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Teachers' Attitudes Towards Solving Mathematical Problems

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KLJUČNE BESEDE: reševanje problemov, vrednotenje, lastna ocena, izzivi

POVZETEK – Cilj raziskave je analizirati in preučiti mnenja učiteljev matematike o reševanju problemov pri pouku matematike, izvajanju reševanja problemov, vrednotenju reševanja problemov, vplivu reševanja problemov na učence in s katerimi izzivi se učitelji soočajo pri reševanju problemov. Udeleženci raziskave so bili učitelji matematike (N = 211) iz štirih hrvaških županij, mesta Zagreb, Ličko-senjske županije, Splitsko-dalmatinske županije in Osječko-baranjske županije. Rezultati te raziskave kažejo, da učitelji izvajajo in ocenjujejo tako formativno kot sumativno in da se pri reševanju nalog pri pouku matematike soočajo z različnimi izzivi, čeprav se zavedajo pomena reševanja nalog za učence. Na podlagi teh podatkov sklepamo, da učitelji poučujejo reševanje problemov pri pouku matematike, se zavedajo izzivov, s katerimi se soočajo oni in učenci, ter pomena reševanja problemov za učence. Učitelji spremljajo in ocenjujejo delo učencev ter pozitivno samoevalvirajo njihov način poučevanja reševanja problemov. Da bi reševanje problemov pri pouku matematike potekalo neobremenjeno, je treba omiliti izzive, s katerimi se srečujejo učitelji. Med njimi sta predvsem pomanjkanje časa in količina snovi, ki jo je treba obdelati.

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KEYWORDS: problem-solving, valuation, self-evaluation, mathematics teaching

ABSTRACT – The aim of this research is to analyse and examine mathematics teachers` attitudes towards problem-solving in mathematics classes, implementing problem-solving, evaluating problem-solving, the impact of problem-solving on students and what challenges teachers face when solving problems. The respondents were teachers of mathematics (N = 211) from four counties in Croatia, the City of Zagreb, Lika-Senj County, Split-Dalmatia County and Osijek-Baranja County. The results of this research show that teachers perform and assess both formatively and summatively, and that they face different challenges when solving problems in mathematics classes, although they are aware of the importance of problem-solving for students. Based on this data, we conclude that teachers teach problem-solving in mathematics classes, are aware of the challenges they and their students face, and recognise the importance of problem-solving for students. Teachers monitor and evaluate students' work and positively evaluate their own way of teaching problem-solving. In order for problem-solving to be carried out in the mathematics classes unencumbered, the challenges teachers face, especially the lack of time and the amount of material to be covered, must be mitigated.

1 Introduction

In the last century, there has been a sudden interest in research involving solving problems in the teaching of mathematics. In the sixties and seventies of the last century, the emphasis was placed on the heuristic approach to problem-solving and the use of heuristic strategies for problem-solving (Polya, 1964; Schoenfeld 1979; Sewerin 1979). Also, for many years, solving problems in mathematics has been considered an important aspect of mathematics, both in learning and teaching mathematics. This paper therefore provides an overview of the previous research on problem-solving in mathematics classes. It then presents the results of the research on mathematics teachers' attitudes towards problem-solving in mathematics classes, the evaluation of implementation, the adoption of problem-solving and students' competencies in mathematics classes.

2 Literature review

Solving problems

Mathematicians have consistently sought effective approaches to problem-solving, emphasizing the importance of self-initiative (Zupančič et al., 2023), fostering innovation (Maksimović et al., 2020), and employing rigorous argumentation (Bone et al., 2021).

However, the turning point in teaching problem-solving was the contribution of mathematician Georg Polya. He believed that problem-solving skills are not innate but can be learned. He classified mathematical problems not according to the topic but according to the method of solving them. After the publication of the book How to solve it, in which he presented 4 steps to solve problems, researchers and scientists began to take an interest in the study of problem-solving. In particular, problem-solving has been a topic of every ICME conference since 1969. In the 1980s and 1990s, researchers and educators dealt with defining a mathematical problem, classifying problems and approaching solving mathematical problems, and recognizing the importance of solving problems for students. From the end of the 1990s until now, various countries around the world have been working on implementing problem-solving into curricula and programs. Also, more and more researchers are interested in teachers' opinions about problem-solving in mathematics classes, how they implement problem-solving in class and what challenges they encounter when teaching problem-solving.

A mathematical problem is a problem that can be presented, analysed and, if possible, solved using mathematical strategies. As such it can be a simpler, real-world problem or a more complex, abstract problem, a purely mathematical problem (Blum & Niss, 1994). A mathematical problem is a task in which the solution is not obvious, as well as the solving strategy itself (Pólya 1981; Blum & Niss, 1991; Nunokawa, 2005). Nunokawa (2005) also states that the problem is what requires deeper thinking, using previous knowledge, transforming the task. Problems are also tasks whose difficulty and complexity make them problematic and non-routine (Xenofontos, 2014, according to Schoenfeld, 1992; Goos et al., 2000).

Problem-solving is generally considered the most important cognitive activity in everyday life (Jonassen, 2000). We can define it as finding an answer to a question in a task, for which there is no known method or procedure (Cindrić, 2014).

Gagne (1980) believes that the central point of education should be how to teach students to think, how to use common sense (to think rationally) and to become better at solving problems (Jonassen, 2000, according to Gagne, 1980). A problem is considered a question that is difficult to solve, a doubtful case, or a complex task that involves doubt and uncertainty (Seel, 2012). Finding the value of the unknown must have some social, cultural or intellectual value (Jonassen, 2000).

Problem-solving in education still has no formal structure, although many people have been rewarded for solving problems. Too little attention is paid to the study of the problem-solving process itself (Jonassen, 2000). Solving problems is a competence that is necessary for everyday life (Cindrć, 2014). Therefore, mathematical problems are a good training ground for problems in everyday life (Cindrić, 2014). Problems can be distinguished and studied according to structure, specificity, abstractness and complexity (Jonassen, 2000).

Most psychologists and educators consider problem-solving to be the most important skill in life, as people are confronted with different problems every day. Unfortunately, problem-solving is not really represented in teaching (Jonassen, 2000), and problem-solving is the most effective means of creating creative thinking (Stojaković, 2005).

Educators and psychologists point out that teaching mathematics should not be reduced only to the implementation of methods, procedures or the application of algorithms. Solving mathematical word problems has been described as the "heart of mathematics", because it connects mathematics with real life, which increases the student's motivation to learn mathematics (Khoshaim, 2020).

Problem-solving in mathematics education has various meanings:

- Goal
- Process
- Basic skill
- Research method
- Mathematical thinking
- Teaching approach (Chapman, 1997).

Kurnik (2002) states that the problem situation created by the teacher himself is of a particular interest because the goal is to increase the efficiency of mathematics teaching and raise the level of students' mathematical education. The same author is of the opinion that it is not enough just to impart certain knowledge, perhaps not even to deal with problem situations in the sense of recognizing and formulating a problem, but that students must learn to solve problems, and Stojaković (2005) also believes that the teacher is a collaborator and coordinator of teaching, and not just a supplier of ready-made knowledge and solutions. Students in problem-based teaching think instead of memorizing mechanically, produce instead of reproducing, create instead of copying (Stojaković, 2005). Furthermore, the lesson is not successful if the students do not actively participate, i.e. if they do not solve problems (Kurnik, 2002; Žakelj et al., 2018). Problem-based teaching serves precisely that, for students to become better thinkers and problem solvers (Nickerson, 1994).

Teaching problem-solving

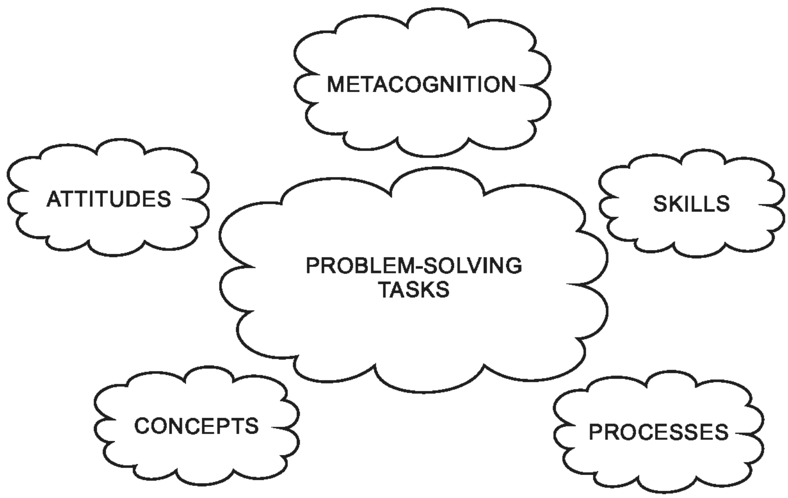
The problem-solving process can be defined as the ability to take certain steps to achieve a certain goal (Hughes & Estrada, 2017). Some problems have one solution, and some have multiple solutions. There are two generally accepted solving methods, the algorithmic one, which requires a series of steps to solve and can be more timeconsuming, and the heuristic method, which reduces the number of cases, and is usually applied in education (Hughes & Estrada, 2017).

In some studies, the authors state the articulation of types of problems, each of which includes different cognitive, affective, and purposeful processes that require focused support (Jonassen, 2000). It is primarily important for teachers to be clear about what they want to achieve by solving a particular problem, which is important for choosing a suitable problem situation (Nunokawa, 2005).

Čižmešija (2015) states that the solving of problem tasks is conditioned by the student's attitudes and skills, as well as metacognition, in order to apply certain processes and connections of mathematical concepts (Figure 1).

Figure 1

Problem-solving as a set of specific skills and attitudes



Over the years of research, it has been shown that problem-solving is not a unique activity, because all problems are not equal in their content, form and solution process (Jonassen, 2000).

We can distinguish problems according to their structure, complexity and abstractness. It should certainly be emphasized that these three elements are not independent, but neither are they equivalent. Classes usually use well-structured problems, more or less complex and without a high level of abstraction. The teacher who presents the students with a problem decides on the characteristics of the problem itself, because the purpose is for the students to find a solution, i.e. to adopt certain outcomes and skills. The complexity of the problem affects the student's ability to solve it, i.e. more complex problems require more cognitive operations than simpler ones (Jonassen, 2000). Problem-solving involves a number of components that the solver has, cognitive abilities, attitudes and behaviour (Nickerson, 1994).

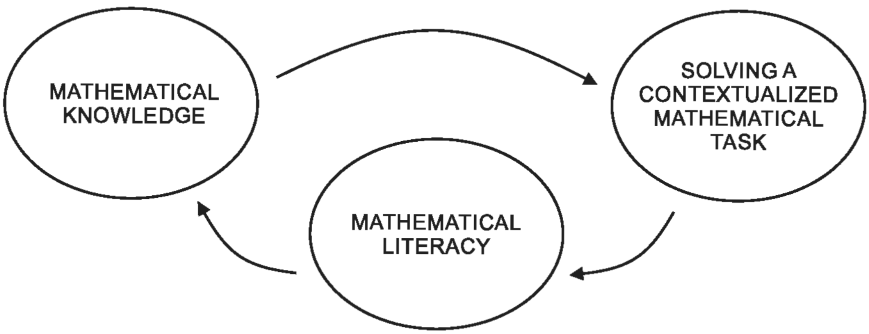
In order to develop certain skills, it is necessary to motivate students, encourage them to communicate and interact. This is best done through activities that encourage discussion (problem-solving) through pair or group work (Radford et al., 1997).

Furthermore, Radford, Netten and Duquette (1997) believe that for the development of mathematical skills, it is necessary to follow the hierarchy, i.e. to classify problems from simpler to more complex.

Similarly, Manfreda Kolar and Hodnik (2021) state that when teaching mathematics, attention should be paid to the complexity of the tasks, i.e. to make the transition from level reproduction to the level of connection and reflection as easy as possible. In other words, tasks should be solved from simpler to more complex. Mathematical knowledge is related to solving contextualized mathematical tasks, which in turn is related to mathematical literacy (Figure 2).

Figure 2

The connection between mathematical literacy, mathematical knowledge and solving a contextualized mathematical problem



Manfreda Kolar, V., & Hodnik, T. (2021). Mathematical literacy from the perspective of solving contextual problems. European Journal of Educational Research, 10(1), 467–483. https://doi.org/10.12973/eu-jer.10.1.467

Radford (1995) mentions the formation of teaching strategies that are based on problem-solving. First, one should start solving simpler problems in order to get to more complex ones, i.e. to the goal. Radford outlined the following procedure:

If students are faced with a problem that is complex in its structure and requires greater mathematical connection, then that problem needs to be simplified and then solved. We then assume how the given problem can be solved with the help of an arithmetic-abstract model, which we finally generalize to an algebraic problem-solving model.

It has been shown that a bad cognitive schema, a bad connection, also causes weaker problem-solving. Cognitive scheme is a way of reasoning, the way an individual perceives a problem situation (Hodnik Čadež & Manfreda Kolar, 2015).

To solve the problem, a developed and well-connected mathematical scheme is required, i.e. a good connection between the problem and the underlying mathematical concept.

We distinguish two types of reasoning:

- □ Inductive reasoning (from individual solutions to a general solution)
- □ Deductive reasoning (from a general solution to a specific solution) (Hodnik Čadež & Manfreda Kolar, 2015).

We also distinguish two types of generalization with regard to the impact on the cognitive scheme:

- □ Expansive generalization (new knowledge is assimilated into the existing cognitive scheme)
- □ Reconstructive generalization (accommodation of the existing scheme, but only well connected) (Hodnik Čadež & Manfreda Kolar, 2015).

In any case, it is necessary that students develop schemes for solving problems, that they set tasks in which they conclude inductively or deductively (Hodnik Čadež & Manfreda Kolar, 2015).

Hodnik and Manfreda Kolar (2022) state that problem-solving and problem setting are interconnected. In a sense, we solve the problem we set, and we set the problem in such a way that we can solve it.

In order to solve the problem, it is necessary to know the basic characteristics of the problem, the mathematical concept with which it is connected, the procedure, and the role it represents. It is necessary to have a well-connected mental scheme. A heuristic approach to teaching, suitable methods and possession of cognitive tools are crucial in solving problems (Hodnik & Manfreda Kolar, 2022). It is also crucial to distinguish between types of generalization (abductive, narrative, naive, arithmetic and algebraic):

When the problem arises, it is necessary to find answers to the following questions:

- □ How to categorize the problem?
- □ How to implement it in research and teaching?
- □ Can it be used in formulating, finding and creating new problems?

It is necessary to reformulate the existing problems or reformulate the existing problems and look at them from a different angle, as well as modelling.

The following two aspects need to be worked out:

- □ Conceptualization
- □ Implementation in the classroom (Hodnik & Manfreda Kolar, 2022).

It is necessary to set the environment and implement the problem in a certain concept. The appropriate role of the teacher and how to assess problem-solving is crucial. It is important to find appropriate problems that are suitable for the age group of students and their abilities, prior knowledge, etc. (Hodnik & Manfreda Kolar, 2022).

Solving problems aims to deepen and apply mathematical knowledge and acquire skills in a changing society (Hodnik & Manfreda Kolar, 2022).

Furthermore, mathematicians always talk about finding new problems, how they pose new problems, and how they formulate new ones from old ones. Mathematicians are aware that problem-solving is an important skill, but also the primary goal of education (Leung, 2013).

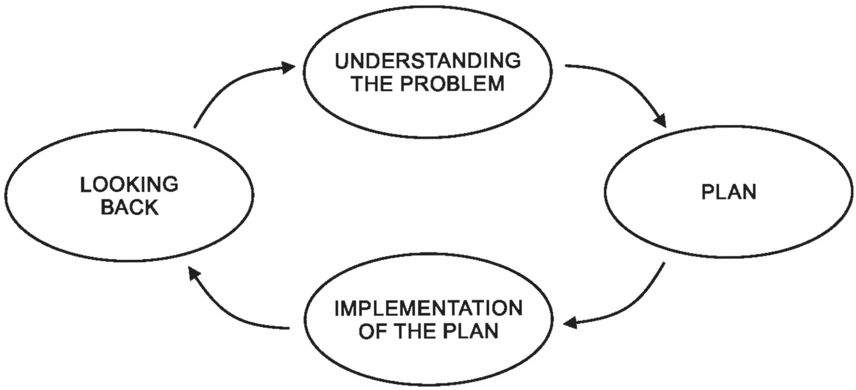
There are four stages of troubleshooting:

- □ Understanding the problem
- □ Coming up with a plan
- □ Implementation of the plan, and
- □ Looking back (Polya, 1945).

Leung (2013) also states that the processes of problem-setting and problem-solving are interconnected; that when one moves from one level to another, one becomes aware of what was done at that stage (Figure 3).

Figure 3

Relationship between problem-setting and problem-solving



Leung, S. S.-K. (2013). Tacher implementing mathematical problem posing in the classroom: challenges and strategies. Educational Studies in Mathematics, 83, 103–116. https://doi.org/10.1007/s10649-012-9436-4

Teachers' attitudes towards problem-solving

The teacher's attitude towards problem-solving in mathematics teaching and the learning process greatly influences the teaching of problem-solving in the classroom. The teacher should primarily have a positive attitude towards mathematics and towards teaching in general (Jukić Matić et al., 2020). The teacher's self-confidence in mathematical abilities, affection, and enthusiasm for teaching influence teaching by solving problems (Harisman et al., 2019).

Mathematicians unconsciously formulated personal metaphors that became the basis of their conceptualization of problems and the design of their teaching (Chapman, 1997).

Primarily, the teaching of mathematics should be directed towards solving problems because it allows students to think about what they are doing (Chapman, 1997). This way of thinking involves the combination and coordination of knowledge, previous experiences, intuition, attitude, beliefs and various abilities, therefore it is not simple.

The effectiveness of the method is studied as it relates to the effectiveness of the teacher, so we must move away from the role of the teacher as a variable if we want to better understand and improve problem-solving (Chapman, 1997).

Attitudes are an important aspect of a teacher's personality, as they are formed over years of experience and are not subject to change. Therefore, it is important to show teachers new aspects of teaching mathematics. Ultimately, the teacher's attitude towards mathematics affects the teaching of mathematics, and therefore the students' achievements (Asempapa, 2022). It is certainly recommended that the teacher, in addition to his knowledge of mathematics, also has the depth of knowledge of mathematics that his student needs for the future (Asempapa, 2022).

Teachers need to develop mathematical resilience – a positive adaptive attitude towards mathematics in order to cope with and face difficulties in teaching. A teacher should set an example for his students to overcome difficulties and limitations in learning mathematics (Ariyanto et al., 2017).

Teachers believe that the greatest problem is a lack of previous mathematical skills, a negative attitude and a decrease in self-confidence in order for students to engage in problem-solving (Khoshaim, 2020). As stated by Khoshaim, (2020), teachers should ensure that the students have certain skills, and that the problem is significant and relevant for the students, before posing a problem to the students.

Based on the previous research, the aim of this research was to find out how mathematics teachers in primary and secondary schools in the Republic of Croatia, in four counties – the City of Zagreb, Lika-Senj County, Split-Dalmatia County and Osijek-Baranja County – implement problem-solving, their opinions and attitudes towards problem-solving, how mathematics teachers value problem-solving and how they evaluate their own teaching of problem-solving.

3 Research methodology

Subject and goal of the research

The aim of this research is to analyse and examine the opinion of mathematics teachers towards problem-solving in teaching mathematics, implementation of problem-solving, evaluation of problem-solving, impact of problem-solving on students and what challenges teachers face when solving problems.

In accordance with the research aim, the following research hypotheses were set:

- H1: There is no statistically significant difference between the genders of mathematics teachers with regard to the method of teaching problem-solving, the importance of problem-solving, challenges when teaching problem-solving, challenges for students when solving problems, and monitoring, evaluating students, and self-evaluation of teachers when solving problems.
- H2: There is no statistically significant difference between mathematics teachers from different counties with regard to the method of teaching problem-solving, the importance of problem-solving, challenges when teaching problem-solving, chal-

lenges for students when solving problems, and monitoring, evaluating students and self-evaluation of teachers when solving problems.

- H3: There is no statistically significant difference between different years of work experience in the education of mathematics teachers with regard to the method of teaching problem-solving, the importance of problem-solving, challenges when teaching problem-solving, challenges for students when solving problems, and monitoring, evaluating students and self-evaluation of teachers when solving problems.
- H4: There is no statistically significant difference between professional advancements regarding the method of teaching problem-solving, the importance of problem-solving, challenges when teaching problem-solving, challenges for students when solving problems, and monitoring, student evaluation, and teacher self-evaluation when solving problems.
- H5: There is no statistically significant difference between the number of schools where teachers work with regard to the method of teaching problem-solving, the importance of problem-solving, challenges when teaching problem-solving, challenges for students when solving problems, and monitoring, evaluating students and self-evaluation of teachers when solving problems.

Measuring instrument

For the purposes of this research, a questionnaire consisting of 3 parts was created.

In the first part of the questionnaire, there were 14 items related to the sociodemographic characteristics of the respondents (gender, type of employment institution, area of work, county of work, age, etc.).

In the second part, there were 10 items, 7 of which were of the Likert type (for example: The following statements refer to your method of teaching problem-solving. "1. I give students a problem, they solve it independently." 1 – never, 2 – almost never, 3 – sometimes, 4 – almost always, 5 – always), two questions related to the teacher's self-assessment on additional education for setting and solving problems, and evaluating the problem-solving element, and one open-ended question as a comment related to the questions of the second part of the questionnaire.

In the third part of the questionnaire, there were 5 items with offered mathematical tasks in which the respondents had to decide whether the task was a problem or not a problem for fifth-grade students.

The data was collected by means of an online survey questionnaire via MS Forms. Content validity was ensured with a careful selection of questions that sought to answer all research questions. First, a pilot study was conducted, after which an effort was made to increase the reliability, validity and applicability of the questionnaire. The pilot study, conducted on a sample of 12 respondents, was designed to obtain information on the clarity of the questions, the attractiveness of the questionnaire, the time taken to complete it, on whether the questionnaire was too long or too short, to obtain information on the response categories from the answers to the closed-ended questions and appropriateness, and if the categories for the closed questions would be generated from the answers to the open-ended questions. The reliability and validity of the instrument increased by selecting a representative, unbiased and not too large or too small sample.

In the research, quantitative data was collected through Likert scales and a scale of self-assessment of knowledge about problem-solving, the use of problem-solving in teaching, by marking the offered answers to questions, how they approach problem-solving, how they evaluate problem-solving. For the reliability of the survey, the Cronbach alpha reliability test was used, which in the pilot study showed a coefficient of $\alpha = .868$, which indicates high reliability.

Space has been left for future research so that respondents can write down the risks and challenges they themselves identify.

Respondents

The respondents were mathematics teachers from four counties in the Republic of Croatia. The sample was a non-random, opportunity sample that represents the meanings of characteristics of the wider population in proportions that can be found in the wider population. According to the data available on the website of the State Bureau of Statistics, there are approximately 250 mathematics teachers in the Osijek-Baranja County, 680 in the City of Zagreb, 350 in the Split-Dalmatia County, and 35 in the Lika-Senj County secondary schools. Finally, the sample consisted of $N = 211$ respondents, of which 59 were from the Osijek-Baranja County, 94 from the City of Zagreb, 49 from the Split-Dalmatia County and 9 from the Lika-Senj County, of which $F = 188$, $M = 23$. The counties were randomly selected from the list where the counties are divided into categories according to the development index, so that one county was selected from each category. This ensures that the data ranges from the least developed to the most developed county. We believe that county development affects the education and professional development of workers. Proximity to larger urban centres and a larger county budget, as well as a larger city or town budget, affect the financial support teachers receive for professional development, such as attending educational workshops, seminars, additional training, and the like.

Procedure

In accordance with the theoretical framework, a survey questionnaire was designed and is attached to this paper. The questionnaire was then converted into an online version in the MS Forms tool, and such was sent to the e-mail addresses of the primary and secondary school principals who were asked to forward it to mathematics teachers as selected in the "Respondents" section. The link to the research was also posted in teacher groups on Facebook with a special note about which counties were included in the research.

At the end of the research, the data was downloaded in the form of an Excel table and processed in IBM SPSS 23.

Twelve respondents participated in the pilot study and it was observed that teachers carry out problem-solving in mathematics classes, that they mostly come up with problems themselves and that problem-solving stimulates students' motivation to learn mathematics, their creativity and connection of knowledge. Teachers value summative and formative problem-solving, but are also aware of the challenges that problem-solving brings. Within the offered mathematical tasks, teachers mostly recognize mathematical problems.

A total of 211 respondents (N = 211) participated in the research, of which 23 were men and 188 were women (M = 23 and F = 188), which was to be expected because there are more females in the education system. For the purposes of the analysis, the reliability of 40 items of the Likert scale was first checked. The Cronbach alpha reliability test showed a coefficient of .747. It can be noted that the reliability of the questionnaire decreased from "very reliable" to "reliable". The variables that were formed in the questionnaire are the following: Method of teaching, Finding problems, Importance for students, Challenges in teaching, Student reaction, Monitoring and evaluation, and Self-evaluation.

The Cronbach alpha reliability test showed the following values for individual variables: Method of teaching ($\alpha = .394$), Finding problems ($\alpha = -.265$), Importance for students ($\alpha = .910$), Challenges in teaching ($\alpha = .764$), Student reaction ($\alpha = .366$), Monitoring and evaluation ($\alpha = .761$), and Self-evaluation ($\alpha = .694$). We can see that the reliability of some variables is low.

Accordingly, a factor analysis was carried out in order to determine the grouping of particles, the factors that explain them and the dispersion of particles itself. Also, factor analysis was used to see which particles "spoil" the reliability of the questionnaire.

KMO (.802) and Bartlett ($p < .001$) indicate that the factor analysis is suitable, and a factor analysis was performed using the method of common factors with the Kaiser extraction criterion and Varimax rotation.

Based on several factor analyses, the new variables Challenges for teachers and Challenges for students were named according to the context they represent. Those two variables were omitted from further factor analysis.

In the factor analysis without the two variables mentioned above, in addition to KMO (.802) and Bartlett ($p < .001$) and the limitation to 3 factors that explain 50.78 % of the variance, it was observed that the particles of the remaining three variables are mostly grouped in one factor. Therefore, according to the context of the particles, two variables Teaching method and Monitoring, evaluation and self-evaluation were formed.

The reliability of the questionnaire after factor analysis was $\alpha = .782$. While the reliability of the particles is as follows: Method of teaching ($\alpha = .634$), Importance for students ($\alpha = .910$), Challenges for the teacher ($\alpha = .786$), Challenges for the student ($\alpha = .782$), Monitoring, evaluation and self-evaluation ($\alpha = .768$). We see that the reliability of all variables is acceptable, so we were able to proceed with further data analysis.

In the next part of the paper, the results of descriptive and inferential statistics will be presented.

Descriptive statistics

In this research, the respondents were mathematics teachers, and the results of some sociodemographic characteristics are shown in Table 1.

Table 1

Descriptive indicators of sociodemographic factors

Characteristic		*N*	*Share [%]*
Respondents		211	x
Sex	Male	23	10.9
	Female	188	89.1
Type of employment institution	Primary school – subject teaching	141	66,8
	Secondary school – grammar school	33	15.6
	Secondary school – vocational or craft occupations	36	17.1
	Other	1	0.5
Professional advancement	Trainee	19	9
	Teacher without professional advancement	133	63
	Supervisor teacher	20	9.5
	Advisor teacher	25	11.8
	Excellent advisor teacher	10	4.7
	Other	4	1.9
County of employment	City of Zagreb	94	44.5
	Lika-Senj County	9	4.3
	Split-Dalmatia County	59	28
	Osijek-Baranja County	49	23.2
Work experience	Less than 5 years	40	19.0
	6–10	35	16.6
	11–20	56	26.5
	21–30	52	24.6
	31–40	27	12.8
	41 and more	1	0.5
Number of schools where teachers work	One school	197	93.4
	Two schools	13	6.2
	Three schools	1	0.5
	Four or more schools	0	0

In accordance with the set research task, we examined teachers' views on teaching methods, importance for students, challenges for teachers and students, monitoring, evaluation and self-evaluation, and the results of the descriptive analysis are shown in Table 2.

Table 2

Descriptive indicators of mathematics teachers' attitudes

Variable		*N*	*Min*	*Max*	*M*	*SD*
Method of teaching	I give the students a problem, they solve it independently.	211	1	5	3.05	0.712
	Students solve the problem in pairs.	211	1	5	2.94	0.622
	Students solve the problem in a small group.	211	1	5	2.71	0.809
	Students solve problems according to their steps.	211	1	5	3.24	0.795
	I use problem-solving to motivate students.	211	1	5	3.60	0.963
Importance for students	Solving problems increases the student's motivation to learn mathematics.	211	1	5	3.71	0.919
	Solving problems motivates students to actively participate in mathematics lessons.	211	1	5	3.77	0.877
	Problem-solving increases students' creativity.	211	1	5	4.21	0.759
	By solving problems, students improve mathematical communication.	211	1	5	4.17	0.802
	Problem-solving increases the application of mathematical knowledge.	211	1	5	4.38	0.729
	By solving problems, students deepen their mathematical knowledge.	211	1	5	4.37	0.747
	By solving problems, students connect individual mathematical concepts.	211	1	5	4.39	0.704
Challenges for teachers	Teaching by solving problems takes a lot of time.	211	2	5	4.22	0.706
	Teaching through problem-solving takes a lot of preparation.	211	2	5	3.99	0.828
	Solving problems takes time during class.	211	1	5	4.00	0.900
	When solving problems, students need more school hours to practice similar examples.	211	1	5	3.94	0.876
	Teaching that includes problem-solving is demanding in preparation.	211	1	5	3.89	0.871
Challenges for students	Only "better" students are successful at solving problems.	211	1	5	3.28	1.088
	Students are not "prepared" to solve problems.	211	1	5	3.33	1.002
	Many of my students give up as soon as they encounter a problem.	211	1	5	3.51	0.963
	Many of my students do not have the necessary prior knowledge and skills to solve problems.	211	1	5	3.50	0.953
	Many of my students lack the confidence to solve problems.	211	1	5	3.64	0.885

Variable		*N*	*Min*	*Max*	*M*	*SD*
Monitoring, evaluation and self-evaluation	I follow the students as they explain their solutions.	211	1	5	4.23	0.623
	I follow the students as they exchange their solutions.	211	1	5	4.06	0.779
	I watch the students communicate if the solutions to the problems differ from group to group.	211	1	5	4.11	0.725
	I conduct a summative evaluation of problem-solving.	211	1	5	3.69	0.998
	I think it is important to often formatively evaluate the element of problem-solving.	211	1	5	3.8	0.846
	When solving a problem, the process is extremely important, and the steps of the process should be evaluated.	211	1	5	3.96	0.726
	I evaluate problem-solving formatively according to the elaborated rubric based on the realization of the outcome of individual components of the mathematical problem.	211	1	5	3.40	1.052
	I think I do a good job of teaching problem-solving.	211	1	5	3.55	0.769
	I am satisfied with my problem-solving teaching.	211	1	5	3.48	0.801
	I think that in the National Curriculum of Mathematics, the element of evaluation is problem-solving.	211	1	5	2.96	0.982

The obtained results show that teachers mostly implement problem-solving so that students work individually (M = 3.05), however, as the mean value of the particles in the Teaching method variable is approximately equal to level 3, teachers neither agree nor disagree with the stated statements except for the statement that they use problem-solving to motivate students (M = 3.60).

Teachers are aware of the importance of problem-solving for students, i.e. problem-solving increases motivation, activity and creativity of students and improves communication, application of knowledge, deepening of knowledge and connection of concepts where they mostly reported level 4.

The teachers also agreed with the statements related to the challenges they face in teaching, that this type of teaching is more demanding in preparation and requires a lot of time, and that this type of teaching requires more time during the lesson and more hours for practice, where they also reported the highest level 4.

Most teachers agreed that students do not have enough prior knowledge, and self-confidence to solve problems and that they give up quickly.

Also, teachers agree with the statements about monitoring, evaluation and self-evaluation when solving problems, except for the statements about the method of formative evaluation, self-evaluation of teaching problem-solving, and the description of the evaluation element in Problem-solving in the National Curriculum of Mathematics, where most reported level 3, that they neither agree nor disagree.

Inferential statistics

In accordance with the set research tasks, a Kruskal-Wallis test was performed for independent samples in order to examine the differences between the genders of teachers with regard to the method of teaching problem-solving, the perception of the importance of problem-solving for students, the assessment of challenges for teachers and students when solving problems, and the perception of teachers on monitoring, evaluation and self-evaluation when solving problems in mathematics classes.

Table 3 shows the results of the Kruskal-Wallis test.

Table 3

Results of the Kruskal-Wallis test of differences with regard to gender, $p < 0.05$

	Method of teaching	*Importance for students*	*Challenges for teachers*	*Challenges for students*	*Monitoring, evaluation, self-evaluation*
χ^2	.808	3.177	4.055	.241	.052
df	1	1	1	1	1
P	.752	.091	.158	.815	.158

The obtained results show that there is no statistically significant difference between the sexes with regard to the method of teaching problem-solving, the importance of problem-solving, challenges when teaching problem-solving, challenges for students when solving problems, and monitoring, evaluating students and self-evaluation of teachers when solving problems, thus confirming the first the null hypothesis.

Since in this paper, we wanted to examine whether there are statistically significant differences between the counties, the Kruskal-Wallis test of differences was performed, and the results are shown in Table 4.

Table 4

Results of the Kruskal-Wallis test of differences between counties, $p < 0.05$

	Method of teaching	*Importance for students*	*Challenges for teachers*	*Challenges for students*	*Monitoring, evaluation, self-evaluation*
χ^2	1.923	2.738	4.149	2.552	4.259
df	3	3	3	3	3
P	.568	.635	.223	.303	.118

It can be seen that there is no statistically significant difference between the counties with regard to the method of teaching problem-solving, the importance of problem-solving, challenges when teaching problem-solving, challenges for students when solving problems, and monitoring, evaluating students and self-evaluation of teachers when solving problems, therefore we confirm the second null hypothesis.

Since we wanted to examine whether there are significant differences between the years of service of teachers with regard to the way of teaching problem-solving, the perception of the importance of problem-solving for students, the assessment of challenges for teachers and students when solving problems, and the perception of teachers about monitoring, evaluation and self-evaluation when solving problems in mathematics classes, we performed the Kruskal-Wallis test for independent samples.

Table 5 shows the results with regard to the years of service.

Table 5

Results of the Kruskal-Wallis test of differences between years of service, $p < 0.05$

	Method of teaching	*Importance for students*	*Challenges for teachers*	*Challenges for students*	*Monitoring, evaluation, self-evaluation*
χ^2	5.226	2.252	3.406	9.577	8.593
df	5	5	5	5	5
p	.330	.715	.342	.143	.344

It is also evident that there is no statistically significant difference between the years of service in education with regard to the method of teaching problem-solving, the importance of problem-solving, challenges when teaching problem-solving, challenges for students when solving problems, and monitoring, evaluating students and self-evaluation of teachers when solving problems, therefore we confirm the third null hypothesis.

We were interested in whether there are differences between teachers who work in one or more schools with regard to the method of teaching problem-solving, the importance of problem-solving, challenges when teaching problem-solving, challenges for students when solving problems, and monitoring, evaluating students and self-evaluation of teachers when problem-solving, so we performed the Kruskal-Wallis test for independent samples.

Table 6 shows the results with regard to the number of schools where teachers work.

Table 6

Results of the Kruskal-Wallis test of differences in the number of schools, p < 0.05

	Method of teaching	*Importance for students*	*Challenges for teachers*	*Challenges for students*	*Monitoring, evaluation, self-evaluation*
χ^2	.070	1.907	1.913	3.032	6.455
df	2	2	2	2	2
p	.342	.601	.336	.633	.356

Based on the results obtained, it was determined that there is no statistically significant difference between the number of schools where teachers work with regard to the method of teaching problem-solving, the importance of problem-solving, challenges when teaching problem-solving, challenges for students when solving problems, and monitoring, evaluating students and self-evaluation teachers when solving problems, thus accepting the fifth null hypothesis.

Table 7

Results of the Kruskal-Wallis test of differences in advancement, p < 0.05

	Method of teaching	*Importance for students*	*Challenges for teachers*	*Challenges for students*	*Monitoring, evaluation, self-evaluation*
χ^2	6.220	9.469	3.162	17.927	17.122
df	5	5	5	5	5
p	.226	.058	.860	.011	.009

Furthermore, when we look at the results of the Kruskal-Wallis test of differences in professional advancement (Table 7) regarding the method of teaching problem-solving, the perception of the importance of problem-solving for students, the assessment of challenges for teachers and students, and the assessment of monitoring, evaluation and self-evaluation, we can conclude that the null hypothesis H4 is partially accepted: there is no statistically significant difference between the advancements in the teacher's profession regarding the teaching method, opinion about the importance of solving problems for students and opinion about the challenges for teachers, while there is a statistically significant difference between the advancements in the teacher's profession regarding the challenges for students when solving problems and monitoring, evaluation and self-evaluation of teachers. The values of rejected parts of the null hypothesis are marked in bold. In accordance with the set research task, we investigated whether there are differences between trainee teachers, teachers without professional advancement, supervisor teachers, advisor teachers, excellent advisor teachers, and teachers who are classified in the category "Other", and a mid-range analysis of variables was carried out with variables Challenges_students and Monitoring_evaluation_self-evaluation by Kruskal-Wallis test of differences, and the results are shown in Table 8.

Table 8

Mean ranks of the variables Challenges_students and Monitoring_evaluation_self-evaluation by Kruskal-Wallis test analysis, p < 0.05

What is your professional level?		*N*	*Mean rank*
Challenges Students	Trainee	19	111.79
	Without professional advancement	133	115.57
	Supervisor teacher	20	94.33
	Advisor teacher	25	76.62
	Excellent advisor teacher	10	77.30
	Other	4	74.13
	Total	211	
Monitoring Evaluation Self-evaluation	Trainee	19	102.58
	Without professional advancement	133	97.22
	Supervisor teacher	20	125.98
	Advisor teacher	25	135.86
	Excellent advisor teacher	10	108.85
	Other	4	120.50
	Total	211	

If we look at the mid-range results, teachers without professional advancement report the greatest challenges for students when solving problems, while teachers classified as "Other" report the least challenges. However, both advisor teachers and excellent advisor teachers report challenges in greater detail. The reason for this may also be that there are only 4 teachers who were classified as "Other". It was to be expected that teachers who advanced in their profession would report fewer challenges that their students face when solving problems.

Furthermore, teachers without professional advancement report the lowest degree of monitoring, evaluation, and self-evaluation when teaching problem-solving, while advisor teachers report the highest degree. It was to be expected that teachers who advanced in their profession reported a greater degree of monitoring, evaluation and self-evaluation when teaching problem-solving.

4 Discussion

This paper aims to examine how teachers teach problem-solving in mathematics classes, how they perceive the importance of problem-solving for students, how they assess the challenges they face and the challenges students face when solving problems, the teachers' perception of monitoring, evaluation and self-evaluation when solving problems in mathematics classes.

Teachers notice the importance of solving problems in mathematics lessons for students, such as connecting mathematical concepts, deepening mathematical knowledge, motivating students, student creativity, and developing communication skills. This finding is consistent with previous research conducted by Radford, Netten and Duquette (1997), who confirmed that communication is key to learning mathematics and acquiring mathematical competence, and communication is best done through interaction and participation in solving problems. Stojaković (2005) states that students think, produce and create when solving problems, and are creative at the same time. Leung (2013) also states that problem-solving is not only an important skill but also a primary goal of education. He conducted research on 60 primary school teachers (N = 60) in which the teachers designed problems, decided when to use the problems in class, and finally systematized them based on student works. The result of this study was the categorization of problem types, 24 categories in accordance with the curriculum, and three teachers who continued to further research the categorization of problems for other classes and other topics with the goal of writing a book with a systematic presentation of mathematical problems for students. Cindrić (2014) states that problem-solving is a skill necessary for everyday life and that solving mathematical problems contributes greatly to this. Hodnik and Manfreda Kolar (2022) state that solving problems deepens and applies mathematical knowledge, and they acquire skills that are important in a changing society.

Teachers state that students mostly solve problems individually and according to their own pace, less in pairs or groups, although researchers state the importance of group work. As mentioned previously, Radford, Netten and Duquette (1997) believe that it is crucial that students solve problems through interaction and cooperation, and this takes place through pair work or group work. Dževahirić, Kukić and Hadžiabdić (2020) conducted research on 12 (N = 12) seventh-grade primary school students, and the results showed that the students working in groups learned mathematics, because in this way they exchange opinions with their colleagues about the assigned tasks and talk about the way of solving them.

As problem-solving is a broad topic that has been researched for a long time, in this research it was observed that teachers are aware of the challenges they face in solving problems in class, as well as the challenges students face when solving problems, such as the lack of time, unpreparedness of students, demandingness in preparing such classes. Similar results were obtained by Khoshaim (2020) and Tomić, (2015). Khoshaim (2020) conducted a research on university professors who teach mathematics courses to students of non-mathematical studies, in two phases. In the first phase, 15 teachers (N = 15) participated, and in the second, interviews were conducted with 4 teachers (N = 4). The results were that the students were not prepared for this way of teaching, that they lacked self-confidence and that they gave up quickly. Tomić (2015) conducted a research on 220 seventh-grade students (N = 220), and the results showed that this type of teaching is more demanding for teachers in preparation and that students are not ready for this type of teaching.

Furthermore, it was determined in this research that teachers report monitoring and evaluating students summatively, but also formatively by monitoring students during problem-solving, the way students communicate, and how they explain their solutions. Teachers also believe that they do their job well when solving problems, which could be compared with the next research after certain time. Rosli, Goldsby and Capraro (2013) report assessing students' problem-solving according to a predetermined rubric. Similarly, Anderson and Puckett (2003) state that the assessment of problem-solving is based on a series of rubrics. There is no research in Croatia, but there are recommendations, for example, Janeš (2022) gave recommendations on how to formatively and summatively evaluate problem-solving, according to an elaborate rubric and similar.

In addition, as the teachers answered the open-ended questions and left their comments, it can be concluded that many of them face challenges in teaching, such as lack of time, 1 hour of mathematics lessons per week in high school, unmotivated students, lack of prior knowledge and quickly dropping out of students and the like. Teachers suggest that problem-solving should be started already in the lower grades of primary school so that over time it develops into something common. Similar results were given by Khoashim (2020), where it was found that teachers believe that solving problems takes a lot of time, that they already have a lot to process without solving problems, and that there is a lack of prior knowledge and lack of motivation among students.

5 Advantages and limitations

Since so far in the Republic of Croatia there has rarely been any research on the opinions of mathematics teachers about problem-solving in mathematics classes, the implementation of problem-solving, the evaluation of problem-solving, the impact of problem-solving on students and what challenges teachers face when solving problems, this paper represents a significant contribution to a better understanding of problem-solving in mathematics classes.

The scientific contribution of this paper is the creation of a measuring instrument with a high-reliability coefficient.

A limitation of this study is the unreliability of the Problem-finding variable. However, with factor analysis, the measuring instrument was improved.

Furthermore, the limitation is also that, although there were only four counties in the research, it is recommended for future research to include respondents from other parts of the Republic of Croatia.

6 Conclusion

The contribution of this study at the level of the Republic of Croatia is that a different sample was used than in previous studies. By randomly selecting counties from the development categories, it was achieved that we can largely infer the rest of the Republic of Croatia as this method covers all categories according to the development index.

The results show that teachers, regardless of their gender, the county they work in, the number of years of service and the number of schools in which they work, are equally likely to teach problem-solving, are equally aware of the importance of problem-solving for students and the challenges they and the students face. They also observe and evaluate the way they teach problem-solving regardless of their gender, the county they work in, the years of service, and the number of schools in which they work.

However, there are also differences in professional advancement with respect to the challenges students face in problem-solving. Teachers without professional advancement consider these challenges to a greater degree than teachers who fall into the Other category. The mean ranks of the Advisor Teacher, Excellent Advisor Teacher and Other categories are very close to each other, and only 4 teachers were classified in the Other category. From this, we can conclude that it is to be expected that teachers who have advanced in the profession report fewer challenges. The same is true for monitoring, student ratings of problem-solving, and teacher self-evaluation. This is because teachers who are classified as advisor teachers are highly likely to monitor and evaluate students in problem-solving and self-evaluation, while teachers without professional advancement indicated the lowest level of monitoring, assessment of students in problem-solving and self-evaluation.

In order to improve the practice of pedagogical work, more and more attention should be paid to problem-solving in mathematics education. This contributes to a better understanding of mathematical concepts and the application of mathematical knowledge as well as mathematical literacy.

In order for problem-solving in mathematics classes to be unencumbered, the challenges faced by teachers, especially the lack of time and the amount of material to be covered, need to be alleviated. The recommendation for this is to increase the weekly workload of mathematics lessons, but not to reduce the teacher's workload, on the contrary, so that teachers can prepare well for these lessons and provide quality feedback to students in a timely manner. Systematic problem-solving, in addition to the points mentioned above, will also reduce the challenges that students face today when solving problems in mathematics classes.

Ivana-Marija Pavković, dr. Anna Alajbeg

Stališča učiteljev o reševanju matematičnih problemov

Reševanje problemov pri pouku matematike dobiva večjo pozornost in pomen od sredine prejšnjega stoletja, nato pa je postalo predmet raziskovanja različnih znanstvenikov. Raziskovalci so raziskovali opredelitev problema, metode in strategije reševanja problemov, kako pomagati učencem, hevristični pristop ter stališča in mnenja učiteljev.

V teoretičnem delu prispevka smo najprej opredelili matematični problem, reševanje problemov, kako mentalne sheme učencev vplivajo na reševanje problemov ter kakšna so stališča in mnenja učiteljev o reševanju problemov pri pouku matematike.

Matematični problem je problem, ki ga je mogoče predstaviti, analizirati in po možnosti rešiti z uporabo matematičnih strategij. Reševanje problemov na splošno velja za najpomembnejšo kognitivno dejavnost v vsakdanjem življenju (Jonassen, 2000). Proces *reševanja problemov lahko definiramo kot sposobnost sprejeti določene korake za dosego določenega cilja (Hughes in Estrada, 2017).*

V letih raziskav se je pokazalo, da reševanje problemov ni edinstvena dejavnost, saj vsi problemi niso enaki po svoji vsebini, obliki in procesu reševanja (Jonassen, 2000).

Predhodne raziskave so pokazale, da je sposobnost reševanja problemov tesno povezana z matematično pismenostjo in znanjem matematike. Prav tako na uspešnost reševanja problemov vplivajo učenčevo poznavanje matematičnih pojmov in procesov, posedovanje določenih veščin, učenčev odnos in metakognicija, to je povezovanje znanja, priklic in refleksija o smiselnosti rešitve naloge in postopka reševanja.

Da bi bili učenci pri tem uspešni, je treba razviti miselno shemo in zastaviti takšne naloge, pri katerih sklepajo induktivno ali deduktivno.

Upoštevati je treba tudi, da so učiteljeva stališča pomemben vidik učiteljeve osebnosti, saj se oblikujejo skozi leta izkušenj in se ne spreminjajo.

Učitelji menijo, da so največje ovire, da bi se učenci vključili v reševanje problemov, pomanjkanje predhodnih matematičnih veščin, negativen odnos in zmanjšanje samozavesti (Khoshaim, 2020).

Glede na navedeno je cilj te raziskave preučiti in analizirati mnenja učiteljev matematike v Republiki Hrvaški o reševanju problemov pri pouku matematike, izvajanju reševanja problemov, vrednotenju reševanja problemov, vplivu reševanja problemov na učence in s katerimi izzivi se učitelji srečujejo pri reševanju problemov. V skladu z zastavljenim ciljem raziskave so bile postavljene naslednje ničelne hipoteze:

- □ *H1: Med spoloma učiteljev matematike ni statistično značilne razlike glede načina poučevanja reševanja problemov, pomembnosti reševanja problemov, izzivov pri poučevanju reševanja problemov, izzivov za učence pri reševanju problemov, spremljanja in vrednotenja učencev ter samoevalvacije učiteljev pri reševanju problemov.*
- □ *H2: Med učitelji matematike iz različnih držav ni statistično značilne razlike glede načina poučevanja reševanja problemov, pomembnosti reševanja problemov, izzivov pri poučevanju reševanja problemov, izzivov za učence pri reševanju problemov, spremljanja in ocenjevanja učencev ter samoevalvacije učiteljev pri reševanju problemov.*
- □ *H3: Glede na različna leta delovne dobe v izobraževanju učiteljev matematike ni statistično značilne razlike glede načina poučevanja reševanja problemov, pomembnosti reševanja problemov, izzivov pri poučevanju reševanja problemov, izzivov za učence pri reševanju problemov, spremljanja in ocenjevanja učencev ter samoevalvacije učiteljev pri reševanju problemov.*
- □ *H4: Ni statistično značilne razlike med poklicnimi napredovanji glede načina poučevanja reševanja problemov, pomembnosti reševanja problemov, izzivov pri poučevanju reševanja problemov, izzivov za učence pri reševanju problemov, spremljanja in vrednotenja študentov ter samoevalvacije učiteljev pri reševanju problemov.*
- □ *H5: Ni statistično značilne razlike glede na število šol, kjer učitelji delajo, pri načinu poučevanja reševanja problemov, pomenu reševanja problemov, izzivih pri poučevanju reševanja problemov, izzivih za učence pri reševanju problemov, spremljanju in ocenjevanju učencev ter samoevalvaciji učiteljev pri reševanju problemov.*

V raziskavi so bile uporabljene kvantitativne metode, za namene raziskave pa je bil izdelan vprašalnik, sestavljen iz 3 delov.

V prvem delu vprašalnika je bilo 14 postavk, povezanih s sociodemografskimi značilnostmi anketirancev.

V drugem delu je bilo 10 postavk, od tega 7 Likertovih, dve vprašanji sta se nanašali na učiteljevo samooceno o dodatnem izobraževanju za postavljanje in reševanje problemov ter evalvacijo elementa reševanje problemov ter eno odprto oz. končano vprašanje kot komentar v zvezi z vprašanji drugega dela vprašalnika.

V tretjem delu vprašalnika je bilo 5 postavk s ponujenimi matematičnimi nalogami, pri katerih so se anketiranci morali odločiti, ali je ta naloga za petošolce problem ali ne.

Podatki so bili zbrani s spletnim vprašalnikom preko MS Forms, tako da so bila na uradne naslove ravnateljev osnovnih in srednjih šol v izbranih županijah poslana elektronska sporočila, ki so vsebovala podatke o raziskavi in raziskovalcu ter povezavo do vprašalnika, ravnatelji pa so bili naprošeni, naj prejeto sporočilo posredujejo učiteljem matematike na šoli. Sporočilo z navedenimi podatki je bilo objavljeno tudi v skupinah učiteljev na Facebooku z dodano opombo, iz katerih okrožij naj učitelji rešujejo anketni vprašalnik.

Najprej je bila izvedena pilotna študija na vzorcu 12 respondentov (N = 12), nato pa smo poskušali povečati zanesljivost, veljavnost in uporabnost vprašalnika. Zanesljivost delcev Likertove lestvice po pilotni študiji je bila α = 0.868, kar kaže na visoko zanesljivost.

V raziskavi je sodelovalo 211 učiteljev matematike (N = 211) iz štirih županij Republike Hrvaške, od tega 59 iz Osječko-baranjske županije, 94 iz mesta Zagreb, 49 iz Splitsko-dalmatinske županije in 9 iz Ličko-senjske županije. Sodelovalo je 188 učiteljic (Ž = 188) in 23 učiteljev (M = 23). Vzorec je bil nenaključen, primeren vzorec. Županije so bile izbrane z naključnim izborom s seznama, v katerem so županije razvrščene v kategorije glede na indeks razvitosti, tako da je bila iz vsake kategorije izbrana ena županija.

Izvedena je bila tudi faktorska analiza postavk Likertove lestvice. Zanesljivost vprašalnika po faktorski analizi je bila: α = 0.782. Medtem ko je zanesljivost delcev naslednja: metoda poučevanja: α = 0.634, pomen za učence: α = 0.910, izzivi za učitelja: α = 0.786, izzivi za učenca: α = 0.782, spremljanje, vrednotenje in samoevalvacija: α = 0.768.

Deskriptivna statistika je pokazala, da je večina anketiranih žensk predmetnih učiteljic v osnovni šoli. Večina učiteljev ni napredovala v svojem poklicu in jih je največ iz mesta Zagreb.

Ugotovljeno je bilo, da učitelji večinoma izvajajo reševanje problemov tako, da učenci delajo individualno.

Učitelji se zavedajo pomena reševanja problemov za učence, tj. da reševanje problemov povečuje motivacijo, aktivnost in ustvarjalnost učencev ter izboljšuje komunikacijo, uporabo znanja, poglabljanje znanja in povezovanje pojmov.

Učitelji med izzivi, s katerimi se soočajo pri reševanju nalog pri pouku matematike, navajajo, da je tovrstni pouk bolj zahteven pri pripravi in zahteva veliko časa, hkrati pa zahteva tudi več časa med poukom in več ur za vajo.

Večina učiteljev se je strinjala, da učenci nimajo dovolj predznanja, samozavesti za reševanje problemov in da hitro obupajo.

V skladu z zastavljenimi raziskovalnimi nalogami je bil za neodvisne vzorce izveden Kruskal-Wallisov test za preverjanje postavljenih ničelnih hipotez. Kruskal-Wallisov test je potrdil prvo, drugo, tretjo in peto ničelno hipotezo ter delno potrdil četrto ničelno hipotezo.

Če povzamemo, učitelji, ne glede na spol, občino, v kateri delajo, število let delovne dobe in število šol, na katerih delajo, enako poučujejo reševanje problemov, se enako zavedajo pomena reševanja problemov za učence in izzivov, s katerimi se soočajo oni in učenci. Enako spremljajo, ocenjujejo učence in samoevalvirajo način poučevanja reševanja problemov ne glede na spol, občino, v kateri delajo, število let delovne dobe in število šol, na katerih delajo.

Obstajajo pa tudi razlike v strokovnem napredovanju glede na izzive, s katerimi se srečujejo učenci pri reševanju problemov. Učitelji brez strokovnega napredovanja te izzive upoštevajo v večji meri kot učitelji, ki spadajo v kategorijo "drugo". Povprečne uvrstitve kategorij učitelj svetovalec, učitelj odličen svetovalec in drugi so si zelo blizu, le 4 učitelji so bili uvrščeni v kategorijo drugo. Zato lahko sklepamo, da je pričakovati, da bodo učitelji, ki so napredovali v poklicu, poročali o manj izzivih. Enako velja za spremljanje, ocenjevanje učencev pri reševanju nalog in samoevalvacijo učiteljev. Učitelji, ki so razvrščeni kot učitelji svetovalci, namreč v veliki meri spremljajo in ocenjujejo učence pri reševanju problemov in samoevalvaciji. Učitelji brez strokovnega izpopolnjevanja pa poročajo o najnižji meri spremljanja, ocenjevanja učencev pri reševanju problemov in samoevalvacije.

Glede na to, da do sedaj v Republiki Hrvaški ni bilo raziskav o mnenjih učiteljev matematike o reševanju problemov pri pouku matematike, izvajanju reševanja problemov, vrednotenju reševanja problemov, vplivu reševanja problemov na učence in o tem, kateri so izzivi, s katerimi se srečujejo učitelji pri reševanju problemov, predstavlja to delo pomemben prispevek k boljšemu razumevanju reševanja problemov pri pouku matematike. Ta dokument vsebuje tudi priporočila o tem, kako izboljšati reševanje problemov pri poučevanju matematike, kar pomeni zmanjšanje izzivov, s katerimi se soočajo učitelji in učenci. Da bi to dosegli, je treba povečati tedensko število ur matematike, vendar ne na račun obremenitve učiteljev, temveč nasprotno, potrebno je zmanjšanje, da se bodo učitelji na takšne ure dobro pripravili in učencem pravočasno podali kakovostno povratno informacijo. S tem bi učitelji pri pouku matematike uvedli več reševanja problemov, kar bi prispevalo k boljšemu uspehu učencev pri matematiki, učenci pa bi pridobili veščine reševanja problemov, ki jih čakajo v vsakdanjem življenju.

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