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Double Charmed Tetraquarks ⋆

Damijan Janc[a] and Mitja Rosina[a,b]

[b] J. Stefan Institute, P.O. Box 3000, 1000 Ljubljana, Slovenia
[a] Faculty of Mathematics and Physics, University of Ljubljana, 1000 Ljubljana, Slovenia

Abstract. We present a detailed four-body calculation in a basis which is also adequate for a weakly bound state of two mesons. The $T_{cc} = cc\bar{u}\bar{d}$ state with quantum numbers $IS = 01$ and positive parity is analyzed. The influence of a weak three-body force is studied.

The bound state of two mesons is now a very hot topic due to new experimental discoveries. The $c\bar{c}$ resonance [1] and the $D_s(2430)$ state [2] detected this year can be explained in the constituent quark model as two-quark two-antiquark bound states. Here we present some numerical results on the $cc\bar{u}\bar{d}$ system. We use a basis which also contains asymptotic channels of two free mesons so that we are able to treat also weakly bound states. The aim of this talk is to explain numerics involved in the calculations, while the motivation for this subject was presented by Mitja Rosina (these Proceedings).

1 Basis

We are interested only in L=0 states, so we expand the orbital part of the tetraquark wave function in terms of gaussians with different widths. We do not use Jacobi coordinates but we rather choose coordinates which are more natural for the two-quark two-antiquark system. This coordinate systems (Fig. 1) were already introduced in [3] but were not fully applied. The use of all systems is important since although the total angular momentum is zero, one can by using e.g. system b) in Fig. 1 have a nonzero relative angular momenta between two quarks l_{12} or between two antiquarks l_{34} resulting in a more complete Hilbert space.

When we have a strong quark mass asymmetry we expect diquark-antidiquark clustering [3] so that the first coordinate system on Fig 1 is more suitable and the dominant color configuration has the diquark in antitriplet and the antidiquark in triplet color state. On the other hand, if the binding is weak, the direct and exchange meson-meson channels are more adequate. For these channels we need also the sextet-antisextet color configuration as can be seen by the recoupling

$$|1_{13}1_{24}\rangle = \sqrt{\tfrac{1}{3}}|\bar{3}_{12}3_{34}\rangle + \sqrt{\tfrac{2}{3}}|6_{12}\bar{6}_{34}\rangle,$$
$$|8_{13}8_{24}\rangle = -\sqrt{\tfrac{2}{3}}|\bar{3}_{12}3_{34}\rangle + \sqrt{\tfrac{1}{3}}|6_{12}\bar{6}_{34}\rangle.$$

⋆ Talk delivered by D. Janc.

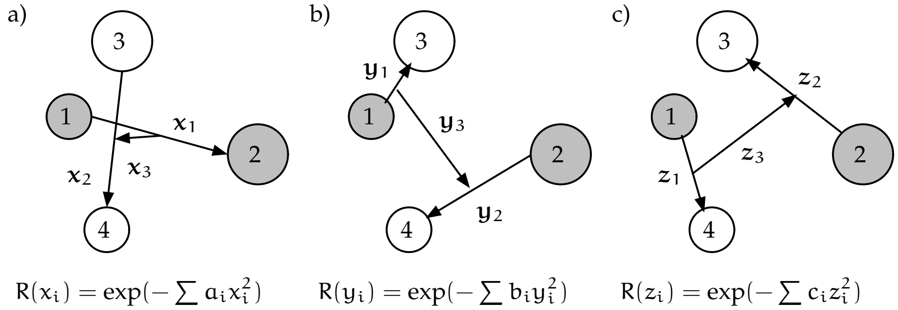

Fig. 1. Two quarks (dashed circles) and two antiquarks (empty circles) in three different relative coordinate systems: a) diquark-antidiquark, b) direct and c) exchange meson-meson channels. The orbital part of the wave function is expanded in Gaussians of relative coordinates $\psi = \sum d_n R^n$ using all three systems.

The important configuration is singlet-singlet while the octet-octet configuration does not make a significant contribution. Similarly one can use different coupling schemes for the spin part of the wave function. To solve the problem as accurately as possible we use all color and spin types of configuration.

2 Binding energy of tetraquarks

We search for solutions of our Hamiltonian with the variational method, where we use a general diagonalization of the Hamiltonian spanned by the nonorthogonal basis functions $R^n = e^{-\sum a_i^n x_i^2}$ or $e^{-\sum b_i^n y_i^2}$ or $e^{-\sum c_i^n z_i^2}$, $n = 1...N$. We have built the basis functions step by step by adding the beat configurations from Fig. 1 with the best color-spin configurations allowed for our quantum numbers (IS=01, positive parity and color singlet) after optimizing the corresponding widths. To obtain 1 MeV accuracy we constructed in this way basis with up to $N_{max} = 40$ functions. This basis states can also accommodate two asymptoticly free mesons if the four-body problem have no bound state.

In our calculations we use nonrelativistic potential model with the Bhaduri potential [4] which is very successful in reproducing the ground state of almost all mesons. The calculation in harmonic oscillator basis [5] has shown that the T_{cc} tetraquark in this model is not bound. Similarly a phenomenological estimate of the mass [6] also suggest that the system is not bound. This estimate is built on the assumption that one can neglect contributions from the sextet-sextet configuration and from direct and exchange meson-meson channels in Fig. 1. In our approach our ground state is a state of two free color singlet mesons so that the mass of the tetraquark is equal to the sum of masses of the D and D^* mesons. For this it is crucial to use in the expansion also states b) and c) from Fig. 1. Since we are interested how the results are changed if we slightly modify the parameters in the Bhaduri potential or add some new weak three-body interaction which would not spoil the meson spectroscopy and will have only minor effects also for baryons. We investigate the possibility of weak binding of T_{cc} and we need a good description also of asymptotic states with respect to which we are calculating the binding energy. This is why we find our basis more suitable than the basis used in [5].

To get some deeper understanding of our four-quark system we calculate the masses of the tetraquarks in a basis where we do not optimize all widths of Gaussian functions, but keep one of them fixed. The most natural choice is to keep the width which define the wave functions between two two-body cluster fixed ($1/d^2$=a_3, b_3, or c_3). If this width are very large the mass of the system should be equal to the sum of the masses of the two mesons. since our basis states do include this asympthotical configurations. On the other hand if the plot of the mass of the tetraquark as a function of this parameter has a local minimum with the mass lower than the asymptotic value, we have a four-body bound state. In this way we can get some information about interaction between two two-body clusters in tetraquark although this mass at fixed d should not be confused with effective potential in Born–Oppenheimer approximation.

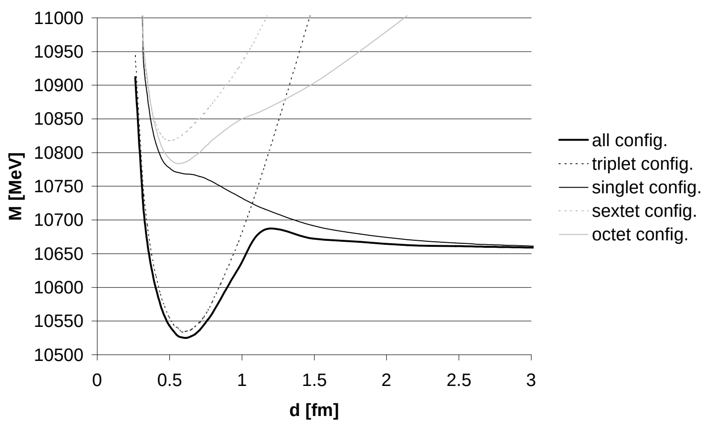

Fig. 2. The mass of T_{bb} as a function of the width between two clusters. Different curves present results of the calculations where only some type of color wave function were used in expansion of the tetraquark wave function (e.g. dotted curve for results with only $|\bar{3}_{12}3_{34}\rangle$ configurations.)

We illustrate this on the $bb\bar{u}\bar{d}$ tetraquark which was already rigorously solved with Bhaduri potential in [5] in harmonic oscillator basis. Results are shown in Fig. 2. The masses of free B and B* mesons obtained with Bhaduri potential are 5301 MeV and 5350 MeV respectively. We see that for large d the energy of the system approaches this value. But at d~0.6 fm we have a minimum which indicate that the T_{bb} is bound in our model. On the same figure are presented the results of calculations with only some type of color wave function used in expansion of the tetraquark wave function. We see that for the minimum at d~0.6 fm the $|\bar{3}_{12}3_{34}\rangle$ configurations are far the most important. Using only this configurations the mass of the tetraquark is 10531 MeV which is only 6 MeV above the energy obtained if we use all color configurations and do minimization without fixing any of the widths. This then means that by ignoring few percents in the binding energy that the ground state of the T_{bb} tetraquark is the antidiquark in color triplet state and the diquark in color antitriplet between which the relative motion can be described by $e^{-x_3^2/(0.6fm)^2}$. Thus the T_{bb} tetraquark can be described as the harmonic oscillator built out of the heavy diquark and light antidiquark.

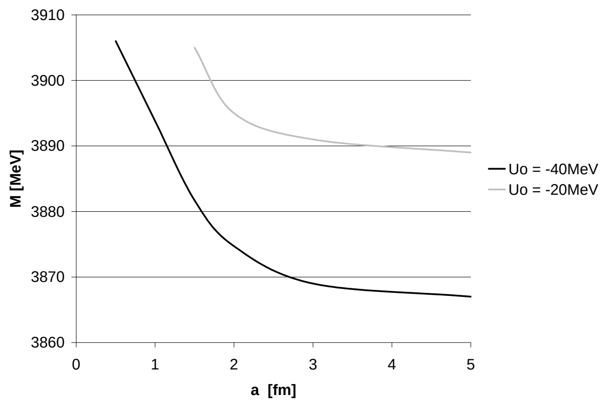

Fig. 3. The mass of T_{cc} as a function of the smearing of three body potential for two different strengths. The asymptotic mass of D plus D* is 3906 MeV in our model.

As expected, we have clustering in color singlet states for large *d* (Fig. 3), while due to confinement the energy of colored configurations rises sharply. The rise for small *d* ($d <$ 0.5 fm) is due to the kinetic energy between two clusters.

3 Three-body interaction

The T_{cc} tetraquark in the nonrelativistic constituent quark model with the Bhaduri potential is above the D D* threshold. But as one can see on Fig 3 that the mass of T_{cc} as a function of the width between two clusters has a significant minimum at $d\sim$ 0.7 fm which indicates a diquark-antidiquark clustering. Now we investigate how close to binding this system is in this model. We do this by introduction a SU(3) color invariant three body interaction. The origin and influence of such interaction on three and four quark state was studied in [7]. We present the results of detailed four-body calculations with Bhaduri potential extended with the tree-body interaction of the form

$$V^{3b}_{qq\bar{q}}(\mathbf{r}_i,\mathbf{r}_j,\mathbf{r}_k) = -\frac{1}{8}d^{abc}\lambda^a_i\lambda^b_j\lambda^{c*}_k U_0 exp[-(r_i^2+r_j^2+r_k^2)/a^2],$$
$$V^{3b}_{q\bar{q}\bar{q}}(\mathbf{r}_i,\mathbf{r}_j,\mathbf{r}_k) = \frac{1}{8}d^{abc}\lambda^a_i\lambda^{b*}_j\lambda^{c*}_k U_0 exp[-(r_i^2+r_j^2+r_k^2)/a^2].$$

Here r_i is the distance of the i-th quark from the center of the triangle formed by i-th, j-th and k-th quark, and similarly for r_j and r_k. λ_a are the Gell-Mann color matrices and d^{abc} are the SU(3) structure constants ($\{\lambda^a,\lambda^b\} = 2d^{abc}\lambda^c$).

The diagonal matrix elements of the color part of the three body interaction between two quarks and an antiquark are -5/18 and 5/9 for $|\bar{3}_{12}3_{34}\rangle$ and $|6_{12}\bar{6}_{3}4\rangle$ color states, respectively. If the strength of this interaction U_0 is negative it will lower the states with diquark-antidiquark configuration. This can be seen on Fig. 4. The dependence of the mass of the T_{cc} tetraquark on the strength of the potential U_0 and on the smearing of this potential is shown in Fig. 3. When $a=$ 3 fm and $U_0 = -20$ MeV the system is bound with the energy of -15 MeV, while as it can be seen on Fig. 4 it is unbound if we fix one of the parameters in orbital wave function. The system still possesses clustering of quarks into diquark and antidiquark but the simple picture where the diquark and the antidiquark form a harmonic oscillator is not accurate anymore. The effective interaction between clusters has now more complicated form. Since $d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c/8$ in color singlet baryons is 10/9 this interaction will lower the masses of the baryons for about U_0 if $a>>$ 1 fm (the size of the baryon) and less for smaller a. Since the Bhaduri potential gives ~ 10 MeV too large masses of baryons this interaction would also improve baryon spectroscopy. But we wish to keep the effect of this new interaction as small as possible, so we prefer weaker three-body force ($U_0 \sim -10$MeV).

The main result therefore is that while T_{cc} is not bound with the Bhaduri potential we can change the situation with a modification of this potential. Just by changing the parameters (strength of confinement, masses) one can not achieve this goal since it is not possible just to reduce the mass of the tetraquark without reducing masses of mesons and thus lowering the threshold. But a weak three-body force whose color factor is zero in the asymptotic channel can lead to the binding.

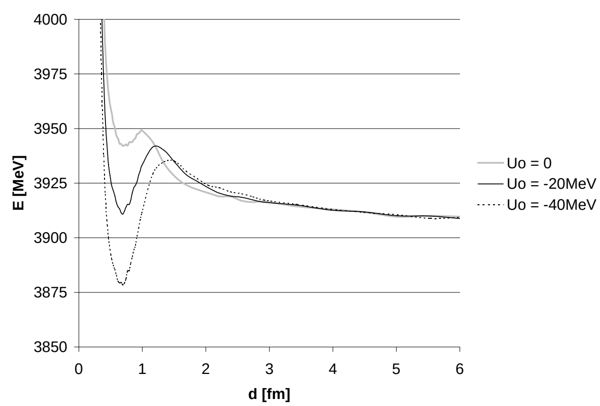

Fig. 4. The mass of T_{cc} as a function of the width between two clusters. The results of the calculations for three different strengths of the tree-body potential are shown. The smearing of this potential is $a = 3$ fm.

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