Bregar, N.S. Mankoč Borštnik, "What scalar fields can be observed on the LHC due to the *spin-charge-family*?", in progress.
30. J. Beringer *et al.*, (Particle Data Group),*Phys. Rev.* D **86** (2012) 010001.
31. A. Hernandez-Galeana, N.S. Mankoč Borštnik, "Symmetries of mass matrices in the *spin-charge-family* theory", in preparation.