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Exact treatment of the Pauli operator in nuclear matter*

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Abstract. Exact formulae are derived for the matrix element of the Pauli operator Q in the Bethe-Goldstone equation and the binding energy per particle in nuclear matter. Numerical calculations are carried out, using the Bonn B potential and the quark model Kyoto-Niigata potential fss2. The exact treatment of the operator Q brings about non-negligible attractive contribution to the binding energy compared with the standard angle average approximation. However the difference is rather small, which quantitatively demonstrates the good quality of the angle average prescription in nuclear matter calculations.

The Pauli principle plays an essential role in the nucleon-nucleon scattering in nuclear medium. It constrains single particle momenta of intermediate two particles to be above the Fermi momentum k_F. The Pauli operator Q is defined as

$$Q = \frac{1}{2}\sum_{\alpha\beta} |\alpha\beta\rangle\langle\alpha\beta|\ \Theta(k_\alpha - k_F)\ \Theta(k_\beta - k_F)\ .$$

The operator Q depends not only on the magnitude of total and relative momenta of scattering two nucleons but also on their relative angles. Properties of this angular dependence, owing to which partial waves with different angular momenta are coupled, were investigated in the early stage of the development of the Brueckner theory. Werner presented explicit coupled equations in 1959 [1]. However, since the numerical calculations are rather involved, the standard angle average approximation has been introduced to avoid the difficulty.

The matrix element of the operator Q between angular-momentum-coupling states is given as

$$\langle \mathbf{K}k(\ell_1 S_1)J_1 M_1 T_1 T_{z1}|Q|\mathbf{K}'k'(\ell_2 S_2)J_2 M_2 T_2 T_{z2}\rangle$$
$$= \delta(\mathbf{K}-\mathbf{K}')\frac{\delta(k-k')}{k^2}\delta_{S_1S_2}\delta_{T_1T_2}\delta_{T_{z1}T_{z2}}Q(\ell_1 J_1 M_1, \ell_2 J_2 M_2 : S_1 T_1 k K\theta_K\phi_K)\ .$$

We derived [2] useful analytic expressions for the $Q(\ell_1 J_1 M_1, \ell_2 J_2 M_2 : S_1 T_1 k K\theta_K\phi_K)$ as

$Q(\ell_1 J_1 M_1, \ell_2 J_2 M_2 : STkK\theta_K\phi_K)$

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$$= f_{\ell_1 ST}\, f_{\ell_2 ST}\ \{ \delta_{\ell_1 \ell_2} \delta_{J_1 J_2} \delta_{M_1 M_2}\, x_0 + \sum_{L>0, L=\text{even}} (-1)^{S+M_1} \frac{\sqrt{4\pi}\hat{\ell_1}\hat{\ell_2}\hat{J_1}\hat{J_2}}{\hat{L}^3}$$

$$\times \langle \ell_1 0 \ell_2 0 | L0 \rangle \langle J_1\ -M_1 J_2 M_2 | LM \rangle Y_{LM}(\theta_K, \phi_K) W(\ell_1 J_1 \ell_2 J_2; SL) [P_{L+1}(x_0) - P_{L-1}(x_0)] \}\ ,$$

where θ_K and ϕ_K are the polar angles of the c.m. momentum $\mathbf{K}$, $\hat{\ell} \equiv \sqrt{2\ell+1}$,

$$f_{\ell ST} \equiv \frac{1-(-1)^{\ell+S+T}}{2} \quad \text{and} \quad x_0 = \begin{cases} 0 & \text{for} \quad k < \sqrt{k_F^2 - K^2/4}, \\ \frac{K^2/4+k^2-k_F^2}{Kk} & \text{for} \quad \sqrt{k_F^2 - K^2/4} < k < k_F + K/2, \\ 1 & \text{otherwise.} \end{cases}$$

We also presented practical expressions for the nucleon single particle potential, based on which numerical calculations of the ground state energy were carried out.

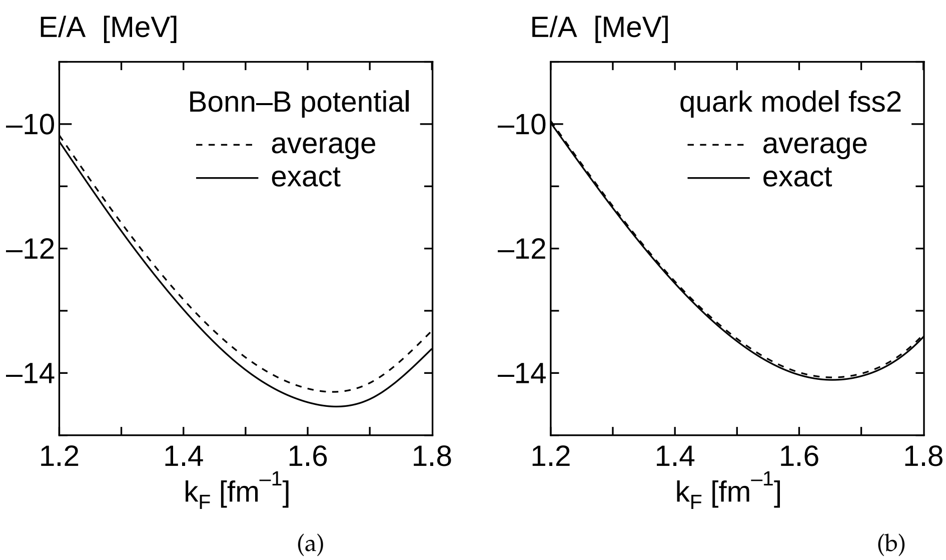

Fig. 1. Energies per nucleon in symmetric nuclear matter as a function of the Fermi momentum k_F: (a) Bonn-B potential [3] and (b) quark model potential fss2 [4].

Calculated saturation curves with the Bonn B potential [3] and the new version of the quark model Kyoto-Niigata potential fss2 [4] are shown in Fig. 1, where results with the exact Pauli operator and the angle averaged one are compared. The exact treatment of the Pauli operator brings about attractive contributions to the binding energy per nucleon at any nuclear densities. However the difference is rather small, although the results somewhat depend on the nucleon-nucleon interaction employed. This quantitatively confirms the good quality of the angle average approximation. The same conclusion was obtained by Schiller, Müther and Czerski [5]. It is suggested that the angle average treatment of the Pauli operator in considering more than three-body correlations is reliable.

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