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PRINCIPLES OF FRACTURE MECHANICS FOR SPACE APPLICATIONS

NA^ELA MEHANIKE LOMA ZA UPORABO V VESOLJU

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Evaluation of the existing and new adhesives may in principle be reduced to the theoretical and/or experimental determination of the material resistance to decohesion as measured by the specific "bonding energy" which must be exceeded via an increase of the external loads and the resulting locally induced state of stress in order to break the bond between two adhesively joined deformable materials. This entity is not merely a material property reflecting simply the strength of the adhesive layer, but it also depends on the elastic moduli of the substrate and the material bonded to it. It is, in fact, a mismatch between the two sets of elastic constants that has an essential influence on the final value of the specific energy of adhesion.

From the theory provided by Nonlinear Mechanics of Fracture it follows that in order to damage the structural integrity of an adhesive bond, it suffices to bring a minute pre-existing crack-like defect to a critical local stress level at which a sustained propagation of fracture becomes thermodynamically feasible – as required by the classic energy balance equation of Griffith. For most loadings and geometrical configurations of the structural component the initiation of crack extension is tantamount to the catastrophic failure which involves an unstable separation and cannot be stopped even when the external loads are reduced to zero.

Intrinsic strength of the bond can also be altered due to variations in the external conditions such as temperature, cyclic loading, an increased rate of loading or the chemically aggressive environment. The state of stress induced in the neighborhood of the crack front contributes substantially to the process of decohesion and it can pose a formidable mathematical problem when fracture propagates within the thin layer of the adhesive placed between two deformable solids with dissimilar elastic and thermal properties. Frequently, the nature of the problem requires an application of techniques and constitutive equations associated with highly developed deformation and fracture process. The nonlinearities encountered here are two-fold: (1) geometrical and (2) physical. The latter involve time-dependent phenomena or plasticity depending on the nature and mechanical properties of the substances involved, the substrate and the adhesive layer. Thus, viscoelasticity so common for a number of commercial adhesives and nonelastic deformation dominated by the irreversible plastic components of the strain tensor requires significant modifications of the constitutive equations. Both viscous and inviscid deformations have to be accounted for by Nonlinear Viscoelasticity and the Theory of Plasticity.

Independently from these studies it is suggested that the fractographic maps of the fracture surfaces are recorded in the post-mortem investigation aimed at direct observation of the Wallner lines and river marks imprinted on the fracture surface while the specimen undergoing fracture is irradiated with ultra-sound waves of various frequencies correlated with the speed of the shock wave which precedes the front of the propagating decohesion zone.

Key words: adhesive bonding, fracture mechanics, constitutive equations, cohezive crack model, materials properties

Oceno obstoje~ih in novih adhezivov lahko izvr{imo s teoreti~no ali/in eksperimentalno dolo~itvijo odpornosti materiala proti dekoheziji, ki jo dolo~a specifi~na vezna energija. Ta mora biti prekora~ena z zunanjo obremenitvijo in lokalno induciranim napetostnim stanjem, ki je potrebno za prelom zveze med dvema adhezivno vezanima preoblikovalnima materialoma. Ta entiteta ni samo lastnost materiala, ki odra`a trdnost adhezivnega sloja, ampak je odvisna tudi od modula elasti~nosti povezanih materialov. Razlika med dvema vrstama elasti~nih konstant ima bistven vpliv na kon~no velikost specifi~ne adhezivne energije.

Iz teorije nelinearne mehanike loma izhaja, da se lahko po{koduje integriteta adhezivne zveze, ~e se majhna, `e obstoje~a razpoka, privede na lokalni kriti~ni nivo napetosti, pri katerem lahko postane propagacija razpoke termodinami~no mogo~a, kot to zahteva klasi~na Griffith-ova ena~ba o ravnote`ju energije. Za ve~ino obremenitev in geometrijskih oblik strukturne komponente je iniciacija rasti razpoke predpogoj za katastrofi~ne po{kodbe, zaradi nestabilne propagacije, ki jih ni mogo~e ustaviti tudi, ko se zunanje breme zmanj{a na ni~.

Specifi~na trdnost zveze se lahko spremeni zaradi spremembe zunanjih pogojev: temperatura, cikli~na obremenitev, pove~ana hitrost obremenitve ali kemi~no agresivno okolje. Stanje napetosti inducirano v okolici ~ela razpoke bistveno prispeva k procesu dekohezije in postane zelo te`ak matemati~ni problem, ko razpoka napreduje v tanki plasti adheziva med dvema trdnima materialoma z razli~nimi elasti~nimi in termi~nimi lastnostmi. ^esto narava problema zahteva uporabo tehnik in konstitutivnih ena~b povezanih z mo~no razvitimi procesi deformacije in preloma. Pri tem naletimo na dvoje vrst nelinearnosti: geometri~ne in fizikalne. Zadnje obsegajo tudi ~asovno odvisne fenomene plasti~nosti, ki so odvisne od narave in mehanskih lastnosti snovi, substrata in plasti adheziva. Zato viskoelasti~nost, zna~ilna za mnoge komercialne adhezive in neelasti~ne deformacije, ki je odvisna od ireverzibilnih plasti~nih komponent tenzorja deformacije, zahteva pomembno spremembo konstitutivnih ena~b. Oboje, viskozno in neviskozne deformacije, je potrebno preveriti na nelinearno viskoelasti~nost in teorijo plasti~nosti.

Neodvisno od teh raziskav se priporo~a, da se zbirajo fraktografske mape prelomnih povr{in pri post-mortem preiskavah z namenom neposrednega opazovanja Wallnerjevih ~rt ter `il, ki nastanejo, ko razpoka napreduje zaradi obsevanja z UZ valovi z razli~no frekvenco odvisno od valovnega {oka, ki napreduje pred ~elom dekohezije.

Klju~ne besede: adhezivna zveza, mehanika loma, konstitutivne ena~be, model kohezivne razpoke, lastnosti materialov

One of the basic assumptions underlying all cohesive crack models used in the description of inelastic fracture has to do with the **shape** of the cohesive force distribution. The exact form of this distribution is unknown, but several very useful clues are provided by the experimental work on fracture at interfaces, cf. Hutchinson[1]. In principle it could be derived from considerations of the molecular forces exchanged between two adjacent planes of atoms which are subject to separation as the leading edge of the crack propagates along the interface.

We shall return to this point after some mathematical preliminaries. The condition of finite stress at the tip of the extended crack, x < a (a visible crack stretches along x < c), valid for the stress boundary conditions

$$p(x) = \begin{cases} \sigma, & 0 < x < c \\ \sigma - S(x), & c < x < a \end{cases} \tag{1}$$

can be set up as follows

$$\begin{aligned} 0 &= K_{TOT}(\sigma, S) - 2\sqrt{\frac{a}{\pi}} \int_0^a \frac{p(x)dx}{\sqrt{a^2 - x^2}} = \\ &= 2\sqrt{\frac{a}{\pi}} \left\{ \int_0^c \frac{\sigma dx}{\sqrt{a^2 - x^2}} + \int_c^a \frac{[\sigma - S(x)]}{\sqrt{a^2 - x^2}} \right\} = \\ &= 2\sqrt{\frac{a}{\pi}} \left\{ \sigma \frac{\pi}{2} \int_c^a \frac{-s(x)dx}{\sqrt{a^2 - x^2}} \right\} \end{aligned} \tag{2}$$

If the stress distribution S(x) is normalized by the reference cohesive stress S_0, say $S(x) = S_0 G(x)$, then Eq. (2) reduces to

$$Q = \int_c^a \frac{G(x)dx}{\sqrt{a^2 - x^2}}, \quad Q = \frac{\pi\sigma}{2S_0} \tag{3}$$

When the variable x is replaced by x_1, $x = x_1 + c$, Eq. (3) reads

$$Q = \int_0^R \frac{G(x_1)dx_1}{\sqrt{a^2 - (x_1 + c)^2}} \tag{4}$$

or, better yet

$$Q = \int_0^1 \frac{G(\lambda)(1-m)d\lambda}{\sqrt{1 - [(1-m)\lambda + m]^2}} \tag{5}$$

Here, $\lambda = x_1/a$ while m is a parameter related to the crack length c and the length of the extended crack, a = c + R, namely, m = c/a. In what follows we shall limit the considerations to the case of R << c, i.e., for $m \to 1$, which is pertinent for "small scale yield condition" met in all cases of practical importance in the context of Materials Science. For this limiting case the integral in Eq. (5) can be simplified as follows:

$$\begin{aligned} [Q(m)]_{m\to 1} &= \\ &= \int_0^1 \frac{1-m}{\sqrt{1-m^2}} \frac{G(\lambda)d\lambda}{\sqrt{1-\lambda}} = \sqrt{\frac{1-m}{2}} \int_0^1 \frac{G(\lambda)d\lambda}{\sqrt{1-\lambda}} \end{aligned} \tag{6}$$

Valuable clues regarding the distribution G(λ) are gained from studies of fracture occurring at the interface between two dissimilar materials joined together either by direct adhesion or by a thin bonding film. In order to account for the experimental data, two main features are expected. First, the stress S should reach a maximum at a certain distance Δ from the crack front. This maximum stress S_{max} may in some cases become substantially larger than the reference stress S_0. It is assumed that S_{max} is attained somewhere within the process zone, most likely at its outer edge, $x_1 = \Delta$. To the left of this point S drops off rapidly to zero to match the boundary condition of stress-free crack at $x_1 = 0$, while to the right of this point S falls down again and levels out at the value S_0, toward the end of the cohesive zone, $x_1 = R$.

In order to account for such behavior we propose a strongly nonlinear function composed of a power function and an exponential. We submit, therefore, a two-parameter distribution function of this form

$$\begin{aligned} S(x_1) &= S_0 \left(\frac{x_1}{R}\right)^n \exp\left[\alpha\left(1 - \frac{x_1}{R}\right)\right], \quad x_1 = R\lambda \\ G(\lambda) &= \lambda^n \exp[\alpha(1-\lambda)], \quad 0 \le \lambda \le 1 \end{aligned} \tag{7}$$

in where α and n are yet undetermined parameters. This function is now substituted into Eq. (6), yielding

$$Q(m) = \frac{\sqrt{R}}{2c} \int_0^1 \frac{\lambda^n \exp[\alpha(1-\lambda)]}{\sqrt{1-\lambda}} \tag{8}$$

Note that for $m \to 1$, the expression (m - 1) can be replaced by R/c, while the integral in Eq. (8) can be cast into a closed form, cf. [2]

$$W(\alpha, n) = \frac{1}{\Gamma\left(\frac{3}{2} + n\right)} \left[\sqrt{\pi} \exp(\alpha)\Gamma(n+1) + {}_1F_1\left(1+n, \frac{3}{2}+n, -\alpha\right) \right] \tag{9}$$

Here the standard notation for the gamma function (Γ) and the hypergeometric function (${}_1F_1$) is used, cf. [3]. Physical interpretation of the integral (9) leads to the energy dissipated within the cohesive zone, hence the symbol W. Finally, combining Eqs. (8) and (9) allows us to define the length of the cohesive zone:

$$R = \frac{\pi}{2W^2} \left(\frac{K_1}{S_0}\right)^2 \tag{10}$$

When K_I attains its critical level K_{Ic}, the Eq. (10) predicts the characteristic microstructural length parameter, $R_{max} = (\pi/2W^2)(K_{Ic}/S_0)^2$.

The primary conclusions of this contribution can be summarized as follows

1. A generalization has been proposed that encompasses all previous cohesive crack models and provides a platform for novel investigations of the influence of the structured nature of the nonlinear zone on the early stages of fracture;

2. By proper choice of parameters α and n we are able to quantify the inner structure of the cohesive zone, the so-called "fine structure", which accounts for the existence of the small process zone of size Δ embedded within the larger R-zone;
3. Microstructure of material is now represented by properties such as the overstress factor, $k = S_{max}/S_0$ and the ductility parameter, $\rho = R_{ini}/\Delta$, in which R_{ini} denotes the threshold value of R associated with the onset of fracture;

For a given k and ρ, the parameters that determine the shape of the S-distribution, α and n, can be evaluated explicitly by matching the ratio $S_{max}/S_0 = (n/\alpha)^n \exp(\alpha - n)$ with the given overstress factor, k. Solving the equation

$$\left(\frac{n}{\alpha}\right)^n \exp[\alpha - n] = k \qquad (11)$$

for the coefficient α, we obtain

$$\alpha = \frac{\rho}{\rho - 1} \ln)(k\rho^n) \qquad (12)$$

Since α/n represents the reciprocal of the coordinate λ at which the maximum in S occurs, we have

$$\frac{\alpha}{n} = \frac{1}{\lambda_{max}} = \frac{R_{ini}}{\Delta} = \rho \qquad (13)$$

Combining it with Eq. (12) results in the transcendental equation

$$\frac{\rho}{\nu(\rho - 1)} \ln(k\rho^n) - \rho = 0 \qquad (14)$$

For any given input set of data, such as specified ρ and k, the other two variables, α and n, can be solved for (numerically, of course). Since the input parameters are deduced from the microstructural data, and can be measured experimentally, the fine structure characteristics α and n are not accessible to an experiment, we have provided a link between the two sets of parameters pertaining to micro-level of fracture. The next step, of course, is to evaluate the macro–level entities such as W and R. Our model makes these calculations possible, too. And thus, we have indeed constructed a bridge between the micro- and macro-scales of fracture representation.

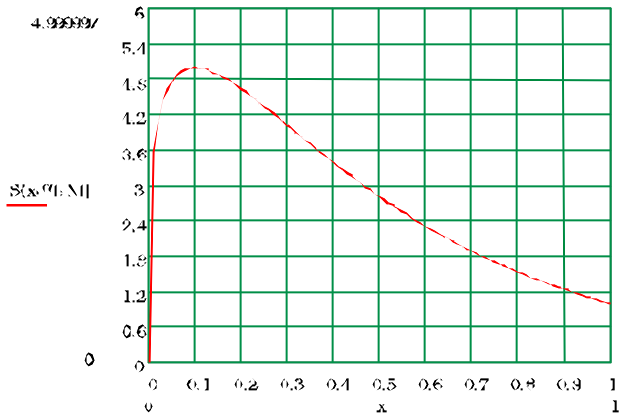

Figure 1: Distribution of the cohesive force $S(\lambda)/S_0$ within the R-zone for the following meso-structural parameters:
- ductility index, $\rho = 10$, and
- overstress factor, k = 5

Slika 1: Porazdelitev kohezivne sile $S(\lambda)/S_0$ v R zoni za naslednje mezo – strukturne parametre:
- index duktilnosti $\rho = 10$ in
- faktor prenapetosti k = 5

To illustrate this statement, we set $\rho = 10$ and k = 5, and then using the equations written above, we obtain n = 0.2403 and $\alpha = \rho n = 2.4031$, while the nondimensional dissipation of energy for those microstructural input data is W (α, n) = 4.4805, and the length of the nonlinear zone is $R_{max} = 0.3506(K_{Ic}/S_0)^2$.

Finally, **Fig. 1** shows the predicted shape of the G-function, which represents a nondimensional cohesive force distribution within the R-zone for the choice of micro-parameters used in our sample calculation.

REFERENCES

[1] J. W. Hutchinson, "The Role of Plasticity in Toughening of Ductile Metals and Interfaces", seminar at Northwestern University in the series "Colloquia on Modern Topics in Mechanics", March **1997**, Evanston, IL

[2] Gradshtein and I. M. Ryzhik, "Tables of Integrals, Series and Products", Academic Press, **1980** (translated from Russian)

[3] "Encyclopedia of Mathematics", Kluwer Academic Press, **1997**, The Netherlands